Fundamentals of Reinforced and Prestressed Concrete

By

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2005

P R E F A C E

Structures should be designed and constructed to fulfill safety, economy and efficiency. When dealing with reinforced and prestressed concrete structures, the designer is facing a combination of two totally different materials, concrete and steel, behaving in a very complicated manner. A rigorous treatment of the inelastic, discontinuous, time-dependent behavior of concrete reinforced by steel is virtually an impossibility. However, extensive research in the field of reinforced and prestressed concrete yielded a good understanding of their behavior and relatively simple theories have been developed for analysis and design. Nevertheless, successful application of these theories needs a good grasp of the assumptions on which they are based and of the extent to which they can be applied. This book is an attempt to present the fundamentals of reinforced and prestressed concrete necessary for economic, safe and efficient design as viewed by the author through forty years of experience in this field.

This edition of the book is divided into four parts, published earlier separately:
Part 1 is composed of three chapters:
Chapter 1 describes the physical and mechanical properties of concrete and reinforcing steel,
Chapter 2 presents the analysis of elements in compression, tension, bending with and without axial forces by making use of basic strength of materials and pertinent simplifications that describe the combined action of concrete and steel,

Chapter 3 discusses safety provisions and different theories of design.

Part 2 contains three more chapters dealing with the design of different reinforced concrete elements:

Chapter 4 gives the design of reinforced concrete beams. It explains the load distribution, internal forces, dimensioning and detailing of beams. Design of sections of different shapes is presented using both the working stress design (WSD) method and the ultimate strength design (USD) method,

Chapter 5 deals with common types of slabs: solid, ribbed, hollow and flat. WSD method is used in solving design examples. Beamless slabs supported on columns of irregular arrangement are also presented,

Chapter 6 is devoted for columns and elements subjected to eccentric forces causing uniaxial and biaxial bending. Both the WSD and USD methods are used in the design. This part was first published in 1976 then together with Part 1 they formed Volume I, published in 1978.

Part 3 contains Chapters 7 to 11 covering the following subjects: stairs, panelled beams, surface and deep foundations, retaining walls and torsion.

Part 4 presents the fundamentals of prestressed concrete. It is divided into ten sections covering the following subjects: materials, prestressing systems, loss of prestress, analysis and dimensioning of sections in bending, shear , bond and bearing stresses, deflections, layout of tendons in statically determinate beams, and finally compression and tension members. This part was first published in 1970, then in combination with Part 3 as Volume II in 1978.

I wish to thank my collegues, my students and the practicing engineers for their enthusiastic acceptance of the different parts of this book. I also wish to apologize for any inconsistency they may

find in the terminology of the different parts. The unexpected speed with which the book is in demand for reprinting gives me virtually no time for revision ! I even think this edition needs some reshuffling and screening of the contents that will hopefully be made in the next edition.

March 1980 M. Hilal

Contents of Part I

Analysis Tables and Sheets

Contents of Part II

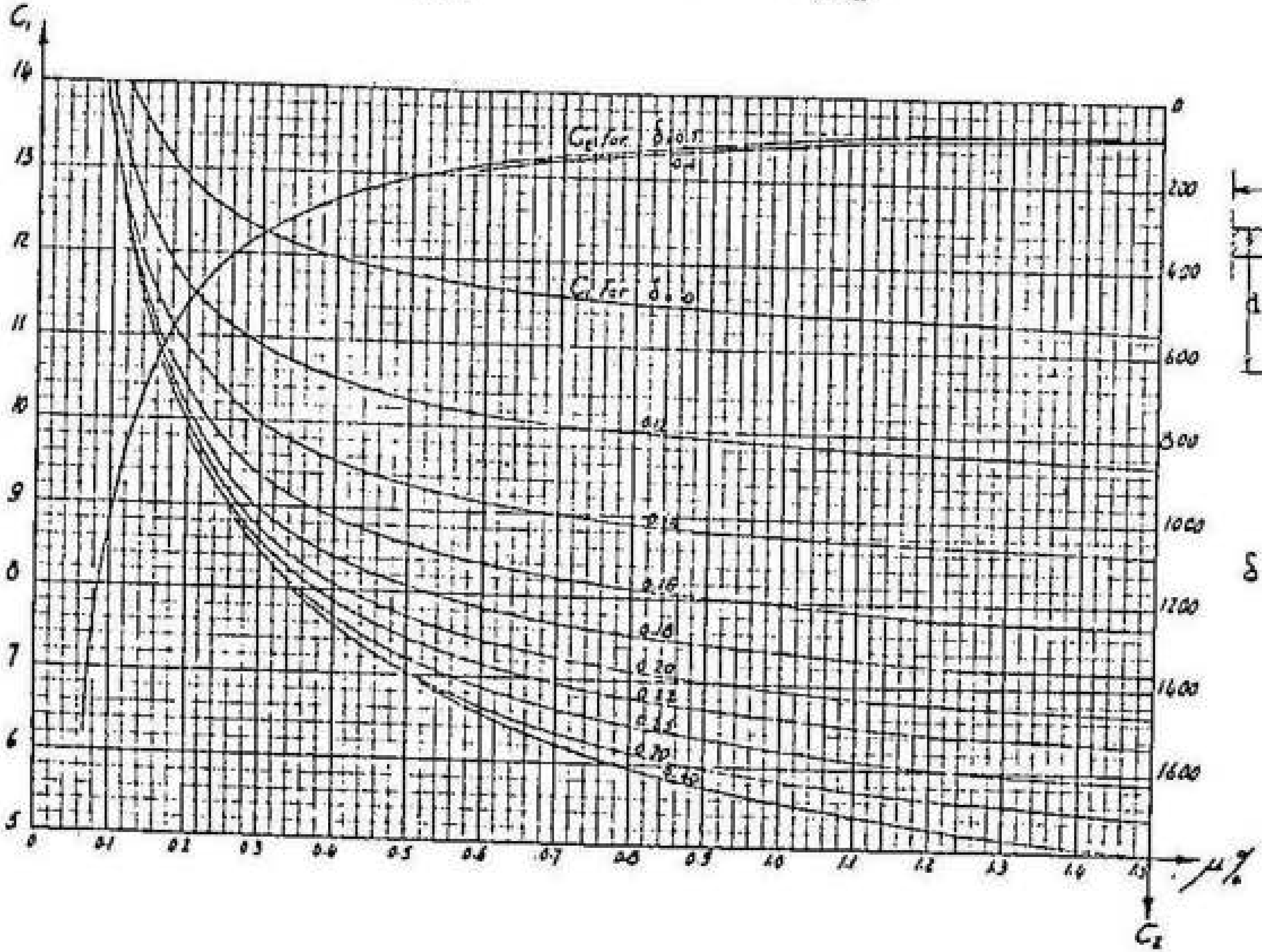
Design Tables and Sheets

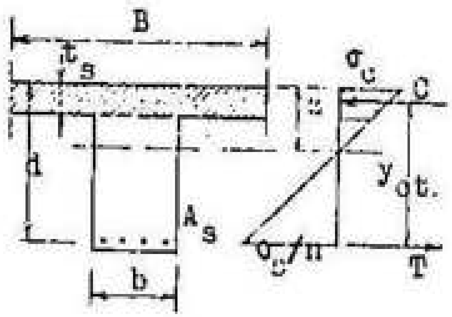
SHEET 4

SIMPLE BENDING

DETERMINATION OF STRESSES IN T-SECTIONS

$\sigma_c = c_1 \frac{M}{B\,d^2}$ $\qquad \sigma_s = c_2 \frac{M}{B\,d^2}$ $\qquad n = 15$





$\delta = t_s/d$ $\qquad \mu = A_s/Bd$

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PART I

MATERIALS AND ANALYSIS

CHAPTER 1

CONCRETE AND REINFORCING STEEL

1.1- INTRODUCTION

Concrete is an artificial stone-like material. It consists of an intimate, carefully proportioned, mixture of inert materials called aggregates (sand, gravel, crushed stone ..... etc.) with cement and water. Because of the action of the cement, the mixture thus obtained begins to harden after a few hours in forms of the shape and dimensions of the required structure and gradually aquires its strength properties. The bulk of the material (~ 85%) consists of fine and coarse aggregates. Cement and water interact chemically to bind the aggregate particles into a solid mass.

The properties of the concrete depend mainly on the following factors:

1. The proportions of the mix, especially the amount of cement and water.
2. The quality and granular composition of the aggregates.
3. The thoroughness with which the various constituents are intermixed.
4. The degree of compactness in the forms.
5. The conditions of humidity and temperature during setting and hardening, i.e. the method of curing.

Good dense concrete possesses high qualities which can only be maintained by a skillful continual quality control throughout the whole process from the proportioning of the mix, through mixing, placing and compaction, until the completion of curing.

The compressive strength of concrete is high while its tensile strengh is low so that its economic use in members subject to axial tension, such as tie rods, or members subject to bending, such as beams is generally not possible.

To overcome this difficulty, it has been found possible to use steel with its high tensile strength to reinforce concrete, mainly in the tension zones where the small tensile strength of the concrete would limit the carrying capacity of the member. The reinforcement, usually round steel rods, is placed in the forms before casting the concrete. When completely surrounded by the hardened concrete mass, it forms an integral part of the member and the two materials statically act together. The resulting concrete element reinforced by steel is called reinforced concrete.

To illustrate this statical action, let us consider the simple beam shown in Fig. 1-1. Under the effect of vertical loads, the beam will be subject to compressive stresses above the neutral axis and tensile stresses below it. If the beam is made of plain concrete, its bearing capacity will be governed by the low tensile strength of concrete. The failure will take place due to the cracking of the tension zone and the high compressive strength of the concrete is not utilized. In order to increase the bearing capacity of the beam, its weak tension zone is reinforced by steel rods placed near the lower extreme fiber as shown in Fig. 1-1b.

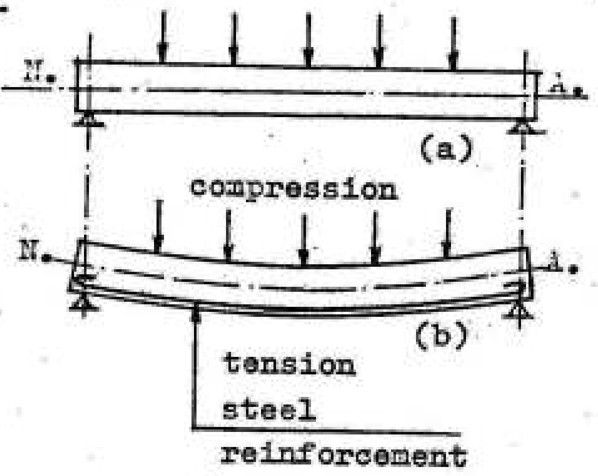


Fig. 1-1

Steel is supposed to be the most convenient material to reinforce the concrete for the following reasons:

1. Bond between steel (especially when deformed) and concrete is relatively high and is generally sufficient to keep the two materials acting together within practically all load stages.
2. Concrete and steel have nearly equal coefficients of thermal expansion, so that no internal stresses take place due to equal temperature changes. The coefficient α for both concrete and steel may, on average be taken as 10^{-5} (0.01 mm/m/degree centigrade).
3. A good dense concrete protects the steel from rusting.

As a building material, reinforced concrete combines many of the advantages of its components - concrete and steel - namely:

A relatively low cost.
A good weather and fire resistance.
A high resistance due to the high compressive strength of the concrete and the high tensile strength, ductility and toughness of the steel.

A good protection of the steel from rusting.

One of the main factors which make reinforced concrete a universal building material is the facility with which it can be formed to almost any practical required shape.

The strength of the concrete required for a reinforced concrete element is limited by the quality and percentage of the steel reinforcements used in the same element. Moreover, the width and distribution of the cracks in reinforced concrete elements depend on the shape and distribution of the reinforcement and the magnitude of the stress acting. So that, for a certain quality of steel, the higher the tensile stress in the steel the wider are the tension cracks.

For these reasons, the compressive strength of the concrete required for reinforced concrete is generally $\leqslant$ 300 kg/cm^2; a bigger strength cannot be effectively utilized, and the required yield stress of the reinforcing steel is limited to $\sim$ 4000 kg/cm^2; the use of better qualities with higher yield and high tensile stress will cause undesirable wide tension cracks.

Due to the remarkable advance of the technique in the recent years, it has been found possible to produce concretes of very high strength (600 kg/cm^2 and eventually 1000 kg/cm^2) and steels having a yield stress of 16000 kg/cm^2 and more. Such high qualities of concrete and steel cannot be effectively used in reinforced concrete.

The use of these high qualities effectively has been found possible in "prestressed concrete". In this type of concrete structures, the steel, mostly in the shape of wires is embedded in the concrete under high tension which is held in equilibrium by compression stresses in the surrounding concrete after hardening. Because of this precompression, the concrete in a flexural member will crack on the tension side at a much larger load than when not precompressed. This reduces radically both the deflections and the tension cracks at service loads in such structures and thereby enables these high-strength materials to be used effectively. Prestressed concrete is particularly suited on a mass-production basis, although it is being used as well without such prefabrication.

We give in the following, some of the principal structural forms of reinforced and prestressed concrete.

Fig. I-2 shows a simple form of a slab, beam and girder floor in which the slab is supported on the secondary beams; the secondary beams are supported on the girders which are supported on the columns.

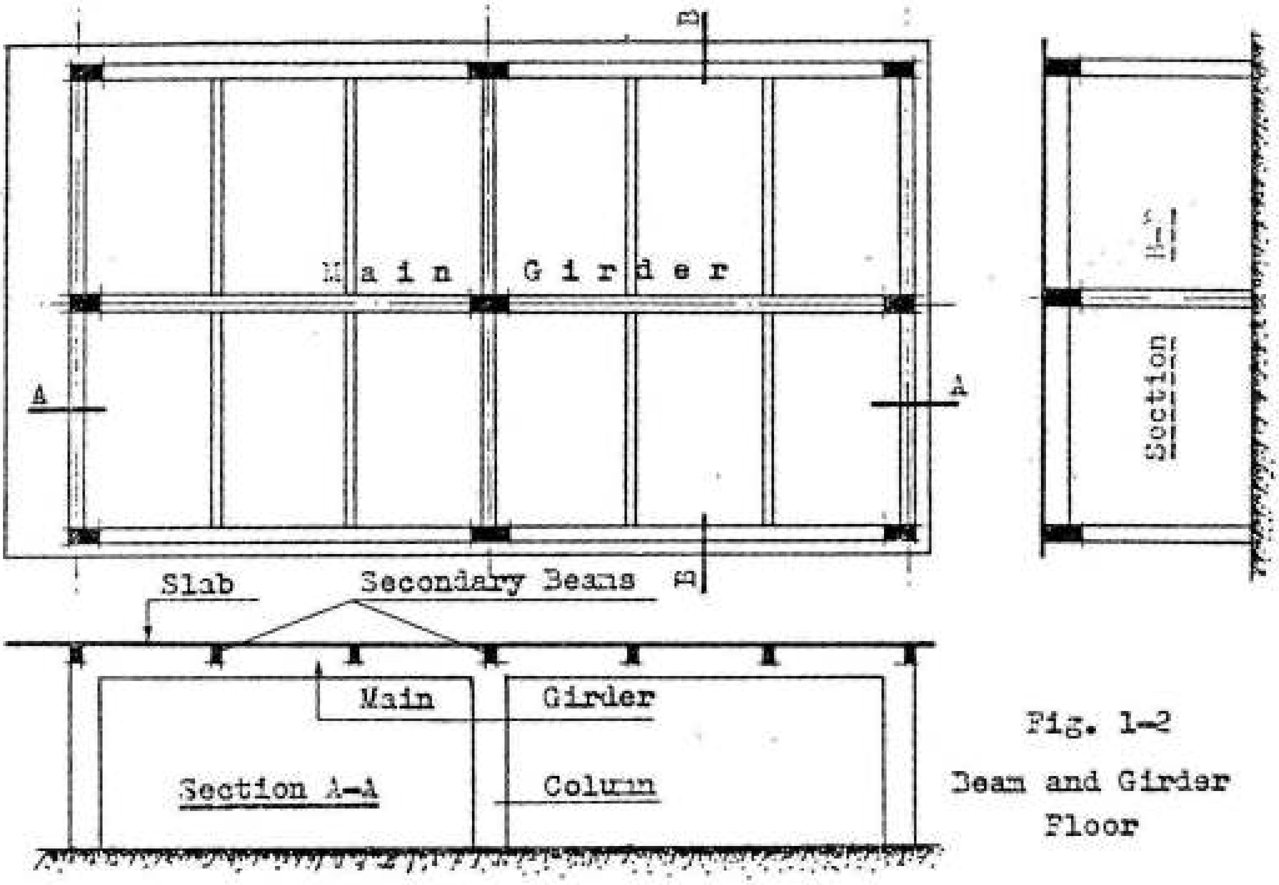


Fig. 1-2
Beam and Girder Floor

Fig.1-3 shows a flat slab without any beams or girders except at the outer edges where one may arrange a marginal beam to support the outside walls. Such floors are frequently used in case of relatively big spans (6 to 10 ms) and heavy loads as in warehouses and garages.

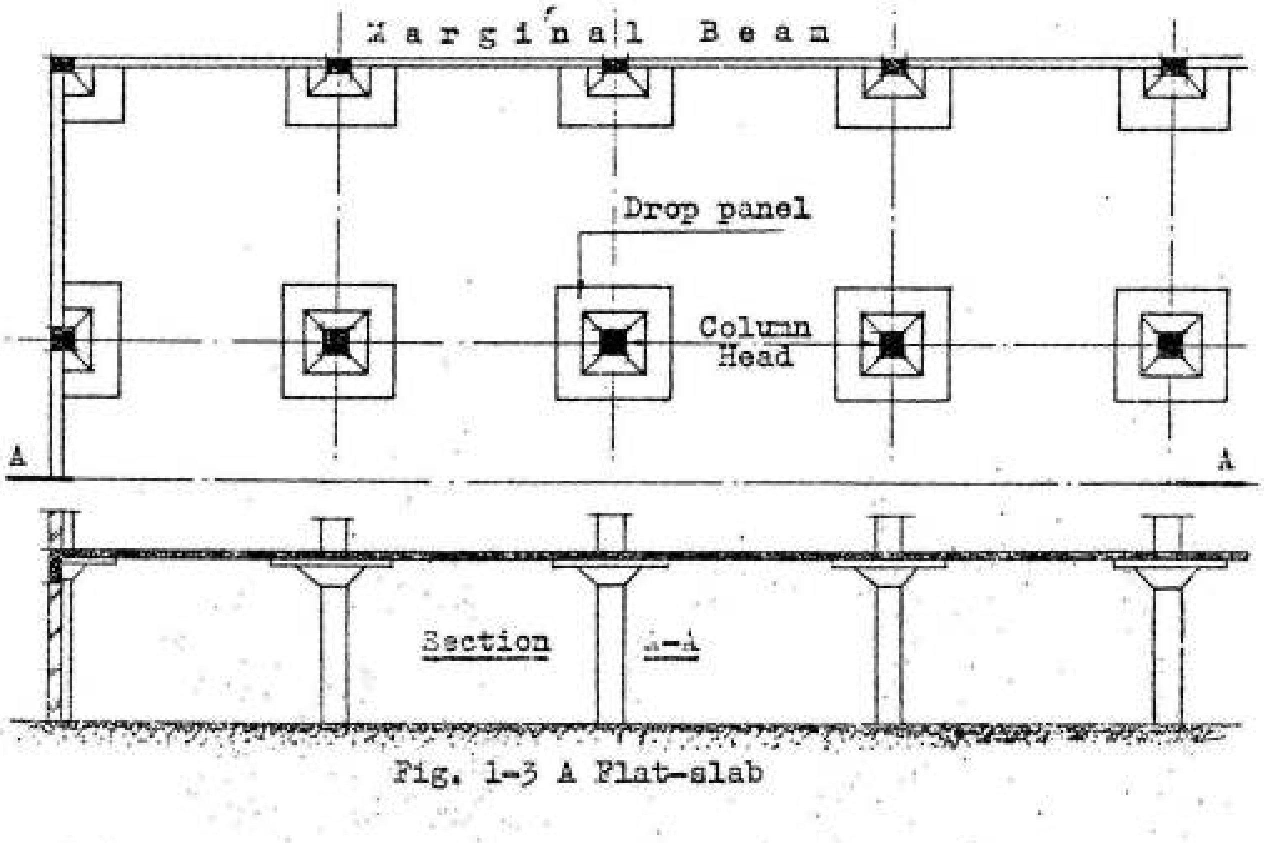


Fig. 1-3 A Flat-slab

Fig. 1-4 shows a one-way ribbed slab which may be used for relatively big spans permitting saving in the concrete quantities and reinforcement steel due to the relatively big depth of the slab. Ribbed slabs may also be two-way as shown in Fig. 1-5.

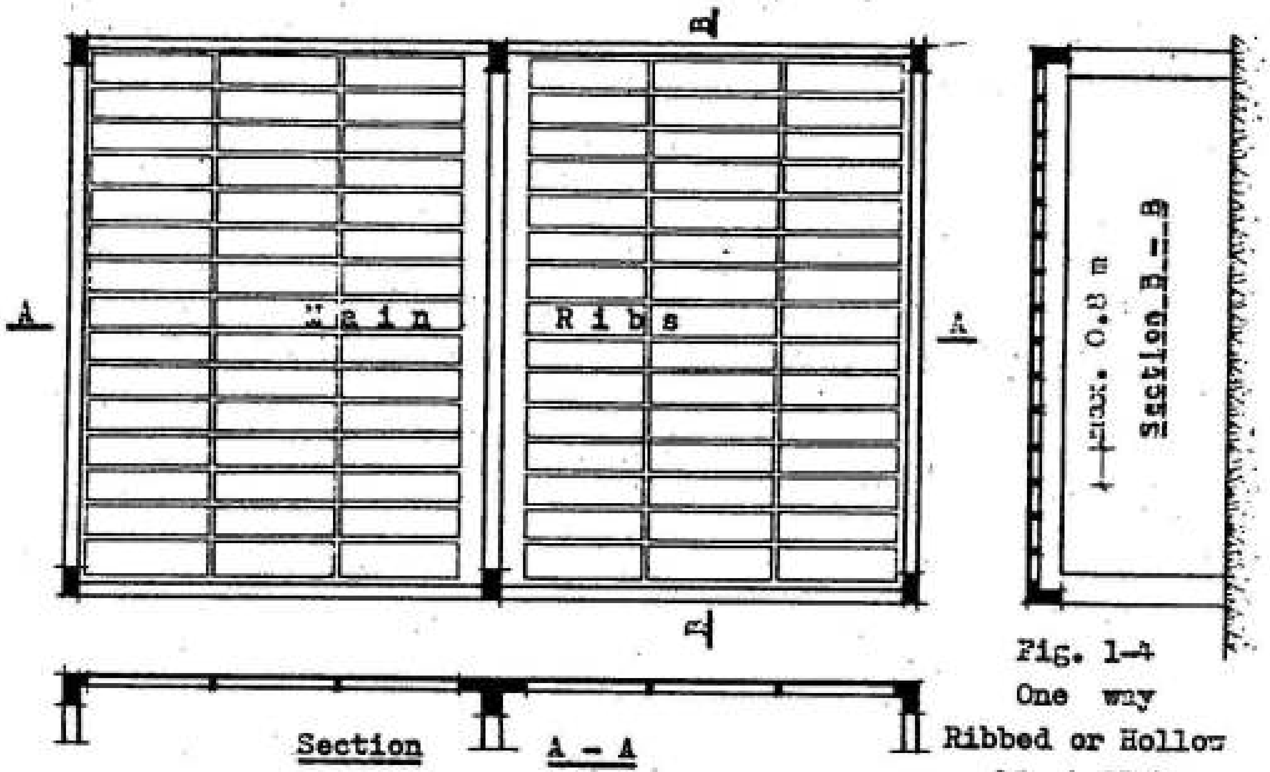


Fig. 1-4
One way
Ribbed or Hollow
block Slab

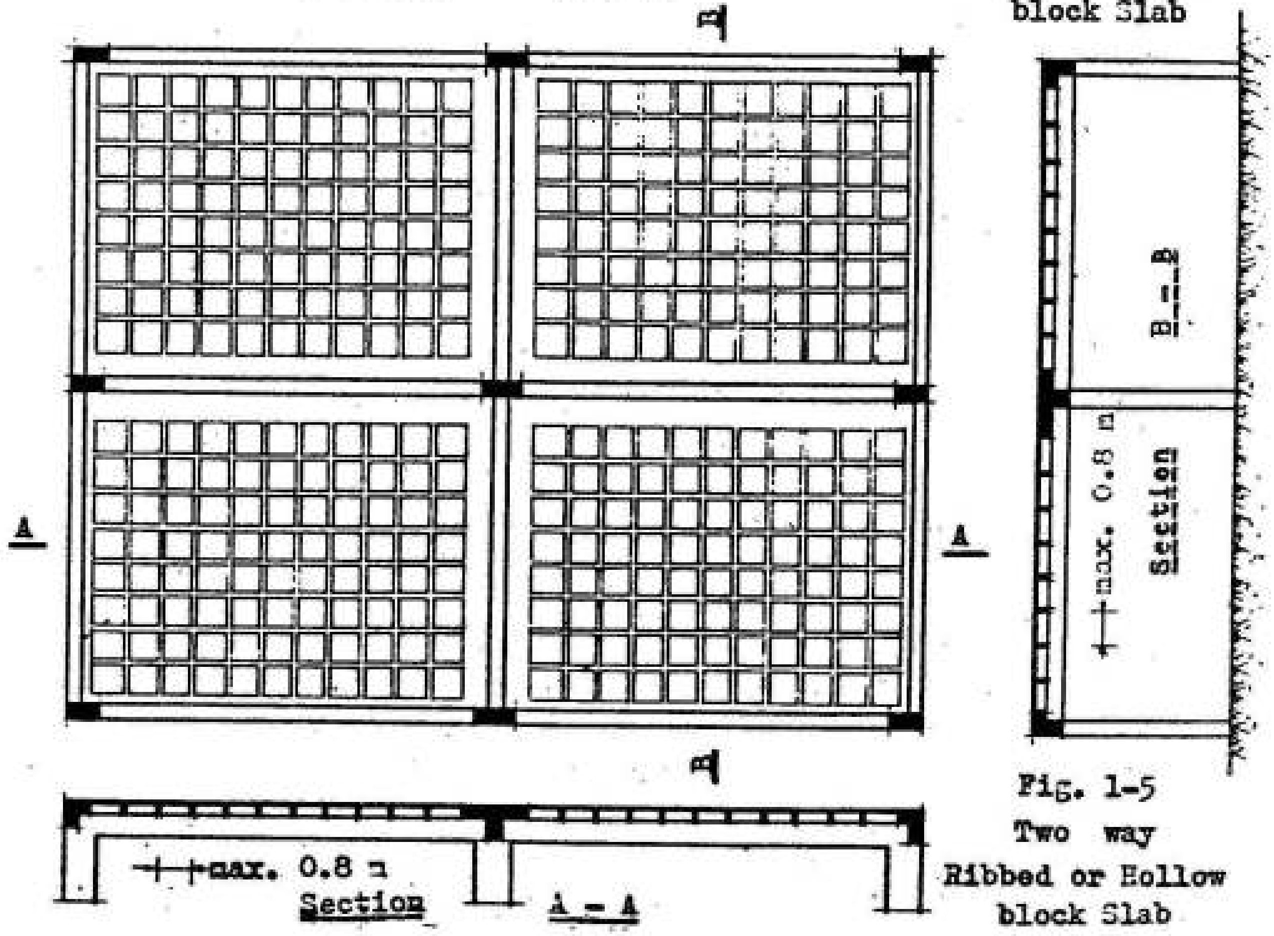


Fig. 1-5
Two way
Ribbed or Hollow
block Slab

One and two way ribbed slabs are generally nice looking; they are not recommended in dusty zones or areas subject to smoke and similar areas. If it is required to have a plane bottom surface for the slab, hollow blocks are arrananged between the ribs shown in Figs. 1-4 and 1-5. This new system adds the cost of the blocks, but it saves most of the bottom shuttering of the slab and increases its insulation against heat and sound.

The use of simple beams as supporting elements is uneconomic as their bending moments are big,-the maximum value $M_{max} = w\, l^2 / 8$ in a simple beam of span l and subject to a uniform load w- and acts at one section -the middle section . The maximum fiber stress at this section governs the design of the beam. For this reason, the use of continuous beams and frames is more convenient as the bending moments are smaller and distributed on many sections. Fig. 1-6 shows the general layout of a part of a hall using a simple rectangular frame as the main supporting element for the roof.

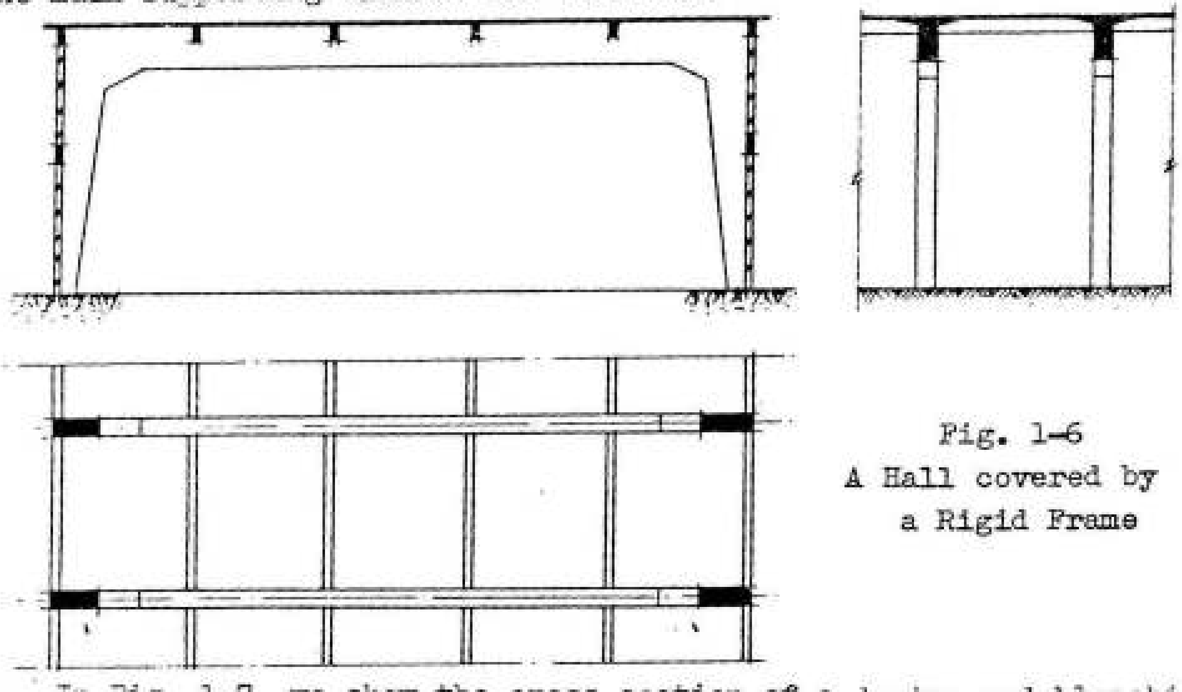


Fig. 1-6
A Hall covered by
a Rigid Frame

In Fig. 1-7, we show the cross section of a dyeing and bleaching hall using a special type of continuous frames.

Arched roofs are conveniently used as reinforced concrete supporting elements in cases where plane surfaces are not specified as they are mainly subject to compressive forces and small bending moments. Figs. 1-8 and 1-9 show two different examples for the use of arched girders.

As a further development of the arch principle, shell surfaces, subject mainly to forces in the plane of shell, provide structurally

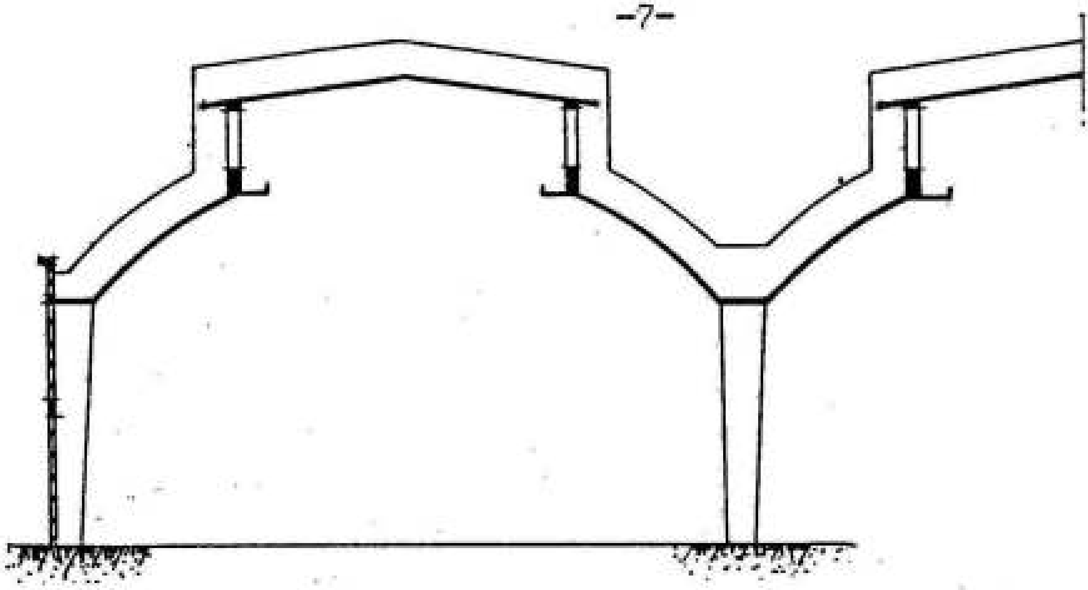


Fig 1-7 A 3-bay Hall in a Factory covered by Rigid Cont. Frames

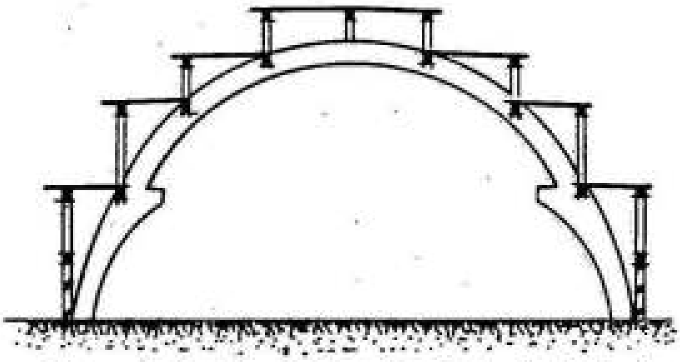


Fig. 1-8 Example 1 of Arch Girder

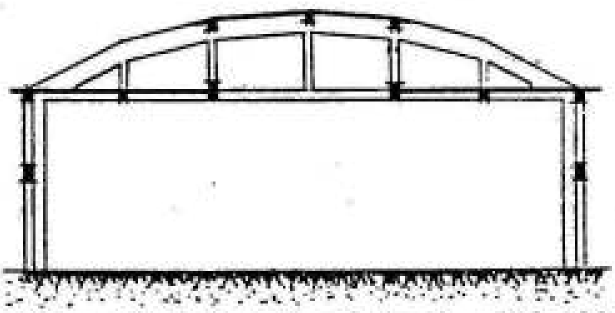


Fig. 1-9 Ex. 2 of Arch with Tie

an efficient solution to the problem of carrying conveniently roof loads over long spans of which we give in Fig. 1-10 the use of a circular dome to cover the upper storey of a building. In Fig. 1-11, the use of a cone to cover a circular tank of a relatively big diameter is shown.

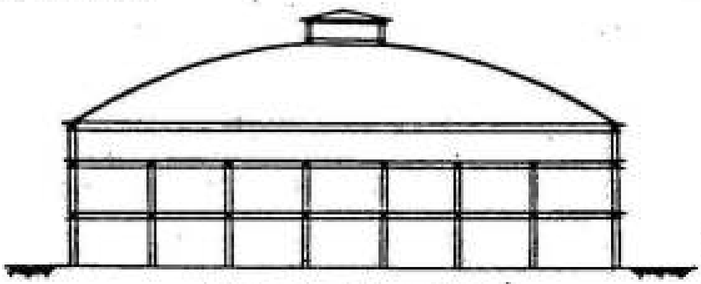


Fig. 1-10 A Dome

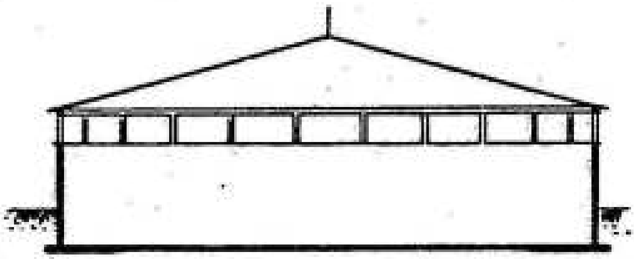


Fig. 1-11 A Cone

Fig. 1-12 shows one example of the convenient and economic use of a circular cylindrical shell as a roof of the barrel form.

In an attempt to simplify the formwork and yet retain many of the advantageous characteristics of cylindrical shells, the folded plates are used. Fig. 1-13 shows an example.

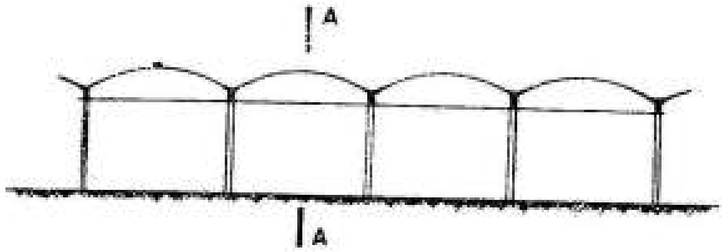
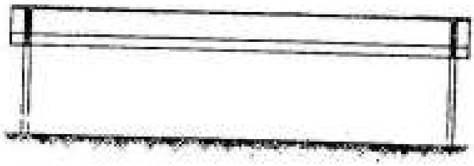


Fig. 1-12 A Barrel cylind. Shell Roof



Section A - A

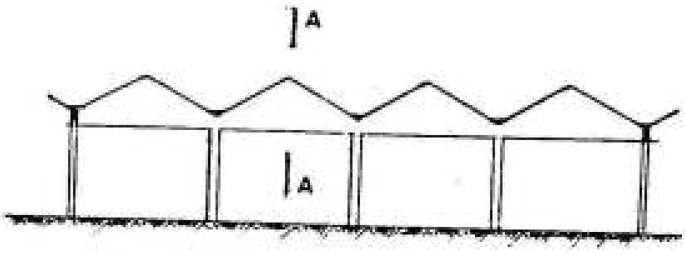
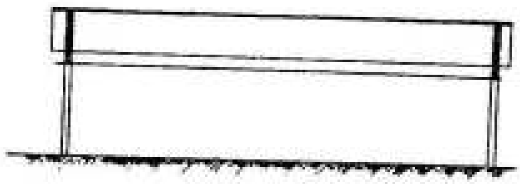


Fig. 1-13 A Folded-plate Roof



Section A - A

Recently, the double curved shells are used ; of which, we give in 1-14 an example of a circular paraboloid and in Fig. 1-15 an example of a conoid. Such forms have proved to be of very high resistance, economic, easy to construct and architecturally very impressive.

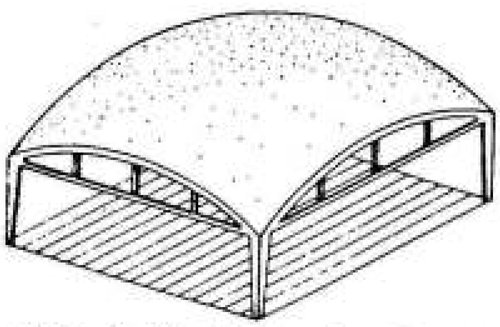


Fig. 1-14 A Circular Paraboloid

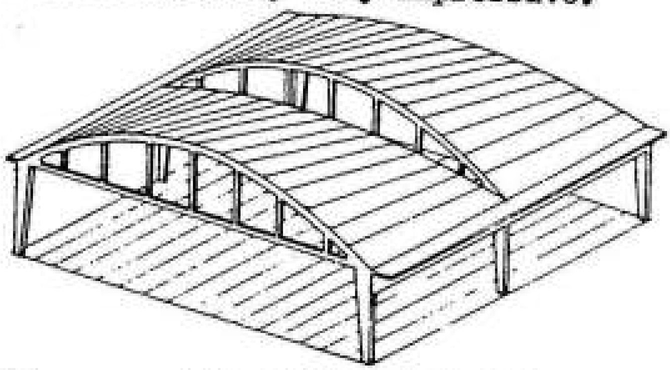


Fig. 1-15 A Coinoid

Moreover, reinforced concrete, and eventually prestressed concrete, are extensively used in approximately all other kinds of engineering structures such as Figs. 1-16 and 1-17 of girder and arch brid-

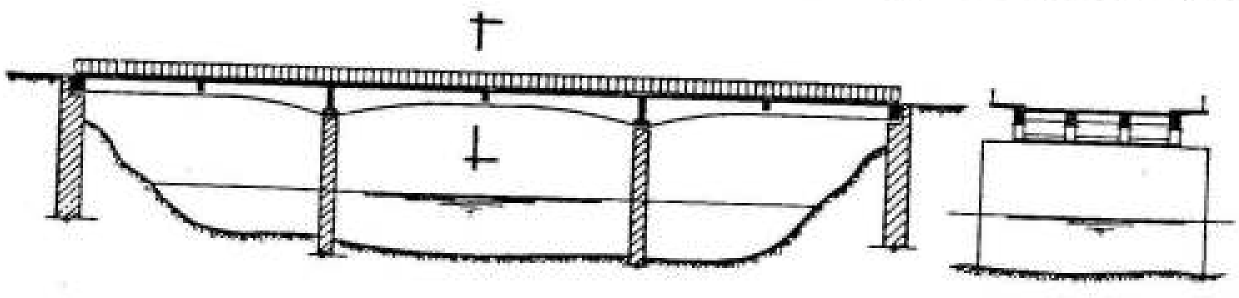


Fig. 1-16 A Girder Bridge

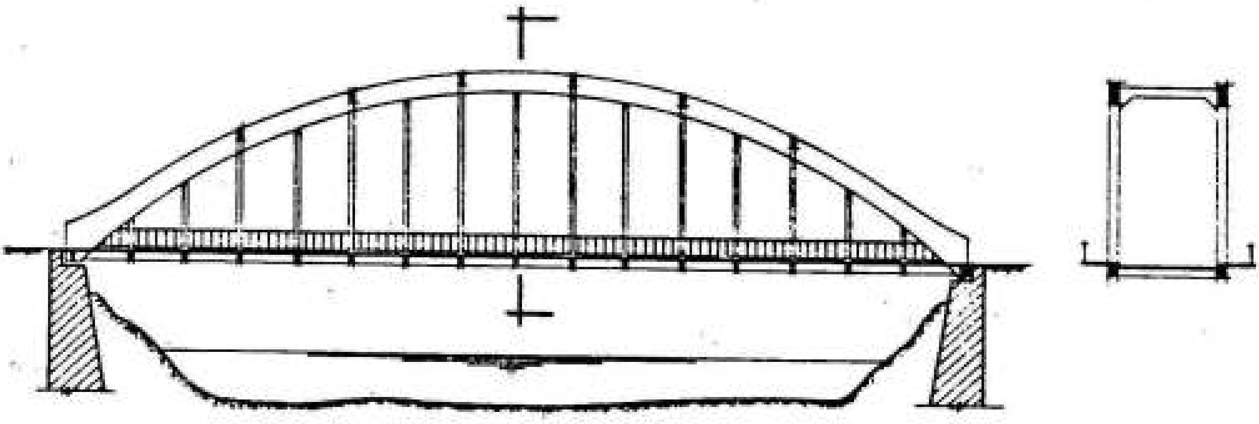


Fig. 1-17 An Arch Bridge

ges, silos, Fig. 1-18, bunkers, Fig. 1-19, tanks and water-towers. Figs. 1-11 and 1-20.

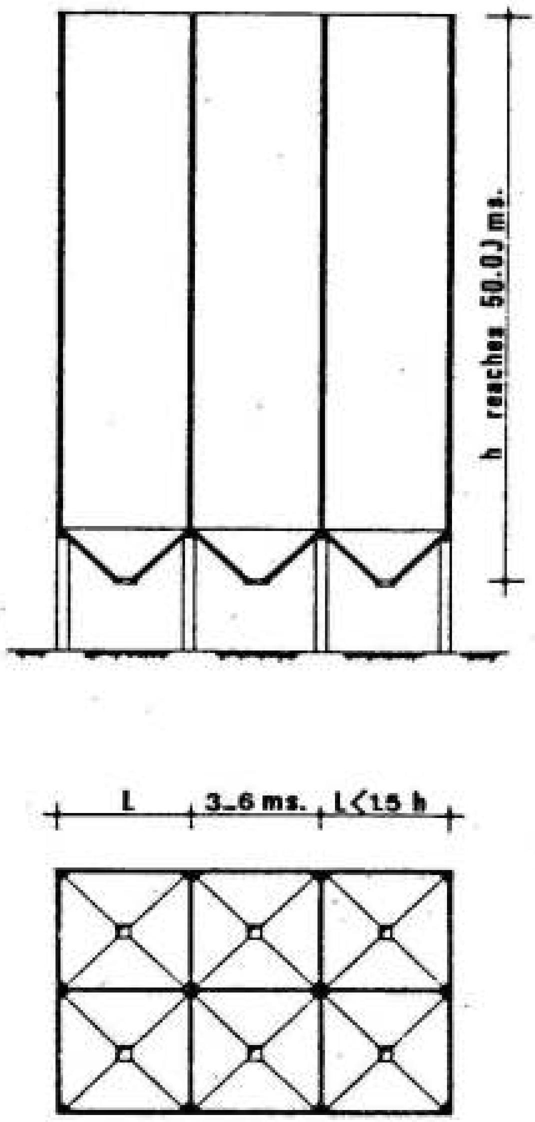


Fig. 1-18 A Silo

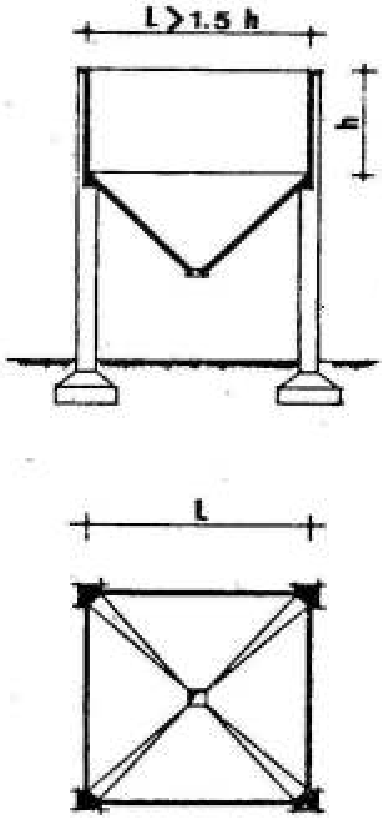


Fig. 1-19 A Bunker

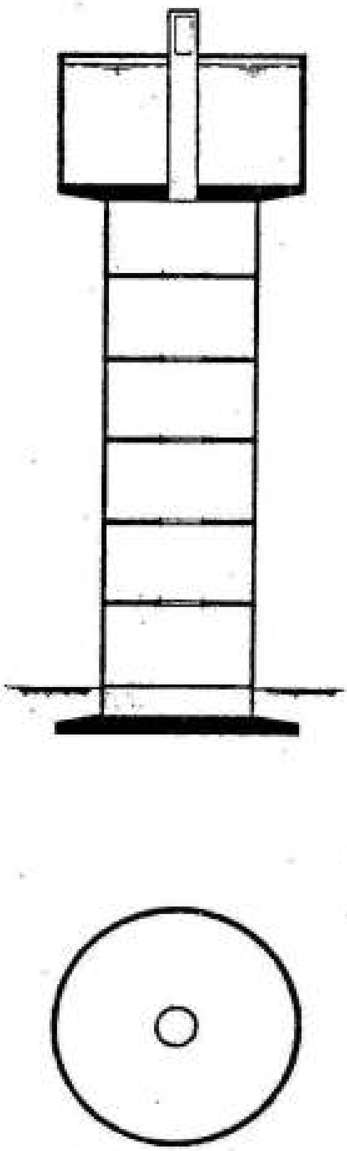


Fig. 1-20
A Water-tower

.2- CONCRETE CONSTITUENTS

1) Cement: Cement is the material which has the adhesive and cohesive properties necessary to bond the aggregates into a solid mass of adequate strength and durability. Water is needed for the chemical process (hydration) in which the cement powder sets and hardens into one solid mass. Of the various portland hydraulic cements, we, in Egypt, distinguish:

1)Ordinary cement which must conform to	E.S/373/1963
2)Rapid-hardening cement (supercrete) which must conform to	,, ,, ,,
3)Low-heat cement which must conform to	E.S/541/1964
4)Blast furnace cement which must conform to	E.S/974/1969
5)Sand-mixed cement (Karnak) 25%	
6)Sulphate resisting cement which must conform to	E.S/583/1970

The main properties are given in the following table:(1-1)

Table 1-1

Kind of Cement	Setting Time		Average Comp. Strength[1] in kg/cm^2 ...... after		
	Initial >	Final <	3 days σ_3 >	7 days[2] σ_7 >	28 days[3] σ_{28} >
Ordinary cement	45 min.	10 hrs.	160	240	
Rapid-hardening cement[4]	45 min.	10 hrs.	210	285	
Low-heat cement	60 min.	10 hrs.	77	140	280
Blast furnace cement 35%[5]	45 min.	10 hrs.	112	210	350
Mixed cement 25%[5]	the same as blast furnace cement 35%				
Sulphate-resisting cement[6]	45 min.	10 hrs.	154	239	

(1) Compression tests to be performed on standard mortar cubes 50 cm^2 section. The values in the table give the average for 3 cubes.
(2) The compressive strength after 7 days must be bigger than that after 3 days.
(3) The compressive strength after 28 days must be bigger than that after 7 days.
(4) The tensile strength (as average of 6 standard mortar test specimens after 24 hours > 21 kg/cm^2. The determination of this value is

optional.
(5) Blast furnace cement 35% (sometimes called slag cement 35%) is produced by grinding together a mixture of normal portland cement clinker and 35% granulated blast furnace slag.

Sand-mixed cement 25% (commercial name Karnak 25%) is produced by grinding together a mixture of normal portland cement clinker and sand.
(6) The British standard specifications B.S.S. 4027/66 specify further:

The average compressive strength for 3 standard concrete cubes

after 3 days $\sigma_3 \geqslant$ 84 kg/cm^2

after 7 days $\sigma_7 \geqslant$ 140 kg/cm^2 and $> \sigma_3$.

The quality of cement should be chosen with due regard to the nature of the structure to be built, its structural characteristics , its purpose and the various requirements that it will have to fulfill more particularly with reference to climatic and local conditions: warm weather, cold weather, presence of aggressive water ... etc.

For ordinary reinforced concrete and prestressed concrete structures ordinary portland cement may be used. However, for prestressed concrete structures in which the tendons (cables) are to be tensioned while the concrete is still young or for structures necessitating early removal of formwork, the use of rapid-hardening cement may be considered. On the other hand, in the case of structures requiring only relatively low mechanical strength,mixed cement (Karnak) can suitably be employed. Slag cement is supposed to be used in structures to be built in aggressive surroundings (in the presence of water with considerable content of calcium sulphate). Egyptian mixed and slag cements may be used in reinforced concrete ordinary structures not subject to high stresses. They can be allowed in heavy reinforced concrete structures when tests prove the possibility of this allowance. Due to some difficulties in the production of uniform granulated blast furnace slag used in producing Egyptian slag cement, it is recommended not to use it in facade mortars and plasters, concrete with surfaces directly exposed to humidity or structural elements subject to abrasion.

In cases where the concrete is used in the form of large masses, cements with very high early strengths should not be used, nor too rich mixes (with more than 300 kgs cement per m^3), as these could cause considerable evolution of heat of hydration; in such cases, low heat portland cement is recommended.

If some of the concrete elements are in contact with liquids containing high ratios of tri-sulphur oxide $300 < S\,O_3 < 2000$ mlg/liter, the use of sulphate resisting cements is essential in these elements. For bigger ratios of $S\,O_3$ special "Super-sulphate"resisting cements must be used.

2) Aggregates: In ordinary structural concretes the aggregates occupy about 70 to 75 per cent of the volume of the hardened mass. The remainder consists of hardened cement paste, uncombined water (i.e., water not involved in the hydration of the cement), and air voids. The latter two evidently do not contribute to the strength of concrete. In general, the more densely the aggregate can be packed, the better are the strength, weather resistance , and economy of the concrete. For this reason the gradation of the particle sizes in the aggregate, to produce close packing, is of considerable importance. It is also important that the aggregate have good strength, durability, and weather resistance, that its surface be free from impurities such as loam, silt, and organic matter which may weaken the bond with the cement paste, and that no unfavorable chemical reaction take place between it and the cement.

According to the UNESCO-Reinforced concrete manual, the granulometric class of an aggregate (sand, gravel, crushed stone) is defined by two dimensions d_o and d_m, corresponding respectively to the smallest and largest of the particles of which that aggregate consists. An aggregate is,by definition, of the class d_o/d_m df, for $d_m > 2d_o$, the following values are obtained:

(a) a residue of less than 10% on the sieve with apertures d_m;
(b) less than 10% passing the sieve with apertures d_o;
(c) less than 3% passing the sieve with apertures $½d_o$.

Aggregates can be subdivided into the categories shown in the following table: (1-2)

Table 1-2

Classification of aggregates	Sieve mesh apertures (in mm)	Diameter of apertures of round-hole screen in mm
Fines (fillers)	<0.080	
Sands: fine	0.080-0.315	
medium	0.315-1.250	
coarse	1.25 -5	
Gravels: fine		6.3-10
medium		10-16
coarse		16-25

The maximum size of gravel coarse-aggregate is, in Egypt, 30 mms.

The grading of the aggregate is defined by its grading curve as determined by sieve analysis. This curve should be within a predetermined zone which defines the permissible grading of the aggregate.

In drawing the grading curve, the percentage of aggregate passing each of the sieves (or round-hole screen) is plotted, as ordinate, against the corresponding sieve mesh aperture (or diameter of aperture in the case of a round-hole screen), relative to the maximum diameter allowed d_m (i.e. d/d_m) as abscissa.

Prof. Dr. A.A. El Arousy* proposes the grading curve shown in Fig. 1-21.

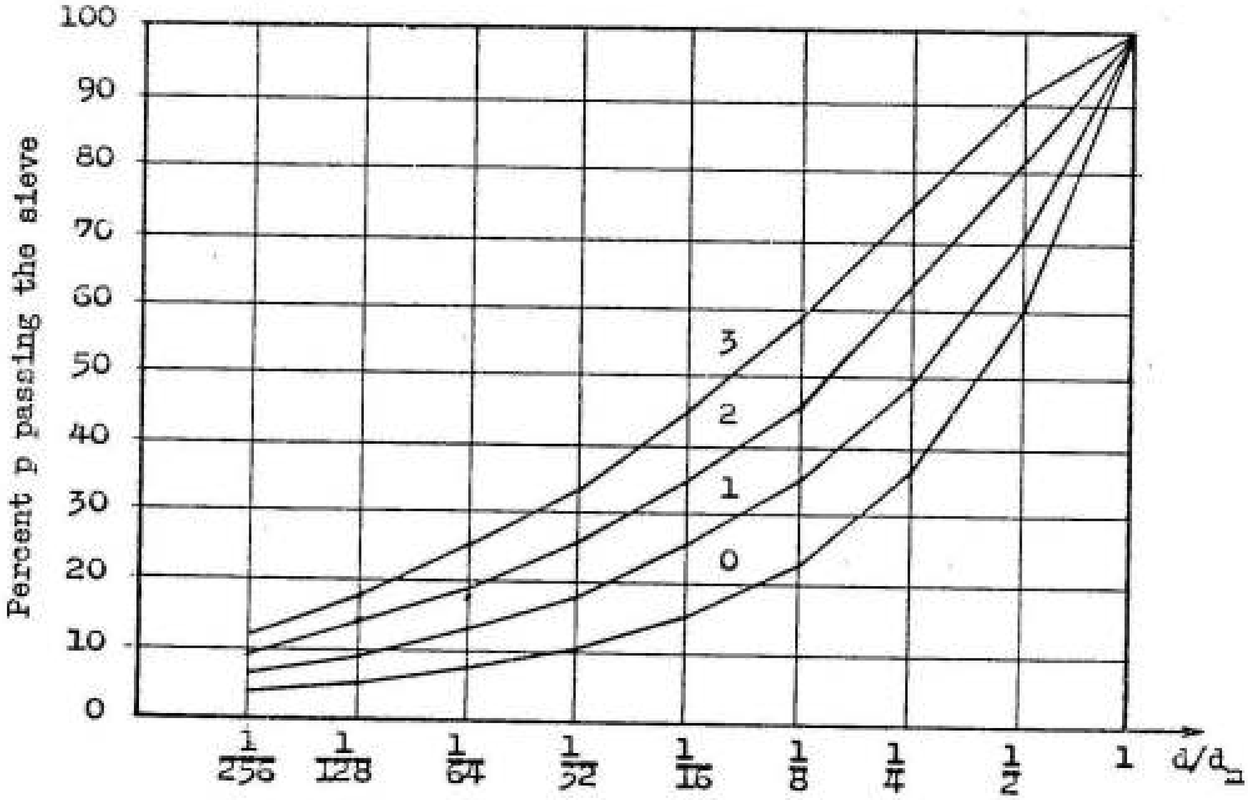


Fig. 1-21 Grading curve of aggregates

The curves shown follow the relations:

Curve 0 $\quad p = 50\sqrt{d/d_{max}} + 50\, d/d_{max}$

Curve 1 $\quad p = 100\sqrt{d/d_{max}}$

Curve 2 $\quad p = 150\sqrt{d/d_{max}} - 50\, d/d_{max}$

* Emeritus professor, faculty of engineering, Ain-Shams university, Cairo.

Curve 3 $p = 200\sqrt{d/d_{max}} - 100\ d/d_{max}$

Zone 1 lies between curves 0 and 1, and grading curves in this zone give a 'very good grading'.

Zone 2 lies between curves 0 and 2, and grading curves in this zone give a 'good grading'.

Zone 3 lies between curves 0 and 3, and grading curves in this zone are 'acceptable'.

The grading curve may be broken; it may have even in some cases a horizontal or a nearly horizontal part between two special diameters which means that we have a gap grading and the aggregates are deficient in these diameters. Such a grading can be assumed very good, good or acceptable according to whether it falls in zone 1, 2 or 3 respectively. Gap grading can however be compensated by intensive compaction.

The grading curve may fall outside curve 0 or curve 3 at certain diameters but it should return again and fall within the required zone. In such cases, lab tests are recommended to determine the most convenient grading.

The following table gives the percentage of the different aggregates required for a concrete mix if maximum aggregate-diameter is 20 and 30 mms.

Diam. of aggregates d mms		Percentage required for a concrete mix		
		Grading: V.G.	Grading: Good	Grading: Accpt.
d_m=30 mm	0 to 3 mm	20 - 30 %	20 - 40 %	20 - 50 %
	3 to 10 mm	25 - 37 %	25 - 48 %	25 - 62 %
	10 to 30 mm	55 - 43 %	55 - 32 %	55 - 18 %
d_m=20 mm	0 to 3 mm	26 - 38 %	26 - 48 %	26 - 60 %
	3 to 10 mm	34 - 44 %	34 - 54 %	34 - 64 %
	10 to 20 mm	40 - 30 %	40 - 20 %	40 - 10 %

The total of the percentages in any mix must be 100%.

The shape of the grading curve gives information on the granulometric composition of an aggregate of the class d_o/d_m, which may have a higher or a lower content of large or small particles or even deficient in certain diameters.

The maximum dimension d_m of the aggregates employed should, on the one hand, be less than the horizontal clear distance between two reinforcing bars (or between a bar and the formwork) and, on the other, be less than one-quarter of the thickness of the member to be concreted.

It is always advantageous to use aggregates with a large maximum dimension, having due regard to the dimensions of the member to be concreted, the shape of the formwork, the arrangement and shape of the reinforcement, etc. For a given cement content the concrete will be denser and stronger according as the aggregate dimension is larger. But this must not be overdone, or else a concrete will be obtained which is difficult to place and work and which will not ensure satisfactory filling of the formwork.

The aggregates used should either be natural sands and gravels, or crushed materials produced from appropriate types of rock. In particular, rock which is too soft or which decomposes in contact with the air or by hydration must not be employed. On the other hand certain blast furnace slags can, when crushed, permissibly be used.

With regard to the geometrical shape of the particles, gravel having flat particles (plates) or excessively elongated ones (needles) should not be used.

In choosing the kind of aggregates, it should be tried, as far as possible, to obtain material of sufficient hardness (which determines the hardness of the concrete) and also sufficient adhesiveness of the cement paste (which is likewise essential to the development of strength). In fact, failure of concrete is generally due either to fracture of the aggregates or to failure of the adhesion of the cement-paste to the aggregate particles.

3) Mixing Water: The water used for mixing should be clean and not contain more than 5 g of matter in suspension per litre (mud, silt, etc.) nor more than 35 g of soluble matter and salts per litre, provided that these dissolved salts entail no risk of harming the durability of the concrete (acids, sulphates, corrosive salts, organic matter, etc.)

It is not permissible to use sea water, unless there are special reasons to justify it. Any water of doubtful quality should be analysed

The use of sea water for the mixing of concrete generally causes a significant lowering of the subsequent strength of the concrete; besides, it promotes corrosion of the reinforcement and is liable to be particularly dangerous in heavily reinforced or in prestressed concrete. In any case it must be taken into consideration that the amount of calcium chloride introduced with the sea water is equivalent to @ 2% (by weight) of the cement.

4) Additives: Additives are special substances which are added in small quantities to mortars and concrete at the beginning of mixing and whose purpose is to modify some of their properties. They are available in the form of powders or liquids which are added at the start of mixing, in order to ensure the uniform distribution of these substances.

In the main, the following kinds of additives are to be distinguished:

1- Plasticisers and wetting agents: By using these additives it is possible to reduce the quantity of water and yet obtain the same plasticity of the mix or, alternatively, to increase the plasticity (and thus achieve better workability of the concrete) without increasing the water content. As a direct result of these two facts, it is possible, by the use of such additives to produce dense concrete of higher strength and water-tightness. Furthermore, they reduce harshness during placing and unreasonable shrinkage during the setting process.

2- Air-entraining agents: By using these additives it is possible to reduce the amount of water required for a mix of specified consistency. Due to such agents, a very large number of small air bubbles are whipped into concrete during the mixing operation, and if they are stabilized so that they cannot collapse or escape, their use gives a good workable concrete with a reduced amount of mixing water. But as organic chemicals they reduce the concrete strength.

It has to be emphasized here that air is the effective additive , that it can be got into the concrete only by mechanical mixing, and that it can be kept there in an effective amount only by the great surface tension created by the foam-stabilizing organic agent.

3- Chemical additives: Such additives are added to concrete for the purpose of modifying the normal plastic behavior of the mixture , or for influencing its rate of gaining hardness and strength. A disadvantage of most chemical additives is that small changes in their amount cause great changes in their action. Furthermore, some may retard one cement and accelerate another. In this connection it should be remembered that cements, even though they may be of successive production lots from the same mill, are not always similar in constitution. Moreover, since chemical agents modify the normal reaction of one or more of the cement constituents, and since they are effective in extremely small amounts, their action is critically influenced by changes in proportioning of mixture. Of this group of chemical additives, we recognize:

Setting retarders: These may be necessary in cases where setting of the concrete has to be retarded (e.g., at construction joints where concreting is temporarily stopped, exposing the aggregate by scrubbing the concrete surface, concreting in very warm weather).

Calcium sulphate in small amounts is a commonly used retarder and yet, with increased amounts, its behavior changes and it becomes a powerful accelerator. Organic material such as gelatin, glue, sugar, and other carbohydrates, even if they are used in extremely small amounts, strongly affect the reaction of cement and water.

Setting accelerators: Additives of this type may have to be used in cases where the formwork must be removed soon or where concreting is done in cold weather.

A commonly used accelerator is calcium chloride, but as it entails a serious risk of corrosion of the reinforcement, its use is strictly limited. If it is used during warm weather, the set may be accelerated to such an extent as to impair finishing, and heat of hydration may be increased to a degree that causes cracking. It may increase the dry shrinkage of hardened concrete. Nevertheless, it definitely improves the early strength of concrete.

However, some chemical additives may either entail a danger of corroding the reinforcement or other metal objects embedded in the concrete; or adversely affect other properties (hardening accelerators cause increased shrinkage, antifreeze agents cause reduction in concrete strength etc.). These dangers should be duly taken into consideration when choosing an additive.

Because of the serious risk of corrosion of the reinforcement, additives containing chlorides are not allowed to be used in:
(a) prestressed concrete structures; (b) tanks and containers;
(c) floors in which heating coils are incorporated; and
(d) concrete members treated by steam curing.

Additives containing chlorides should be protected from moisture during storage. These substances should never be incorporated directly into mortars or concretes. However, the necessary steps must be taken so that such additives are completely dissolved and that their concentration in the concrete is as uniform as possible. It is also recommended not to use them in conjunction with another additive, as this may give rise to chemical reactions liable to cancel the desired effect.

1.3- PROPORTIONING OF CONCRETE MIX

The concrete is proportioned by defining the quantities of the various constituents making up one cubic meter of finished concrete.

The amount of cement / m^3 concrete is preferably measured by weight while the aggregates are generally measured by volume.

The concrete mix should normally contain between 250 and 450 kg of cement per cubic meter of finished concrete. Departure from this requirement will call for special justification. For ordinary reinforced concrete structures the concrete should contain 300 kg of cement per cubic meter. For reinforced concrete structures which are required to have special properties of water-tightness, strength and density, and, for prestressed concrete structures, the concrete should contain 350-450 kg of cement per cubic meter.

The cement content most commonly adopted for reinforced concrete structures exposed to the weather is 350 kg/m^3. However, this can permissibly be varied on the basis of the following considerations:

(a) the strength of a concrete will be higher according as the cement content is higher;

(b) increasing the cement content will increase the risk of shrinkage and cracking of the concrete and will cause more heat of hydration to be evolved during setting.

(c) for equal strength of the concrete, the cement content can be reduced if the dimension d of the aggregates is increased; this reduction may be applied proportionally to $\sqrt[5]{d}$.

As an empirical rule, the cement content of reinforced concrete should not be less than $550 / \sqrt[5]{d}$ in kg/m^3 finished concrete, i.e.:

350 kg/m^3 for d = 10 mm
300 kg/m^3 for d = 25 mm
250 kg/m^3 for d = 50 mm

The quantity of aggregates (sand + gravel) required to give one cubic meter of finished concrete is $\sim$ 1.2 m^3 because the sand fills the voids between the gravel particles.

The choice of the proportions of the aggregates can be determined by the following G/S - coefficient method. The method is purely empirical and is based on a large number of tests performed on a wide variety of concretes made in the laboratory using sand and gravel of the same specific gravity. However, the relative proportions of sand and gravel should be such that the concrete has satisfactory density and homogeneity without any risk of segregation.

In the most frequent case of a concrete mix comprising coarse aggregate (gravel, volume G) and fine aggregate (sand, volume S) the proportions are determined by the ratio G/S, once the cement content has been decided on the basis of the information given in the preceding section.

In general, the value normally adopted for G/S is 2.0. It may, however, be varied between 1.5 and 2.4, having regard to the following considerations:

(a) The higher G/S is, the higher will be the mechanical strength of the concrete; against this, however, the concrete is more liable to segregate and present difficulties in placing because of deficient workability.

(b) For a very plastic concrete mix, with high content of mortar and good workability, giving surfaces that present a good appearance on removal of the formwork, but not producing a concrete of exceptional strength, the following values may be adopted:

$$1.5 < G/S < 1.7$$

(c) For normal concrete, as used in ordinary reinforced concrete construction, with plasticity that may be varied according to the nature of the structure by varying the amount of water in the mix, of fair workability and giving good strengths, the following values may be adopted:

$$1.8 < G/S < 2.0$$

(d) For high-density concrete, of stiff consistency when freshly mixed, producing high strengths but liable to segregate and requiring certain precautions at the time of concreting (compaction by powerful vibration, in particular), the following values may be adopted:

$$2.0 < G/S < 2.2 \text{ (and exceptionally, 2.4)}$$

(e) In the case of rounded aggregates the above values of G/S are valid, but if crushed stone aggregates are employed, slightly lower values should be adopted to G/S: for example, the above values should be reduced by: 0.1 in the case of rounded sand and crushed coarse aggregate; 0.2 in the case of crushed fine aggregate and crushed coarse aggregate.

Water content and consistency of the mix: The mix should contain enough water to give the plasticity compatible with good workability, but not too much water, as the strength of the concrete becomes lower with increasing water content of the mix. On no account must water be added to a concrete mix that is considered to be too dry after leaving the mixer.

The desired consistency can be defined on the basis of a standard slump test.

The water content can be defined only in terms of the desired consistency of the mix.

For mixes containing a quantity of cement $C = 300$ to $400\ kg/m^3$, a total water content (assuming dry aggregates) may be adopted so as to obtain a water/cement ratio (W/C) within the following limits:

$$0.4 < W/C < 0.6, \text{ with an average value } W/C = 0.5$$

A value of $W/C < 0.5$ is adopted if it is desired to have a stiff or very stiff mix, or if the sand is somewhat deficient in fine particles, or if the gravel consists for the most part of large particles and is very porous, or for values of $G/S > 2$, or if an additive is used (plasticizer or wetting agent). In other cases a value of $W/C > 0.5$ is recommended.

The values of the slump (required to define the consistency of the concrete) given in the following table can be taken as applicable to concretes ordinarily employed: (Table 1-3)

Consistency of the concrete mix	stiff	plastic	very plastic
slump	0-2 cm	3-7 cm	8-15 cm

A consistency corresponding to a slump of 5-7 cm is generally very suitable for normally vibrated concretes.

1.4- MIXING, CONVEYING, PLACING, COMPACTING AND CURING OF CONCRETE

The principal purpose of mixing is to produce an intimate mixture of cement, water, fine and coarse aggregate, and possible additives of uniform consistency throughout each mix. Mixing should preferably be done in a vertical-axis mixer of the revolving-drum type. For a medium-size mixer running at 15-20 rev/min the minimum mixing time can be taken as 2 minutes. Mixing can be continued for a considerable time without adverse effect. This fact is particularly important in connection with ready-mixed concrete.

The constituent materials of the concrete should be put into the mixer in the following order: gravel, cement, sand. Water must not be added until these three materials have first been mixed in the dry condition for a time.

In certain cases it is to be recommended that first a portion of the coarse aggregates and water be put into the mixer and to run the latter briefly so as to wet the walls of the mixing drum or pan and thus prevent possible sticking of the mortar to them.

The concrete should be handled, transported and conveyed in such a way as not to entail any risk of segregation nor of setting before it has been placed in the formwork.

On large projects, it is recommended to use ready-mixed concrete which is batched in a stationary plant and then transported to the site in trucks in one of three ways: (1) mixed completely at the stationary plant and transported in a truck agitator, (2) batched at the plant but mixed in a truck mixer, or (3) partially mixed at the plant with mixing completed in a truck mixer. Concrete should be discharged from the mixer or agitator within at most one hour after the water is added to the batch.

The concrete mix must be conveyed from the mixer to the forms immediately in such a way that no segregation, excessive loss of ingredients or intrusion of foreign matter are liable to occur.

If the conveying time exceeds 30 minutes, it is advisable, especially in warm weather, to check by means of laboratory tests whether this is indeed a permissible length of time.

Placing is the process of transferring the fresh concrete from the conveying device to its final place in the forms. Before placing, loose rust must be removed from reinforcement, forms must be cleaned, and hardened surfaces of previous concrete lifts must be cleaned. Placing and compacting have a big and critical effect on the final quality of the concrete.

Concrete should always be placed and compacted by vibration unless special justification to waive this requirement is provided.

Vibrators used in compacting concrete in the forms are high-frequency power-driven tools. The following types of vibrators can be distinguished:

(a) Internal vibrators(also known as immersion vibrators) of varying thickness are inserted into the concrete. A vibrator of this kind consists of a tube containing a pneumatically powered high speed turbine driving a slightly eccentrically mounted vibrating unit. They should be able to penetrate into every part of the formwork, so that, they can reach and compact the whole of the concrete. The vibrators should be held vertically, moved axially and withdrawn very slowly, in such a way that the cavity left in the concrete can suitably fill up.

(b) External vibrators: the vibrators are attached to the formwork, which should therefore be of strong construction. This method is seldom used on construction sites.

(c) Surface vibrators: The compaction by the use of this type of vibrators is effected by means of vibrating surface compactors, generally on relatively large areas of concrete: precast panels, slabs, pavings, etc.
The thickness of layers of concrete compacted by surface vibration should not exceed 20 cm.

Vibration gives the concrete its maximum density by eliminating air voids and ensuring perfect filling of the formwork. It considerably reduces the internal friction of the particles of the constituents of the concrete and tends to give them liquid properties.

Vibration should not be overdone, especially with concrete of very plastic consistency, because the liquefaction of the concrete causes the larger aggregate particles to sink to the bottom, with the result that at the surface the mix will contain an excess of fine constituents and water. It is preferable to apply vibration for short periods, but at a large number of points located sufficiently close together. The internal vibrators should be withdrawn slowly from the concrete, before vibration is stopped, so as to avoid leaving holes in the concrete, as these would subsequently fill up with mortar, laitance or water.

Hot weather concreting

The production of concrete in hot weather poses the basic problem of keeping evaporation of the mixing water at the lowest possible rate. Slowing down the loss of water is accomplished by keeping the concrete as cool as possible and by helping the moisture to remain in the concrete for as long a period as is possible during the chemical hydration process combining water and cement.

However, special precautions should be taken in handling, transporting and placing the concrete and during its setting and hardening; in particular, curing is to be considered indispensable.

The special precautions which may have to be taken, depending on he temperature and humidity of the ambient air, are the following:

(a) stop concreting during the hottest hours of the day; if necessary, concreting may have to be done at night;
(b) use cold water or, possibly, water which has been cooled by having ice added to it in advance (ice must never be put directly into the mixer);
(c) protect the aggregates from the sun and spray them with water;
(d) never use hot cements;
(e) do not make too dry mixes;
(f) cover the skips in which the concrete is trasported;
(g) wet the external surfaces of the formwork, by spraying them with water, before and after concreting;
(h) place the concrete in the formwork as quickly as possible after mixing;
(i) use a setting retarder (in exceptional circumstances).

Curing of concrete

The object of curing is to keep the concrete sufficiently moist for satisfactory hardening to take place; it is indispensable in dry and warm weather.

Curing should start as soon as initial setting of the concrete occurs, for a delay of several hours may significantly impair its effectiveness: it should be continued for a week in normal cases and for two weeks if the weather is very dry and very warm.

Curing can be done either by wetting the concrete or by means of an impermeable temporary coating applied to it.

Wet curing consists in spraying the exposed concrete surfaces and the timber formwork with water two or three times daily, depending on the temperature and the atmospheric humidity. Exposed surfaces (i.e., not covered by the formwork) are most vulnerable, and it is advisable to cover them with straw mats or sacking which should be kept permanently moist by spraying at appropriate intervals. Alternatively, horizontal surfaces may be covered with a layer of sand.

If impermeable formwork is used, such as steel formwork, there is no need to wet the concrete surfaces in contact with such formwork so long as the latter has not been removed.

When concrete is cured by means of an impermeable temporary coating (membrane curing), a special compound is sprayed on the concrete surfaces to be protected. The compound forms a coating on the concrete and, being impermeable, prevents the evaporation of water from the concrete. These curing compounds are generally resin emulsions which break up as soon as they come into contact with the freshly placed concrete. The thin film of resin which is thus formed is the protective membrane. The compound should be slightly coloured so as to enable the continuity and evenness of application to be judged.

1.5- MECHANICAL PROPERTIES OF CONCRETE

The determination of the quality of a structural material should, in theory, encounter the study of all properties (chemical, mechanical, ...etc.) at all levels of measurements (microscopic and macroscopic). Due to practical limitations, however, the mechanical properties at the macroscopic level (so large as to be visible to the naked eye) are conventionally chosen as the sole measure of the quality as far as the structural behavior is concerned. Determination of the mechanical properties involves the selection of test specimens of standard shapes and dimensions as well as the procedure for preparing and testing such specimens. It is also conventional to select "the most significant" parameter(s) (ultimate compressive strength of concrete, yield point of steel, ...) and designate the material accordingly. In this manner, material designation normally serves as a name or a trade mark that reflects a nominal value of a certain property of the structural material. It is therefore of utmost importance for the designer to identify the designation of a structural material and realize the underlying assumptions before he can utilize such material efficiently.

Aside from statistical strength variations, measures of concrete quality vary with cement content, aggregate-to-cement ratio, age, relative humidity, size of test specimen, type and rate of loading, as well as many other factors. Although measures of concrete quality are numerous, the data presented hereafter are limited to measures of the mechanical properties pertinent to design.

Traditionally, the most significant measure of concrete quality is the maximum compressive stress that can be resisted by standard specimens (e.g., cubes, prisms, or cylinders) concentrically loaded at a specified loading rate when the concrete reaches a certain age (normally 28 days).

1) Compression Stress-Strain Relationship

Force-displacement response of a structural element constitutes a major branch of the serviceability requirements. This response is derived normally from the stress-strain relationship for the material of which the structural element is made. Knowledge of the stress-strain relationship is also essential in order to evaluate the ultimate strength. A typical set of compression stress-strain curves is shown in Fig. 1-21. These curves represent various qualities of concrete at the

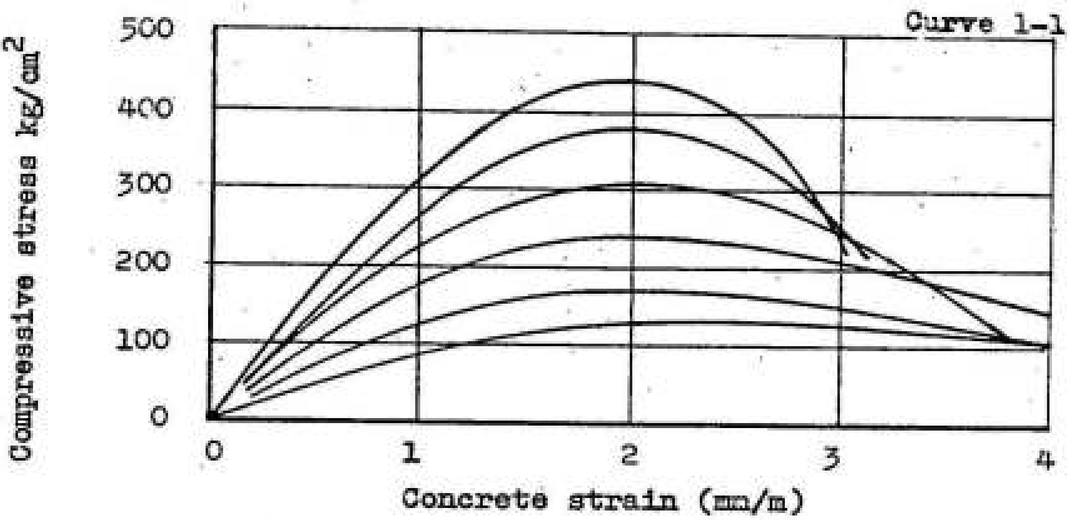


Fig. 1-21 Typical stress-strain curves at an age of 28 days from compression tests on concrete prisms

age of 28 days obtained from compression tests performed by concentrically loading concrete prisms at normal testing speeds (order of few minutes). All curves share somewhat similar characteristics; each curve consists of an initial relatively linear portion. After reaching about half the maximum value of stress, each curve deviates horizontally until reaching that maximum value at a strain of 0.002 (or 2 mm/m); thereafter, a descending branch takes place. One may notice that higher grade concretes are generally more brittle, i.e., fracture at a smaller value of maximum strains.

The maximum value of stress of any curve, Fig. 1-21, is a universally accepted measure of concrete strength under compression and is called prism strength of concrete. Cube and cylinder strengths are defined similarly, that is, by adapting a standard specimen in the form of a cube or a cylinder. Prism and cylinder strengths as defined above yield almost the same value, which is about 80 percent of cube strength for concretes of normal quality; this variation reflects the phenomenon of size effect.

Compressive strength varies with age; for normal concretes at moderate temperatures, the ratio of the compressive strength at 28 days to that at a given concrete age may be estimated from the values given in the following table 1-4.

Table 1-4

Type of Portland Cement	Ratio of Concrete compressive strength at the Age of 28 Days to that at the Age (Days) of :				
	3	7	28	90	360
Normal	2.50	1.50	1.00	0.85	0.75
Rapid Hardening	1.80	1.30	1.00	0.90	0.85

Experimental investigations showed that the concrete compressive strength is influenced by the rate of loading: strength decreases with the increase of load duration. For design purposes, one may assume the value of concrete compressive strength for sustained loading (i.e.,for loads with long duration) to be equal to 85 percent of that for short duration loading.

The concrete strength f_c chosen as a basis for the design must be the minimum guaranteed value which is obtained from the analysis of a sufficient number of test specimens. Such an analysis can however be done in many ways:

1) By statistical analysis of the test results which is defined by the relation

$$f_c = f_{cm} (1 - k\delta)$$

where f_{cm} denotes the arithmetic mean of test results, δ denotes the relative standard deviation, and k denotes a coefficient depending on the number of test results defining f_c and on the probability of the test results being below the value of f_c :

Assuming that the test results are f_{c1}, f_{c2}, f_{c3} .... f_{cn} where n is the number of test results, then

$$f_{cm} = \frac{f_{c1} + f_{c2} + f_{c3} + \dots f_{cn}}{n} = \frac{\Sigma f_c}{n}$$

If it is allowed that 5 - 10 % only of the test results are below the required guaranteed value of f_c: this probability of 5 - 10 % entails k = 1.64 - 1.28.

The relative standard deviation δ can be calculated from the relation:

$$\delta = \frac{1}{f_{cm}} \sqrt{\frac{\Sigma (\Delta f_c)^2}{n}}$$

in which

$$\Delta f_c = f_{c1, 2, 3 \dots n} - f_{cm}$$

The minimum value of δ to be included in the design is 10%, thus

$$f_c \leqslant f_{cm}\,(1 - 0.1\,k)$$

Then

for $k = 1.64$, $f_c = 0.836\,f_{cm}$ & for $k = 1.28$, $f_c = 0.872\,f_{cm}$

In concrete structures, if no sufficient tests ($n < 20$) are carried out, one may assume $\delta = 0.2$. In which case:

$$f_c \leqslant f_{cm}\,(1 - 0.2\,k)$$

and

for $k = 1.64$, $f_c = 0.672\,f_{cm}$ & for $k = 1.28$, $f_c = 0.744\,f_{cm}$

Illustrative example

Assume that the results of 20 concrete compression tests in kg/cm^2 arranged in decreasing order are:

235	234	231	228	225	223	215	212	208	205	f_{cm} =
200	200	198	195	191	189	185	180	178	168	205 kg/cm^2

$$\delta = \frac{1}{205}\sqrt{\frac{(235-205)^2 + (234-205)^2 + \ldots\ldots + (178-205)^2 + (168-205)^2}{20}}$$

which means that $\delta = 0.095 < 0.1$

Assuming $k = 1.64$, then $f_c = 0.836\,f_{cm} = 0.836 \times 205 = 171.35$ kg/cm^2

2) This second method is an approximation which yields very much the same value (for $k = 1.64$) but which avoids having to calculate the standard deviation. It consists in taking for the minimum guaranteed strength twice the mean value of half the results below the median, minus the mean value of all the results.

For the previous example, we have $f_{cm} = 205$ kg/cm^2. Therefore

$$f_c = (200 + 200 + 198 + 195 + 191 + 189 + 185 + 180 + 178 + 168)\,2\,/\,10 - 205 = 376.8 - 205 = 171.8 \text{ kg/cm}^2$$

which is approximately the same result.

3) The following method given in the German Code, DIN 1045, article 7-4-5-2 may also be used.

Strength requirements are assumed satisfied when the average compressive strength of every series of three cubes, from three different batches, is bigger than the minimum values given in column 3 of the following table and, the compressive strength of every cube is minimum equal to the values given in column 2.

For any concrete quality of the same composition, the strength of not more than one cube out of nine cubes in three series is to be smaller by more than 20% of the values of column 2. However, the average of a series of three cubes must be bigger than the values given in column 3.

Table 1-5

1	2	3	4
conc. grade	min. strength of every cube kg/cm^2	av. strength of 3 succ. cubes kg/cm^2	use
C50	50	80	pl. conc. only
C100	100	150	for pl. and rfd conc.
C150	150	200	
C250	250	300	
C350	350	400	
C450	450	500	
C550	550	600	

2) Modulus of Elasticity

The instantaneous modulus of elasticity of concrete in compression, denoted by E_c, is defined as the slope of the initially straight portion of the stress-strain curves such as these shown in Fig. 1-21. The word instantaneous identifies the modulus to refer to short duration loading (one or two minutes). This instantaneous modulus of elasticity E_{cj}, may be estimated from the following empirical formula for concretes at the age j days:

$$E_{cj} = 21{,}000 \sqrt{f_{cj}} \quad kg/cm^2$$

where f_{cj} is the concrete prism strength at j days in kg/cm^2.

In order to account for the combined effects of shrinkage and creep (long duration of 24 hours), the designer may assume a reduced modulus, E_c , that equals one third the value of the instantaneous modulus. Or

$$E_c = E_{cj} / 3$$

In case of ordinary buildings where the live load (instantaneous load) is about one third of dead loads, the modulus of elasticity E_{cj} may be assumed according to the relation

$$E_{cj} = 15{,}000 \sqrt{f_{cj}} \quad kg/cm^2$$

Egyptian standards adapt the cube compressive strength as the measure for concrete quality and give a designation accordingly. A designation C200, for example, refers to concrete having a 28-day compressive strength of 200 kg/cm^2 obtained from a testing cube with a side dimension of 15.8 cm (area 250 cm^2) when loads are applied at normal testing speeds.

3) Tensile Strength

The ultimate resistance of concrete subjected to tensile stresses (or the tensile strength of concrete) is so important that it called for adaptation of an accepted measure(s) of strength. The importance stems from the fact that tensile stresses govern significant behavioral phenomena such as: (a) initiation and propagation of tesile cracking, (b) shear and torsion distress, and (c) interface (or bond) failure.

Because of experimental difficulties and the statistical nature of strength variation, different tests were proposed in the hope to arrive at a stable measure of tensile strength of concrete. Different tests produce, on the average, different values of mean tensile strength. All such tests, however, have concluded that strength of concrete in tension is very limited (about 10 to 15 percent) compared with that in compression. This fact is the sole reason for provision of steel to reinforce concrete in order to produce the efficient structural material known as Reinforced Concrete (R.C.).

The various tests differ mainly in the choice of the type of external loading that induces internal tensile stresses which are high enough to be the primary cause of failure. The most common tests are as follows: (1) direct tension test (axial tensile force), (2) split-cylinder or split-cube test (compressive force), and (3) modulus of rupture test (flexural force). The last two types of tests are more common than the first one mainly because of the suitability of the loading arrangements to the testing machines.

Split-cylinder tests are performed, generally at an age of 28 - days, by application of two equal compressive forces along diametrically opposite generating lines of a cylindrical specimen. The cylinder measures 15 cm in diameter and 30 cm in length, Fig. 1-22(a).

The tensile strength, denoted by f_t, is assumed equal to 0.85 the stress f_{sp}, at failure of the specimen of the split test and is given by the formula:

$$f_t = 0.85\ f_{sp} = 0.54\ P\ /\ d\ l$$

where d and l refer to the diameter and length of the cylindrical specimen, and P is the load at which the concrete cylinder fails by split-

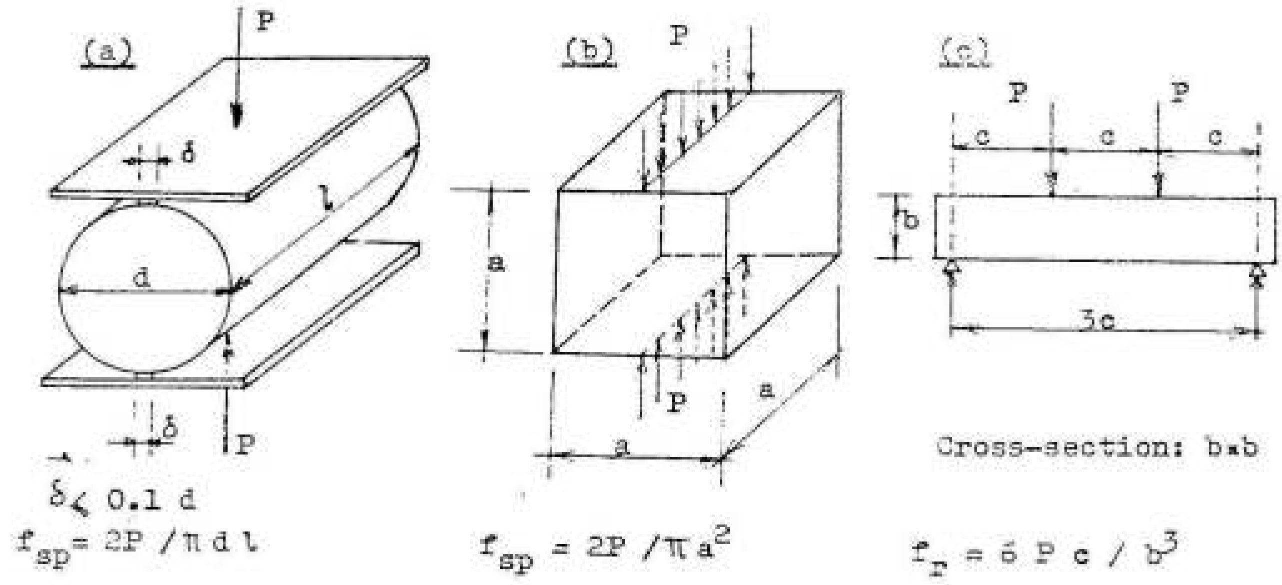


Fig. 1-22. Types of specimens for tensile strength of concrete.
(a) Split-cylinder test. (b) Split-cube test.
(c) Modulus of rupture test.

ting. Upon substituting for the values of d and l, this formula reduces to

$$f_t = 1.2 P$$

where P is measured in tons and f_t in kg/cm^2

In case of split-cube tests, Fig. 1-22 (b), the tensile strength is expressed as follows:

$$f_t = 0.54 P / a^2$$

where a stands for the side length of the cube. When the value of the side length is standarized as 15.8 cm, this expression takes the form:

$$f_t = 2.16 P$$

where P is measured in tons and f_t in kg/cm^2.

The modulus of rupture tests are conducted, generally at an age of 28 days, by the application of two concentrated loads at the middle third points of the span of a prismatic specimen. This specimen has a 10 x 10 cm square cross-section, and overall length of 50 cms, and a span of 45 cm, Fig. 1-22 (c). The value of the tensile strength is, conventionally taken as the three-fifths of the modulus of rupture, or symbolically: $f_t = 0.6 f_r$; and after substituting for the value of b and c, one gets

$$f_t = 54 P$$

where P is measured in tons and f_t in kg/cm^2.

Experiments showed that the tensile strength of concrete increases with age: the ratio of the tensile strength of concrete at an age of 28 days to that at a given age follows the trend given by the table: 1-6

Type of Portland Cement	Ratio of Concrete Tensile Strength at the Age of 28 Days to that at the Age (Days) of:				
	3	7	28	90	360
Normal	2.00	1.40	1.00	0.95	0.90
Rapid Hardening	1.50	1.20	1.00	0.95	0.90

For design purposes, many investigators have tried to express the tensile strength of concrete as a function of its compressive strength. For moderate-strength concretes (C160 to C400), the following empirical formula was recommended for the estimation of the tensile strength f_t in terms of the 28-day prism compressive strength f_{cp}:

$$f_t = 1.4\sqrt{f_{cp}}$$

For various concrete designations, the following table gives the corresponding values of the cube and the prism strengths as well as the modulus of elasticity in normal cases assuming an age of 28 days:

1-7 Mechanical Properties for Different Concrete Designations

Concrete Designation	Cube Strength f_{c28} kg/cm^2	Prism Strength f_{cp} kg/cm^2	Tensile Strength f_t kg/cm^2	Mod. of Elasticity E_{c28} ton/cm^2
C 160	160	135	16	175
C 180	180	150	17	185
C 200	200	165	18	195
C 225	225	185	19	205
C 250	250	200	20	210
C 275	275	220	21	220
C 300	300	240	22	230
C 350	350	280	23	250
C 400	400	310	24.5	265
C 450	450	345	26	280
C 500	500	380	27	290
C 550	550	415	28.5	305
C 600	600	450	30	320

4) <u>Shear and Torsion Strength</u>

Because of shear and torsion stresses, with or without normal stresses, principal tensile and compressive stresses normally develop. In such cases, the most reasonable failure hypothesis is: concrete is liable to fail when the principal tensile stress exceeds the tensile strength.

5) <u>Poisson's Ratio</u>

In cases where the transverse strains are of importance, a Poissons ratio of 0.15 may be assumed.

6) <u>Coefficient of Thermal Expansion</u>

The coefficient of thermal expansion depends on many variables such as the nature of cement and aggregate, the cement content, and the relative humidity. In the absence of an experimentally determined value, the coefficient of thermal expansion may be taken as 10^{-5}.

7) <u>Creep</u>

The property of a material to deform with time when subject to a constant stress is known as creep. Creep deformation increases with : (1) time, (2) the magnitude of the sustained stress, and (3) the water cement ratio. Creep deformations also increase when concrete is loaded at an early age or if it is exposed to drying conditions.

Influence of creep deformations on the mechanical properties of concrete varies considerably with the type of measure. Compared with

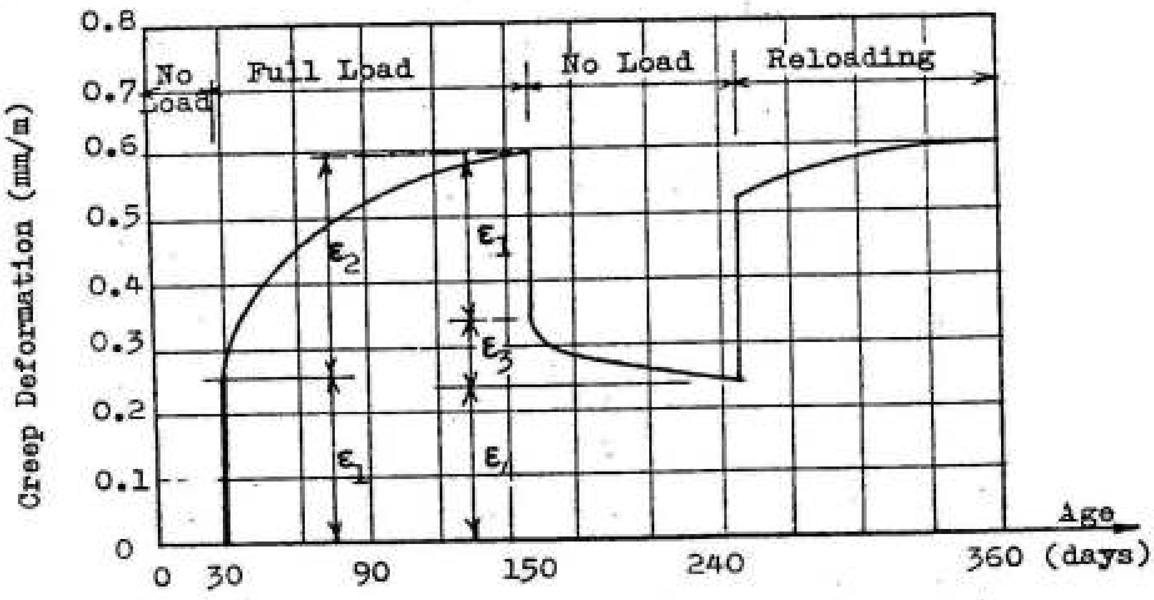


Fig. 1-23 Creep deformations: Influence of age and loading sequence

values for instantaneous loading, the compressive strength decreases only about 15 percent; whereas, the modulus of elasticity drops to about one-third of its instantaneous value.

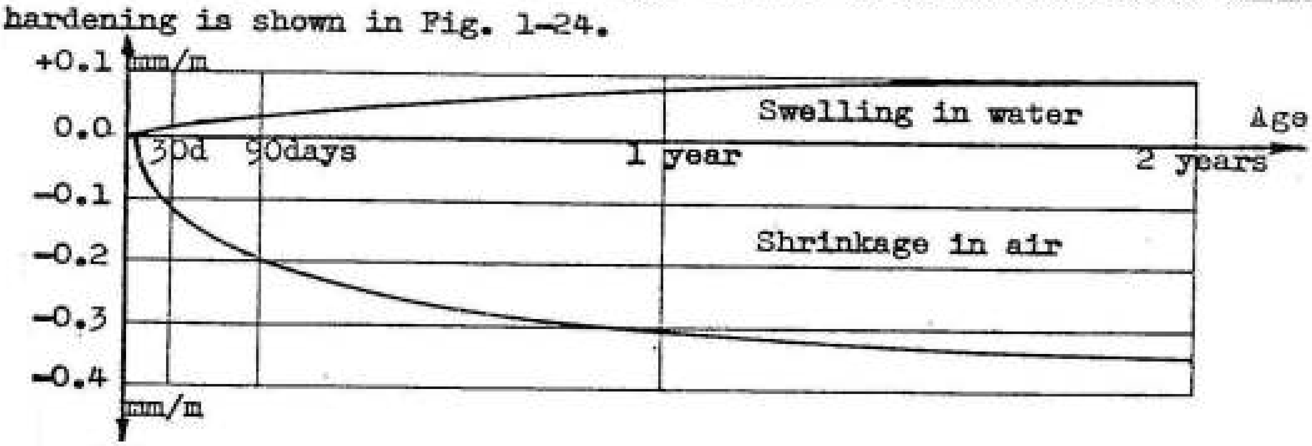
A typical plot showing the influence of time and loading sequence on creep deformations is given in Fig. 1-23. Upon loading to a stress as high as one-half the value of the compressive strength, an instantaneous elastic deformation, ϵ_1, occurs. If the load is sustained for a period of time, creep deformation, ϵ_2, takes place. Upon removal of the full load, an instantaneous elastic deformation ϵ_1 is recovered followed by a time-dependent creep recovery, ϵ_3; thereby, a considerable part of the deformation, ϵ_4, is normally left unrecovered. If the specimen is reloaded, similar response is indicated; the instantaneous elastic deformation in each case is proportional to the magnitude of the load.

8) <u>Shrinkage</u>

The phenomenon of reduction in size of concrete due to loss of moisture is known as shrinkage. Shrinkage is a time-dependent reversible phenomenon in the sense that it is a function of time and that immersing concrete in water, after shrinkage, results in an expansion to nearly the original size. Shrinkage increases: (a) significantly with the amount of mixing water, (b) measurably with the humidity conditions , (c) noticeably with the presence of aggregates of high absorptive characteristics, and (d) slightly with the cement content.

The value of strain associated with shrinkage varies from about 0.0002 (or 0.2 mm/m) to about 0.0007 (or 0.7 mm/m). The low strain values correspond to low amounts of mixing water and very humid environment; whereas, high shrinkage strains occur in cases of large amounts of mixing water and very dry conditions.

Shrinkage in air and swelling in water of normal concretes during hardening is shown in Fig. 1-24.



g. 1-24 Shrinkage in air and swelling in water during hardening

1.6- REINFORCING STEEL

In comparison with concrete, steel is a high strength material. The usable strength (or the strength at yield in compression or in tension) of ordinary reinforcing steel is in the order of ten times the compressive strength of common structural concrete. The relatively high cost of steel discourages to use it liberally to resist compressive stresses. Concrete, on the other hand, is much cheaper but its tensile strength is very limited. An economic structural material results when concrete and steel are combined properly: the concrete provides resistance to compressive forces while tensile stresses are assigned to the reinforcing steel.

An effective combined action (or bonding) of the concrete and the reinforcing steel is essential for the two materials to afford monolithic behavior. In order to furnish such action (or bond), reinforcing steel commonly takes the form of bars. The use of high strength steels called for introduction of projections (or deformations) that are rolled on the bar surface; this makes the bars look deformed and are called deformed bars. Bars without such deformations are called ordinary bars, smooth bars, or plain bars. The fact that deformed bars were originally introduced for high strength reinforcing steel should not overshadow the intended purpose; that is, deformed bars are used to improve the bond between the reinforcing bars and the surrounding concrete. Deformed bars have become so popular that many countries restrict the use of plain bars to those having a diameter of no more than 10 mm regardless of the type of steel used.

Reinforcing steel bars may be classified according to two distnct schemes, namely: (a) steel characteristics, and (b) bar profile. The first scheme, which deals only with the type of steel regardless of bar shape, is as follows: (1) normal mild steel (ϕ), (2) high grade mild steel (Φ), and (3) cold-twisted steel ($\ddagger$). According to the second scheme, the bars are: (1) plain bars, or (2) deformed bars.

1) Stress-Strain Relationship

The stress-strain relationship is obtained from results of tension tests of the reiforcing steel bar. The specimen is simply a piece of the bar with a sufficient length. The tensile force is applied, at a specified rate, at the ends of the specimen that elongates until failure by separating into two parts.

Strain is defined herein as the ratio of the stretch (or elongation) between two conveniently established gauge points to the gauge ngth, which is the distance between the gauge points prior to loading. Whereas, the stress is determined by dividing any particular applied load by the specimen's original cross-sectional area. The area of plain bars presents no special problem as it is that derived directly from the bar diameter. The area of a deformed bar, however, is a nominal area based on a nominal diameter defined as the diameter of an equivalent smooth bar having the same weight per unit length. That is, the deformations are included in the calculations of stresses although they are not really helping in carrying the load.

Typical stress-strain curves for bars classified according to steel chracteristics are shown in Fig. 1-25. All curves possess an initially straight segment the slope of which is defined as the elastic modulus , E_s . For all types of reinforcing steel, the elastic modulus is constant and equals 2100 ton/cm^2. Stress-strain curves for mild steels (both normal and high grade) differ from that for cold-twisted steel bars in that the former exhibits a 'sharp break' from the straight-line condition representing the linearly elastic behavior. Immed-

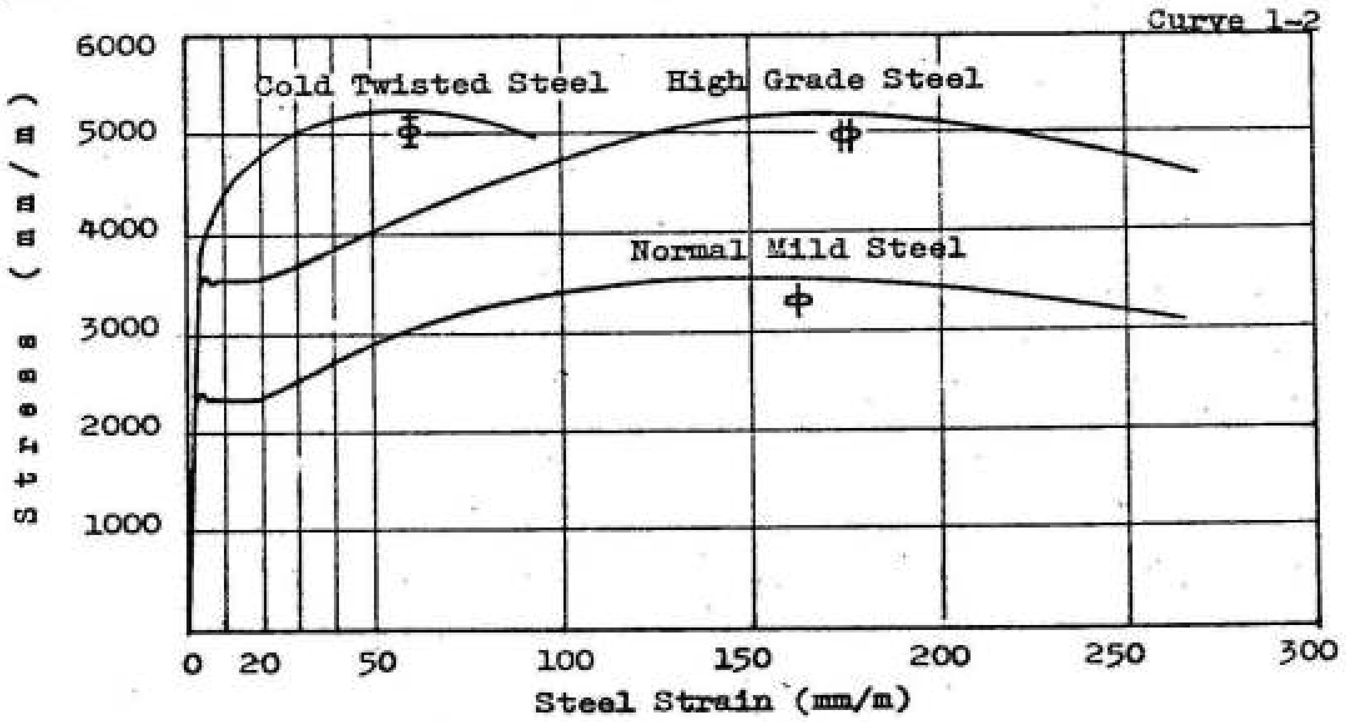
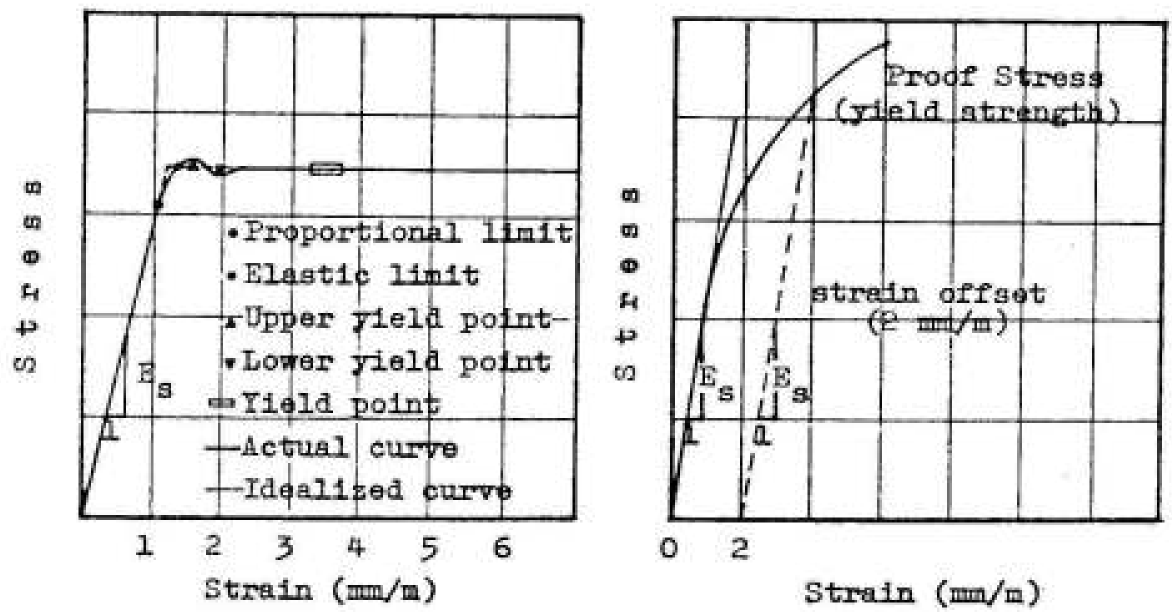


Fig. 1-25 Typical stress-strain curves for reinforcing bars of different steel characteristics

iatly before this so-called sharp break, the strain ceases to be proportional to the stress; the stress at this point is defined as the proportional limit. Past the proportional limit, the material remains elastic but nonlinearly elastic. That is, unloading does not produce permanent strains; meanwhile, strains are not proportional to the stresses . As the stress increases, the material reaches a limit for this elastic behavior (or the elastic limit). A further increase in the load brings the material stress to a local maximum value defined as the upper-yield-point then to a local minimum stress defined as the lower-yield-point. Further loading results in an increase in the strain without any noticeable increase in the stress; the material exhibiting such characteristic is said to be a ductile material. The stress at this stage is defined as the yield point of the material.

A detailed study of the stress-strain diagram shows that the proportional limit, the elastic limit, the upper and lower yield points are all so near in value to the yield point that for design purposes all are considered as one, namely the yield point, Fig. 1-26(a). The value of the stress at yield (or the yield point) is of utmost importance in reinforced concrete design according to the ultimate limit state



(a) Mild-steel diagram (b) Cold-twisted steel diag.

Fig. 1-26 Stress-strain diagrams: identification of yield point and yield strength

The stress-strain diagram for cold-twisted steel bars does not possess a sharp break or a well-defined yield point. For design purposes, a nominal value is conventionally chosen to represent a fictitious yield point; this is done by means of the so-called offset method. According to this method, the material's proof stress (also known as the yield strength) is defined as the stress determined by the intersection of the stress-strain curve and a line parallel to the elastic portion at a specified strain offset (or shift). This specified strain offset, Fig. 1-26(b), is conventionally taken as 0.2 percent (or 2 mm/m).

The tensile strength of a reinforcing bar is defined as the uppermost stress that can be resisted by the test specimen. The values of tensile strength for different types of steel are given in the following table together with the values of yield point, proof stress, and strain at failure.

Table 1-8

Types of steel	Minimum yield point kg/cm^2	Minimum proof stress kg/cm^2	Minimum tensile strength kg/cm^2	Minimum strain at failure mm/m
Mild steels:				
Normal	2300	-	3500	200
High grade	3600	-	5200	180
Cold-twisted steel	-	4000	5000	100

It is of interest to note that the value of the strain at the initiation of yielding of mild steel is about 1.0 to 1.3 mm/m, which is about one-twentieth of that at the beginning of the strain-hardening region.

2) Poisson's Ratio

The Poisson's ratio of steel is in the neighbourhood of 0.25.

3) Coefficient of Thermal Expansion

The value of the coefficient of expansion of steel is approximately 1.2 x 10^{-5} cm per cm per one degree centigrade, which is slightly higher than that for concrete. Fortunately, the difference is not high enough, under normal temperatures, to cause concern.

CHAPTER 2

ANALYSIS OF REINFORCED CONCRETE AT DIFFERENT LOAD STAGES

2.1- DESIGN AND ANALYSIS

The chief task of a structural engineer is the design of structures. By design it is meant the determination of the general shape and all specific dimensions of a particular structure so that it will perform the function for which it is constructed and will safely withstand the actions which will act on it during construction and through its useful lifetime. These actions are primarily the loads and other forces which may act, as well as indirect actions such as temperature changes, settlement of supports .... etc.

In order to be able to make this design efficiently, one should be capable of analysing its elements under different stages of loading. Conventionally, the analysis of a reinforced concrete section, for example, involves the following task: for a given cross section of known dimensions, material properties, and reinforcements, it is required either to find the stress distribution due to known external forces and moments, or to determine the external moments and forces corresponding to a certain stress or strain profile.

2.2- BASIC ASSUMPTIONS AND CONDITIONS OF EQUILIBRIUM

The fundamental principles on which the analysis of reinforced concrete is based are as follows:

1) The internal forces such as bending moments and shearing forces and the corresponding normal and shear stresses at any section of a member are in equilibrium with the effects of the external loads at that section.

2) It is assumed that the bond between steel and concrete is sufficient to keep them acting together under the different load stages i.e., no slip can occur between the two materials and the strain in an embedded reinforcing bar is the same as that of the surrounding

concrete.

3) In elements subject to bending or eccentric forces, plane sections before loading remain plane after loading i.e., the strains are proportional to the distance from the neutral axis. This assumption is not sufficiently accurate at stages close to failure but the deviations are usually small and the results of theory based on this assumption check well with test information.

4) Concrete in tension is generally cracked (except for small loads) and is neglected in the design. In some special cases, this assumption is dispensed with and advantage is taken of the small tension strength that the concrete can develop.

5) The theory is based on the actual stress-strain relationship and strength properties of the two constituent materials, taking the plastic behavior at higher stress levels in consideration.

2.3- AXIAL COMPRESSION

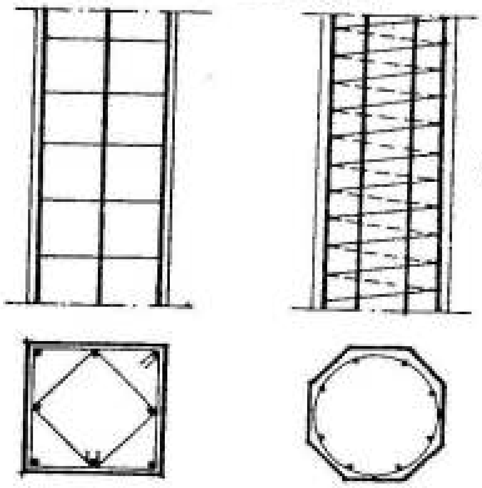
It is more economical to design an axially compressed member so that concrete carries most of the load. However, it is necessary to introduce reinforcements in such members in order to:

1) resist the eventual bending moments that are liable to be acting because very few members are truly axially loaded,

2) increase the safety. Plain concrete and tied members fail suddenly; whereas provision of spiral reinforcement causes cracks to develop long before failure,

3) reduce the dimensions of the member because a part of the load will be carried by the steel with its much greater strength.

The two chief forms of reinforced concrete members designed to carry axial loads are : the tied columns and the spirally reinforced ones, Fig. 2-1. In tied columns, Fig. 2-1a, the longitudinal bars serve as main reinforcement; they are held in place by transverse ties which prevent displacement of main bars during construction operations and prevent the buckling of the longitudinal bars under loading. In spirally reinforced



(a) Tied column

(b) Spirally reinforced column

Fig. 2-1 Chief forms of columns

columns, Fig. 2-1b, the longitudinal bars are surrounded by a closely spaced spiral which serves the same purpose as the more widely spaced ties and prevents the lateral strain of the concrete within it, thereby increasing its resistance to axial compression.

1) Elastic Analysis of Columns

Under working stresses, ($\sigma_c < f_{cp}/2$), and neglecting the effect of shrinkage and creep, the concrete behaves nearly elastically, i.e. stresses and strains are quite closely proportional so that an axial normal force N acting on a section of the form shown in Fig. 2-1a is resisted by both concrete and steel, hence:

$$N = A_c \sigma_c + A_s \sigma_s \qquad (2\text{-}1)$$

in which

A_c = area of concrete section of element
A_s = cross sectional area of longitudinal reinforcement
σ_c = uniform compressive stresses in concrete due to N
σ_s = compressive stress in longitudinal steel due to N

Both materials being subject to the same strain ϵ, then

$$\epsilon_c = \epsilon_s$$

In the elastic range, we have

$$\epsilon_c = \sigma_c/E_c = \epsilon_s = \sigma_s/E_s$$

Accordingly, we get

$$\underline{\sigma_s} = (E_s/E_c)\ \sigma_c = \underline{n\ \sigma_c} \qquad (2\text{-}2)$$

where $n = E_s/E_c$ is known as the modular ratio and is assumed = 15.

Hence, we have

$$\left.\begin{aligned} N &= A_c\ \sigma_c + A_s\ n\ \sigma_c && \text{or} \\ N &= \sigma_c\ (A_c + n\ A_s) && \text{i.e.} \\ \sigma_c &= N\ /\ (A_c + n\ A_s) && \\ &\text{Assuming } A_s/A_c = \mu \text{, one can write} && \\ N &= \sigma_c\ A_c\ (1 + n\mu) && \text{and} \\ \sigma_c &= N\ /\ A_c\ (1 + n\mu) && \text{i.e.} \end{aligned}\right\} \qquad (2\text{-}3)$$

2) Effectiveness of Spirals

If an elastic element of length unity, Fig. 2-2, is subjected to a longitudinal compressive stress σ_1 and a corresponding strain ϵ_1, transverse tensile stresses σ_2 and strains ϵ_2 will be developed and

$$\epsilon_2 = \epsilon_1\ /\ m$$

or $\sigma_2 = \sigma_1 / m$

in which

$1/m$ = Poisson's ratio = 1/5 for reinforced conc.

If the tensile stresses are resisted by spirals having an area of cross section = A_{sp}, the tension in the spirals T_{sp} can be evaluated from the relation:

$$T_{sp} = A_{sp}\,\sigma_s' = \frac{d_k}{2}\,\sigma_2\,s$$

where

d_k = diameter of concrete core inside spirals,

s = pich of spirals, and

σ_s' = allowable tensile stress in spirals.

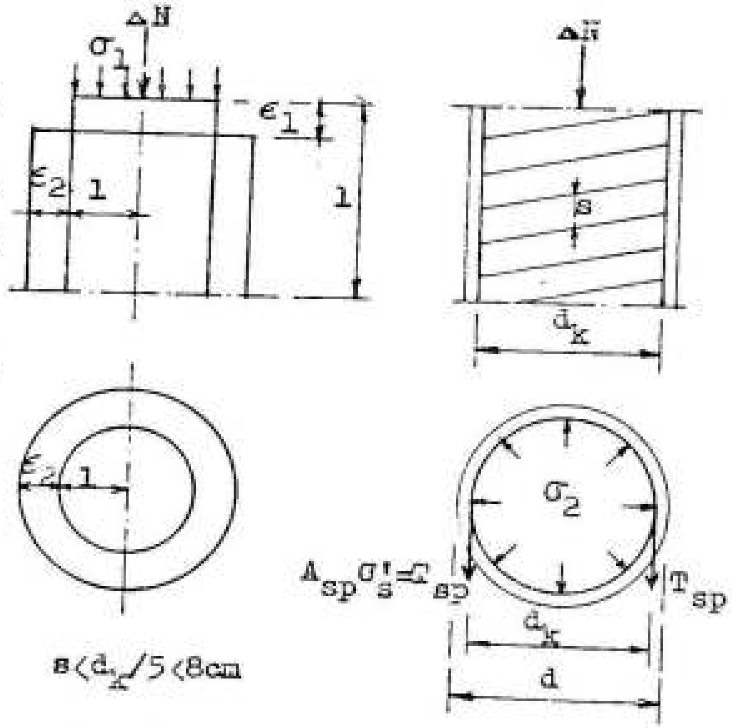


Fig. 2-2 Resistance of spirals

Substituting σ_2 by σ_1/m, we get

$$A_{sp}\ \sigma_s' = \frac{d_k}{2}\,\frac{\sigma_1}{m}\,s$$

and

$$\sigma_1 = \frac{2\,m\,A_{sp}\,\sigma_s'}{d_k\,s}$$

or

$$\sigma_1 = \frac{\pi\,d_k}{\pi\,d_k}\cdot\frac{2\,m\,A_{sp}\ \sigma_s'}{d_k\ s}$$

assuming

$A_s' = \pi\,d_k\,A_{sp} / s$ = cross-sectional area of an imaginary longitudi- reinforcement having the same volume as the spirals, and

$A_k = \pi\,d_k^2 / 4$ = area of core of section, we get

$$\sigma_1 = \frac{m}{2}\,\frac{A_s'\,\sigma_s'}{A_k}$$

and the corresponding load that can be resisted by the spirals:

$$\Delta N_{sp} = \sigma_1\,A_k = \frac{m}{2}\,A_s'\,\sigma_s' = 2.5\,A_s'\,\sigma_s' \qquad (2\text{-}4)$$

Spirals are significantly effective only at high loads and generally after the failure of the concrete cover, so that only the area of the core A_k may be effective when calculating the column load. Hence

$$N_{sp} = A_k\,\sigma_c + a_s\,\sigma_s + \frac{m}{2}\,A_s'\,\sigma_s' \qquad (2\text{-}5)$$

in which

N_{sp} = axial load carried by a spirally reinforced section,

A_k = area of core of section = $\pi\, d_k^2 / 4$,

σ_c = working compressive stress in concrete section,

A_s = cross sectional area of longitudinal reinforcement,

σ_s = working compressive stress in long. reinforcement = $n\, \sigma_c$,

σ_s' = working tensile stress in spirals,

$A_s' = A_{sp}\, \pi\, d_k / s$ and $m/2 = 2.5$

So that we have:

$$N_{sp} = \sigma_c\,(A_k + n\,A_s) + 2.5\,A_s'\,\sigma_s' \qquad (2\text{-}5')$$

In order to have sufficient safety against failure of cover, we should have:

$$\sigma_c\,(A_k + n\,A_s) + 2.5\,A_s'\,\sigma_s' \leq 2\,\sigma_c\,(A_c + n\,A_s) \qquad (2\text{-}6)$$

which means that the working load of a spirally reinforced column, calculated according to equation 2-5 must not be assumed more than twice the working load calculated according to equation 2-3.

It has been found that cross-reinforced members, Fig. 2-3, can be calculated according to the equations given for spirally reinforced elements. In which case, A_s' = volume of cross mesh per unit length and $s < d_k / 5 < 8$ cm

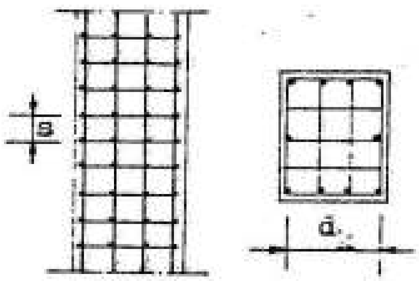


Fig. 2-3 Cross-reinforced member

3) Evaluation of the Effects of Shrinkage or Creep

Shrinkage and creep are plastic phenomenons. However, it is tried to evaluate their effect in an elastic manner approximately in the following manner:

Let Fig. 2-4a represent a symmetrically reinforced concrete element of unit length. If the element were not reinforced as in Fig. 2-4b, it would have shortened a distance ϵ_{sh} = the shrinkage strain of

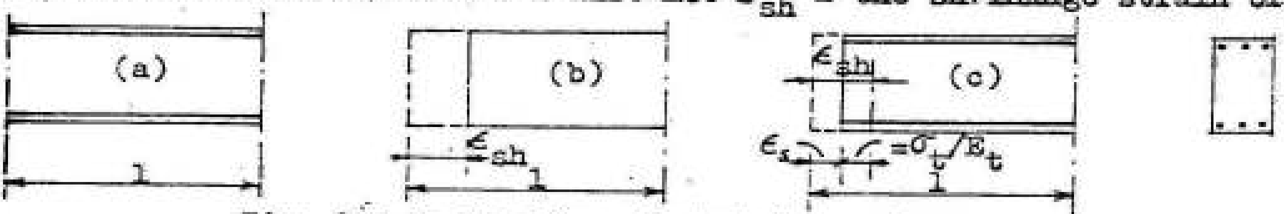


Fig. 2-4 Evaluation of shrinkage stresses

concrete. Due to the presence of the steel reinforcement, which does not shrink, the shortening will be $\epsilon_s < \epsilon_{sh}$ only, i.e., the steel reinforcement shortens a distance ϵ_s and hence is subject to compressive stresses σ_s while the concrete elongates a distance equal to ϵ_{sh} minus ϵ_s and hence is subject to tensile stresses σ_t. The stresses in steel and concrete can be evaluated in the following manner:

strain due to shrinkage $= \epsilon_{sh}$

final strain in steel $= \epsilon_s = \sigma_s / E_s$

final strain in concrete $= \epsilon_c = \epsilon_{sh} - \sigma_t / E_c$

both materials are subject to the same strain, hence

$$\epsilon_s = \epsilon_c \qquad \text{or}$$

$$\sigma_s / E_s = \epsilon_{sh} - \sigma_t / E_c \qquad \text{or}$$

$$\sigma_s = \epsilon_{sh} E_s - n\,\sigma_t \qquad \text{(a)}$$

As no external forces act on the element, then

$$\sigma_t A_c - \sigma_s A_s = 0 \qquad \text{(b)}$$

Equations (a) and (b) give

$$\sigma_t A_c - (\epsilon_{sh} E_s - n\,\sigma_t)\,A_s = 0 \qquad \text{or}$$

$$\underline{\sigma_t = \frac{\epsilon_{sh} E_s A_s}{A_c + n A_s}} \qquad \text{tension} \qquad (2\text{-}7)$$

Equation (b) gives

$$\underline{\sigma_s = \sigma_t A_c / A_s = \sigma_t / \mu} \qquad \text{comp.} \qquad (2\text{-}8)$$

where μ = the ratio of the steel in the section.

σ_s can also be given in the form:

$$\underline{\sigma_s = \frac{\epsilon_{sh} E_s A_c}{A_c + n A_s}} \qquad \text{comp.} \qquad (2\text{-}8')$$

Stresses due to creep can be calculated from the same equations 2-7 and 2-8 if ϵ_{sh} is replaced by ϵ_{cr}. According to these relations, one can see that shrinkage and creep have a very big effect on the stresses both in concrete and steel as can be seen from the illustrative example shown under 6).

4) Columns Tests

It has been verified by tests that actual working stresses could not be determined or estimated with any reasonable accuracy, they also indicated that ultimate column strength did not vary appreciably with the history of loading. If steel yields first, the concrete deforms with additional loading until it reaches its ultimate strength. Thus regardless of loading history, a tied axially loaded column reaches its ultimate load N_u when it is equal to the sustained maximum strength of concrete ($0.85\ f_{cp}\ A_c$) plus the yield strength of the longitudinal steel ($f_y\ A_s$) . i.e.,

$$\underline{N_u = 0.85\ f_{cp}\ A_c + f_y\ A_s} \qquad (2-9)$$

At this load, the concrete immediately fails by crushing and shearing outward along inclined planes, and the longitudinal steel buckles between ties as shown in Fig. 2-5.

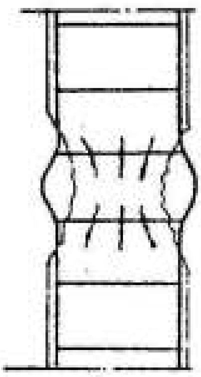


Fig. 2-5 Failure of a tied column

The failure of a spirally reinforced column is different. Upon reaching the ultimate load of a tied column having the same area and longitudinal reinforcement, the presence of spirals (with narrow pitch) confines concrete and steel inside the core and delays failure. The concrete cover, however, not being so confined does fail, i.e., the outer cover fails when the load N_u (equ. 2-9) is reached. It is at this stage that the confining action of the spirals takes effect, and, if adequate spirals are provided, the failure load of the column can be increased.

It has been found that a given amount of spiral reinforcement per unit length of a column is about 2.5 times as effective as the same amount of steel used in the form of longitudinal bars. A convenient amount of spirals prevents the instantaneous crushing of concrete and buckling of steel; that is , it produces a more gradual and ductile failure. The situation is best understood from Fig. 2-6 which compares the performance of a tied column with that of a spiral column whose

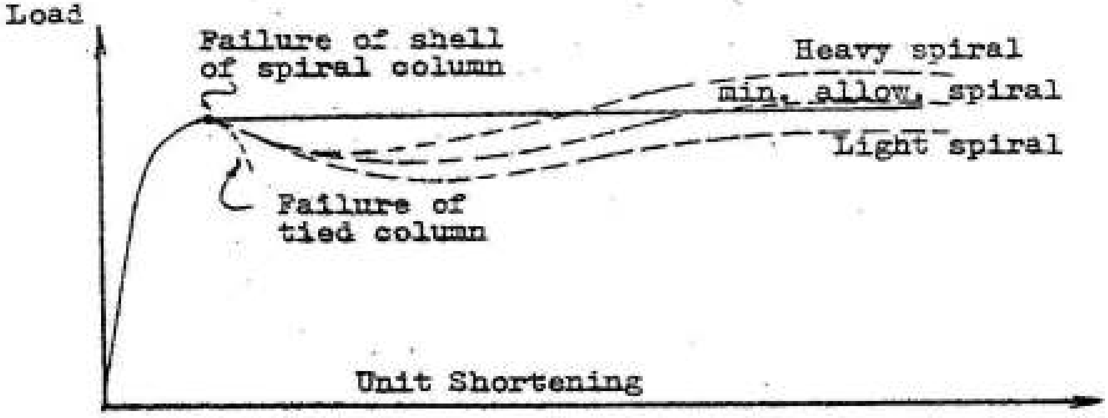


Fig. 2-6 Comparison of strains in tied & spiral columns

load at failure of cover is equal to the ultimate load of the tied column. The failure of the tied column is abrupt and complete. This is true, to almost the same degree, of a spiral column with a spiral so light that its strength contribution is considerably less than the spalled cover. The minimum amount of spirals should be that whose strength contribution is at least equal to that of the spalled cover.

5) Ultimate Strength of Short Axially Loaded Columns

From the previous investigation it is easy to see that the ultimate load N_u of an axially loaded tied column is given by:

$$\underline{N_u = 0.85\ f_{cp}\ A_c + f_y\ A_s} \qquad (2\text{-}9)$$

and of a sirally reinforced column by the relation:

$$\underline{N_u = 0.85\ f_{cp}\ A_k + f_y\ A_s + 2.5\ f_y'\ A_s'} \qquad (2\text{-}10)$$

The minimum amount of spirals giving a strength contribution equal to that of the spalled concrete can be determined in the following manner:

$$2.5\ A_s'\ f_y' = 0.85\ f_{cp}\ (A_c - A_k) \qquad (2\text{-}11)$$

or

$$\frac{A_s'}{A_k} = \mu' = 0.34\ \frac{f_{cp}}{f_y'}\ (\frac{A_c}{A_k} - 1)$$

In order to have a reasonable margin of safety, it is recommended to choose:

$$\min \mu' = 0.4\ \frac{f_{cp}}{f_y'}\ (\frac{A_c}{A_k} - 1) \qquad (2\text{-}12)$$

The ACI assumes:

$$2\ A_s'\ f_y' = 0.85\ f_{cp}\ (A_c - A_k) \qquad (2\text{-}11')$$

leading to

$$\frac{A_s'}{A_k} = \mu' = 0.425\ \frac{f_{cp}}{f_y'}\ (\frac{A_c}{A_k} - 1)$$

For safety, one may assume:

$$\min \mu' = 0.45\ \frac{f_{cp}}{f_y'}\ (\frac{A_c}{A_k} - 1) \qquad (2\text{-}12')$$

In order to have sufficient safety against failure of cover, it is specified in the German code 1045 art. 17-2-5 that:

$$A_s' \leqslant (0.42\ A_c - 0.28\ A_k)\ \frac{f_{c28}}{f_y'} + 0.2\ A_s$$

Assuming $f_{c28} \approx 1.2\ f_{cp} \approx 0.85\ f_{cp}\ /\ 0.7$, one can prove that:

$$0.85\ f_{cp}\ A_k + f_y\ A_s + 2.5\ f_y'\ A_s' \leqslant \frac{3}{2}\ (0.85\ f_{cp}\ A_c + f_y\ A_s) \qquad (2\text{-}13)$$

The equation means that the ultimate load of a spirally reinforced column calculated according to equation 2-10 should not exeed 1.5

times the ultimate load based on equation 2-9.

6) Illustrative Example

In order to show the effect of shrinkage and creep on the working stresses according to the elastic theory and the justified use of the theory of ultimate strength, we give the following illustrative example.

It is required to determine the stresses and strains both in concrete and steel at different load stages due to axial loads acting on a short column 40x40 cms, reinforced by 8 Φ 22 mm if the concrete strength is f_{cp}=160 kg/cm^2 and the longitudinal reinforcement is normal mild steel having a yield stress f_y = 2300 kg/cm^2. Assume E_s = 2100000 kg/cm^2 and n = 15.

Data:

Area of concrete section	$A_c = 40 \times 40$	= 1600		cm^2
Area of longitudinal steel	A_s = 8 Φ 22	= 30.5		cm^2
Ratio of longit. steel	$\mu = A_s/A_c = 30.5/1600$	= 0.019	1.9 %	
Prism strength of concrete	= f_{cp}	= 160		kg/cm^2
Sustained prism strength of concrete	= $0.85\ f_{cp}$	= 135		kg/cm^2
Allowed concrete comp. stress $f_{cp}/3$	σ_c	= 55		kg/cm^2

Working load according to elastic theory assuming n = 15 is given by:

$N = \sigma_c\ (A_c + n\ A_s) = 55\ (1600 + 15 \times 30.5) = 55 \times 2057 = \underline{114\ \text{tons}}$

This load will be applied in two stages, a first stage of 57 tons in which the column will be under the effect of shrinkage (ϵ_{sh} = 0.25 mm/m) and creep (ϵ_{cr} = 0.5 mm/m) and a second stage of 57 tons in which the column will be under the effect of an extra creep of 0.25 mm/m. Then, the load on the column will be increased in two further stages until failure in which case the stress in the steel equals its yield stress $\sigma_s = f_y$ = 2300 kg/cm^2 and the stress in the concrete equals its sustained prism strength of 135 kg/cm^2.

The stresses and strains in concrete and steel at different stages of loading are as follows:

1. Shrinkage in unloaded stage: $\epsilon_{sh} = 0.25 \times 10^{-3}$

$$\sigma_t = \frac{\epsilon_{sh}\ E_s\ A_s}{A_c + n\ A_s} = \frac{0.25 \times 10^{-3} \times 2100\,000 \times 30.5}{1600 + 15 \times 30.5} \quad \text{or}$$

$$\sigma_t = \frac{16012.5}{2057} = 7.8\ \text{kg/cm}^2 \quad \text{tension}$$

$\sigma_s = \sigma_t / \mu = 7.8 / 0.019 = 410\ kg/cm^2$ comp.

$\varepsilon_s = \sigma_s / E_s = 410 / 2100\,000 = 0.000195 = 0.195\ mm/m$

2. Permanently acting load of 57 tons

$\sigma_c = N / (A_c + n\,A_s) = 57000 / 2057 = 28.2\ kg/cm^2$ comp

$\sigma_s = n\,\sigma_c = 15 \times 28.2 = 423\ kg/cm^2$ comp

$\varepsilon_s = \sigma_s / E_s = 423 / 2100\,000 = 0.000215 = 0.215\ mm/m$

3. Creep of 0.5 mm/m (causing double the shrinkage stresses & strains)

$\sigma_t = 2 \times 7.8 = 15.6\ kg/cm^2$ tension and

$\sigma_s = 2 \times 410 = 820\ kg/cm^2$ compression

4. Additional load of 57 tons to reach the max. working load of 114 t

$\sigma_c = 28.2\ kg/cm^2$ comp. and $\sigma_s = 423\ kg/cm^2$ comp.

5. Additional creep of 0.25 mm/m to the end of this phenomenon

$\sigma_t = 7.8\ kg/cm^2$ tension and $\sigma_s = 410\ kg/cm^2$ comp.

The stresses and strains in concrete and steel at the different stages of loading are given in the following table:

Case of loading	ΔN tons	N tons	σ_s kg/cm^2	σ_c kg/cm^2	ε mm/m
1. Shrinkage 0.25 mm/m	0	0	410	-7.8	0.195
2. Loading 57 ton	57	57	422	28.2	0.215
Cases (1) + (2)	-	57	832	20.4	0.410
3. Creep 0.50 mm/m	-	-	820	-15.6	0.390
Cases (1) to (3)	-	57	1652	4.8	0.800
4. Add. Load 57 ton	57	-	422	28.2	0.215
Cases (1) to (4)	-	114	2074	33.0	1.015
5. Add. Creep .25 mm/m	-	-	410	-7.8	0.195
Cases (1) to (5)	-	114	2484	25.1	1.210
Same and $\sigma_s = f_y$	-	114	2300	27.4	1.100
6. Add. Load 86 ton	86	-	-	53.8	
Cases (1) to (6)	-	200	2300	81.2	*
7. Add. Load 86 ton to Failure	86	-	-	53.8	
Cases (1) to (7)	-	286	2300	135	*

* At these stages of loading, the concrete is in a plastic state, hence, the strains are to be determined from the actual stress-strain relationship of the concrete.

From the given table, one can deduce the following:

- Under a load of 57 tons, we have $\sigma_c = 28.2$ and $\sigma_s = 422$ kg/cm^2.
- Due to shrinkage, the concrete stress is reduced to 20.4 kg/cm^2 and the steel stress increases to 832 kg/cm^2.
- Because of creep, the concrete stress is further reduced to 4.8 kg/ cm^2 and the steel stress increases to 1652 kg/cm^2. In this stage, the total column load is approximately carried by the steel reinforcement only.
- Increasing the load to the allowed value of 114 ton, the concrete stress is 32.9 kg/cm^2 and the steel stress becomes 2074 kg/cm^2.
- For an additional creep of 0.25 mm/m, the concrete stress will be 25.1 kg/cm^2 and the calculated steel stress is 2484 kg/cm^2.
- The maximum that can be resisted by steel is $f_y = 2300$ kg/cm^2 . Hence, the corresponding maximum load, N_s, is given by $N_s = f_y A_s = 2.3 \times 30.5 = 70.15$ tons. The remainder, N_c, will be resisted by concrete. Therefore, $N_c = 114 - 70.15 = 43.85$ tons. The corresponding concrete stress is $\sigma_c = N_c / A_c = 43.85 \times 1000 / 1600 = 27.4$ kg/cm^2.
- It is interesting to see that the stress in steel reaches the yield limit under the working load of 114 tons.
- Calculating the stress in concrete and steel due to the working load of N = 114 ton according to the elastic theory with n = 15, we find that $\sigma_c = 55$ kg/cm^2, i.e., more than twice the actual stress, whereas $\sigma_s = 825$ kg/cm^2 which is less than one third its actual value.
- This means that the elastic theory gives a wrong idea about the stresses both in concrete and steel.
- Any further increase of loading will be carried by concrete only until σ_c reaches the maximum resistance of concrete under sus - tained loads which is equal to $0.85 f_{cp} = 135$ kg/cm^2 because steel in yield cannot resist any stress more than $f_y = 2300$ kg/cm^2.
- The effect of shrinkage and creep diminishes with increasing loading and disappears completely at failure.

However, no columns are likely to be purely axially loaded. Even if design calculations show a compression member to be free from any bending moments, imperfections of construction, such as slight beam eccentricities and deviation from straightness or verticality, will cause unintentional bending moments. For this reason, it is recommended to design compression members for an adequate minimum eccentricity.

2.4- AXIAL TENSION

Elastic analysis: The tensile strength of concrete being low, reinforced concrete is not well suited for tension members. Still, reinforced concrete may be coveniently used in tension; e.g in water tanks and tie rods of arches.

In a circular water tank, the walls are mainly subject to axial tension. In order to have sufficient water-tightness and to protect the steel from rusting, the thickness of the walls must be so chosen that the tensile stress in concrete σ_t is smaller than its tensile strength f_t. In this situation, expression 2-3 derived for axial compression will be valid i.e.,

$$N = \sigma_t\ (A_c + n\ A_s) \qquad (2\text{-}14)$$

and

$$\sigma_t = N\ /\ (A_c + n\ A_s) \qquad (2\text{-}14')$$

in which

$n = E_s/E_c$ = modular ratio (here, generally chosen equal to 10)

Sections subject to axial tension are generally symmetrically reinforced and shrinkage causes tensile stresses in concrete and compressive stresses in steel according to equations 2-7 and 2-8.

Accordingly, the total tensile stress in concrete is given by:

$$\sigma_t = (N + \epsilon_{sh}\ E_s\ A_s)\ /\ (A_c + n\ A_s) \qquad (2\text{-}15)$$

in which ϵ_{sh} = shrinkage strain (generally 0.25×10^{-3} for design purposes)

In spite of that, the steel alone must be sufficient to resist all the tensile force N. So that,

$$A_s = N\ /\ \sigma_s \qquad \text{or} \qquad \sigma_s = N\ /\ A_s \qquad (2\text{-}16)$$

In normal cases, such as in tie rods, tension cracks, with a maximum specified width, are allowed. When this happens, the concrete section is neglected in the design and the steel alone must be sufficient to resist all the tensile force. In this case, equation (2-16) applies.

Ultimate strength analysis: The ultimate tensile force of a tension member is that force which will just cause the steel stress to reach the yield point. That is,

$$N_u = f_y\ A_s \qquad (2\text{-}17)$$

2.5- SIMPLE BENDING

a) Bending of homogeneous beams

Reinforced concrete beams are nonhomogeneous in that they are made of two entirely different materials. The methods used in the analysis of reinforced concrete beams are therefore different from those used in the design or investigation of beams composed entirely of any other structural homogeneous material such as steel. The fundamental principles involved are, however, essentially the same as those relating to homogeneous beams. Briefly, these principles are as follows:

At any cross section there exist internal forces which may be resolved into components normal and tangetial to the section. Those components which are normal to the section are the bending stresses. Their function is to resist the bending moment at the section. The tangential components are known as the shear stresses, and they resist the transverse or shear forces.

The fundamental assumptions on which the analysis is based are the following:

1. Plane sections before bending remain plane after bending which means that the strains in any section are proportional to the distance from the neutral axis.

2. The bending stress 'σ' at any point depends on the strain ϵ at that point in a manner given by the stress-strain diagram of the material. If the beam is made of a homogeneous material whose stress - strain diagramin tension and compression is that of Fig. 2-7a, the strains and the corresponding stresses at different load stages are shown in Fig. 2-7 b and c.

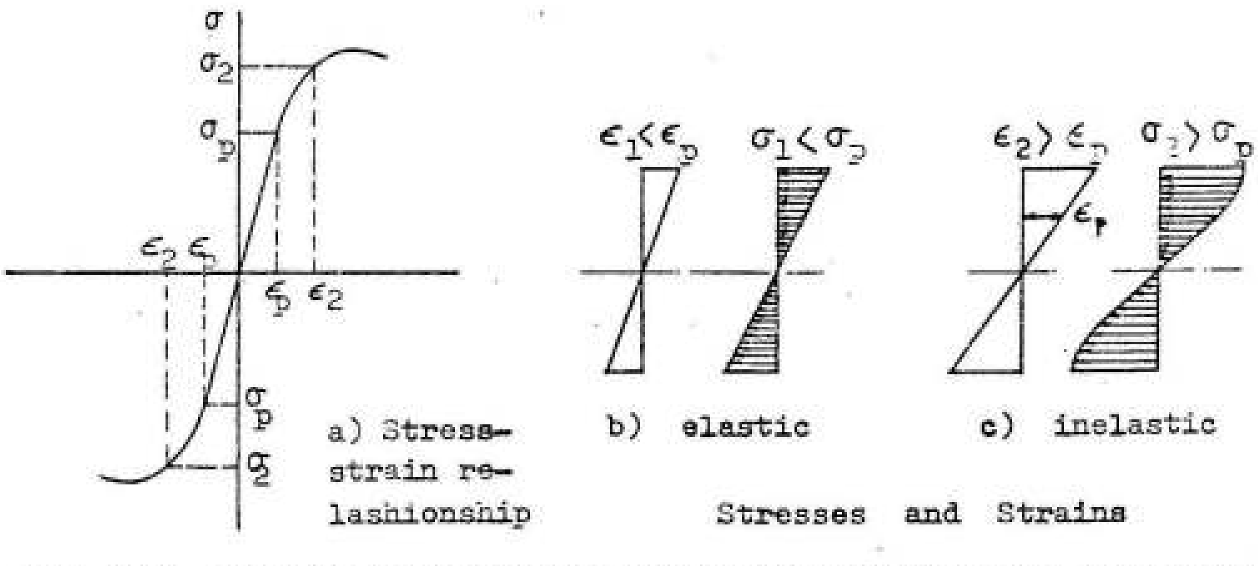
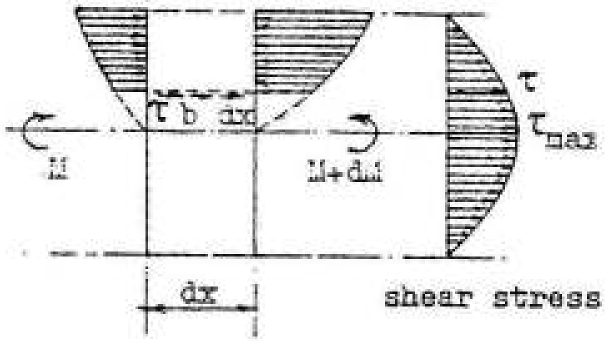


Fig. 2-7 Elastic and inelastic stress distribution in a homogeneous beam



b = breadth of beam

Fig. 2-8 Shear stress distribution in a homogeneous beam

3. The shear stress acting on any fiber of an element dx of a beam subject to varying bending moments (M and M + dM) is the result of the difference between the normal stresses acting on both sides of the element as shown in Fig. 2-8.

The distribution of the shear stresses τ over the depth of the section depends on the shape of the cross section and of the stress-strain diagram. These shear stresses are largest at the neutral axis and equal to zero at the outer fibers. The shear stresses on horizontal and vertical planes through any point are equal otherwise, the element which includes the point rotates around itself

4. Owing to the combined action of shear stresses (horizontal and vertical) and flexure stresses, at any point in a beam there are inclined stresses of tension and compression, the largest of which form an angle of 90° with each other. The intensity of the inclined maximum or principal stress at any point is given by the equation

$$\sigma_{\frac{1}{2}} = \frac{\sigma}{2} \pm \sqrt{\frac{\sigma^2}{4} + \tau^2} \qquad 2\text{-}18$$

where

σ = intensity of normal fiber stress

τ = intensity of tangential shearing stress

The inclined stress makes an angle α with the horizontal such that

$$\tan 2\alpha = 2\tau / \sigma \qquad 2\text{-}19$$

5. Since the horizontal and vertical shearing stresses are equal and the flexural stresses are zero at the neutral plane, the inclined tensile and compressive stresses at any point in that plane form an angle of 45° with the horizontal, the intensity of each being equal to the unit shear at that point.

6. When the stresses in the outer fibers are smaller than the proportional limit σ_p, the beam behaves elastically as shown in Fig.2-7b. In this case we have:

a. The neutral axis passes through the center of gravity of the cross section.

b. The intensity of the bending stress normal to the section increases directly with the distance from the neutral axis and is maximum at the extreme fibers. The stress at any given point in the cross section is given by the equation

$$\sigma = M y / I \qquad 2\text{-}20$$

where σ = bending stress at a distance y from neutral axis

M = external bending moment at section

I = moment of inertia of cross section about neutral axis.

The maximum bending stress occurs at the outer fibers and is equal to

$$\sigma_{max} = M z / I = M / Z \qquad 2\text{-}21$$

where z = distance from neutral axis to outer fiber

Z = I / z = section modulus of cross section.

c. The shear stress (longitudinal or transverse) at any point in the cross section is given by

$$\tau = Q S / I b \qquad 2\text{-}22$$

where Q = total shear at section

S = statical moment about neutral axis of that portion of cross section lying between a line through point in question parallel to neutral axis and nearest face (upper or lower) of beam

I = moment of inertia of cross section about neutral axis

b = width of beam at given point.

d. The intensity of shear along a vertical cross section in a rectangular beam varies as the ordinates of a parabola, the intensity being zero at the top and bottom of the beam and a maximum at the neutral axis. The maximum is 3/2 the average.

b) Bending of reinforced concrete beams: stress distribution at various stages of loading

The distribution of compressive stresses in a reinforced concrete section depends on many variables: (1) The magnitude of the external forces, (2) the rate of loading, (3) the history of shrinkage, creep, and possibility of differential temperature, (4) the location relative to the position of a tension crack, and (5) most important, the stress strain relationship of concrete.

Neglecting effects of shrinkage, creep and temperature changes , one may correlate between the stress distribution of concrete in compression and the stage of loading in the following manner, Fig.2-9 :

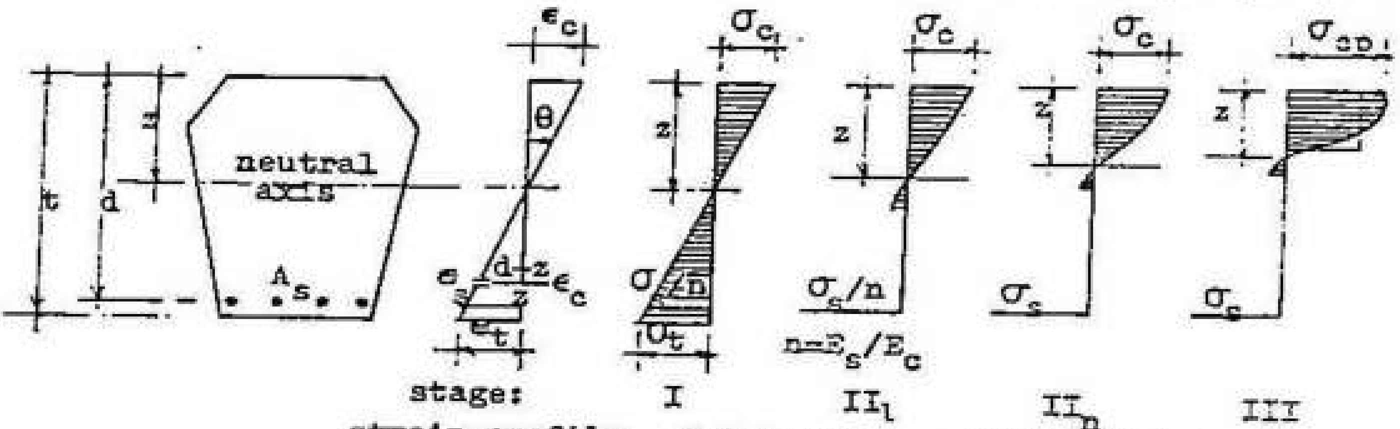


Fig. 2-9 Distribution of strains and stresses for various stages of loading

Stage 1 (noncracked, linear stage). In this stage where loads are small, compressive stresses are very low (strains much less than 0.50 mm/m) and the maximum tesile stress of concrete is less than its rupture strength (strains less than about 0.1 to 0.15 mm/m). Accordingly, the assumption stating that, "in the tension zone, the concrete strength is negligible" is an underestimation of the observed strength. Analysis in this stage could disregard that assumption. Concrete can reasonably be assumed as linearly elastic material; the modulus of elasticity in tension may be assumed equal to that in compression. The analytical expression for the distance to the neutral axis, z, is in a quadratic but separable form.

This stage should be considered as the basis for calculating the cracking moment when the maximum concrete tensile strain is taken as 0.12 mm/m. It is also taken as a basis for the design of sections that should be free from cracks (e.g. sections of water structures when the tensile stresses are on the water side).

Stage II_l (cracked, linear stage). In this stage, higher loads cause tension cracks to develop (maximum tensile strain in concrete is in excess of 0.12 mm/m). Neglecting strength of concrete in tension is realistic; the steel alone resists the entire tension without rupture. The maximum stress of concrete in compression does not exceed one half the prism compressive strength (i.e., strains are less than about 0.5 mm/m). Accordingly, an assumption of a linearly elastic behavior is justifiable; therefore, compressive stress distribution can be approximated by a triangle. It is usually hypothesized that reinforced concrete sections under working loads belong to this stage. Again, the analytical expression for the distance to the neutral axis, z, is in a quadratic but separable form.

Stage II_n (cracked, nonlinear stage). For loads greater than these producing stage II_l, the maximum compressive stress in concrete exceeds one half the prism compressive strength. however, concrete in compression has not crushed (i.e., the value of the maximum compressive strain in concrete has exceeded about 0.5 mm/m but is still below 3.0 mm/m). Although strains are assumed to remain proportional to the distance from the neutral axis, stresses are not. Concrete is assumed to behave in a nonlinear inelastic fashion; meanwhile, steel or concrete or both are about to reach their limiting strength. Because of the nonlinear behavior, the expression for the distance to the neutral axis, z, is in a general nonlinear nonseparable form, which requires an iterative scheme for solution. This stage is the most difficult to analyse and the solution may necessitate the use of an electronic machine.

Stage III (ultimate strength). The load has reached the maximum value that can be resisted by the reinforced concrete section. The concrete strains are still assumed to be proportional to the distance from the neutral axis with a maximum value of about 3.0 mm/m occurring at the extreme fiber of the section. Three types of flexural failures are possible, namely: (1) tension failure, (2) balanced failure, or (3) compression failure; this depends on whether the concrete strain reaches its maximum value when the tension steel has: (1) already yielded, (2) just yielded, or (3) not yielded yet, in that order. In all three cases the shape of the compressive stress distribution is the same but the position of the neutral axis varies significantly. Tension failures, which occur in cases of pure bending or in cases of normal forces with large eccentricities, produce a small value of z, i.e., the neutral axis is near the most compressed fiber. For compression failures, which occur in cases of normal forces with small or no eccentricities, the neutral axis could be well below the mid-depth of a rectangular cross section. The balanced failure is, obviously , an intermediate case with relatively intermediate values of z and generally occurs in cases of normal forces with "medium eccentricities". In case of a rectangular section, the expression for z is simply linear.

It is clear from the previous discussion that when relatively moderate amounts of reinforcement are employed, at some value of the load the steel will reach its yield point. At that stress the reinforcement yields suddenly and stretches a large amount, and the tension cracks in the concrete widen visibly and propagate upward, with simultaneous significant deflection of the beam. When this happens , the strains in the remaining compression zone of the concrete increase to such a degree that crushing of the concrete, the so-called secondary compression failure, takes place at a load only slightly larger than that which caused the steel to yield. Effectively, therefore, attainment of the yield point in the steel determines the carrying capacity of moderately reinforced beams. Such yield failure is gradual and is preceded by visible signs of distress, such as the widening & lengthening of cracks and the marked increase in deflection.

On the other hand, if large amounts of reinforcement or normal amounts of steel of very high strength are employed, the compression strength of the concrete may be exhausted before the steel starts yielding. Concrete fails by crushing when strains become so large that they disrupt the integrity of the concrete. Exact criteria for this occurance are not yet known, but it has been observed that rectangular beams fail in compression when the concrete strains reach values of about 3 to 4×10^{-3} for concretes with prism strengths varying between 350 and 150 kg/cm^2. Compression failure through crushing of the concrete is sudden, of an almost explosive nature, and occurs without warning. For this reason it is good practice to dimension beams in such a manner that, should they be overloaded, failure will be initiated by yielding of the steel rather than by crushing of the concrete.

The major factors influencing the type of failure are: (1) the shape of the cross section, (2) the geometrical percentage of the reinforcement (smallest for tension failure), (3) the value of the yield point or the proof stress of the steel, and (4) the compressive strength of concrete. It should be emphasized that the rotational capacity of a section (measured by the angle of curvature θ in Fig. 2-9) is much larger for tension failures as compared with balanced or compression failures. The rotational capacity gives an indication to the nature of ductility (ability to deform, past the elastic range, before failure) of a reinforced concrete member. This property plays a major role in limit state design because it, among other factors, guarantees safety. Safety in the sense that large deflections without total collapse give warning and ample time for evacuation of personnel and equipments.

Accordingly, one should expect codes of practice to favor designs producing tension failures in case of excessive loading; at the same time, brittle or sudden failures such as the balanced and compression failures should be prevented as much as possible.

c) Noncracked linear stage

In this stage, the strain and stress distribution is essentially the same as in an elastic, homogeneous beam. The only difference is the presence of the steel reinforcement.

Due to bond between steel and concrete, both materials will be subject to the same strain and the stress in the steel will be n times that of the concrete. This fact can be taken into account in the calculation by replacing the actual steel-and-concrete cross section with a virtual section composed of the concrete section and n times the steel area located at the level of the steel.

However, the forces resisted by the steel reinforcement are small compared to those resisted by the concrete, that the section may be treated as an elastic noncracked plain concrete section without making an appreciable error.

If the steel reinforcement is taken into cosideration with n times its area, n may be assumed equal to 10 and the stresses can be computed with equations 2.20 to 2.22. in which y, z, S and I are to be determined for the virtual area.

d) Elastic analysis. Cracked linear stage

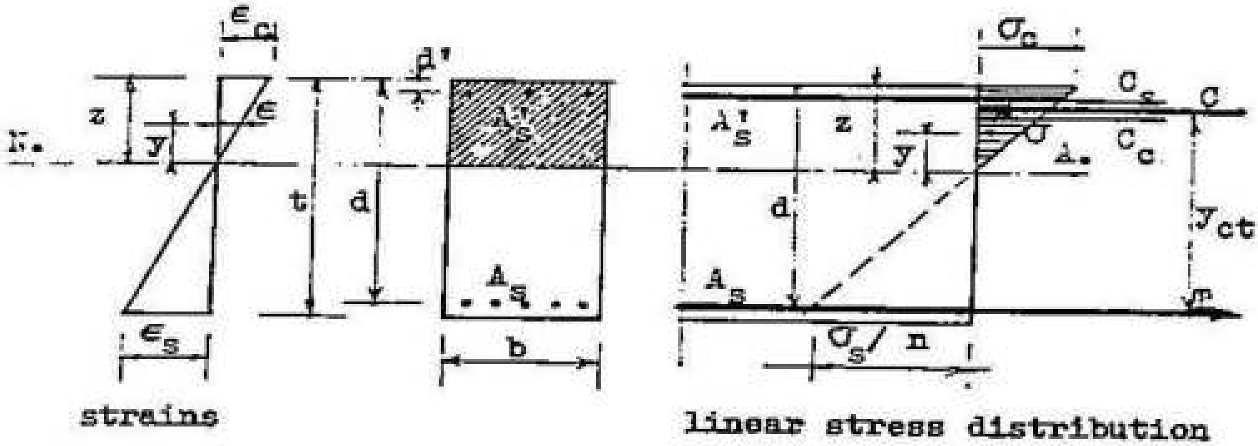


Fig. 2-10 Linear stress distribution in a reinforced concrete rectangular section subject to simple bending

1. N o t a t i o n

- b = breadth of cross section
- t = total depth of cross section
- d = theoretical depth of cross section from center of tension steel to outside fiber of compression zone
- d' = distance from compression reinforcement to outside fiber of compression zone
- z = distance from neutral axis to outside fiber of compression zone
- y_{ct} = arm of resisting moment of internal forces
- A_s = cross sectional area of tension reinforcements
- A'_s = cross sectional area of compression reinforcements
- M = external moment acting on section
- C_c = compressive force on concrete
- C_s = compressive force on compression steel
- C = $C_c + C_s$ = total compression on section
- T = tensile force in tension steel
- ϵ_c = strain of outside fiber of concrete in compression
- ϵ_s = strain of tension eteel
- σ_c = maximum compressive stress of concrete in compression
- σ_s = tesile stress of steel in tension
- σ'_s = compressive stress of steel in compression
- E_c, E_t = modulus of elasticity of concrete in compression and tension respectively
- E_s = modulus of elasticity of steel

$n = E_s / E_c$ = modular ratio

$\zeta = z / d$ $\beta = d' / d$ $r = \sigma_s / \sigma_c$ $\mu = A_s / b\,d$ $\mu' = A'_s / bd$

2. Assumptions

(1) The tensile strength of concrete in tension is small and for this reason it is generally assumed that hair cracks take place in the tension zone under working loads so that the concrete in tension does not statically act and all the tensile stresses are resisted by the tension steel only.

(2) Plane sections before bending remain plane after bending; i.e. the strains are proportional to the distance from the neutral axis or

$$\epsilon_c / \epsilon_s = z / (d - z) \qquad 2\text{-}23$$

(3) Under working loads, concrete and steel act as elastic materials, hence

$$\epsilon_c = \sigma_c / E_c \quad \text{and} \quad \epsilon_s = \sigma_s / E_s$$

so that

$$\epsilon_c / \epsilon_s = \frac{\sigma_c}{\sigma_s} \cdot \frac{E_s}{E_c}$$

Assuming $E_s / E_c = n$ then,

$$\epsilon_c / \epsilon_s = \frac{\sigma_c}{\sigma_s / n} \qquad 2\text{-}24$$

Equations 2-23 and 2-24 give

$$\frac{\sigma_c}{\sigma_s / n} = z / (d - z) \qquad 2\text{-}25$$

Assuming $z = \zeta d$ and $r = \sigma_s/\sigma_c$ we get:

$$\zeta = \frac{z}{d} = \frac{n}{r + n} \qquad 2\text{-}26$$

3. Conditions of equilibrium

The section being subject to simple bending, then

(1) The sum of the stresses acting on the section must be equal to zero, i. e.

$$\int \sigma \, dA = 0 \quad \text{but} \quad \sigma = \epsilon E \quad \text{then}$$

$$\int \epsilon E \, dA = 0$$

According to assumption 2 $\epsilon = c\,y$ where c = constant. Hence, we get

$c \int y\, E\, dA = 0$ or $c\, E_c \int y \frac{E}{E_c}\, dA = 0$ where

E/E_c = 1 for concrete in compression,
= 0 for concrete in tension, and
= n (generally 15) for steel reinforcements.

This equation means that the statical moment of the virtual area about the neutral axis is equal to zero; i.e. the neutral axis passes through the center of gravity of the virtual area.

Assuming further that:
The total compression acting on the section is equal to C , and the total tension acting on the section is equal to T , then

$$C = T \qquad 2\text{-}27$$

(2) The internal moment of resistance must be equal to the external moment M acting on the section . Hence

$$C\, y_{ct} = T\, y_{ct} = M \qquad 2\text{-}28$$

4. A p p l i c a t i o n s

(1) Rectangular sections with tension reinforcements only.
Fig. 2-11

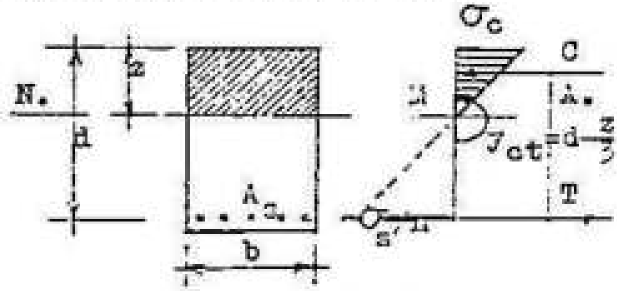


Fig. 2-11

Given M, b, d and A_s.
Required σ_c and σ_s.

The neutral axis passes through the center of gravity of the virtual area, hence

$b\, z^2 / 2 = n\, A_s\, (d - z)$ or

$$z = \frac{n\, A_s}{b} \left(-1 + \sqrt{1 + \frac{2\, d\, b}{n\, A_s}}\right) \qquad 2\text{-}29$$

According to equation 2-28: $C\, y_{ct} = T\, y_{ct} = M$ or

$\sigma_c \frac{z\, b}{2} (d - \frac{z}{3}) = A_s\, \sigma_s\, (d - \frac{z}{3}) = M$ or

$$\left.\begin{aligned} \sigma_c &= \frac{2\, M}{b\, z\, (d - \frac{z}{3})} \quad \ldots\ldots\ldots a \\ \sigma_s &= \frac{M}{A_s\, (d - \frac{z}{3})} \quad \ldots\ldots\ldots b \end{aligned}\right\} \qquad 2\text{-}30$$

Replacing z by $\zeta\, d$ and A_s by $\mu\, b\, d$, the neutral axis can be conputed by the relation:

$$\zeta = z / d = n \mu \left(-1 + \sqrt{1 + \frac{2}{n\mu}}\right) \qquad 2\text{-}31$$

The arm of the internal forces $y_{ct} = \eta d$ is therefore given by:

$$y_{ct} = d - \frac{z}{3} \quad \text{or} \quad \eta = y_{ct} / d = (1 - \frac{\zeta}{3})$$

The stresses in concrete and steel can be computed from equations 2-30 as follows:

$$\sigma_c = \frac{2M}{b\, z\, (d - \frac{z}{3})} = \frac{2M}{b\, \zeta\, d^2\, (1 - \frac{\zeta}{3})} \qquad \text{giving}$$

$$\underline{\sigma_c = c_1 \frac{M}{b\, d^2}} \qquad 2\text{-}32$$

where

$$c_1 = \frac{2}{\zeta\, (1 - \frac{\zeta}{3})}$$

and

$$\sigma_s = \frac{M}{A_s\, (d - \frac{z}{3})} = \frac{M}{\mu\, b\, d^2\, (1 - \frac{\zeta}{3})} \qquad \text{giving}$$

$$\underline{\sigma_s = c_2 \frac{M}{b\, d^2}} \qquad 2\text{-}33$$

where

$$c_2 = \frac{1}{\mu (1 - \frac{\zeta}{3})}$$

Table 1 gives, for a rectangular section with tension reinforcements only, the values of ζ, η, c_1 and c_2 for different μ-values.

(2) <u>Rectangular sections with double reinforcements.</u> Fig. 2-12

Given M, b, d, A_s and A_s'.

Required σ_c and σ_s.

The neutral axis passes through the center of gravity of the virtual area; hence

$$\frac{b\,z^2}{2} + n\, A_s' \,(z - d') = n\, A_s\, (d-z)$$

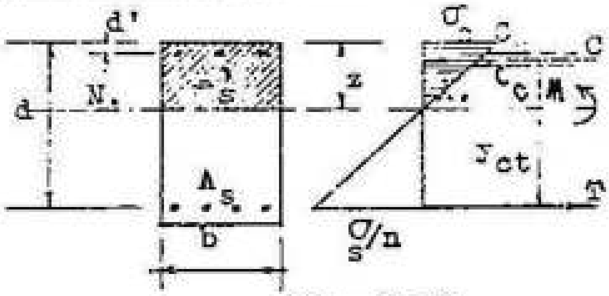


Fig. 2-12

which, when solved for z, gives:

$$z = \frac{-n\,(A_s + A_s')}{b} + \sqrt{\left[\frac{n\,(A_s + A_s')}{b}\right]^2 + \frac{2n}{b}\,(A_s\, d + A_s'\, d')} \qquad 2\text{-}34$$

Further, $C = C_c + C_s$ and $C_c\,(d - \frac{z}{3}) + C_s\,(d - d') = M$ or

$$\sigma_c \frac{z\, b}{2}\,(d - \frac{z}{3}) + A_s'\, \sigma_s'\,(d - d') = M$$

Table 1: Elastic analysis. Stresses in rectangular sections with ,tension reinforcements only subject to simple bending

$$\sigma_c = c_1 \frac{M}{b\ d^2} \qquad \sigma_s = c_2 \frac{M}{b\ d^2} \qquad n = 15$$

Values of $\zeta = \frac{z}{d}$, $\eta = \frac{y_{ct}}{d}$, c_1 and c_2 for different μ-values

100μ	ζ	η	c_1	c_2	100μ	ζ	η	c_1	c_2
0.20	0.217	0.928	9.942	538.8	1.20	0.446	0.851	5.263	97.9
22	226	925	9.564	491.3	24	451	850	5.213	94.9
24	235	922	9.234	451.6	28	456	848	5.165	92.1
26	243	919	8.952	418.3	32	462	846	5.120	89.6
28	251	916	8.694	389.5	36	467	844	5.077	87.1
0.30	0.258	0.914	8.472	364.8	1.40	0.471	0.843	5.035	84.7
32	265	912	8.268	343.1	44	475	842	4.995	82.6
34	272	909	8.082	323.8	48	480	840	4.957	80.5
36	279	907	7.914	307.3	52	484	839	4.921	78.5
38	285	905	7.752	291.1	56	489	837	4.886	76.6
0.40	0.292	0.903	7.596	276.9	1.60	0.493	0.836	4.853	74.8
42	298	901	7.458	264.2	64	497	834	4.821	73.1
44	303	899	7.332	252.7	68	501	833	4.790	71.4
46	309	897	7.212	242.2	72	505	832	4.760	69.9
48	314	895	7.104	232.7	76	509	830	4.731	68.5
0.50	0.319	0.894	7.008	222.9	1.80	0.513	0.829	4.703	67.0
52	324	892	6.906	215.5	84	517	828	4.677	65.6
54	329	890	6.810	207.8	88	521	826	4.551	64.3
56	333	889	6.726	200.8	92	524	825	4.627	63.1
58	338	887	6.642	194.3	96	528	824	4.603	61.9
0.60	0.343	0.886	6.570	188.2	2.00	0.531	0.823	4.579	60.8
62	348	884	6.498	182.5	2.10	539	820	4.523	58.0
64	353	882	6.432	177.1	2.20	547	818	4.472	55.6
66	357	881	6.366	172.0	2.30	554	815	4.424	53.5
68	361	880	6.300	167.2	2.40	562	813	4.381	51.2
0.70	0.365	0.878	6.240	162.7	2.50	0.569	0.810	4.339	49.4
72	369	877	6.174	158.4	2.60	575	808	4.300	47.6
74	373	876	6.120	154.4	2.70	582	806	4.264	46.0
76	377	874	6.066	150.5	2.80	588	804	4.230	44.4
78	381	873	6.018	146.8	2.90	594	802	4.197	43.0
0.80	0.384	0.872	5.968	143.4	3.00	0.600	0.800	4.166	41.6
82	388	871	5.920	140.1	3.10	605	798	4.138	40.4
84	392	869	5.875	136.9	3.20	611	796	4.110	39.2
86	396	868	5.831	133.9	3.30	617	794	4.084	38.1
88	399	867	5.788	131.0	3.40	622	793	4.059	37.1
0.90	0.402	0.866	5.746	128.3	3.50	0.627	0.791	4.036	36.1
92	405	865	5.706	125.7	3.60	632	789	4.014	35.2
94	408	864	5.667	123.1	3.70	636	788	3.992	34.3
96	412	863	5.629	120.7	3.80	641	786	3.971	33.4
98	415	862	5.594	118.3	3.90	645	785	3.952	32.6
1.00	0.418	0.861	5.560	116.1	4.00	0.649	0.784	3.932	31.9
04	424	859	5.494	112.0	4.10	653	782	3.914	31.2
08	430	857	5.431	108.1	4.20	657	781	3.897	30.5
12	436	855	5.372	104.5	4.30	661	780	3.881	29.8
16	441	853	5.316	101.1	4.40	665	778	3.865	29.2
1.20	0.446	0.851	5.263	97.9	4.50	0.669	0.777	3.849	28.6

But $$\frac{\sigma_s' / n}{\sigma_c} = \frac{z - d'}{z}$$ therefore

$$\sigma_c \frac{z\,b}{2} (d' - \frac{z}{3}) + A_s' \, n \, \sigma_c \frac{z - d'}{z} (d - d') = M$$ giving

$$\sigma_c = \frac{M}{\frac{b\,z}{2} (d - \frac{z}{3}) + n \, A_s' \frac{z - d'}{z} (d - d')} \qquad 2\text{-}35$$

Having determined σ_c and z, then σ_s can be calculated from equation 2-25, thus

$$\sigma_s = n \, \sigma_c \frac{d - z}{z} \qquad 2\text{-}36$$

Assuming $z = \zeta d$, $A_s = \mu b d$, $A_s' = \mu' b d$, $\alpha = \mu'/\mu$, and $\beta = d'/d$ the equation of the neutral axis can be given in the form:

$$\zeta = n \, \mu \, (1 + \alpha) \left[\sqrt{1 + \frac{2\,(1 + \alpha\beta)}{n \, \mu (1 + \alpha)^2}} - 1 \right] \qquad 2\text{-}37$$

The arm of the internal forces $y_{ct} = \eta d$, and the stresses σ_c and σ_s can be determined in the following manner:

$$C \, (d - \frac{z}{3}) + C_s \, (d - d') = C \, y_{ct}$$ or

$$\sigma_c \frac{z\,b}{2} (d - \frac{z}{3}) + A_s' \sigma_s' (d - d') = (\sigma_c \frac{z\,b}{2} + A_s' \sigma_s') \, y_{ct}$$

but $$\sigma_s' = n \, \sigma_c \frac{z - d'}{z}$$ therefore

$$\sigma_c \frac{z\,b}{2} (d - \frac{z}{3}) + nA_s' \sigma_c \frac{z - d'}{z} (d - d') = (\sigma_c \frac{z\,b}{2} + nA_s' \sigma_c \frac{z - d'}{z}) \, y_{ct}$$

Substituting $z = \zeta d$, $y_{ct} = \eta d$, $d' = \beta d$, $A_s = \mu b d$, $A_s' = \mu' bd$ and $\alpha = \mu'/\mu$, we get

$$\zeta \frac{d^2 b}{2} (1 - \frac{\zeta}{3}) + n \, \alpha \, \mu \, b \, d^2 \frac{\zeta - \beta}{\zeta} (1 - \beta) = (\zeta \frac{d\,b}{2} + n \, \alpha \, \mu \, b \, d \frac{\zeta - \beta}{\zeta} \, \eta \, d)$$

or

$$\eta = \frac{y_{ct}}{d} = \frac{\frac{\zeta}{2} (1 - \frac{\zeta}{3}) + n \, \alpha \, \mu \, \frac{\zeta - \beta}{\zeta} (1 - \beta)}{\frac{\zeta}{2} + n \, \alpha \, \mu \, \frac{\zeta - \beta}{\zeta}} \qquad 2\text{-}38$$

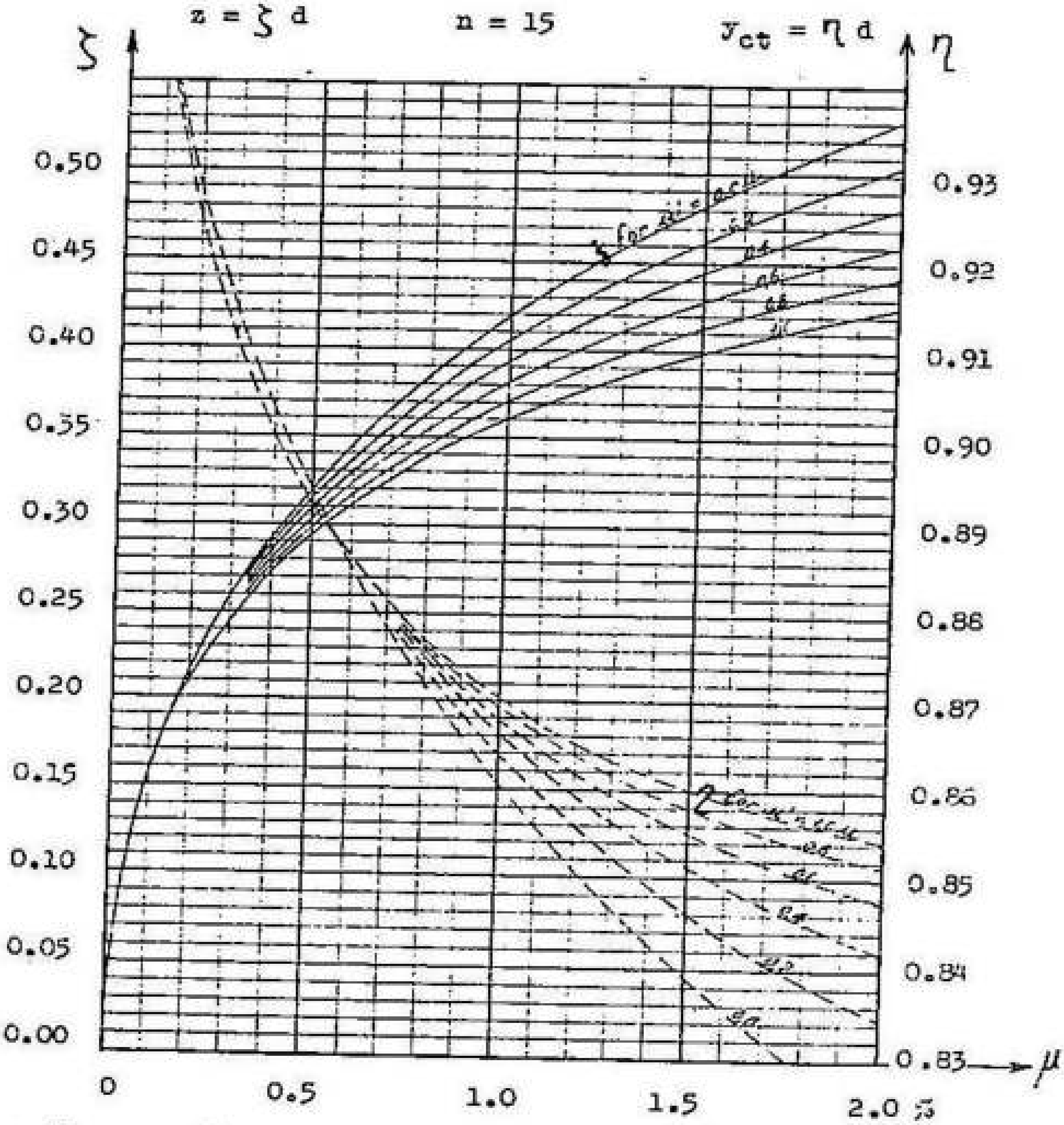
Equation 2-28 gives $$C \, y_{ct} = M$$ or

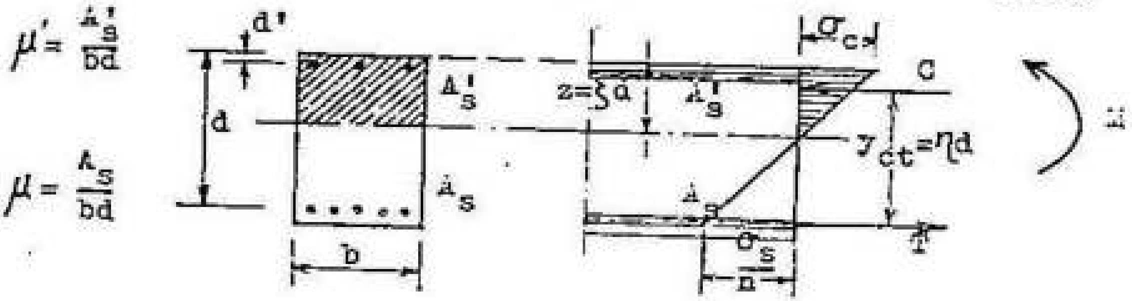
$$(\sigma_c \frac{z\,b}{2} + A_s' \;\; {}_s') \, y_{ct} = M$$ therefore

SHEET 1

SIMPLE BENDING

POSITION OF NEUTRAL AXIS

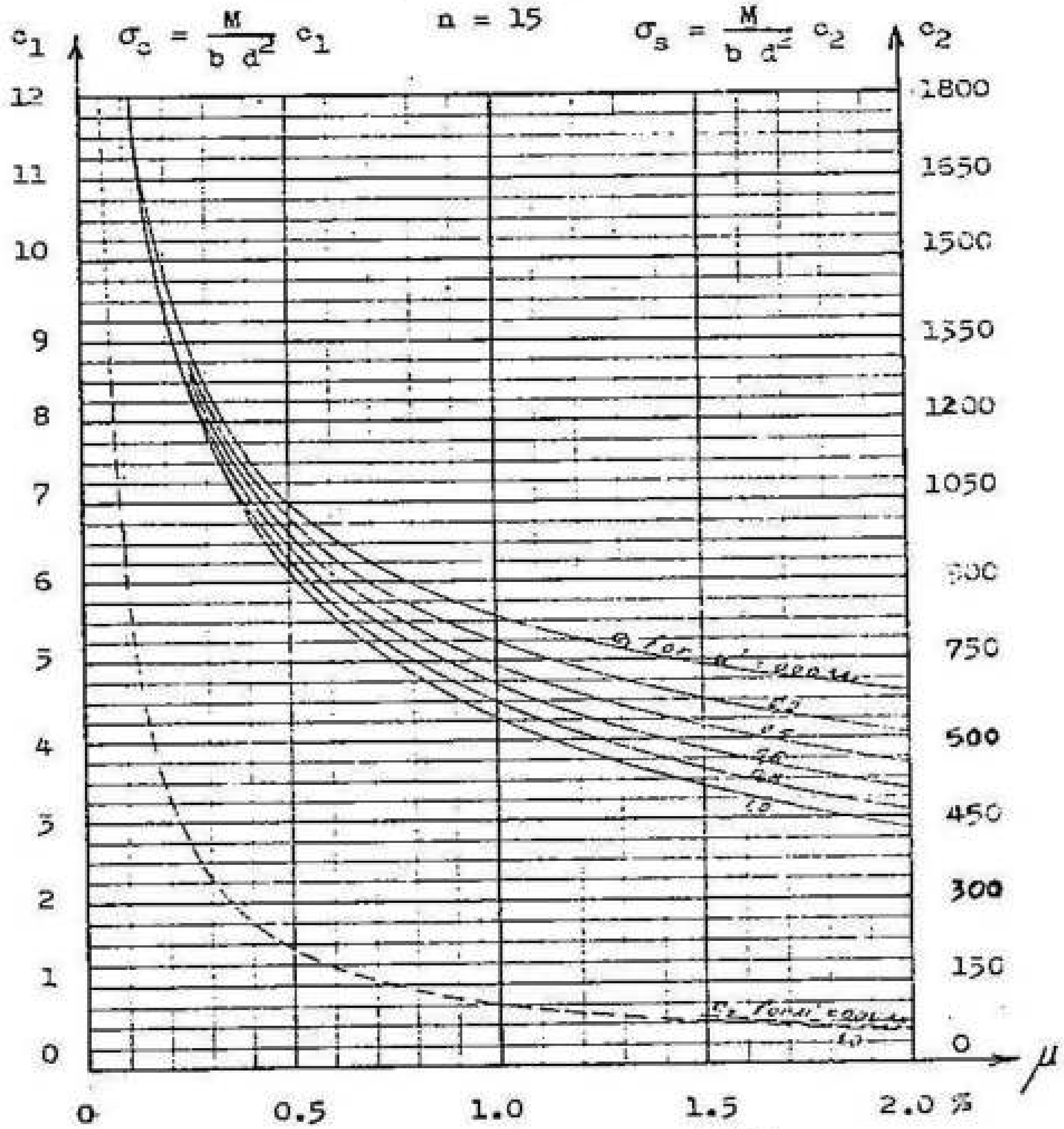


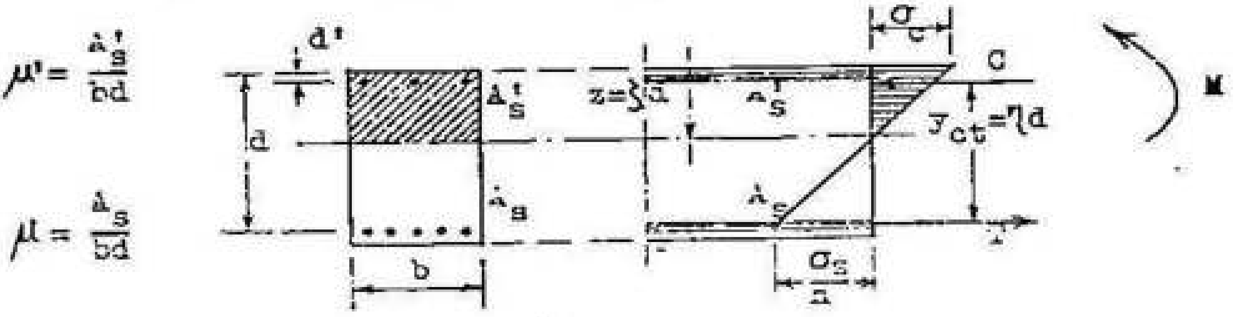


SHEET 2

SIMPLE BENDING

DETERMINATION OF MAXIMUM STRESSES





$$\left(\sigma_c \frac{b\,d}{2}\xi + n\,\alpha\,\mu\,b\,d\,\sigma_c \frac{\xi-\beta}{\xi}\right)\eta\,d = M$$

or

$$\underline{\sigma_c = c_1 \frac{M}{b\,d^2}} \qquad 2\text{-}39$$

where

$$c_1 = \frac{1}{\eta\left(\frac{\xi}{2} + n\,\alpha\,\mu\,\frac{\xi-\beta}{\xi}\right)} = \frac{1}{\frac{\xi}{2}\left(1-\frac{\xi}{3}\right) + n\,\alpha\,\mu\,\frac{\xi-\beta}{\xi}(1-\beta)}$$

Having determined z and σ_c, then σ_s can be computed from the relation:

$$\frac{\sigma_s/n}{\sigma_c} = \frac{d-z}{z} = \frac{1-\xi}{\xi}$$

or

$$\sigma_s = n\;\sigma_c\,\frac{1-\xi}{\xi}$$

so that

$$\underline{\sigma_s = c_2 \frac{M}{b\,d^2}} \qquad 2\text{-}40$$

where

$$c_2 = c_1\,n\,\frac{1-\xi}{\xi} = \frac{n\,(1-\xi)}{\frac{\xi^2}{2}\left(1-\frac{\xi}{3}\right) + n\,\alpha\,\mu\,(\xi-\beta)\,(1-\beta)}$$

Values of ξ and η are given in sheet 1 whereas values of c_1 and c_2 are given in sheet 2.

However, it is recommended to determine σ_s from the following relations:

$$\underline{\sigma_s = \frac{M}{A_s\,y_{ct}} = \frac{1}{\mu\eta}\frac{M}{b\,d^2}} \qquad 2\text{-}41$$

which means that $\qquad c_2 = \frac{1}{\mu\eta}$

Illustrative examples:

1) Given b = 40 cms, d = 100cms, $A_s = 40\ cm^2$ and M = 50 mt.
   Required σ_c and σ_s:

Equation 2-29 gives: $\quad z = \frac{15 \times 40}{40}\left(-1 + \sqrt{1 + \frac{2 \times 100 \times 40}{15 \times 40}}\right) = 41.8$ cms

Equation 2-30a gives: $\sigma_c = \frac{2 \times 50 \times 10^5}{40 \times 41.8\,(100 - \frac{41.8}{3})} = 69.5\ kg/cm^2$

Equation 2-30b gives: $\sigma_s = \frac{50 \times 10^5}{40\,(100 - \frac{41.8}{3})} = 1450\ kg/cm^2$

Table 1 gives: for $\mu\,\% = \frac{A_s}{b\,d} \times 100 = \frac{40 \times 100}{40 \times 100} = 1\ \%$

$\zeta = 0.418 \quad \eta = 0.861 \quad c_1 = 5.56$ and $c_2 = 116.1$

The stresses can therefore be computed by equations 2-32 and 2-33. i.e.

$$\sigma_c = 5.56 \times \frac{50 \times 10^5}{40 \times 100^2} = 69.5 \qquad \sigma_s = 116.1 \times \frac{50 \times 10^5}{40 \times 100^2} = 1450 \text{ kg per cm}^2.$$

Further, according to sheet 2, we have:

for $\mu = 1\%$, $c_1 = 5.5$ and $c_2 = 115$; so that $\sigma_c = 69$ and $\sigma_s = 1440$ kg/cm^2

Using equation 2-41, we get $\sigma_s = \frac{M}{A_s\, y_{ct}} = \frac{50 \times 10^5}{40 \times 86.1} = 1450$ kg/cm^2

2) If the same section is reinforced with $A_s = 50$ cm^2 and $A_s' = 10$ cm^2 and is subject to $M = 60$ mt, find σ_c and σ_s assuming d' = 5 cms.

The neutral axis can be determined from equation 2-34. Hence

$$z = -\frac{15\,(50 + 10)}{40} + \sqrt{\left[\frac{15\,(50 + 10)}{40}\right]^2 + \frac{2 \times 15}{40}\,(50 \times 100 + 10 \times 50)}$$

$$= - \quad 22.5 \quad + \quad 65.5 \quad = 43 \text{ cms}$$

Equation 2-35 gives σ_c, thus:

$$\sigma_c = \frac{60 \times 10^5}{\frac{40 \times 43}{2}\,(100 - \frac{43}{3}) + 15 \times 10 \times \frac{43 - 5}{43}\,(100 - 5)} = 70 \quad \text{kg/cm}^2$$

Equation 2-36 gives σ_s, thus:

$$\sigma_s = 15 \times 70 \times \frac{100 - 43}{43} = 1390 \quad \text{kg/cm}^2$$

Using sheet 1, we get:

For $\mu = \frac{50}{40 \times 100} \times 100 = 1.25$ % and $\mu' = \frac{10}{50}\mu = 0.2\mu$

$\zeta = 0.435$ i.e. $z = 43.5$ cms and $\eta = 0.849$ i.e. $y_{ct} = 84.9$ cms

Using sheet 2, we get: $c_1 = 4.80$ and $c_2 = 93$. Therefore,

$$\sigma_c = 4.80 \times \frac{60 \times 10^5}{40 \times 100^2} = 72 \qquad \sigma_s = 93 \times \frac{60 \times 10^5}{40 \times 100^2} = 1390 \quad \text{kg/cm}^2$$

Using equation 2-41, we get: $\sigma_s = \frac{M}{A_s\, y_{ct}} = \frac{60 \times 10^5}{50 \times 84.9} = 1410$ kg/cm

The results obtained by the different equations, tables or curves are approximately the same.

(3) T-sections with tension reinforcements only

If the neutral axis lies inside the compression flange i.e. $z < t_s$, the section behaves as a rectangular section with breadth B. Hence, the equation of the neutral axis is given by:

$$z = \frac{n A_s}{B} \left(-1 + \sqrt{1 + \frac{2 d B}{n A_s}}\right)$$

and the stresses can be computed from the relations:

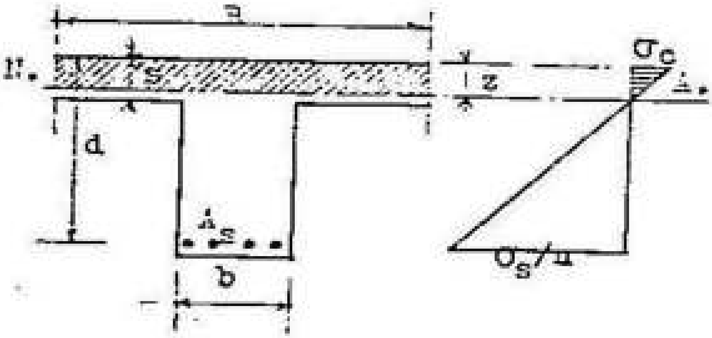


Fig. 2-14

$$\sigma_c = \frac{2 M}{B z \left(d - \frac{z}{3}\right)} \quad \text{and} \quad \sigma_s = \frac{M}{A_s \left(d - \frac{z}{3}\right)}$$

(Refer to equations 2-29 and 2-30 a and b.)

If the neutral axis lies outside the compression flange, i.e. $z > t_s$, one can proceed as follows:

In this case, the small compressive stresses resisted by the web between the bottom of the flange and the neutral axis may be neglected without making an appreciable error.

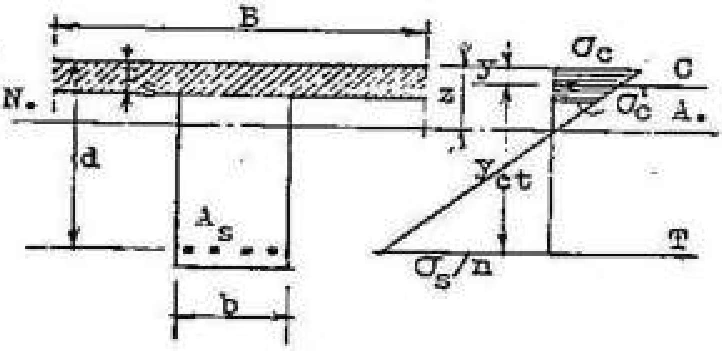


Fig. 2-15

The neutral axis passes through the center of gravity of the virtual area composed of the assumed concrete compression zone (equals area of flange) and n times the area of the tension steel; hence

$$B\, t_s \left(z - \frac{t_s}{2}\right) = n A_s (d - z) \qquad \text{or}$$

$$z = \frac{B \frac{t_s^2}{2} + n A_s d}{B t_s + n A_s} \qquad 2\text{-}42$$

The center of gravity of the assumed trapezoidal compressive stresses is given by:

$$y = \frac{t_s}{3} \cdot \frac{\sigma_c + 2 \sigma_c'}{\sigma_c + \sigma_c'}$$

but $$\sigma_c' = \sigma_c \frac{z - t_s}{z}$$ then

$$y = \frac{t_s}{3} \cdot \frac{\sigma_c + 2\sigma_c \frac{z - t_s}{z}}{\sigma_c + \sigma_c \frac{z - t_s}{z}} = \frac{t_s}{3} \cdot \frac{3z - 2t_s}{2z - t_s}$$

The arm of the internal forces is therefore given by:

$$y_{ct} = d - y = d - \frac{t_s}{3} \cdot \frac{3z - 2t_s}{2z - t_s} = d - \frac{t_s}{2} + \frac{t_s^2}{6(2z - t_s)} \qquad 2\text{-}43$$

In cases where t_s is small compared to z, one can assume:

$$y_{ct} = d - \frac{t_s}{2}$$

The stresses can accordingly be determined from the relations:

$$\sigma_s = \frac{m}{A_s} = \frac{M}{A_s\, y_{ct}} \qquad \text{and} \qquad \sigma_c = \frac{\sigma_s}{n} \cdot \frac{z}{d - z} \qquad 2\text{-}44$$

Assuming $z = \zeta d$, $t_s = \delta d$, and $A_s = \mu B d$, the above relations can be given in the form:

$$z = \zeta d = \frac{\frac{\delta^2}{2} + n\mu}{\delta + n\mu}\, d \qquad 2\text{-}45$$

$$y_{tc} = \eta d = \frac{d}{\mu c_2} \qquad 2\text{-}46$$

$$\underline{\sigma_c = c_1 \frac{M}{B d^2}} \qquad \text{and} \qquad \underline{\sigma_s = c_2 \frac{M}{B d^2} = \frac{M}{A_s \eta d}} \qquad 2\text{-}47$$

in which

$$c_1 = \frac{\zeta}{\frac{\delta^3}{12} + \delta(\zeta - \frac{\delta}{2})^2 + n\mu(1 - \zeta)^2}$$

and

$$c_2 = \frac{n(1 - \zeta)}{\frac{\delta^3}{12} + \delta(\zeta - \frac{\delta}{2})^2 + n\mu(1 - \zeta)^2}$$

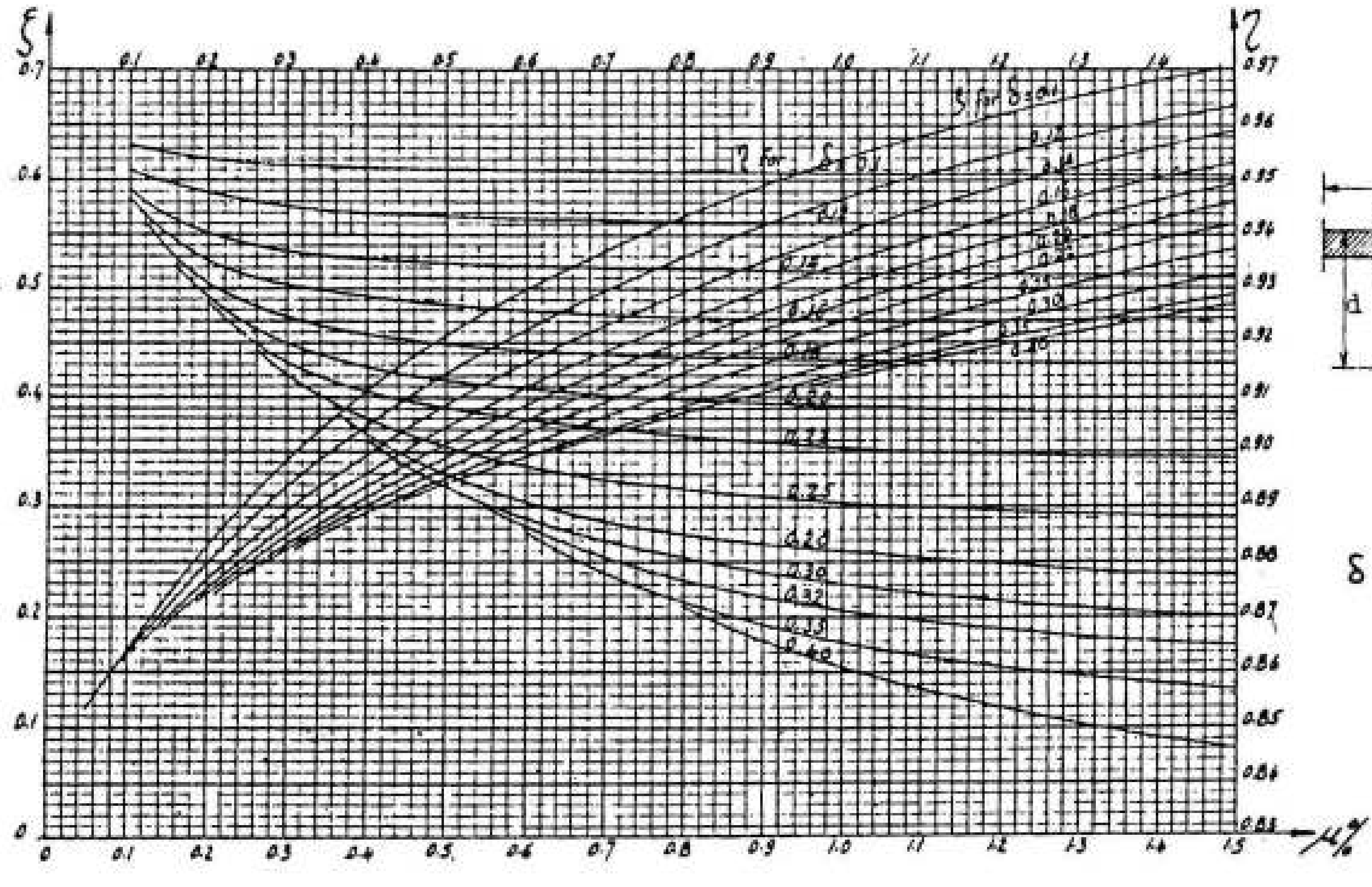
Values of ζ and η are given in sheet 3 whereas values of c_1 and c_2 are given in sheet 4.

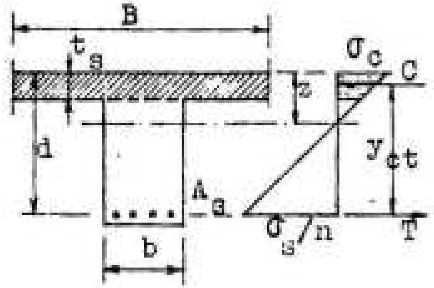
SHEET 3

SIMPLE BENDING

POSITION OF NEUTRAL AXIS AND ARM OF INTERNAL FORCES IN T-SECTIONS

$\zeta = z/d$ $\eta = y_{ct}/d$ $n = 15$





$\delta = t_s/d$ $\mu = A_s/Bd$

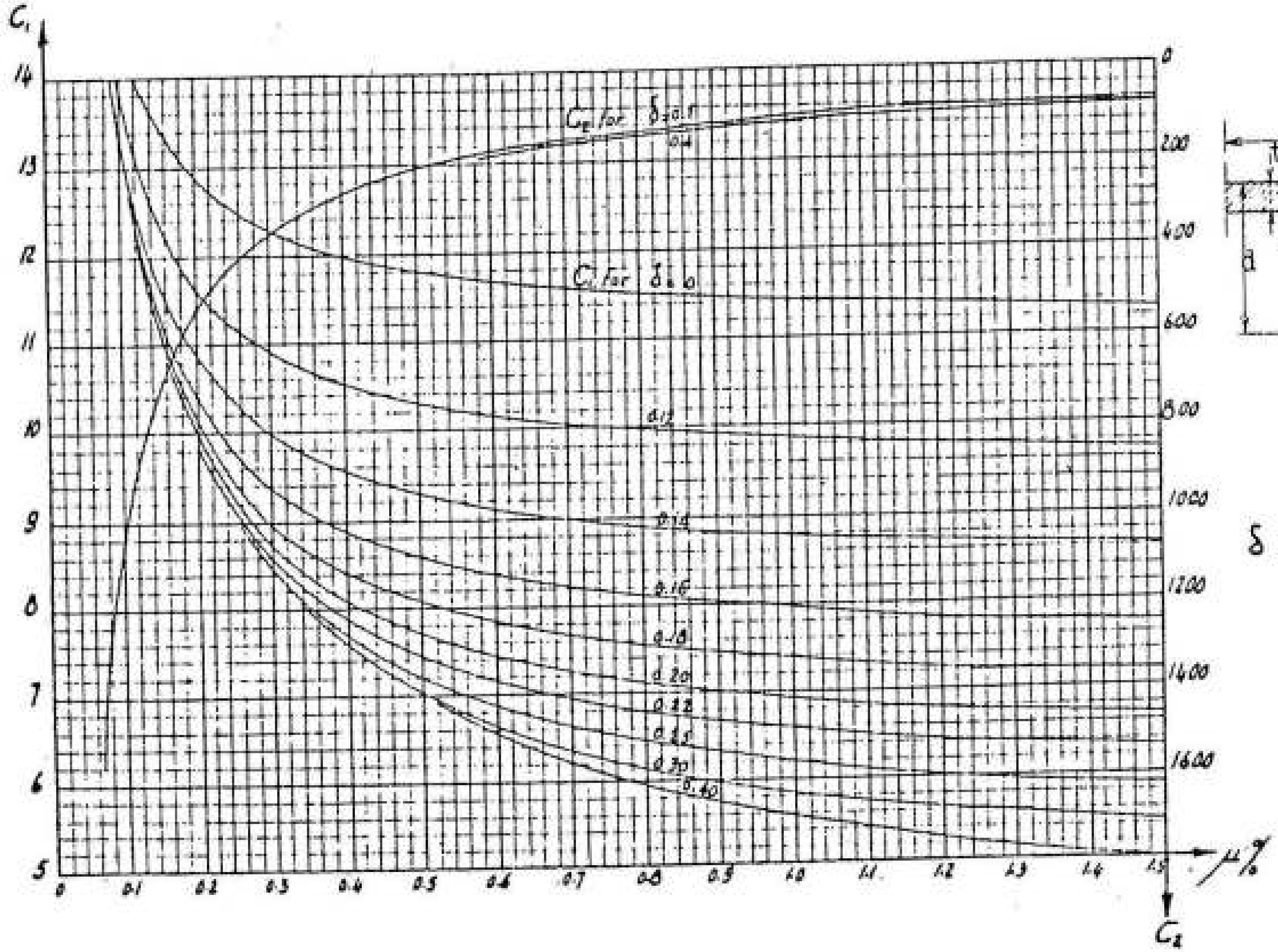
SHEET 4

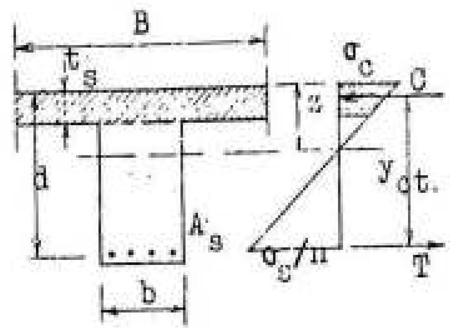
SIMPLE BENDING

DETERMINATION OF STRESSES IN T-SECTIONS

$n = 15$

$$\sigma_c = c_1 \frac{M}{B\,d^2} \qquad \sigma_s = c_2 \frac{M}{B\,d^2}$$





$\delta = t_s/d \qquad \mu = A_s/Bd$

(4) ⌐-sections with tension reinforcements. Fig. 2 -15

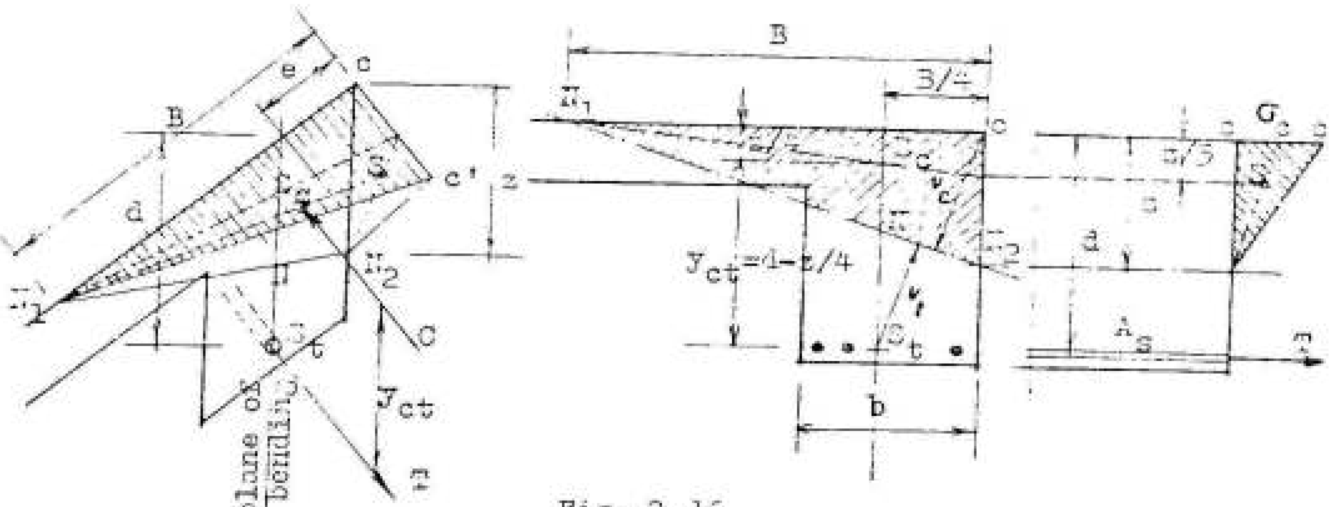


Fig. 2-16

Assumptions:

1) The rib lies in the plane of bending which passes through the center of gravity of the steel reinforcement S_t.
2) The stress in the center of gravity of the steel reinforcement is taken as a measure for the tensile strength of the section.

As a result of the first assumption, the center of compression S_c will lie in the plane of bending. Further, the neutral axis will be inclined and the maximum concrete stress σ_c takes place at the corner c. The stress distribution has the shape of a pyramid whose base is the triangle N_1 N_2 c and height σ_c.

The compression C is equal to the volume of the pyramid and acts in its center of gravity which lies at B/4 from c N_2 and z/4 from c N_1 i.e., $e = B/4$ or $B = 4\ e$.

We have further: $C = T$ or $\frac{B\ z}{2} \cdot \frac{\sigma_c}{3} = A_s\ \sigma_s$ and

$$\frac{\sigma_c}{\sigma_s / n} = \frac{v_c}{v_t} = \frac{c\ N_2}{N\ S_t} = \frac{z}{d - \frac{3}{4}z}$$

These two relations give:

$$z = \frac{9\ n\ A_s}{4B}\left(-1 + \sqrt{\frac{32\ B\ d}{27\ nA_s} + 1}\right) \qquad \text{2-48 a}$$

Assuming $B = 2b$, then

$$z = \frac{9\ n\ A_s}{8b}\left(-1 + \sqrt{\frac{b\ d}{3\ n\ A_s} + 1}\right) \qquad \text{2-48 b}$$

The arm of the internal forces is given by: $y_{ct} = d - \frac{z}{4}$ 2-49

The stresses are therefore:

$$\sigma_s = \frac{M}{A_s\, y_{ct}} \qquad \text{and} \qquad \sigma_c = \frac{\sigma_s}{n} \cdot \frac{z}{d - \frac{3}{4} z} \qquad 2\text{-}50$$

(5) Triangular sections with tension reinforcements. Fig. 2-17

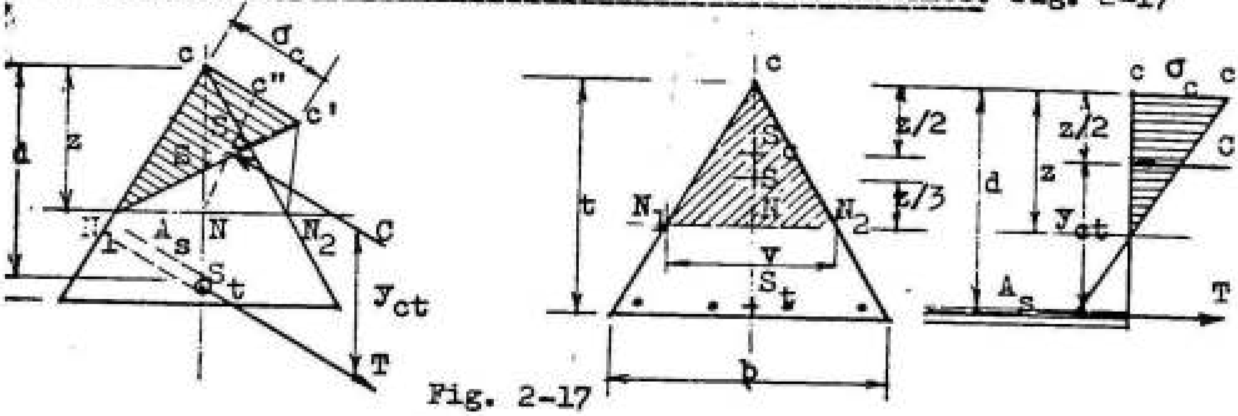


Fig. 2-17

The compressive stresses are distributed over the area :

$N_1 N_2 c = \frac{v\, z}{2}$ where $v = \frac{b\, z}{t}$ i.e., $N_1 N_2 c = \frac{b\, z^2}{2\, t}$

The total compression C is given by:

$$C = \frac{b\, z^2}{2\, t} \cdot \frac{\sigma_c}{3} = \frac{b\, z^2}{6\, t}\, \sigma_c$$

while the total tension T is:

$$T = A_s\, \sigma_s = C$$

The neutral axis passes through the center of gravity of the virtual area. Thus

$$\frac{b\, z^2}{2\, t} \cdot \frac{z}{3} = n\, A_s\, (d - z) \qquad 2\text{-}51$$

from which one can determine z.

The center of compression S_c lies on the center of the stress triangular upright pyramid $N_1 N_2 c c'$. It lies on the middle line $N c''$ at height $\sigma_c/4$ from the base $N_1 N_2 c$.

The arm of the internal forces is therefore given by:

$$y_{ct} = d - \frac{z}{2} \qquad 2\text{-}52$$

We have further:

$$M = T\, y_{ct} = A_s\, \sigma_s\, (d - \frac{z}{2}) \qquad \text{or}$$

$$\sigma_s = \frac{M}{A_s\, (d - \frac{z}{2})} \qquad 2\text{-}53$$

and $$M = C\, y_{ct} = \frac{b\, z^2\, \sigma_c\, (d - \frac{z}{3})}{6\, t} \qquad \text{or}$$

$$\sigma_c = \frac{6\ M\ t}{b\ z^3\ (d - \frac{z}{3})} \qquad 2\text{-}54$$

If the triangular section is acting such that its base is the outside fiber of the compression zone as shown in Fig. 2-18, then the compression zone will be trapezoidal and the compression C is given by:

$$C = \frac{b\ z\ (t - \frac{z}{3})\ \sigma_c}{2\ t}$$

The center of compression is at a distance y from the base of the triangle; where

$$y = \frac{t - \frac{z}{2}}{t - \frac{z}{3}} \cdot \frac{z}{3} - \frac{z}{3} \qquad 2\text{-}55$$

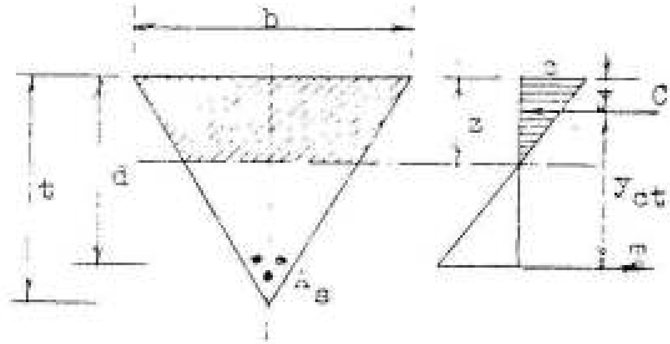


Fig. 2-18

The arm of the internal forces, y_{ct}, is given by:

$$y_{ct} = d - y \qquad 2\text{-}56$$

The stresses can therefore be calculated from the basic equations:

$$\sigma_s = \frac{M}{A_s\ y_{ct}} \qquad \sigma_c = \frac{2\ M\ t}{b\ z\ (t - \frac{z}{3})\ y_{ct}} \qquad 2\text{-}57$$

(6) Graphical determination of stresses

In complicated forms of sections, it is generally not possible to derive easy relations for determining the stresses. In such cases, graphical methods may be used. Fig. 2-19

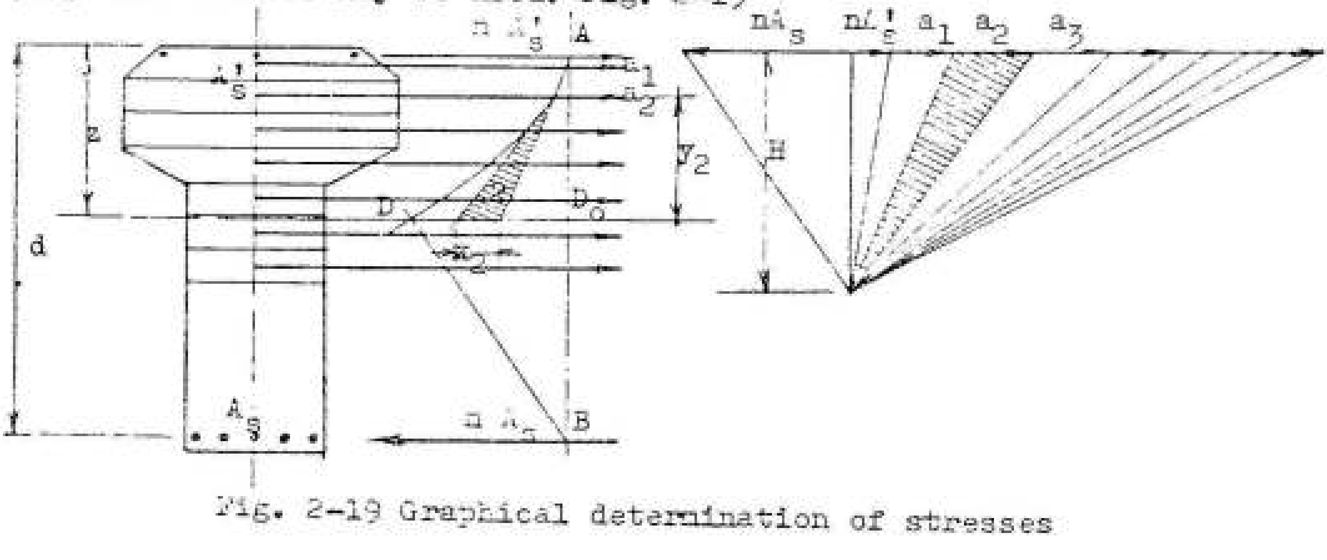


Fig. 2-19 Graphical determination of stresses

In order to determine the position of the neutral axis of a symmetrical section, one can proceed as follows:

a) Divide the area of the section into small strips of areas a_1, a_2, a_3 .... etc.

b) Consider the area of each strip as a force acting in its center of gravity and parallel to the neutral axis; the steel is to be introduced with n times its area.

c) Draw the force polygon, polar diagram and link polygon in the manner shown in Fig. 2-19.

The neutral axis passes through the point of intersection of the last rays of the link polygon of the compression and tension elemental areas i.e., point D.

The statical moment of the concrete and n times the steel in compression as well as the statical moment of n times the steel in tension about the neutral axis is, according to the shown construction, equal to: $H \times \overline{D D_o}$.

Having determined the position of the neutral axis, the stresses in concrete and steel can be determined from the following relations:

$$\sigma_c = \frac{M\ z}{I_{nv}} \qquad \sigma_s = n\,\frac{M\ (d - z)}{I_{nv}} \qquad 2\text{-}58$$

in which

I_{nv} = the moment of inertia of the virtual area about the neutral axis = 2 H x area A B D.

The proof for the above statements is as follows:

Statical moment of any elemental area (say a_2) about the neutral axis is given by: $S_2 = a_2\ y_2$.

The two hatched triangles being similar, then

$a_2 / H = x_2 / y_2$ or $x_2\ H = a_2\ y_2 = S_2$

Therefore, the statical moment of the virtual compression zone is given by:

$$S_{nv} = H \cdot \overline{D D_o} \qquad 2\text{-}59$$

Moment of inertia of any elemental area (say a_2) about the neutral axis is given by: $I_2 = a_2\ y_2^2$. But $a_2\ y_2 = x_2\ H$; then

$I_2 = x_2\ y_2\ H = 2\ H$. area of hatched triangle (2) and

$$I_{nv} = 2\ H\ .\ \text{area}\ \overline{ABD} \qquad 2\text{-}60$$

e) Ultimate strength analysis

The analysis considered in this section is limited to that normally adapted by the modern design methods. That is, the stress distribution of concrete in compression is as given for stage III, which is the stage representing the ultimate resistance of a section or its "ultimate strength".

As mentioned before, analysis is independent of the design methods and may vary only with the extent of the idializations made to simplify the observed material properties.

(1) Idealized properties of concrete and reinforcing steel

(a) Reinforcing steel

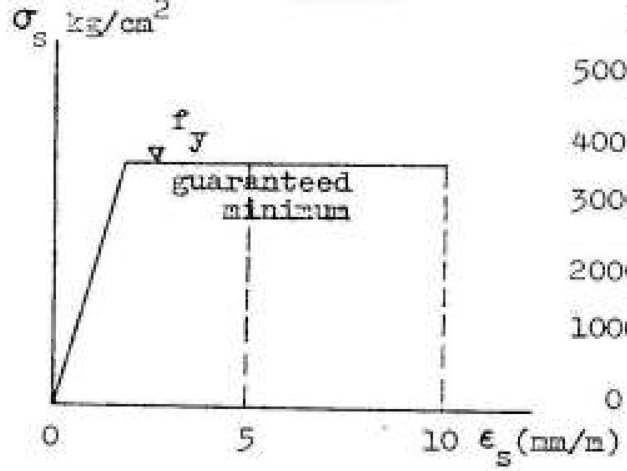


Fig. 2-20a Stress-strain relationship according to CEB

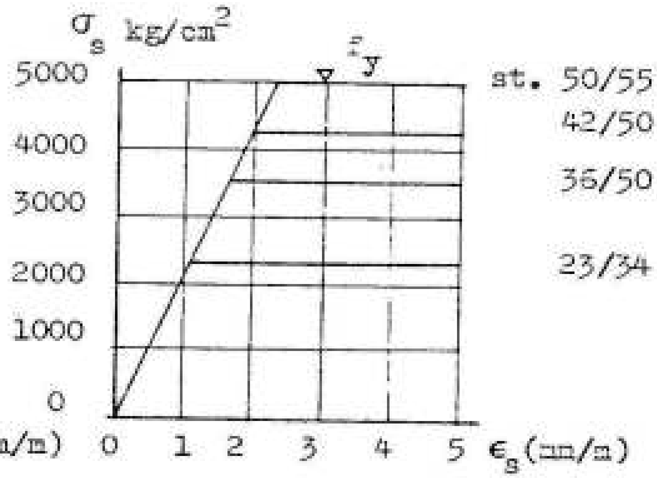


Fig. 2-20b Stress-strain relationship according to DIN

Characteristic strength, for calculations according to CEB-recommendations, is referred to the yield stress, f_y, of the steel (apparent limit for mild steel or 0.2% proof stress for cold-drawn steels).

For all steels currently used in reinforced concrete construction, the stress-strain relationship may be of the form shown in Fig.2-20a.

The new DIN 1045 (published 1972) give the characteristic relationships shown in Fig. 2-20b. It is assumed here that the maximum critical strain is $\epsilon_y = 5‰$.

(b) Concrete

According to the new DIN 1045, the stress-strain relationship of the compressed zone to be considered in the design may either be assumed of the parabolic rectangular form, Fig. 2-21a, or of the simplified form shown in Fig. 2-21b.

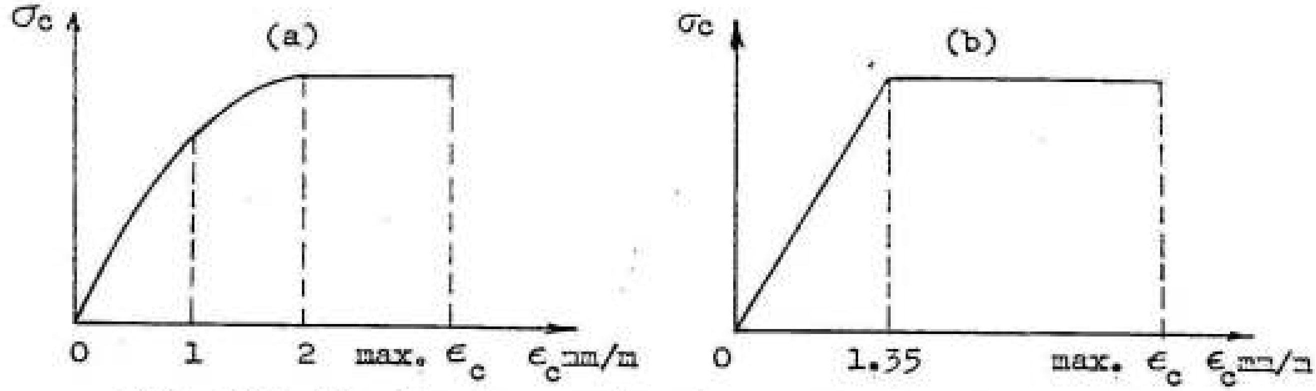


Fig. 2-21 Idealized concrete stress-strain relationship

The maximum compressive strain (max. ϵ_c) is assumed 3.5‰ for cases of simple bending and eccentric forces with big eccentricity and 2‰ for cases of axial compression.

According to the ACI (318-71), the maximum compressive strain is assumed: max. ϵ_c = 3.0‰.

The UNESCO assumes: max. ϵ_c = 2.0‰.

(2) General analysis. Application to rectangular sections with tension reinforcements only.

It has been stated before that at ultimate loads the stresses and strains are no longer proportional and that their distribution on the compression side of the section is of the same shape as the stress - strain curve.

The failure of the section can be caused in one of two ways:

(a) If moderate amounts of tension steel, corresponding generally to economic design, are used, the failure will take place due to yielding of the tension steel. At that stage, the reinforcement yields suddenly and stretches a large amount, and the tension cracks in the concrete widen visibly and propagate upwards with significant deformation of the member under consideration. When this happens, the height of the compression zone is reduced and the stresses are increased to such a degree that crushing of the concrete takes place. Such yielding failure is gradual and is preceeded by sufficient warning such as the widening and lengthening of cracks and the marked decrease in deflection.

In this case, the use of high grade concrete or compression steel does not increase the moment of resistance of the section and that at failure the stress in the tension steel is equal to its yield stress, f_y, and at the same time the maximum compressive stress is equal to the maximum resistance of concrete in compression, f_{cp}.

(b) If large amounts of reinforcement or normal amounts of steel of high strength are used, the compression strength of concrete is reached before the steel starts yielding and the concrete fails by crushing.

Rectangular beams fail in compression when the maximum concrete strain ϵ_c reaches 2-4 ‰.

Compression failure through crushing of concrete is sudden, of an almost "explosive" nature, and occurs without warning; hence it is to be avoided.

The analysis of stresses and ultimate strength of reinforced concrete sections has been treated in different ways. One of them, recommended by the ACI, is shown in the following:

f_{cp} = prism strength of concrete

$\alpha = 0.72$

$\beta = 0.425$ and

for $f_{cp} < 280$ kg/cm^2

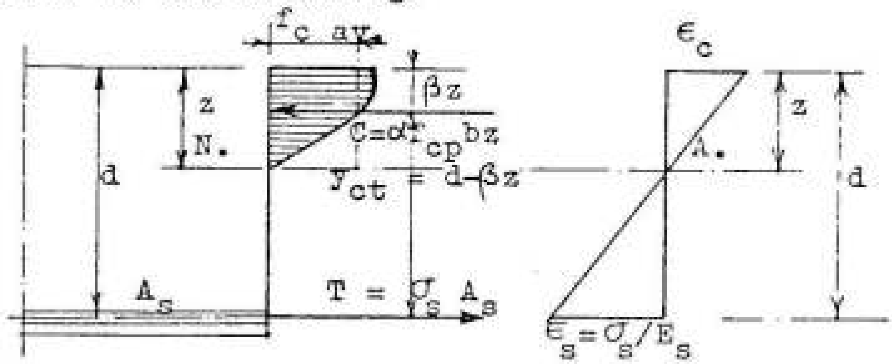


Fig. 2-22 Distribution of stresses and strains at failure

Let Fig. 2-22 represent the distribution of the internal stresses and strains when the section is about to fail. It is required to determine the ultimate moment M_u at which the section will fail either by yielding of the tension steel or by crushing of the concrete in the outer compression fiber.

The section fails by yielding when $\sigma_s = f_y$ which is accompanied by crushing of the compression zone when $\epsilon_c = \epsilon_{cu} = 0.003$.

It is however not necessary to know the exact shape of the concrete stress distribution; what should be known, for a given distance z of the neutral axis, is (1) the total resultant compression C and (2) its location, i.e., its distance from the outer compression fiber.

In a rectangular section, the area in compression is b z, and the total compressive force on that area can be expressed as:

$C = f_{c\ av.}\ b\ z$, where $f_{c\ av.}$ is the average compressive stress over b z.

Assuming $f_{c\ av.} / f_{cp} = \alpha$ then $C = \alpha\ f_{cp}\ b\ z$

For a given distance z to the neutral axis, the location of C can be defined as some fraction β of this distance. Thus, as indicated in Fig. 2-22, for a concrete of given strength, f_{cp}, it is only necessary to know α and β in order to define completely the effect of the concrete compressive stresses.

For concrete grades below C350 (f_{cp} = 280 kg/cm^2), we have:

$$\alpha = 0.72 \quad \text{and} \quad \beta = 0.425$$

The ultimate strength can be calculated from the laws of equilibrium and the strain compatibility relation (plane sections before bending remain plane after bending).

In considering simple bending, equilibrium requires:

$$C = T \qquad \text{or} \qquad \alpha f_{cp}\, b\, z = A_s \sigma_s \qquad 2\text{-}61$$

External moment equals moment of internal stresses, hence,

$$M = T\, y_{ct} = A_s \sigma_s (d - \beta z) \qquad 2\text{-}62$$

and

$$M = C\, y_{ct} = \alpha f_{cp}\, b\, z\, (d - \beta z) \qquad 2\text{-}63$$

The condition for tension failure by yielding is $\sigma_s = f_y$ with $\epsilon_s > \epsilon_y$. Substituting this value in equation 2-61, we get:

$$z = A_s f_y / \alpha f_{cp} b \qquad 2\text{-}64$$

For convenience, the following non-dimensional parameters are introduced:

$$f_y / f_{cp} = \rho \qquad A_s / b d = \mu \qquad 2\text{-}65$$

then, equation 2-64 can be rewritten in the form:

$$z = \mu \rho d / \alpha \qquad 2\text{-}66$$

The ultimate moment for tension failure can now be determined by eliminating z from equations 2-62 and 2-66; thus,

$$M_u = A_s f_y d (1 - \beta \rho \mu / \alpha) \qquad 2\text{-}67$$

For $\alpha = 0.72$ and $\beta = 0.425$, this expression becomes

$$M_u = A_s f_y d (1 - 0.59 \rho \mu) \qquad 2\text{-}67a$$

For compression failure, the compressive strain in the concrete is also given by $\epsilon_{cu} = 0.003$; meanwhile, the steel stress σ_s remains proportional to the steel strain ϵ_s, i.e.,

$$z = \frac{\epsilon_{cu}}{\epsilon_s + \epsilon_{cu}} d = \frac{0.003}{\sigma_s / E_s + 0.003} d \qquad 2\text{-}68$$

The ultimate moment of a reinforced concrete section reinforced so heavily that failure occurs by compression in the concrete can be then obtained by solving Eqn. 2-68 and 2-61 to obtain the value of z then substitute that value into Eqn. 2-63.

If it is required to determine the balanced steel ratio μ_b giving the amount of reinforcement necessary to make the beam fail by crushing of concrete at the same load which causes the steel to yield, one needs to equate the right hand side of equation 2-66 to that of equation 2-68, one gets

$$\mu_b = \frac{\alpha}{\beta} \cdot \frac{0.003}{f_y / E_s + 0.003} \qquad 2\text{-}69$$

Substituting $\alpha = 0.72$ and $E_s = 2100\ 000\ kg/cm^2$, equation 2-69 can be rewritten in the form:

$$\mu_b = \frac{0.72}{\beta} \cdot \frac{6300}{6300 + f_y} \qquad 2\text{-}69a$$

In order to ensure yielding-type (ductile) failure, it is generally specified that the tension-steel-ratio should be smaller than 0.75 the balanced ratio μ_b.

(3) Equivalent rectangular stress block

It was noted in the previous investigation that the actual geometrical shape of the concrete compression-stress distribution varies considerably and that, in fact, one needs not know this shape exactly, provided one does know two things: (1) the magnitude of C and (2) its location. Information on these two quantities was obtained from results of experimental research and expressed in two parameters α and β.

The actual stress-distribution will be replaced by an equivalent one of some simple geometrical shape, provided that it gives the same magnitude of C applied at the same location as in the acual member, when it is on the point of failure. The rectangular distribution proposed by Whitney is shown in Fig. 2-22' for a beam of rectangular cross section made of concrete $< C350$ with $f_{cp} \leqslant 280\ kg/cm^2$.

It is seen that the actual distribution is replaced by an equivalent one of simple rectangular outline. The intensity of this uniform and constant equivalent compression stress is $0.85\ f_{cp}$ acting over a part of the section of depth $y = 0.85\ z$, where z is the distance to the acual neutral axis. It is in no way maintained that the compression stresses are actually distributed in this most unlikely manner. It is maintained, however, that this equivalent distribution gives the answers just derived for the actual situation.

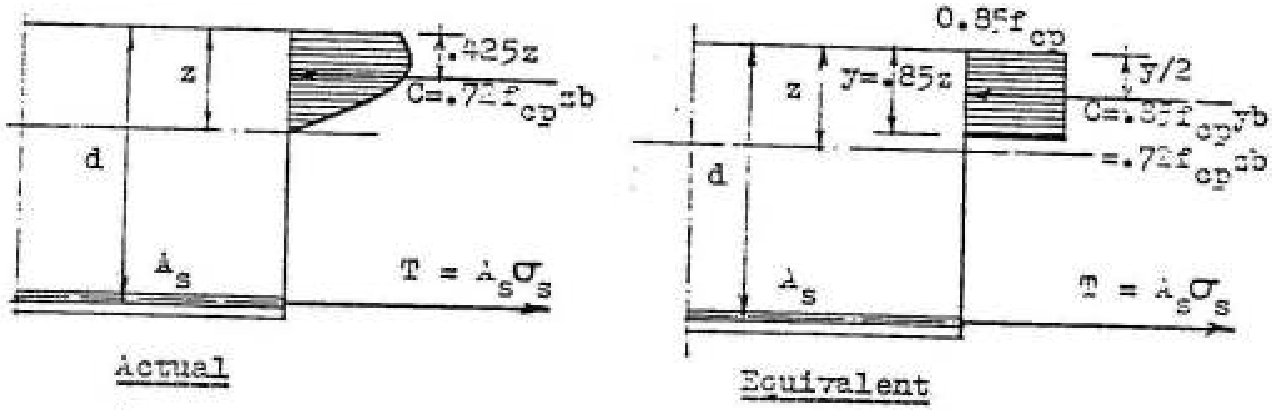


Fig. 2-22' Stress-distribution at ultimate load (for $f_{cp} < 280$ kg/cm^2)

In a rectangular beam of width b, the equivalent uniform compression stress, 0.85 f_{cp}, acts on an area y b, so that the total compression force is

$$C = 0.85\ f_{cp}\ y\ b \qquad 2\text{-}70$$

If we express the depth y of the rectangular stress block in terms of the distance z to the neutral axis as assumed, that is $y = 0.85\ z$ we obtain for the compression force the magnitude:

$$C = 0.85\ f_{cp} \cdot 0.85\ z\ b = 0.72\ f_{cp}\ z\ b \qquad 2\text{-}70a$$

which is the same as calculated from the actual diagram for $\alpha = 0.72$. Further, since the centroid of a rectangle is at mid-depth, the distance from the compression edge to C is evidently y/2. Again the distance from top fiber to compression resultant is given by $0.5 \times 0.85\ z = 0.425\ z$, which is the correct distance for $\beta = 0.425$.

This rectangular stress distribution is easily visualized and its geometrical properties are extremely simple, many calculations can be directly carried out without referance to the previously given equations as can be seen in the following example.

Illustrative example: Fig. 2-23

It is required to determine the ultimate failure moment M_u of a beam 25 cms wide and 65 cms deep . It is reinforced in tension only by 15 cm^2 high grade steel with $f_y = 3600$ kg/cm^2; the concrete quality is C250.

Data: b = 25 cms   t = 65 cms   d = 60 cms $A_s = 15$ cm^2

$f_{cp} = 0.80 \times 250 = 200$ kg/cm^2 and $f_y = 3600$ kg/cm^2

$\mu = A_s / b\ d = 15/25 \times 60 = 0.01$ $E_s = 2100$ t/cm^2

Solution

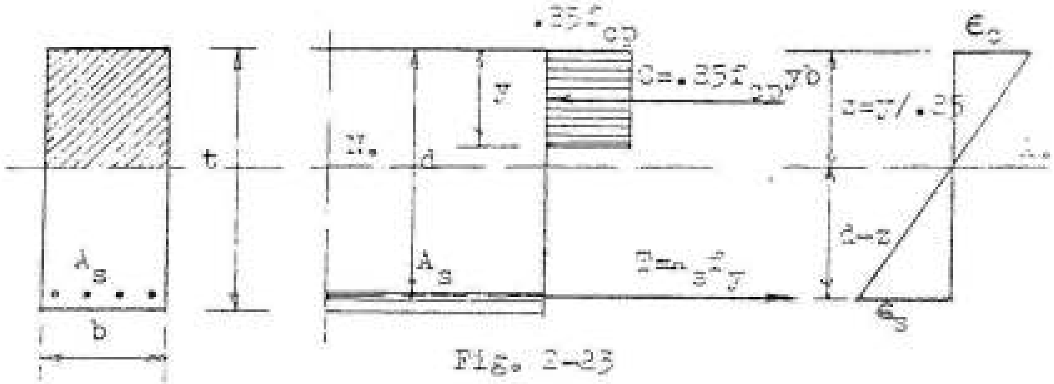


Fig. 2-23

The depth y is to be determined from the condition C = T. Hence

$0.85\ f_{cp}\ y\ b = A_s\ f_y$ or $0.85 \times 200 \times 25\ y = 15 \times 3600$

giving $y = 12.7$ cms i.e. $z = y/0.85 = 12.7/0.85 = 15$ cms

In order to know whether the beam fails by yielding of the tension steel or crushing of the compression concrete one has to see whether ϵ_c is smaller or larger than ϵ_{cu} (0.003) at the instant the steel starts yielding. At that point, the strain in steel is:

$\epsilon_y = f_y / E_s = 3600 / 2100\ 000 = 0.00172$

From the similar triangles of the strain diagram, we get:

$\epsilon_c = \epsilon_y\ z / (d - z) = 0.00172 \times 15 / (60 - 15) = 0.000544 < 0.003$

Hence, failure will be initiated by yielding of the tension steel.

The ultimate moment M_u is therefore given by:

$M_u = A_s\ f_y\ (d - y/2) = 15 \times 3600\ (60 - 12.7/2) = 29 \times 10^5$ kg cm

The balanced steel ratio μ_b can be calculated from the condition that at the moment of yielding $\epsilon_s = \epsilon_y = f_y / E_s = 0.00172$ and the strain of concrete $\epsilon_c = \epsilon_{cu} = 0.003$. Hence, from the similar triangles of the strain distribution, we get:

$z / d - z = 0.003 / 0.00172$ or $z = 38$ cms

and $y = 0.85\ z$ or $y = 0.85 \times 38 = 32.3$ cms

The corresponding internal tension or compression is given by:

$T = C = 0.85\ f_{cp}\ y\ b = 0.85 \times 200 \times 32.3 \times 25 = 137\ 000$ kgs

The corresponding area of steel is $A_s = 137\ 000 / 3600 = 38$ cm^2

i.e. the balanced steel ratio is $\mu_b = 38 / 25 \times 60 = 0.0253$

(4) Upper limit of resisting moment

In order to have a ductile failure, the ultimate moment due to the compressive stresses in the concrete is limited to the value that can be resisted by a balanced section.

For a rectangular section, this moment can be calculated in the following manner:

At moment of failure, the maximum strains in concrete and steel are $\epsilon_c = 0.003$ and $\epsilon_s = \epsilon_y$.

According to equation 2-68, the position of the neutral axis is given by:

$$z_b = \frac{0.003}{f_y / E_s + 0.003} d = \zeta d$$

where $E_s = 2100\,000 \text{ kg/cm}^2$

Knowing further that: $y_b = 0.85\, z_b = 0.85\, \zeta\, d$ and

$$(y_{ct})_b = d\,(1 - 0.425\, \zeta) = \eta\, d$$

we get:

$$C_b = 0.85\; f_{cp}\, y_b\, b = 0.85\; f_{cp} \times 0.85\, \zeta\, d\, b = 0.72\, f_{cp}\, \zeta\, d\, b \quad \text{and}$$

$$M_{ub} = 0.72\, \zeta\, \eta\, f_{cp}\, b\, d^2$$

In the following table, we give the resisting moment of a balanced section M_{ub} for different qualities of steel:

f_y kg/cm^2	$\zeta = z/d$	y/d	$\eta = y_{ct}/d$	C_b	M_{ub}
2400	0.724	0.611	0.695	$0.519 f_{cp} bd$	$0.361 f_{cp} bd^2$
3600	0.636	0.541	0.730	0.460 ,,	0.335 ,,

Whitney assumed a balanced beam would carry: $M_{ub} = 0.330\; f_{cp}\; b\; d^2$ independent of f_y, provided $f_{cp} > 200 \text{ kg/cm}^2$.

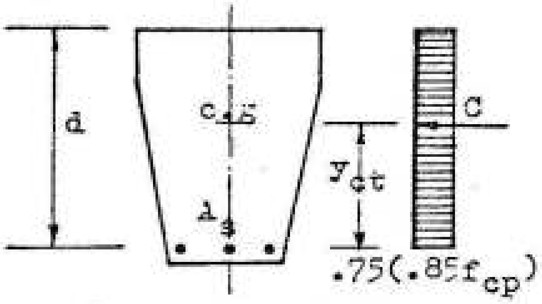


Fig. 2-24

The CEB assumes that the value of the ultimate moment due to the compressive stresses in the concrete (apart from the contribution of any compressive reinforcement provided) is limited to the value of the moment (with respect to the reinforcing bars in tension) of the force acting upon the total effective section assumed to be subjected to a uniform stress equal to 0.75 of the maximum average compressive strength of the concrete ($0.85\ f_{cp}$).

The introduction of an upper limit to the ultimate moment is equivalent to gradually reducing the concrete stress from the average compressive strength ($0.85\ f_{cp}$) to 0.75 its value according as the depth of the compressive zone increases from a certain limit value (at which this upper limit of the moment is reached) until it becomes equal to the effective depth d (see Fig. 2-24).

The upper limit of the ultimate moment is equal to $C\ y_{ct}$, so that, for a rectangular section , this becomes:

$$\underline{M_{upper}} = 0.75\ (0.85\ f_{cp})\ b\ d^2\ /\ 2 = \underline{0.32\ f_{cp}\ b\ d^2} \qquad 2\text{-}71$$

(5) Rectangular sections with double reinforcements

In sections of limited depth, it may happen that the concrete cannot develop the compression force required to resist the given moment. In this exceptional case, compression reinforcement may be added.

For increasing the ultimate moment of a section, compression reinforcement is required only when the ratio of the tension steel is larger than the value $0.75\ \mu_b$. However, there are situations in which it is used for reasons other than strength. It has been found, for example, that the introduction of some compression reinforcement reduces the long-time deflections of beams. In addition, in some cases, bars will be placed in the compression zone for some cases of heavy live and rolling loads or as stirrup hangers. It is often desirable to have a small amount of such reinforcements in flexural design.

If in a beam with double reinforcements, the ratio of the tension reinforcement μ is smaller than or equal to $0.75\ \mu_b$, the effect of the compression reinforcement on the ultimate moment can be neglected. The strength of such sections will be controlled by tesile yielding.

If the ratio of the tension steel is larger than $0.75\ \mu_b$, the ultimate moment of a rectangular section with double reinforcements can be determined in the following manner: Fig. 2-25.

Assume for the moment that both A_s and A'_s are stressed to f_y at failure, and that the ultimate moment is composed of two parts. The first part, M_{u1}, is provided by the couple consisting of the force in the compression steel A'_s and the force in an equal area of tension steel. Thus

$$M_{u1} = A'_s\ f_y\ (d - d') \qquad 2\text{-}72$$

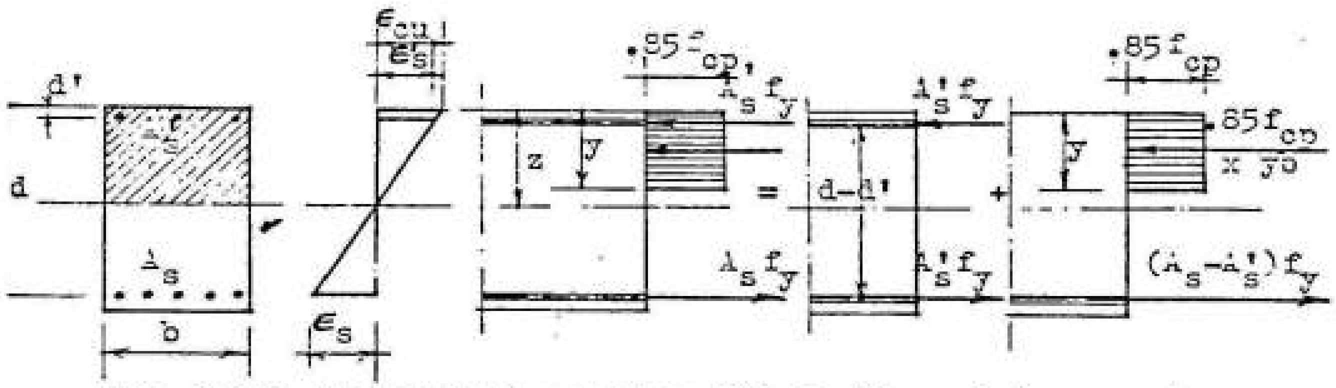


Fig. 2-25 Rectangular section with double reinforcement

The second part, M_{u2}, is the contribution of the remaining tension steel $(A_s - A'_s)$ acting with the compression concrete. Hence

$$M_{u2} = (A_s - A'_s)\, f_y\, (d - \frac{y}{2}) \qquad 2\text{-}73$$

The depth of the stress block y is determined in the normal way from the relation: $C = T$ or $0.85\, f_{cp}\, y\, b = (A_s - A'_s)\, f_y$ i.e.

$$y = \frac{(A_s - A'_s)}{b}\, \frac{f_y}{0.85\, f_{cp}} \qquad 2\text{-}74$$

Assuming $\mu = A_s / bd$,, $\mu' = A'_s / bd$ and $\rho = f_y / f_{cp}$, y can be rewritten as:

$$y = \frac{(\mu - \mu')\,\rho}{0.85}\,.\, d \qquad 2\text{-}74a$$

The total moment M_u is therefore

$$M_u = M_{u1} + M_{u2} \qquad 2\text{-}75$$

This development is predicted on the assumption that both tension and compression steel are at yield when the beam fails. It is highly desirable that failure be caused by tensile yielding rather than by the crushing of the concrete. In order to ensure the same margin against brittle-concrete failure in a beam with double reinforcement as in a beam with single reinforcement, one should have

$$(\mu - \mu') \leqslant 0.75\, \mu_b \qquad 2\text{-}76$$

Whether or not the compression steel will have yielded at failure can be determined, according to Fig. 2-25, as follows:

Having determined $z = y/0.85$ and assuming $\epsilon_{cu} = 0.003$, the strain in the compression steel can be computed from the strain diagram, thus

$$\epsilon'_s = \epsilon_{cu}\, \frac{z - d'}{z}$$

If ϵ'_s is bigger than f'_y/E_s then the compression steel will be yielding at failure, otherwise, the contribution of the compression steel could conservatively be neglected in the design.

If y is bigger than 2d', the compression steel will generally be yielding at failure.

In order to simplify the calculations, the values of $0.75\,\mu_b$ - where μ_b is given by equation 2-69a - are given in table 2 for the normally used qualities of concrete and steel.

Table 2. Ultimate strength analysis. Balanced ratios of tension steel

Yield of steel f_y kg/cm^2	Quality of concrete	f_{cp} kg/cm^2	$\rho = \frac{f_y}{f_{cp}}$	μ_b	$0.75\,\mu_b$
2300	C200	165	13.9	0.038	0.0285
	C250	200	11.5	0.046	0.0345
3600	C200	165	21.8	0.021	0.0157
	C250	200	18.0	0.0254	0.0189

Illustrative examples:

(1) A rectangular section with double reinforcements is made of concrete C200; it has b = 30 cms, t = 60 cms, d = 55 cms, d' = 3.5 cms. It is reinforced by $A_s = 40$ cm^2 and $A'_s = 12$ cm^2 normal mild steel with $f_y = 2300$ kg/cm^2. Determine the ultimate moment of the section.

Solution

$\mu = A_s / b\,d = 40 / 30 \times 55 = 0.0242$

Table 2 gives: $0.75\,\mu_b = 0.0285 > \mu$

Hence, failure will be initiated by yielding of the tension steel and the contribution of the compression reinforcement is neglected.

We have further $C = T$ or $0.85\, f_{cp}\, b\, y = A_s\, f_y$ i.e.

$0.85 \times 165 \times 30\, y = 40 \times 2300$ or $y = 21.8$ cms and

$M_u = A_s\, f_y\, (d - \frac{y}{2}) = 40 \times 2300\, (55 - \frac{21.8}{2}) = 40.5 \times 10^5$ kg cm

(2) If the same beam were reinforced by $A_s = 30$ cm^2 and $A'_s = 10$ cm^2 high grade steel, determine the ultimate moment.

Solution

$\mu = 30 / 30 \times 55 = 0.0182$ and $0.75\,\mu_b = 0.0157 < \mu$

$\mu' = 10 / 30 \times 55 = 0.0061$ so that $\mu - \mu' = 0.0121 < 0.75\mu_b$

Hence, failure will be initiated by yielding of the tension steel and the contribution of the compression reinforcement will be considered.

$M_{u1} = A'_s f_y (d - d') = 10 \times 3600 (55 - 3.5) = 18 \times 10^5$ kg cm

Further $C = T$ or $0.85 f_{cp} b y = (A_s - A'_s) f_y$ i.e. giving

$0.85 \times 165 \times 30 y = (30 - 10) \times 3600$

$y = 17.1$ cms and $z = y / 0.85 = 17.1 / 0.85 = 20.1$ cms

and

$M_{u2} = (A_s - A'_s) f_y (d - \frac{y}{2}) = 20 \times 3600 (55 - \frac{17.1}{2}) = 33.4 \times 10^5$ kg cm

Total $M_u = M_{u1} + M_{u2} = 18 + 33.4 = 51.4$ mt

As y is much bigger than $2d'$ then the compression steel will be yielding at failure because in this case ϵ'_s will be much bigger than f'_y / E_s as can be seen from the following:

$\epsilon'_s = \epsilon_{cu} \frac{z - d'}{z} = 0.003 \frac{20.1 - 3.5}{20.1} = 0.00247$

$f'_y / E_s = 3600 / 2100\,000 = \qquad = 0.00172 < \epsilon'_s$

(6) Analysis of T-sections with tension reinforcements

The analysis of a T-section depends on the ratio of the flange-thickness t_s to the total thickness t. The following three cases are distinguished:

1) $t_s/t < 1/12$ or $t_s < 8$ cms

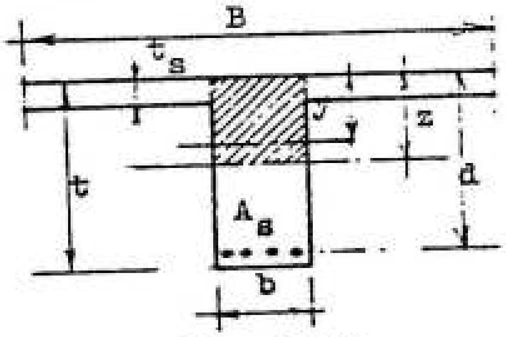


Fig. 2-26

In this case, Fig. 2-26, the stresses resisted by the flange may be neglected and the section can be calculated as rectangular with breadth b.

2) $t_s/t > 1/12$ and the neutral axis is inside the flange i.e., $z < t_s$

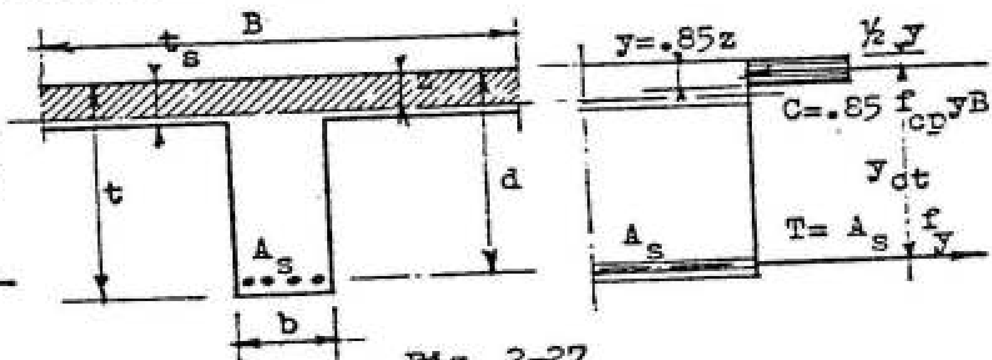


Fig. 2-27

In this case, Fig. 2-27, we have:

$C = 0.85 f_{cp}\, y\, B$

or $C = 0.85\ f_{cp}\ 0.85\ z\ B = 0.72\ f_{cp}\ z\ B$ and $T = ^{*}f_y\ A_s$

This means that this case can be applied if:

$$f_y\ A_s < 0.72\ f_{cp}\ t_s\ B \qquad 2\text{-}77$$

The section is to be treated as rectangular with breadth B.

3) $t_s/t > 1/12$ and the neutral axis is outside the flange i.e., $z > t_s$

The first step in this case, Fig. 2-28, is to determine the position of the neutral axis.

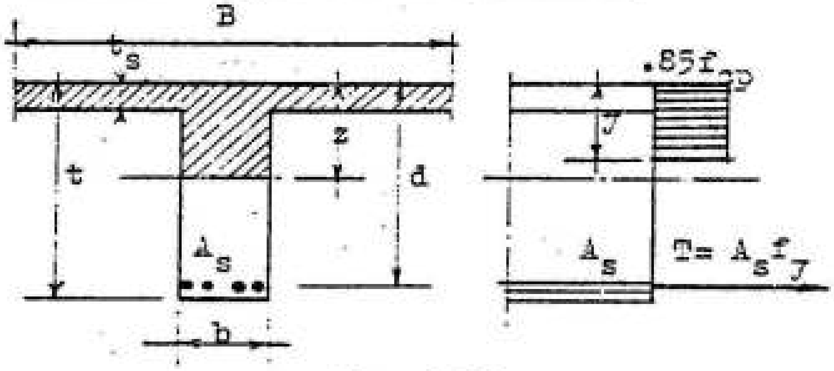


Fig. 2-28

Assuming for trial purposes that it lies in the compression flange, then

$$0.85\ z = y = \frac{A_s\ f_y}{0.85\ f_{cp}\ B}$$

Assuming further $A_s = \bar{\mu}\ B\ d$, z can be given in the form:

$$z = \frac{\bar{\mu}\ f_y\ d}{0.72\ f_{cp}} \qquad 2\text{-}78$$

If z is equal to or smaller than t_s, then the section behaves as rectangular of width B and depth d. If z is greater than t_s, one can proceed as follows:

Divide the total tensile steel into two parts. The first part A_{s1} represents the steel area which, when stressed to f_y, is required to balance the compressive force in the overhanging portions of the flange. Thus

$$A_{s1} = \frac{0.85\ f_{cp}\ (B - b)\ t_s}{f_y} \qquad 2\text{-}79$$

The force $A_{s1}\ f_y = 0.85\ f_{cp}\ (B - b)\ t_s$ acts with a lever arm $(d - ½t_s)$ to provide the resisting moment

$$M_{u1} = A_{s1}\ f_y\ (d - ½t_s) \qquad 2\text{-}80$$

* It has been assumed here that the strength of the T-section is controlled by yielding of the tension steel. This will usually be the case, because of the large compressive concrete area provided by the flange.

The remaining steel area A_{s2}, at a stress f_y, is balanced by the compression in the rectangular portion of the beam. The depth of the equivalent rectangular stress block in this zone is found from the equation of equilibrium

$$0.85\ f_{cp}\ b\ y = A_{s2}\ f_y$$

where

$$A_{s2} = A_s - A_{s1}$$

hence

$$y = \frac{(A_s - A_{s1})\ f_y}{0.85\ f_{cp}\ b} \qquad 2\text{-}81$$

The force $(A_s - A_{s1})\ f_y = 0.85\ f_{cp}\ y\ b$ acting with a lever arm $(d-\frac{1}{2}y)$ provides the resisting moment:

$$M_{u2} = (A_s - A_{s1})\ f_y\ (d - \tfrac{1}{2}\ y) \qquad 2\text{-}82$$

The total resisting moment is:

$$M_u = M_{u1} + M_{u2} \qquad 2\text{-}83$$

or

$$M_u = A_{s1}\ f_y\ (d - \tfrac{1}{2}\ t_s) + (A_s - A_{s1})\ f_y\ (d - \tfrac{1}{2}\ y) \qquad 2\text{-}83a$$

It is desired here also to ensure that the tensile steel will be yielding prior to sudden crushing of the compression concrete. hence if

$$\mu = A_s\ /\ b\ d \qquad \text{and} \qquad \mu_1 = A_{s1}\ /\ b\ d$$

we should have:

$$\mu - \mu_1 < 0.75\,\mu_b \qquad 2\text{-}84$$

(For values of $0.75\,\mu_b$ refer to table 2).

Illustrative example

It is required to determine the ultimate moment of the T-section shown in Fig. 2-29 if it is made of concrete C200 with f_{cp} = 165 kg/cm^2 and reinforced in tension only by 40 cm^2 high grade steel with f_y = 3600 kg/cm^2. The beam is continuous at its both ends and has a span of 6.0 ms. The distance center to center ribs is 2.5 meters.

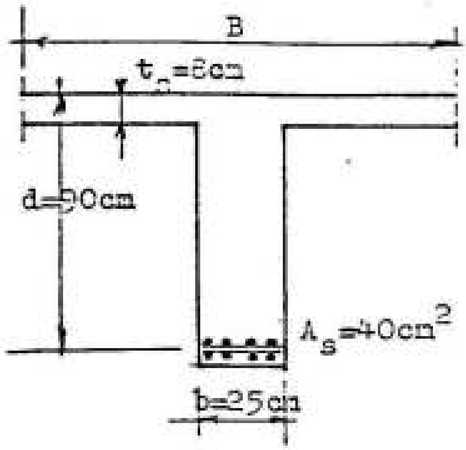


Fig. 2-29

Solution

According to the code of practice, the effective breadth B is the minimum of:

$12\ t_s + b = 12 \times 8 + 25 = 121$ cms

Distance C.L to C.L ribs= 250 cms

1/3 distance between points of zero bending moments; or

$1/3 \times 0.76 \times 600 \qquad = 152$ cms

The flange breadth is therefore chosen equal to B = 121 cms.

Assume first that the neutral axis lies inside the flange and that

$\bar{\mu} = A_s / B\ d = 40 / 121 \times 90 = 0.00367$ so that

$$z = \frac{\bar{\mu}\ f_y\ d}{0.72\ f_{cp}} = \frac{0.00367 \times 3600 \times 90}{0.72 \times 165} = 10 \text{ cms} > t_s = 8 \text{ cms.}$$

$$A_{s1} = \frac{0.35\ f_{cp}\ (B - b)\ t_s}{f_y} = \frac{0.35 \times 165 \times 96 \times 8}{3600} = 30 \text{ cm}^2$$ and

$$M_{u1} = A_{s1}\ f_y\ (d - \frac{t_s}{2}) = 30 \times 3600 \times 86 = 92.9 \times 10^5 \text{ kg cm}$$

$$y = \frac{(A_s - A_{s1})\ f_y}{0.85\ f_{cp}\ b} = \frac{(40 - 30) \times 3600}{0.85 \times 165 \times 25} = 10.3 \text{ cms}$$

$$M_{u2} = (A_s - A_{s1})\ f_y\ (d - \frac{y}{2}) = (40 - 30) \times 3600 \times (90 - \frac{10.3}{2})$$

$$= 10 \times 3600 \times 84.85 = 30.5 \times 10^5 \text{ kg cm}$$

The total ultimate moment is therefore:

$M_u = M_{u1} + M_{u2} = 92.9 + 30.5 = 123.4$ mt

Check for ductile failure:

$$\mu - \mu_1 < 0.75\ \mu_b$$

$\mu = A_s / b\ d = 40 / 25 \times 90 = 0.0178$

$\mu_1 = A_{s1} / b\ d = 30 / 25 \times 90 = 0.0133$

Table 2 gives: $0.75\ \mu_b \qquad = 0.0157$

Therefore, ductile failure is ensured.

2.6- SHEAR AND DIAGONAL TENSION

The distribution of shear stresses and the corresponding principal stresses due to the combined action of shear and flexure in homogenous elastic bodies has been shown in section 2.5 a.

Such distribution in reinforced concrete depends, among other things, on the stage of loading, the ratio of the shearing force to bending moment, the ratio of tension steel, ... etc.

Shear stresses with or without normal stresses cause principal compressive and tensile inclined stresses. We are most interested in the distribution and magnitude of the principal inclined tensile stresses as they cause the failure of the concrete if their magnitude exceeds the tensile strength.

a) Elastic analysis. Noncracked linear stage.

At this stage , the stresses in tension and compression are low, no cracks are developed in the tension zone so that concrete in tension can be assumed as statically acting and its modulus of elasticity in tension and compression is the same.

Accordingly, the normal and shear stresses may be assumed the same as those of elastic homogeneous bodies. The steel reinforcements are either to be neglected or to be introduced with n-times their area (where $n = E_s/E_c$ assumed here, in stage I, equal to 10). Hence the normal stresses are given by: $\sigma = M\,y / I_v$ where I_v = the moment of inertia of the virtual area about the neutral axis.

For determining the shear stresses, consider an element dx of a beam subject to varying bending moments M at a-a and M+dM at b-b. Fig.

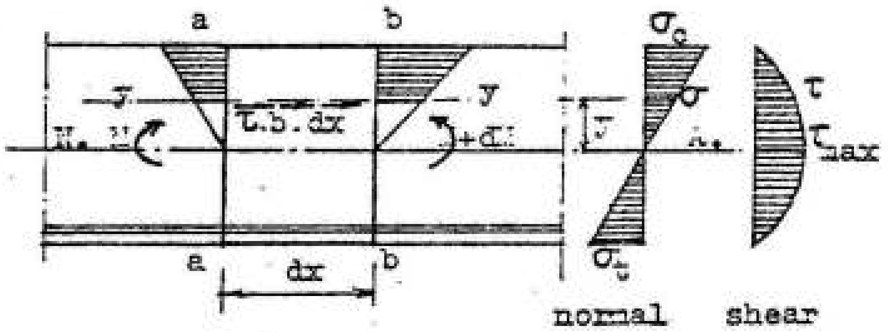


Fig. 2-30
Normal and shear stresses in non-cracked linear stage

2-30. The normal stresses on sections a-a and b-b will generally be not equal. Any horizontal plane y-y above the neutral axis will be subject to horizontal shear τ (equal to the vertical shear) due to the difference between the normal stresses on b-b and a-a above plane y-y and is given by: $\tau = Q\, S_v / I_v\, b$ where

S_v = the statical moment of the virtual cross-sectional area above y-y about the neutral axis, and

b = breadth of section at y-y

S_v is equal to zero at the outside fibers of the section and approaches its maximum value at the neutral axis, and hence the shear stress distribution will be as shown in Fig. 2-30.

The principal inclined tensile and compressive stresses can be calculated by equations 2-18 and 2-19.

b) Elastic analysis. Cracked linear stage.

At higher load stages, cracks are developed in the tension zone which will be neglected when computing the normal stresses. All the tensile stresses are assumed to be resisted by the tension steel. Accordingly, the distribution of the shear stresses in reinforced concrete cracked elements will vary from that of homogeneous or reinforced concrete noncracked elements in the manner shown in Fig. 2-31.

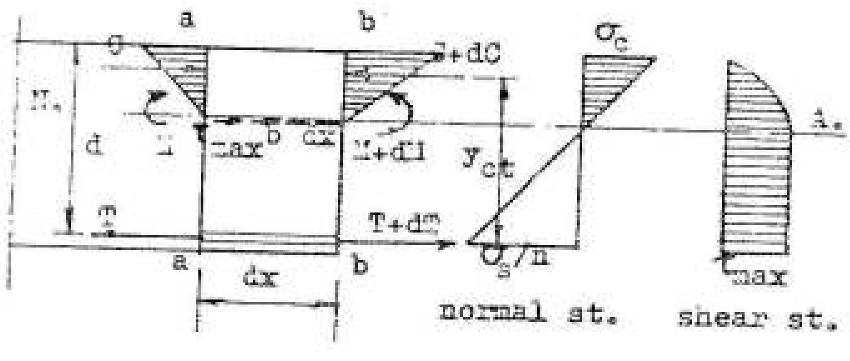


Fig. 2-31 normal and shear stresses in cracked linear stage

The basic equation for calculating the shear stresses in a noncracked section can be applied to cracked sections; one has only to neglect the cracked concrete in tension when calculating S_v and I_v. (The modular ratio n in this case, stage II, is assumed equal to 15)

Thus, the statical moment S_v will approach its maximum value at the neutral axis and remains constant below it, so that the part of the cross section between the neutral axis and the tension steel is subject to shear stresses equal to τ_{max} only because the normal stresses in this zone are assumed equal to zero.

The maximum shear stress in a rectangular section of constant breadth b and depth d subject to bending moment M and shearing force

Q can be directly calculated as follows:

$$\tau_{max}\, b\, dx = dT \qquad \text{but} \qquad T = \frac{M}{y_{ct}} \qquad \text{and}$$

$$\frac{dT}{dx} = \frac{1}{y_{ct}}\frac{dM}{dx} = \frac{Q}{y_{ct}}$$

therefore

$$\underline{\tau_{max} = \frac{Q}{y_{ct}\, b}} \qquad 2\text{-}85$$

In a rectangular section, one may assume $y_{ct} = 0.87$ d. So that

$$\underline{\tau_{max} = \frac{Q}{0.87\, d\, b}} \qquad 2\text{-}85a$$

The principal inclined tensile and compressive stresses can be calculated from the relation

$$\sigma_{\frac{1}{2}} = \frac{\sigma}{2} \pm \sqrt{\frac{\sigma^2}{4} + \tau^2}$$

their inclination α with the horizontal is given by

$$\tan 2\alpha = 2\,\tau/\sigma$$

At upper fiber $\tau = 0$ and $\sigma = \sigma_c$; so that $\sigma_1 = \sigma_c$ acting at an angle $\alpha = 0$, and $\sigma_2 = 0$ acting at an angle $\alpha = 90°$.

At the neutral axis and below it $\tau = \tau_{max}$ and $\sigma = 0$; so that $\sigma_1 = + \tau_{max}$ and $\sigma_2 = - \tau_{max}$ acting at $\alpha = 45°$ and $135°$.

This means that between the neutral axis and the tension reinforcement the element will be subject to simple shear causing principal tensile and compressive stresses acting in diagonal directions.

The principal diagonal tensile stress is equal in magnitude to the vertical or horizontal shear. In reinforced concrete design, it is generally called diagonal tension or diagonal shear and can be calculated from the relation 2-85 .

The diagonal tension acts in a direction parallel to the resultant of the shear stresses acting on a cubic element at the position under consideration as shown in Fig. 2-32.

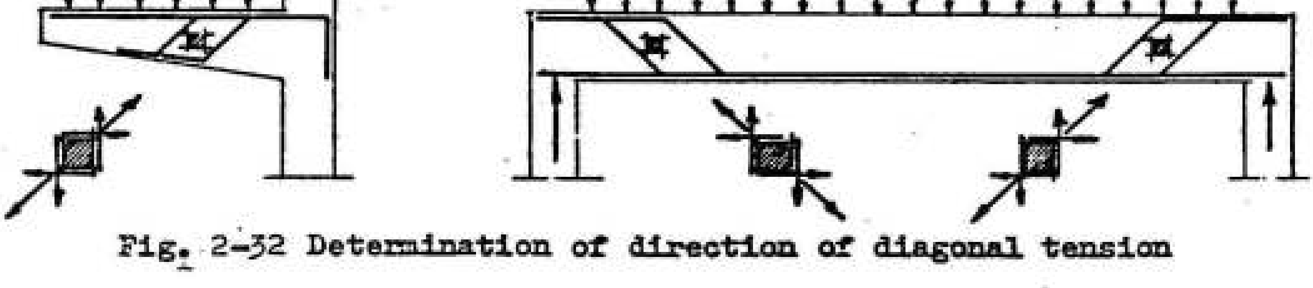


Fig. 2-32 Determination of direction of diagonal tension

For beams of variable depth, the shear stresses can be calculated in the following manner: Fig. 2-33

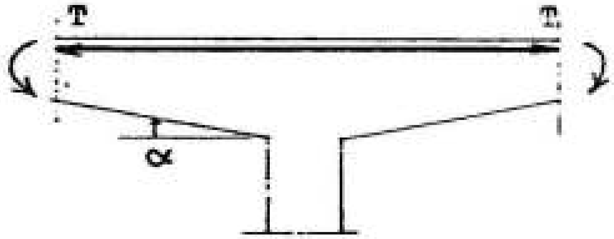


Fig. 2-33 Beams of variable depth

It has been proved that:

$\tau_{max}\, b\, dx = dT$ or

$\tau_{max} = \frac{1}{b} \frac{dT}{dx}$ and $T = \frac{M}{y_{ct}}$

M and y_{ct} being not constant, hence, we get:

$$dT = \frac{y_{ct}\, dM - M\, dy_{ct}}{y_{ct}^2} \quad \text{and} \quad \frac{dT}{dx} = \frac{1}{y_{ct}} \frac{dM}{dx} - \frac{M}{y_{ct}^2} \frac{dy_{ct}}{dx}$$

but

$$\frac{dM}{dx} = Q \quad \text{and} \quad \frac{dy_{ct}}{dx} = \frac{dd}{dx} = \tan\alpha$$

so that

$$\frac{dT}{dx} = \frac{Q}{y_{ct}} - \frac{M}{y_{ct}^2} \tan\alpha = \frac{1}{y_{ct}} \left(Q - \frac{M \tan\alpha}{y_{ct}}\right) = \frac{Q_{red}}{y_{ct}}$$

where

$$Q_{red} = Q - \frac{M}{y_{ct}} \tan\alpha \qquad 2\text{-}86$$

It is supposed in this equation that y_{ct} (and d) increases with the increase of M; if it happens that the depth decreases with the increase of M, the negative sign must be replaced by a positive one. Fig. 2-34. Hence

$$\underline{\tau_{max} = \frac{Q_{red}}{b\, y_{ct}}} \qquad 2\text{-}87$$

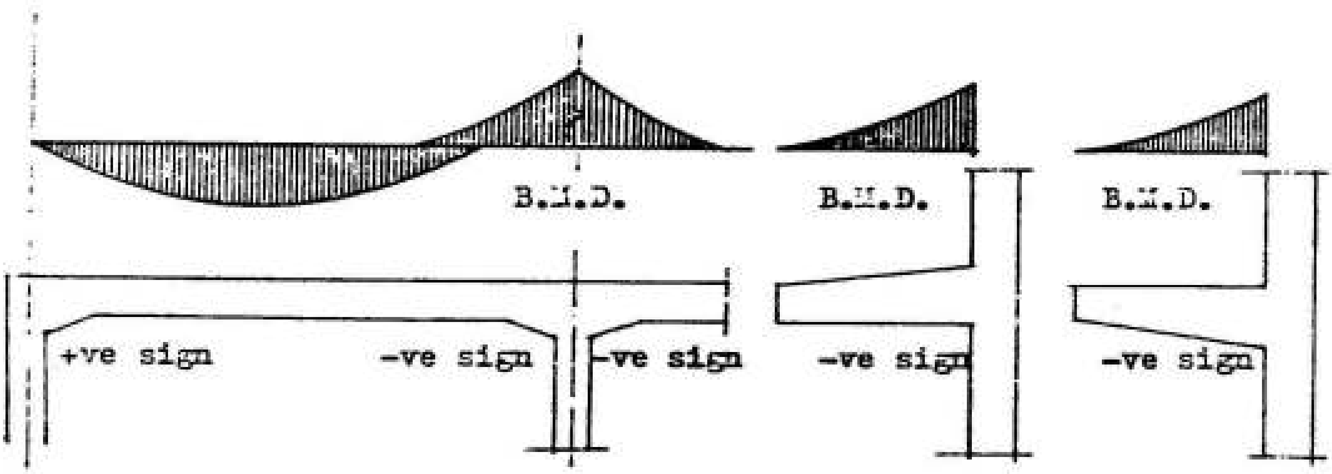


Fig. 2-34 Sign of reduced shearing force

c) Formation of cracks and results of tests.
Behavior of diagonally cracked beams.

The behavior of diagonally cracked beams for different ratios of beding moment and shearing force and the results of extensive tests carried out in the USA have been illustrated in "Design of Concrete Structures" by Winter, Urquhart, O'Rourke and Nilson as follows:

The diagonal tension σ_1 depends on both σ and τ or on M and Q. Where Q is big and M small there will be little flexural cracking, if any, prior to the development of a diagonal tension crack. The average shear stress prior to crack formation is

$$\tau = Q / b\,d \qquad 2\text{-}88$$

If σ is small, the diagonal tension stresses are inclined at $\sim 45°$ and are numerically equal to τ with a maximum value at the neutral axis . Consequently, diagonal tension cracks form mostly at or near the neutral axis and are expected to form when the diagonal tension stress at the vicinity of the neutral axis becomes equal to the tensile strength σ_t of concrete.i.e., in this case (big Q and small M), diagonal tension cracks form at an average critical shear stress τ_{cr} of about

$$\tau_{cr} = Q_{cr} / b\,d = \sigma_t \qquad 2\text{-}89$$

where Q_{cr} is that shear force at which the formation of the crack was observed.

Where Q and M have big values, flexural tension cracks form first in a well proportioned and reinforced beam. The width and length of such cracks is well controlled and kept small by the presence of longitudinal reinforcements . However, when the diagonal tension stress at the upper end of one or more of these cracks exceeds the tensile strength of the concrete, the crack bends in a diagonal direction and continues to grow in length and width. At the instant at which a diagonal tension crack of this type develops, the average shear stress is larger than that given by equation 2-88 because the preexisting tension crack has reduced the area of the uncracked concrete which is available to resist shear to a value smaller than that of the uncracked area b d used in equation 2-88. Furthermore, σ combines with τ to further increase σ_1. No way has been found to calculate reliable values of the diagonal tension stress under these conditions.

Tests have shown that in the presence of large moments (for which adequate longitudinal reinforcement has been provided), the nominal shear stress at which diagonal tension cracks form and propagate is $\sim$ half the value given by equation 2-89, i.e.

$$\tau_{cr} = Q_{cr} / b\, d = 0.5\ f_t \qquad 2\text{-}90$$

It is evident, then, that the shear at which diagonal cracks develop depends on the ratio of shearing force to bending moment, Q/M, or in nondimensional terms, on Qd/M. Equation 2-89 gives the cracking shear for very large values of Qd/M, and equation 2-90 for very small values.

Apart from this influence of Qd/M, it has been found that increasing amounts of tension reinforcements have a benefitial affect in that they increase the shear at which diagonal cracks develop. This is so because larger amounts of longitudinal steel result in smaller and narrower flexural tension cracks prior to the formation of diagonal cracking, leaving a larger area of uncracked concrete available to resist shear.

Behavior of diagonally cracked reinforced concrete beams.

In beams with longitudinal reinforcement and without web reinforcement, flexural cracks are permitted and are in no way detrimental to the strength of the member. If members without shear reinforcement are used, the diagonal cracks that are liable to develop are much more decisive in subsequent performance and strength of the beam than the flexural cracks. Two types of behavior have been observed:

(1) In shallow beams with span to depth ratios > 8, the diagonal crack once formed, spreads either immediately or at only slightly higher load, traversing the entire beam from the tension reinforcement to the compression face, splitting it into two and failing the beam in a sudden mode.

(2) In deeper beams, the diagonal crack, once formed, spreads toward and partially into the compression zone but stops short of penetrating to the compression face. No sudden collapse occurs and the failure load is much higher than the previous case as shown in Fig. 2-35.

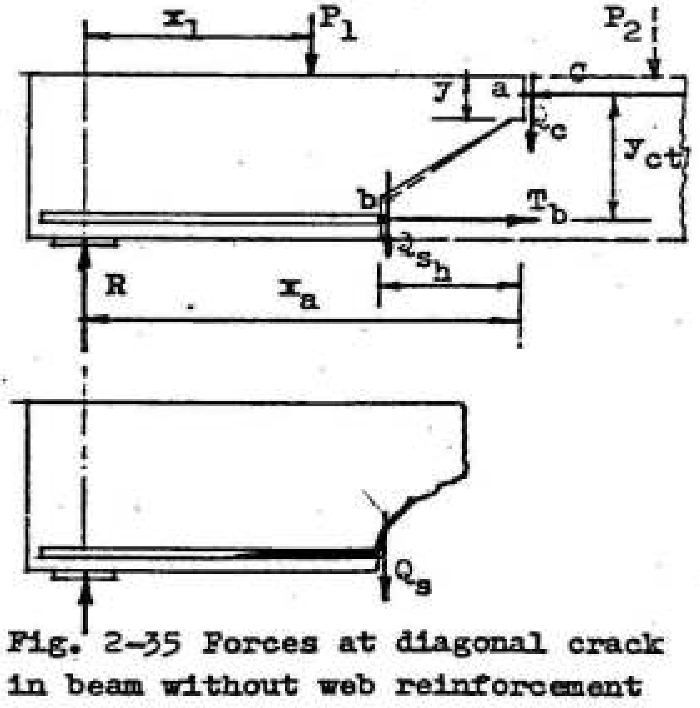


Fig. 2-35 Forces at diagonal crack in beam without web reinforcement

Considering the part to the left of the crack, we get:

External shear $Q_{ext} = R - P_1$

Internal shear $Q_{int} = Q_c + Q_s$

But external shear must be equal to internal shear; so that

$$Q_{ext} = Q_{int}$$

and therefore

Shear resisted by concrete: $Q_c = Q_{int} - Q_s = Q_{ext} - Q_s$

Q_s is very small as it is resisted by cover only . It often causes splitting of the concrete along the tension reinforcements as shown in Fig. 2-35. Accordingly, we may write:

$$Q_c = Q_{ext} \qquad 2\text{-}91$$

Further, we have:

$$(M_{ext})_a = R\, x_a - P_1(x_a - x_1)$$

and

$$(M_{int})_a = T_b\, y_{ct} + Q_{sb}\, h$$

But $M_{ext} = M_{int}$ and Q_{sb} is small that it may be neglected; therefore:

$$T_b = (\, M_{ext}\,)_a \,/\, y_{ct} \qquad 2\text{-}92.$$

The formation of the diagonal crack, then, is seen to produce the following redistribution of internal forces and stresses:

1) In the vertical section through a the average shear stress before crack formation was Q_{ext} / b d. After crack formation that same shear force is almost entirely resisted by the much smaller area b y of the remaining uncracked concrete. Hence, the average shear stress in the concrete has now increased to: Q_{ext} / b y

2) The diagonal crack usually rises above the neutral axis and traverses some part of the compression zone before it is arrested by the compressive stresses. Consequently, the compression force C also acts on an area b y smaller than that on which it acted before the crack was formed, i.e., formation of the crack has increased the compression stresses in the remaining uncracked concrete.

3) Prior to diagonal cracking, the tension force in the steel at point b was caused by and proportional to the bending moment in a vertical section through the same point b. As a consequence of the diagonal crack, however, the tension in the steel at b is now caused by and is proportional to the bending moment at a. Since the moment at a is evidently larger than at b, formation of the crack has caused a sudden increase in the steel stress at b.

4) The dowel action of Q_s causes bearing and tension stresses which contribute to the tendency of the concrete to split along the bar to-

ward the nearby support.

If the two materials are capable of resisting these increased stresses, equilibrium will establish itself after internal redistribution, and further load can be applied before failure occurs. Such failure can then develop in various ways:

1) If steel at b is sufficient only to resist moment at that section, the increase of the steel force , in 3) above, will cause the steel to yield because of the larger moment at a and thus failing the beam.

2) If the beam is properly designed to prevent this occurance, the concrete at the head of the crack may crush due to the high compressive and shear stresses.

3) Splitting along the tension reinforcement weakens bond between steel and concrete to such a degree that the reinforcement may pull loose.

In beams with no special shear reinforcement, the design is generally based on the shear force Q_{cr} or the shear stress τ_{cr} at which a diagonal crack must be expected. (Refer to equation 2-90).

Reinforced concrete beams with web reinforcements.

Economy of design demands, in most cases, that a flexural member be capable of developing its full moment capacity rather than having its strength limited by premature shear failure. Sudden shear failures are not desirable. If the available shear strength as given by equation 2-90 (or as specified by a code) is not adequate, web reinforcement is used to increase this strength.

Web reinforcement may either be consisting of vertical (or inclined) stirrups or bent bars as shown in Fig. 2-36.

Fig. 2-36 Web reinforcements of reinforced concrete beams

Measurements have shown that such reinforcement is free from stress prior to crack formation. After crack formation, web reinforcements affect the shear resistance of a beam in three seperate ways:
1) Part of the shear force is resisted by the web reinforcements which traverse a particular crack.
2) The presence of these bars restricts growth of diagonal cracks and reduces their penetration in the compression zone; hence increas-

ing the uncracked concrete at the head of the crack to resist the combined action of shear and compression.

3) Stirrups tie the longitudinal reinforcements with the bulk of concrete; this provides some measure of restraint against the splitting of concrete along the longitudinal reinforcement and increases the dowel action.

The behavior of a beam after the formation of diagonal cracks is quite complex and depends on crack configuration (length, inclination, and location of the main or critical cracks). Present design practice are based partly on rational analysis, partly on test evidence, and partly on longtime successful experience. Research has furnished important experimental data for recent improvements in shear design and analysis but has not yet resulted in a completely consistent rational analysis of shear behavior.

Beams with vertical stirrups:

The web reinforcement being ineffective in the balanced beam, the magnitude of the shear force or stress which causes cracking to occur is the same as in a beam without web reinforcement (as is given by equation 2-90). Equilibrium of the forces in the cracked section in the vertical direction requires:

$$Q_c + \sum_n A_{st}\,\sigma_{st} + Q_s = Q_{ext}$$

where n is the number of stirrups traversing a crack.

Assuming s = spacing between stirrups. Then (Fig. 2-37)

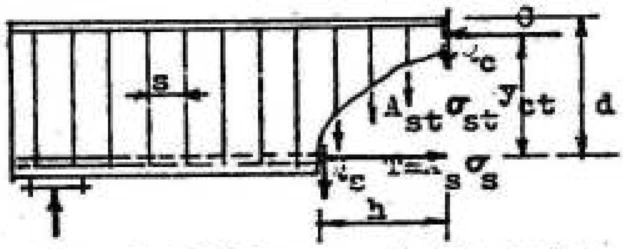


Fig. 2-37 Forces at diagonal crack in beam with vert. stir.

$$n = h / s$$

At failure $\sigma_{st} = f_y$. Neglecting further Q_s, we get:

$$Q_u = Q_c + n\,A_{st}\,f_y$$

Q_c may be neglected. The ACI-code assumes it the same as that which caused the diagonal crack to develop (given by equation 2-90) i.e.,

$$Q_c = Q_{cr}$$

Assuming further h = d, then

$$Q_u = Q_{cr} + \frac{A_{st}\,f_y\,d}{s}$$

and

$$\tau_u = \frac{Q_u}{b\,d} = \tau_{cr} + \frac{A_s\,f_y}{b\,s} \qquad \text{2-93}$$

Beams with inclined stirrups.

Proceeding in the same way as in the previous case, it is possible to prove that: (Fig. 2-38)

The ultimate shearing force and the ultimate diagonal tension are given by the relations:

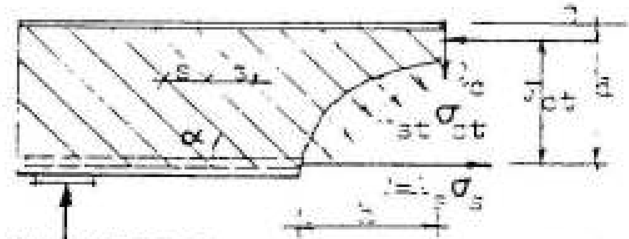


Fig. 2-38 Forces at diag. crack in beam with inclined web rfmt.

$$Q_u = Q_{cr} + \frac{A_{st}\ f_y\ d}{s} (\sin\alpha + \cos\alpha)$$

and

$$\tau_u = \frac{Q_u}{b\ d} = \tau_{cr} + \frac{A_{st}\ f_y}{b\ s} (\sin\alpha + \cos\alpha) \qquad 2\text{-}94$$

For $\alpha = 45°$, we get

$$Q_u = Q_{cr} + \frac{A_{st}\ f_y\ d}{s\ /\sqrt{2}}$$

and

$$\tau_u = \tau_{cr} + \frac{A_{st}\ f_y}{b\ s\ /\sqrt{2}} \qquad 2\text{-}94a$$

τ_{cr} may be assumed equal to the shear resisted by the compression zone or equal to zero in preservative countries in which case, the web reinforcement will be over-dimensioned, or any convenient value between these two limits. Leonhardt* proposes that $\tau_{cr} = f_{cp}/20$ for simple beams and $= f_{cp}/30$ for continuous beams.

Accordingly, we get:

a) Under working loads:

In case of vertical stirrups: $\tau = \tau_c + \dfrac{A_s\ \sigma_s}{b\ s}$ 2-95

In case of bent bars or inclined stirrups at an angle of 45°: $\tau = \tau_c + \dfrac{A_s\ \sigma_s}{b\ s\ \sqrt{2}}$ 2-96

in which

$$\tau_c = 0 \quad \text{or not more than} \quad 0.25\sqrt{f_{cp}}$$

b) At ultimate strength:

In case of vertical stirrups: $\tau_u = \tau_{cr} + \dfrac{A_s\ f_y}{b\ s}$ 2-97

In case of bent bars or stirrups at 45°: $\tau_u = \tau_{cr} + \dfrac{A_s\ f_y}{b\ s\ \sqrt{2}}$ 2-98

in which

$$\tau_{cr} = 0 \quad \text{or not more than} \quad 0.40\sqrt{f_{cp}}$$

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* F. Leonhardt "Über die Kunst des Bewehrens von Stahlbetontragwerke" Beton und Stahlbetonbau, Heft 8 und 9 / 1965.

2.7- ECCENTRIC COMPRESSION

The analysis of a reinforced concrete section subject to eccentric compression varies according to the eccentricity and stage of loading. We give in the following, the analysis of the normal, shear and principal stresses in the different stages:

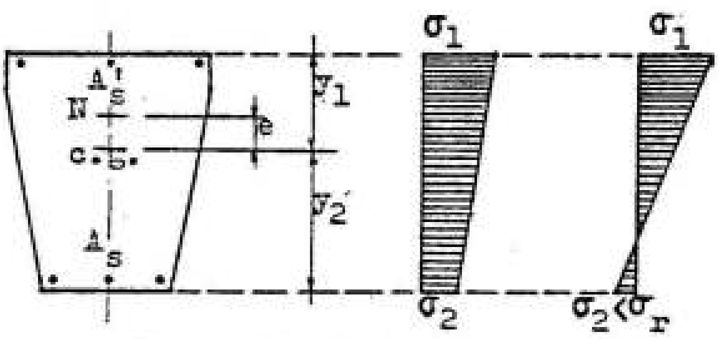
a) Elastic analysis. Noncracked linear stage in eccentric compression with small eccentricity

The following analysis applies to sections subject to eccentric compression with small eccentricity and the section is not cracked. This will be the case if the normal force N lies inside the core of the section because the whole section will be subject to compression stresses, or if N lies outside the core of the section but not far from it. In this new case, the section will be subject to small tensile stresses; if these stresses are smaller than the rupture strength of concrete, the section will not be cracked.

In both cases, the whole section can be assumed as statically acting. The normal stresses can be computed from the equations of Navier, namely: (Fig. 2-39)

$$\sigma_1 = -\frac{N}{A_v} - \frac{N\,e\,y_1}{I_v} \quad \text{and} \quad \sigma_2 = -\frac{N}{A_v} + \frac{N\,e\,y_2}{I_v} \qquad 2\text{-}99$$

Fig. 2-39



Stresses in a section subject to eccentric comp. with small ecc.

Where

A_v = The virtual area of the section = $A_c + n\,(A_s + A'_s)$

I_v = The moment of inertia of the virtual area about the c.g.-axis of the section

N = Normal force acting on the section

e = eccentricity of normal force from c.g.-axis

y_1 and y_2 are the distances of the extreme fibers from the c.g.-axis.

Application to rectangular sections

A rectangular section can be treated according to the above method if N lies inside the middle two thirds of the section; in which case, we have:

for $e = t/3$ $\sigma_2 = -\sigma_1/3$

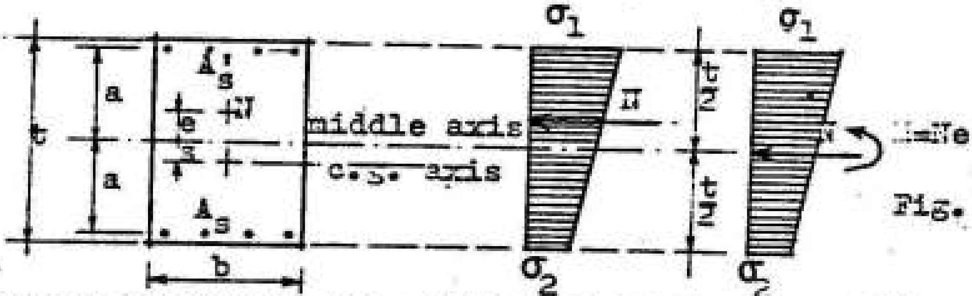


Fig. 2-40

Stresses in a rect. sec. subject to ecc. comp. with small ecc.

Applying equation 2-99 to the rectangular section shown in Fig. 2-40, we get

$$\sigma_1 = \frac{N}{A_v} + \frac{N\,(e + s)\,(\frac{t}{2} + s)}{I_v} \quad \text{and} \quad \sigma_2 = \frac{N}{A_v} - \frac{N\,(e + s)\,(\frac{t}{2} - s)}{I_v}$$

in which

$$s = \frac{S_v}{A_v} = \frac{n\,(A_s - A'_s)\,a}{b\,t + n\,(A_s + A'_s)}$$

where

S_v = statical moment of virtual area about the middle axis

Assuming: $\mu = \frac{A_s}{b\,t}$, $\mu' = \frac{A'_s}{b\,t}$, $\varsigma = \frac{s}{t}$, $\lambda = \frac{a}{t}$ and $\alpha = \frac{\mu'}{\mu}$, we get

$$A_v = A_c + n\,(A_s + A'_s) = \left[1 + n\,\mu\,(1 + \alpha)\right] b\,t$$

$$S_v = n\,(A_s - A'_s)\,a = n\,\mu\,\lambda\,(1 - \alpha)\,b\,t^2$$

therefore

$$\varsigma = \frac{S_v}{A_v\,t} = \frac{n\,\mu\,\lambda\,(1 - \alpha)}{1 + n\,\mu\,(1 + \alpha)}$$

and

$$I_v = \frac{b\,t^3}{12} + b\,t\,s^2 + n\,A_s\,(a - s)^2 + n\,A'_s\,(a + s)^2$$

$$= b\,t^3\left[\frac{1}{12} + \varsigma^2 + n\,\mu\,(\lambda - \varsigma)^2 + n\,\mu\,\alpha\,(\lambda + \varsigma)^2\right]$$

In reinforced concrete design, the bending moments are generally computed with respect to the middle axis; so that $M = N\,e$. The stresses can accordingly be given in the form:

$$\sigma_1 = \frac{N}{A_v} + N\,(e + s)\,\frac{\frac{t}{2} + s}{I_v} = N\left[\frac{1}{A_v} + \frac{s\,(\frac{t}{2} + s)}{I_v}\right] + M\,\frac{\frac{t}{2} + s}{I_v}$$

$$\sigma_2 = \frac{N}{A_v} - N\,(e + s)\,\frac{\frac{t}{2} - s}{I_v} = N\left[\frac{1}{A_v} - \frac{s\,(\frac{t}{2} - s)}{I_v}\right] - \ldots\frac{\frac{t}{2} - s}{A_v}$$

Substituting for A_v, I_v and s the values given in the above relations, we get:

$$\left.\begin{aligned} \sigma_1 &= c_1\,\frac{N}{b\,t} + c_1'\,\frac{M}{b\,t^2} \\ &\text{and} \\ \sigma_2 &= c_2\,\frac{N}{b\,t} - c_2'\,\frac{M}{b\,t^2} \end{aligned}\right\} \qquad 2\text{-}100$$

The values of c_1, c_1', c_2 and c_2'* are given in sheet 5 for μ = 0 to to 2%, μ' = 0 to 1μ, a = 0.4 t and n = 15.

b) Elastic analysis. Cracked linear stage in eccentric compression with big eccentricity

In case of big eccentricity, the section will generally be subject to high tensile stresses, and therefore, the assumptions made in case of simple bending will be valid. Fig. 2-41.

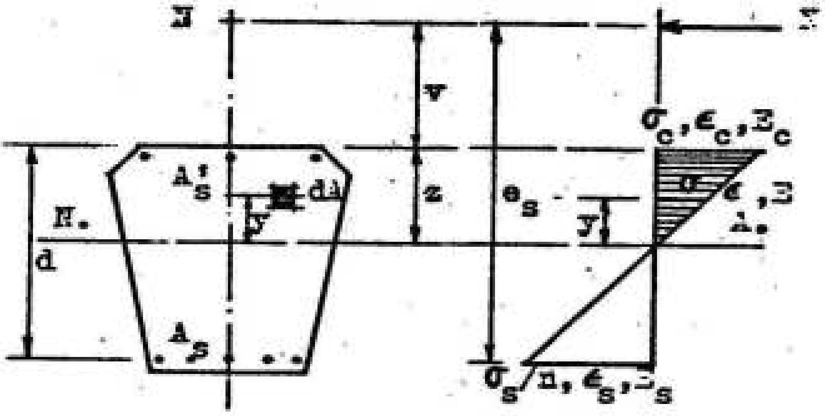


Fig. 2-41

Stresses in a section subject to ecc. comp. with big ecc.

Accordingly, we have

1) The sum of the stresses must be equal to the external normal force, or

$$\int_A \sigma\, dA = N \qquad \text{(a)}$$

2) The moment of the stresses about the neutral axis is equal to the moment of the external normal force about the same axis; or

$$\int_A \sigma\, y\, dA = N\,(v + z) \qquad \text{(b)}$$

* For c_1, c_1', c_2 and c_2' refer to M. Ritter 'Tabellen zur Berechnung von Eisenbetonkonstruktionen'. Verlag Leemann & Co. Zurich

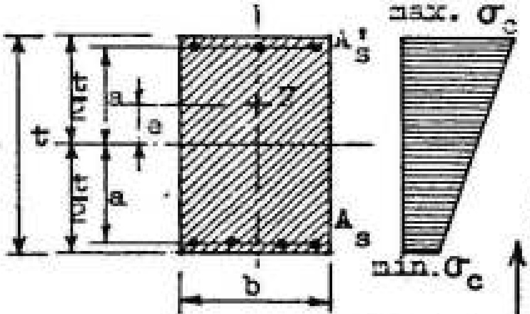
SHEET 5

ECCENTRIC COMPRESSION WITH SMALL ECCENTRICITY

MAXIMUM STRESSES IN RECTANGULAR SECTIONS

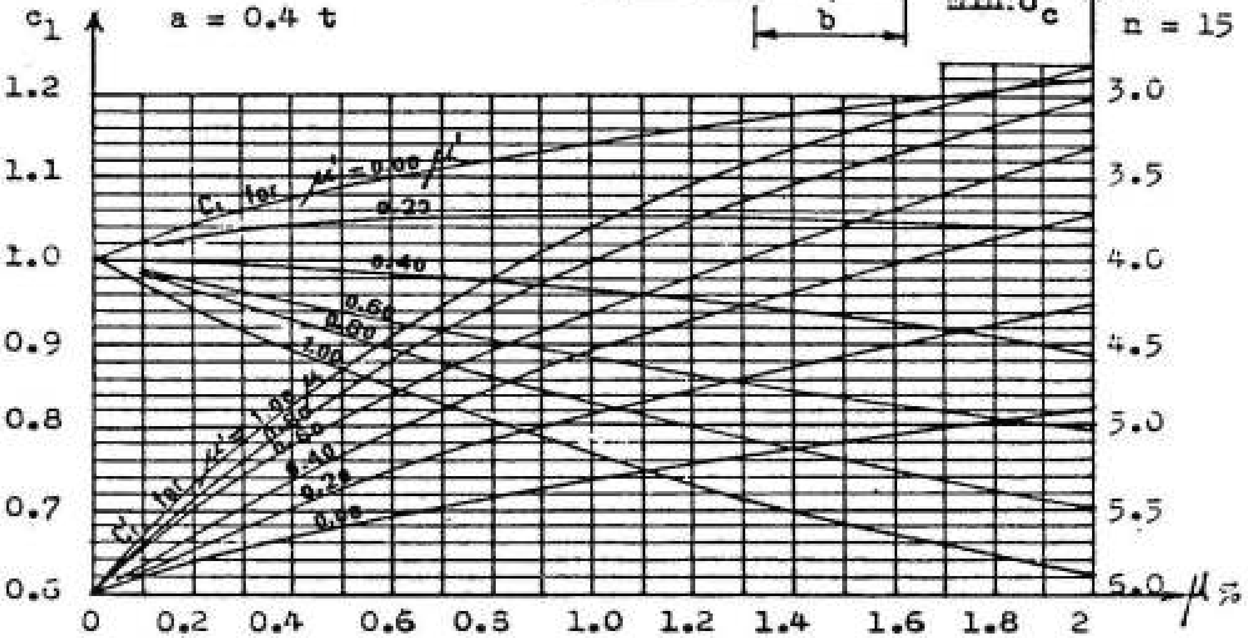
$$\text{Max}\ \sigma_c = c_1 \frac{N}{b\,t} + c_1' \frac{M}{b\,t^2}$$

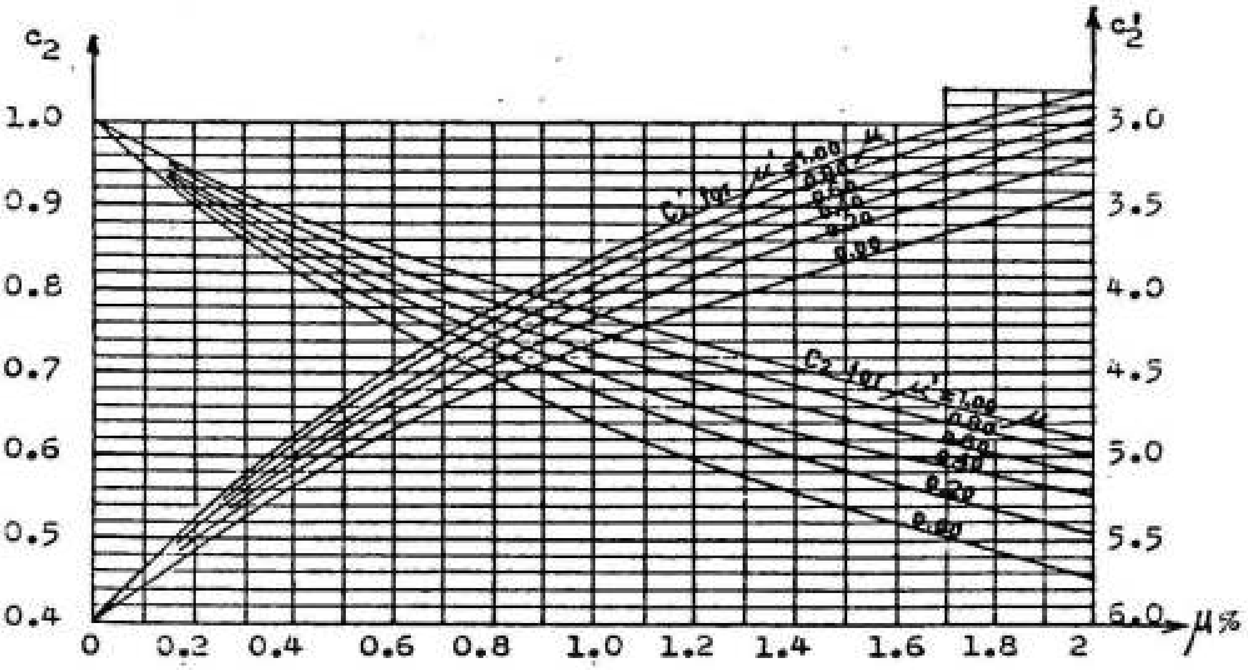
$$\text{Min}\ \sigma_c = c_2 \frac{N}{b\,t} - c_2' \frac{M}{b\,t^2}$$



$$\mu = \frac{A_s}{bt}$$

$$\mu' = \frac{A_s'}{bt}$$





3) The assumption of Bernoulli being valid, we get:

$$\frac{\epsilon}{\epsilon_c} = \frac{y}{z} \quad \text{or} \quad \frac{E_c}{\sigma_c} \cdot \frac{\sigma}{E} = \frac{y}{z} \quad \text{or} \quad \sigma = \frac{E}{E_c}\,\sigma_c\,\frac{y}{z} \qquad \text{(c)}$$

Equations a and c give:

$$N = \int_A \frac{E}{E_c}\,\sigma_c\,\frac{y}{z}\,dA = \frac{\sigma_c}{z}\int_A \frac{E}{E_c}\,y\,dA = \frac{\sigma_c}{z}\,S_{nv} \qquad 2\text{-}101$$

in which

S_{nv} = statical moment of virtual area about neutral axis

Equations b and c give:

$$N\,(v + z) = \int_A \frac{E}{E_c}\,\sigma_c\,\frac{y^2}{z}\,dA = \frac{\sigma_c}{z}\int_A \frac{E}{E_c}\,y^2\,dA = \frac{\sigma_c}{z}\,I_{nv} \qquad 2\text{-}102$$

in which

I_{nv} = moment of inertia of virtual area about neutral axis.

Dividing equation 2-102 by 2-101, we get:

$$v + z = I_{nv} / S_{nv} \qquad 2\text{-}103$$

Having determined the position of the neutral axis, the stresses can be computed by taking moments of the external and internal forces about the tension and compression reinforcements.

<u>Application to rectangular sections</u>

Given N, v, A_s, A_s', d and b. Required σ_c and σ_s. Refer to Fig. 2-42.

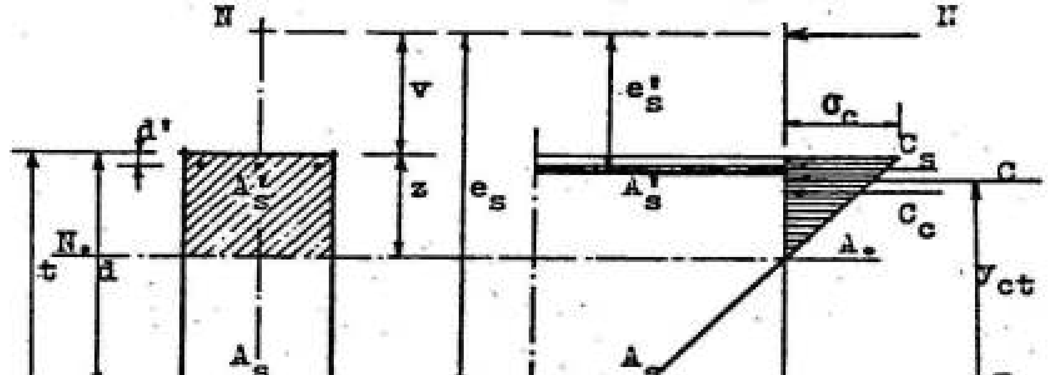


Fig. 2-42 Stresses in a rectangular section subject to eccentric compression with big eccentricity.

The position of the neutral axis can be directly calculated from equation 2-103. Thus

$$v + z = \frac{I_{nv}}{S_{nv}} = \frac{\dfrac{b\,z^3}{3} + n\,A_s'\,(z - d')^2 + n\,A_s\,(d - z)^2}{\dfrac{b\,z^2}{2} + n\,A_s'\,(z - d') - n\,A_s\,(d - z)} \qquad 2\text{-}103$$

Assuming $v + d = e_s$ and $v + d' = e'_s$ we get:

$$z^3 + z^2.3(e_s - d) + z.\frac{6n}{b}(A_s e_s + A'_s e'_s) - \frac{6n}{b}(A_s d e_s + A'_s d' e'_s) = 0 \qquad \text{2-103b}$$

This equation gives z and can be solved by trial.

Taking moments about the tension steel, we get:

$$\sigma_c \, b \, \frac{z}{2} \, (d - \frac{z}{3}) + A'_s \, \sigma'_s \, (d - d') = N \, e_s = M_s$$

but

$$\sigma'_s = n \, \sigma_c \, \frac{z - d'}{z}$$

therefore

$$\sigma_c = \frac{M_s \, z}{\frac{b \, z^2}{2} \, (d - \frac{z}{3}) + n \, A'_s \, (z - d') \, (d - d')} \qquad \text{2-104}$$

σ_s can be determined from the relation:

$$\sigma_s = n \, \sigma_c \, \frac{d - z}{z} \qquad \text{2-105}$$

Assuming $\mu = \frac{A_s}{bd}$, $\mu' = \frac{A'_s}{bd}$, $\alpha = \frac{\mu'}{\mu}$, $\zeta = \frac{z}{d}$, and $\beta = \frac{d'}{d}$,

the equation of the neutral axis can be given in the form:

$$\zeta^3 + \zeta^2.3(\frac{e_s}{d} - 1) + \zeta.6n\mu(\frac{e_s}{d} + \alpha\frac{e'_s}{d}) - 6n\mu(\frac{e_s}{d} + \alpha\beta\frac{e'_s}{d}) = 0 \qquad \text{2-103c}$$

The stresses σ_c and σ_s can, according to equations 2-104 and 2-105, be given in the form:

$$\underline{\sigma_c = c_1 \frac{M_s}{b \, d^2}} \qquad \text{and} \qquad \underline{\sigma_s = c_2 \frac{M_s}{b \, d^2}} \qquad \text{2-106}$$

where

$$c_1 = \frac{\zeta}{\frac{\zeta^2}{2} \, (1 - \frac{\zeta}{3}) + n \, \mu \, \alpha \, (\zeta - \beta) \, (1 - \beta)}$$

and

$$c_2 = \frac{n \, (1 - \zeta)}{\frac{\zeta^2}{2} \, (1 - \frac{\zeta}{3}) + n \, \mu \, \alpha \, (\zeta - \beta) \, (1 - \beta)}$$

Values of c_1 and c_2 are given in sheets 6, 7 and 8 for μ' varying from 0 to μ and $e_s/d = 0.8$ (begin of big eccentricity) to ∞ (simple bending), assuming $n = 15$ and $\beta = 0.1$.

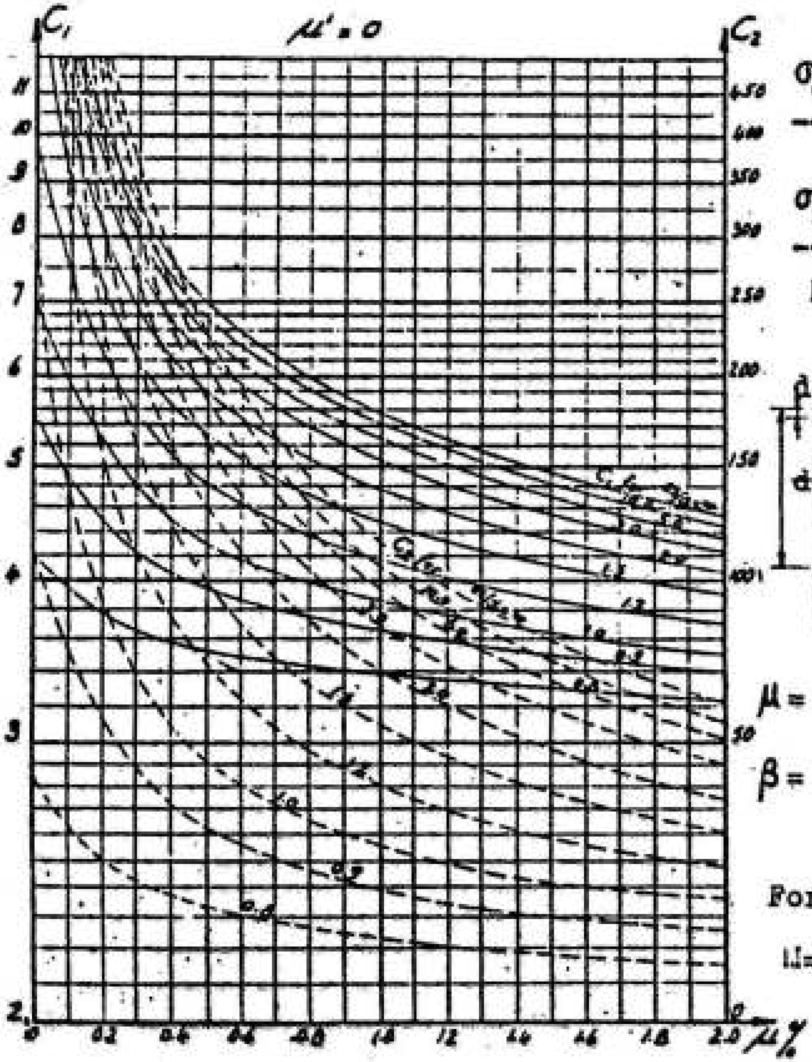
Stresses in sections subject to simple bending can be determined from these curves. In this case $M_s = M$ and $e_s/d = \infty$.

SHEET 6

ECCENTRIC COMPRESSION WITH BIG ECCENTRICITY

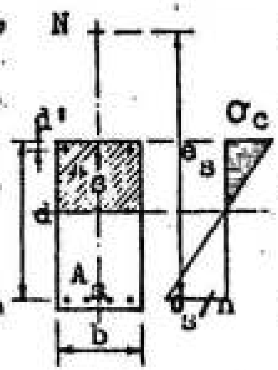
DETERMINATION OF MAXIMUM STRESSES IN RECTANGULAR SECTIONS

n = 15



$$\sigma_c = c_1 \frac{M_s}{b\,d^2}$$

$$\sigma_s = c_2 \frac{M_s}{b\,d^2}$$



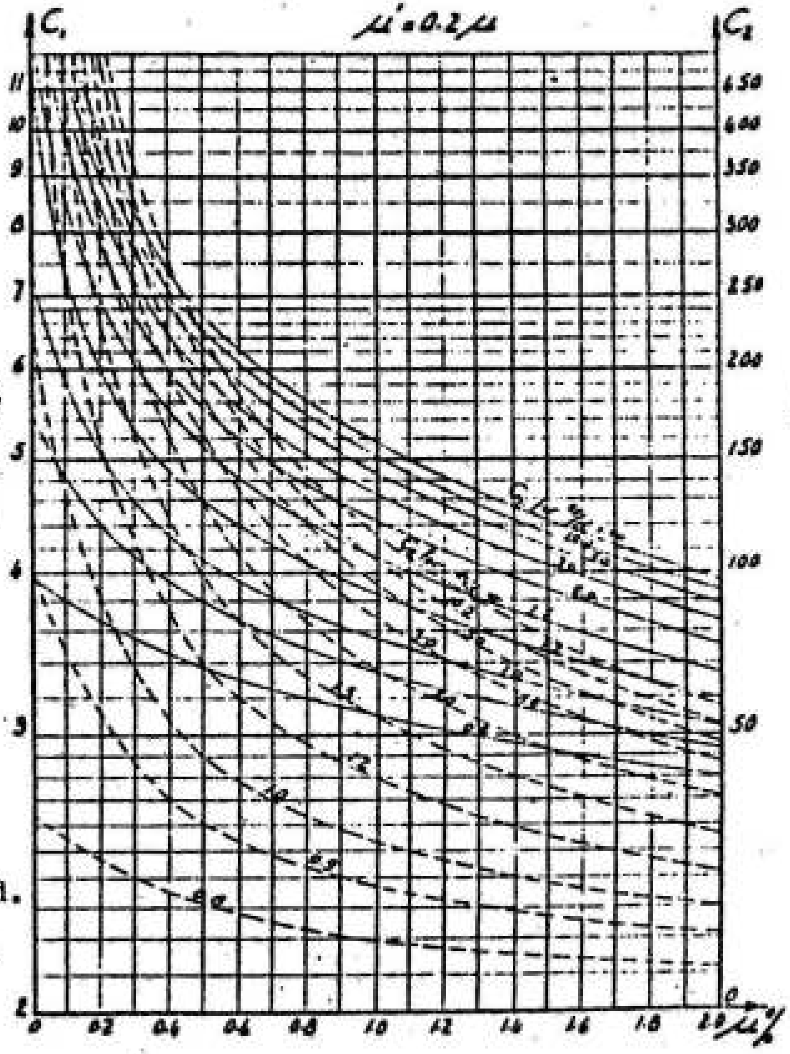
$\mu = \frac{A_s}{bd}$ $\mu' = \frac{A'_s}{bd}$

$\beta = d'/d = 0.1$

$M_s = N\,e_s$

For simple bend.

$M = M_s$ & $e_s = \infty$

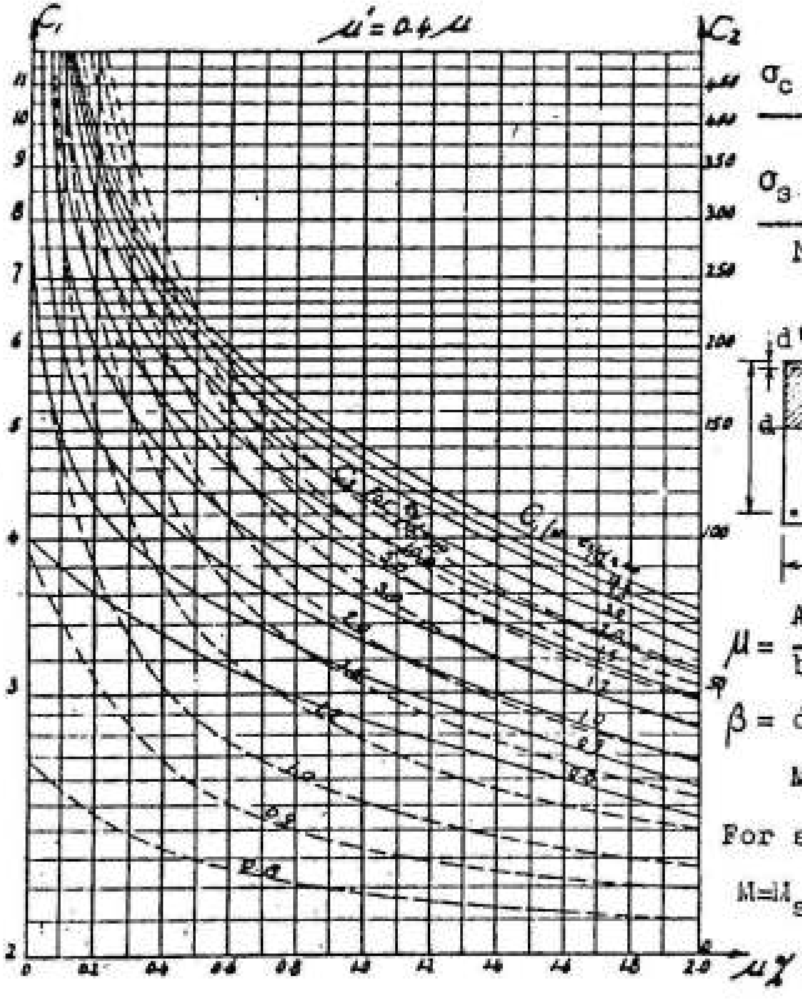


SHEET 7

ECCENTRIC COMPRESSION WITH BIG ECCENTRICITY

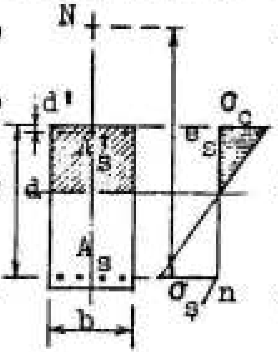
DETERMINATION OF MAXIMUM STRESSES IN RECTANGULAR SECTIONS

n = 15



$$\sigma_c = c_1 \frac{M_s}{b\ d^2}$$

$$\sigma_s = c_2 \frac{M_s}{b\ d^2}$$



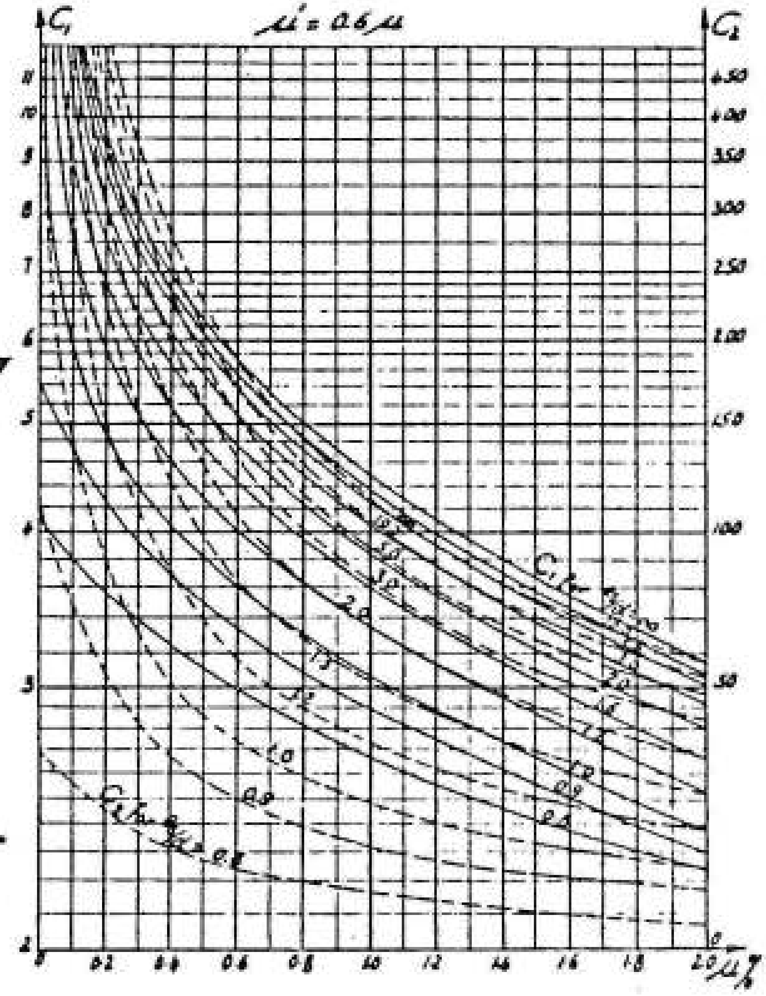
$\mu = \frac{A_s}{bd}$ $\mu' = \frac{A'_s}{bd}$

$\beta = d'/d = 0.1$

$M_s = N\ e_s$

For simple bend.

$M = M_s$ & $e_s = \infty$

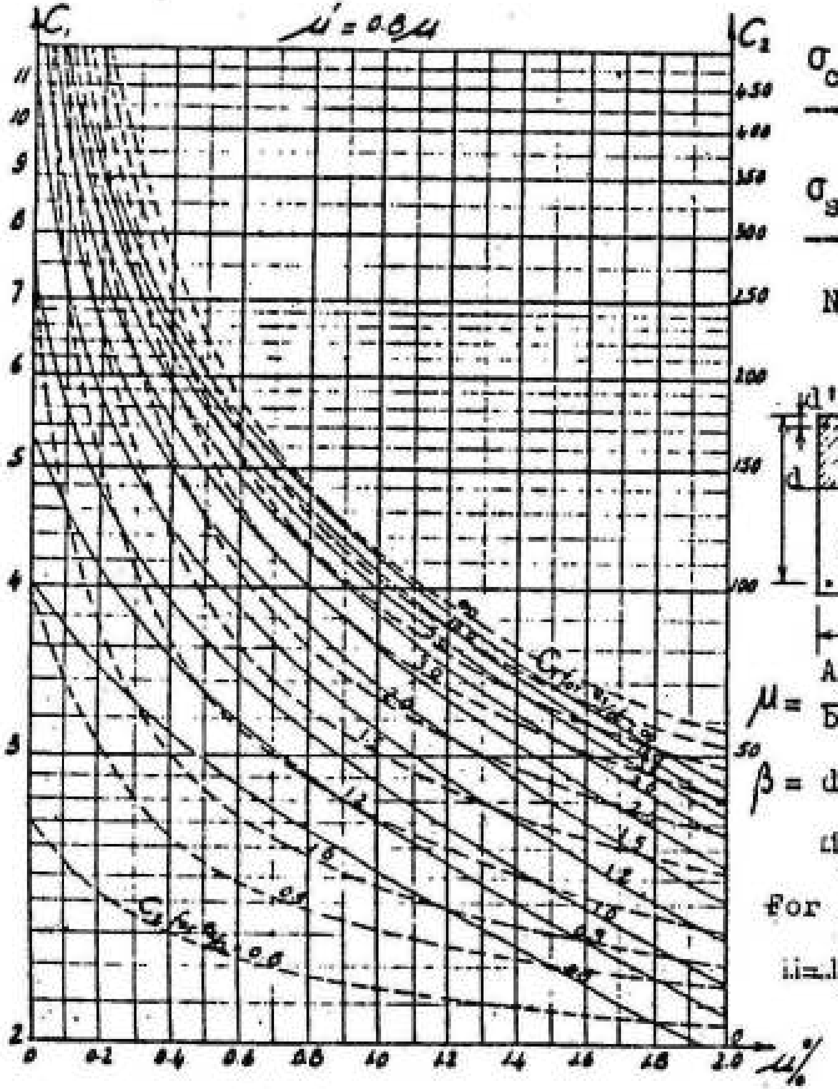


SHEET 8

ECCENTRIC COMPRESSION WITH BIG ECCENTRICITY

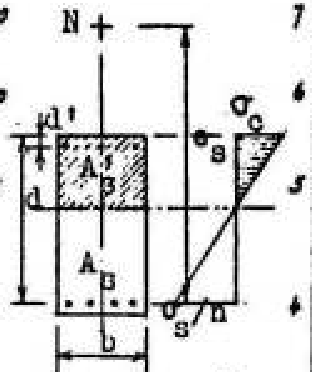
DETERMINATION OF MAXIMUM STRESSES IN RECTANGULAR SECTIONS

n = 15



$$\sigma_c = c_1 \frac{M_s}{b\, d^2}$$

$$\sigma_s = c_2 \frac{M_s}{b\, d^2}$$



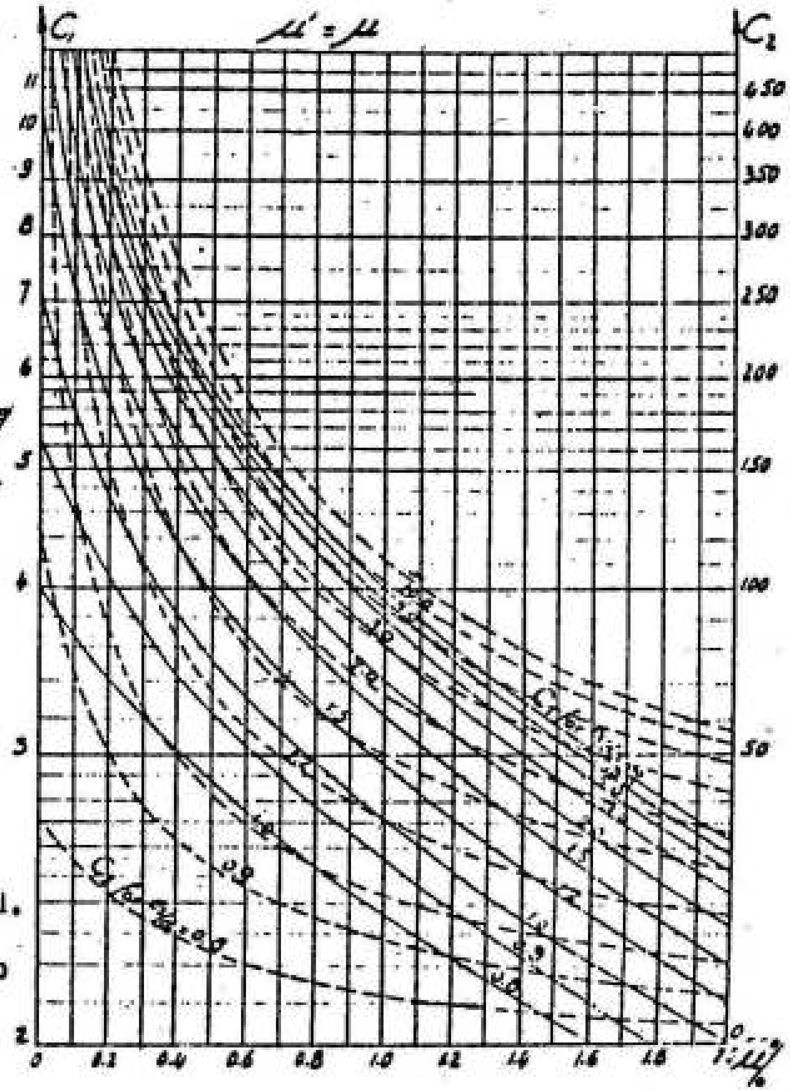
$\mu = \frac{A_s}{bd}$ $\mu' = \frac{A'_s}{bd}$

$\beta = d'/d = 0.1$

$M_s = N\, e_s$

For simple bend.

$M = M_s$ & $e_s = \infty$



Application to T-sections

When analysing the stresses in a T-section, one has to proceed as follows:

1. If the whole section is subject to compressive stresses only or compressive stresses and small allowed tensile stresses, the equation of Navier can be used.

2. If the section is subject to eccentric compression with big eccentricity and the neutral axis falls inside the flange, the stresses can be determined according to the methods, formulae and curves given for rectangular sections if the breadth of the web b is replaced by the flange-breadth B.

3. If the neutral axis falls outside the flange, the compressive stresses between bottom of flange and neutral axis may be neglected. The strésses can, in this case, be calculated as follows: Fig. 2-43.

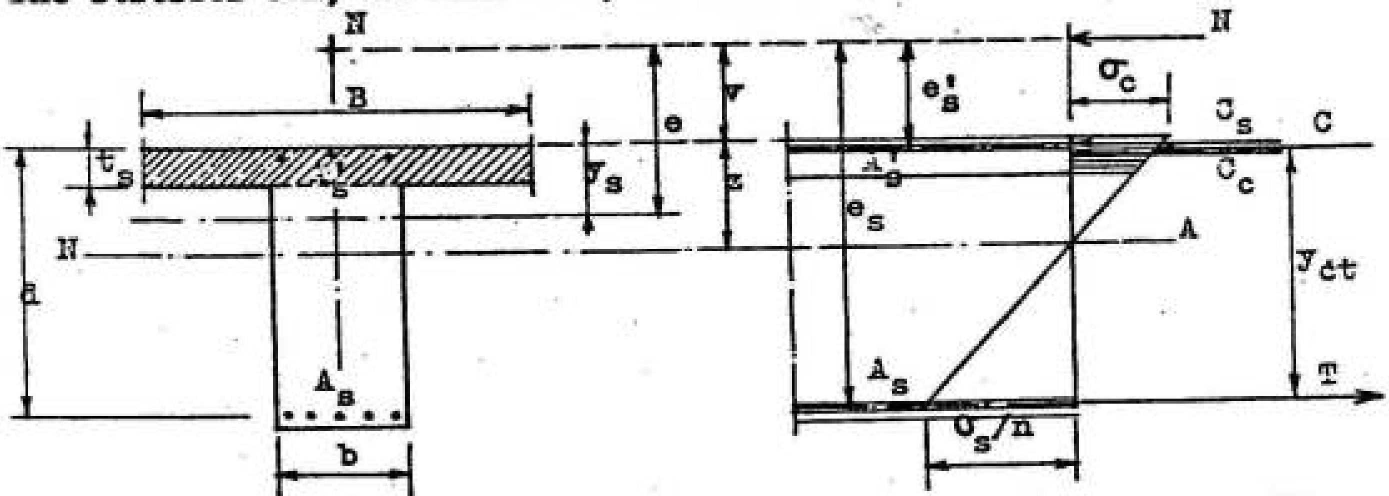


Fig. 2-43 Stresses in a T-section subject to eccentric forces with big eccentricity

The virtual area is given by:

$$A_v = B\ t_s + n\ (A_s + A'_s) \qquad 2\text{-}107$$

The center of gravity of the virtual area falls at a distance y_s from the upper surface of the compression flange. Thus

$$y_s = \frac{\frac{B}{2}\ t_s^2 + n\ (A_s\ d + A'_s\ d')}{A_v} \qquad 2\text{-}108$$

The moment of inertia of the virtual area about the c.g.-axis is given by:

$$I_{sv} = \frac{B\ t_s^2}{3} + n\ (A_s\ d^2 + A'_s\ d'^2) - A_v\ y_s^2 \qquad 2\text{-}109$$

The stresses are therefore given by:

$$\left.\begin{aligned}\sigma_c &= \frac{N}{A_v} + \frac{N\,e}{I_v}\,y_s \\ \sigma_s &= n\left[\frac{N}{A_v} - \frac{N\,e}{I_v}\,(d - y_s)\right] \\ \sigma_s' &= n\left[\frac{N}{A_v} + \frac{N\,e}{I_v}\,(y_s - d')\right]\end{aligned}\right\} \qquad 2\text{-}110$$

In case of deep T-sections, with tension reinforcements only, one can approximately assume that the center of compression falls at the center line of the flange, i.e.

$$y_{ct} = d - \frac{t_s}{2}$$

Assuming further that σ_{ca} = average concrete stress in compression-flange, we get:

By taking moments about the tension steel:

$$N\,e_s = M_s = \sigma_{ca}\,B\,t_s\,y_{ct}$$

or

$$\sigma_{ca} = \frac{M_s}{B\,t_s\,(d - t_s/2)} \qquad 2\text{-}111$$

and by taking moments about the center line of the flange:

$N\,e' = M' = \sigma_s\,A_s\,y_{ct}$ where $e' = v + t_s/2$ we get:

$$\sigma_s = \frac{M'}{A_s\,(d - t_s/2)} \qquad 2\text{-}112$$

Graphical method for determining the stresses in sections subject to eccentric compression

The method is based on the condition of equilibrium stating that the moment of the stresses about any axis is equal to the moment of the external normal force about the same axis, i.e. the moment of the stresses about an axis passing through the point of application of the external normal force N is equal to zero. Hence

$$\int \sigma\,r\,dA = 0$$

but

$$\sigma = \sigma_c\,\frac{E}{E_c}\cdot\frac{y}{z}$$

then

$$\frac{\sigma_c}{z}\int \frac{E}{E_c}\,y\,r\,dA = 0$$

but $\frac{E}{E_c}\,dA = dA_v$ then $\int y\,r\,dA_v = 0$

Assuming further $r\,dA_v = dw$, we get $\int y\,dw = 0.$

This relation means that the statical moment of the vectors w (assumed as elastic weights) about the neutral axis is equal to zero.

The method can therefore be illustrated in the following steps: Fig. 2-44.

1- Divide the concrete area into strips parallel to the neutral axis; determine the area 'a' of each strip.
2- Assume the statical moment of the areas of these strips (and n-times the areas of the steel reinforcements) about the point of application of the normal force as elastic weights acting in the centers of gravity of each strip. i.e.,

$w_1 = n\, A'_s\, r_1$ $\quad w_2 = a_2\, r_2$ $\quad w_3 = a_3\, r_3$ ...etc

$w'_1 = n\, A_{s1}\, r'_1$ $\quad w'_2 = n\, A_{s2}\, r'_2$ ...etc

3- Starting from point O, draw a force polygon for the elastic weights w in one direction, and for w' in an opposite direction.
4- Choosing the first ray through O vertical, draw a polar diagram with the polar distance H.
5- Draw the corresponding link polygon. The neutral axis passes through the point of intersection D.

The stresses can be calculated from equation 2-101. Thus

$$\sigma_c = \frac{N\,z}{S_{nv}} \qquad \text{and} \qquad \sigma_s = n\,\frac{N\,(d - z)}{S_{nv}}$$

in which

$$S_{nv} = \int_A \frac{E}{E_c}\, y\, dA = \int_A \frac{E}{E_c}\,(Z - r)\, dA = Z\int_A \frac{E}{E_c}\, dA - \int_A \frac{E}{E_c}\, r\, dA \qquad \text{or}$$

$$S_{nv} = Z\, A_v - \text{effective elastic weights } (w + w')$$

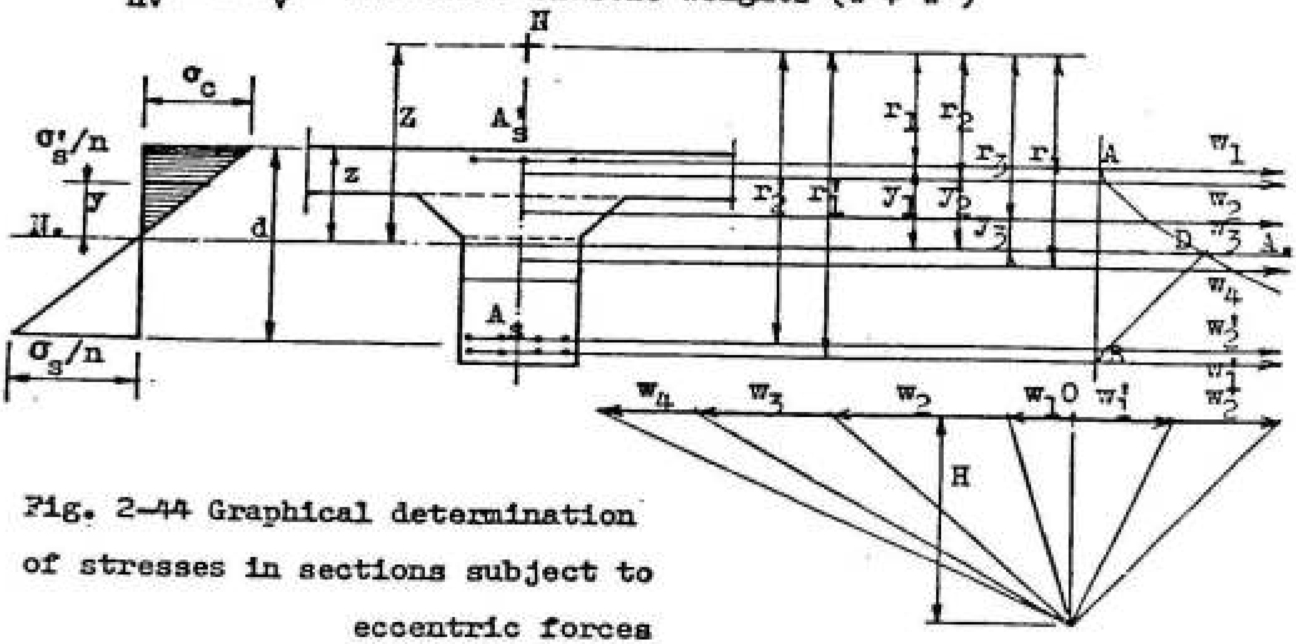


Fig. 2-44 Graphical determination of stresses in sections subject to eccentric forces

c) Ultimate strength analysis

In eccentrically compressed members, at loads approaching the ultimate, a nonelastic redistribution of stresses takes place. In compression members, even more than in beams and girders, we are interested in the carrying capacity i.e., the ultimate strength. Stresses and rigidity at service loads, which govern deflections of flexural members, are of little interest for compression members, whose deflections or other service behavior need rarely be computed. The elastic stresses under service loads are, according to the elastic theory, taken as a measure of carrying capacity by limiting them to appropriate fractions of f_{cp} and f_y. Experimental studies showed, however, that satisfactory correlation of actual strength with elastically computed stresses could not be obtained. However, ways of calculating ultimate strength on the basis of inelastic behavior have now been developed for eccentrically compressed members as for other members. Their results are in satisfactory agreement with extensive test evidence. Increasingly, in engineering practice, they are replacing the elastic, internally incosistent methods.

If a rectangular section as that shown in Fig. 2-45a is compressed by an eccentric force N, the steel and concrete stresses just prior to failure, i.e., at ultimate load, are distributed in the manner shown in Fig. 2-45b. That is, with plane sections assumed to remain plane the strains are those of Fig. 2-45c, which result in the shown stress distribution. Just as for simple bending, this actual distribution can be replaced, for purposes of calculation, with the equivalent rectangular stress distribution (Fig. 2-45d). A large number of tests on columns of a variety of shapes have shown that the ultimate strengths computed on this basis are in satisfactory agreement with test results.

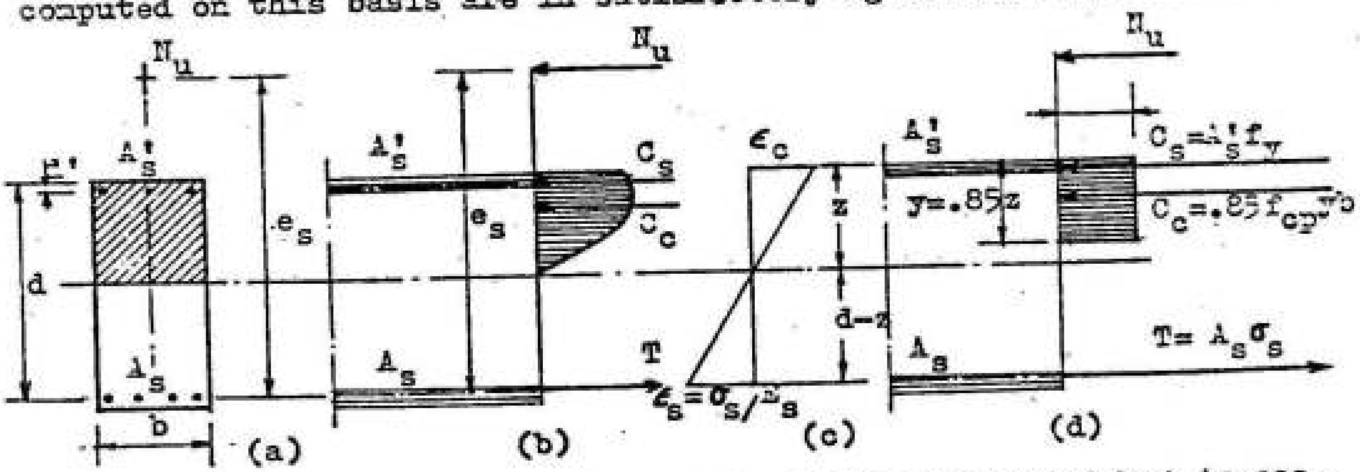


Fig. 2-45 Ultimate stresses in a rectangular section subject to eccentric compression

It is assumed in Fig. 2-45d that at ultimate load, when the concrete fails in compression, i.e., when $\epsilon_c=\epsilon_u=0.003$, the compression steel is yielding i.e., $\sigma_s'=f_y$ as shown. For ordinary conditions, this is always the case.

Equilibrium between external and internal forces requires that:

$$N_u = C_c + C_s - T$$

or

$$N_u = 0.85\ f_{cp}\ y\ b + A_s'\ f_y - A_s\ \sigma_s \qquad 2\text{-}113$$

Also, the moment of the external and internal forces about the tension steel must be equal. Hence

$$N_u\ e_s = 0.85\ f_{cp}\ y\ b\ (d - y/2) + A_s'\ f_y\ (d - d') \qquad 2\text{-}114$$

These are the two basic equilibrium equations for rectangular eccentrically compressed members.

For big eccentricities, failure will be initiated by yielding of the tension steel, followed by a shift of the neutral axis toward the compression side until crushing of the concrete causes a secondary compression failure. Conversely, for small eccentricities, the concrete may crush while the tension steel may be far from yielding.

For any cross section of given dimensions and internal strength values, there is one specific eccentricity e_b such that a force applied at that distance will cause failure by simultaneous yielding of the tension steel and crushing of the concrete. For this balanced condition, the concrete strain is:

$$\epsilon_c = \epsilon_u = 0.003$$

and simultaneously the strain in the tension steel at the instant at which yielding commences is:

$$\epsilon_s = \epsilon_y = f_y/E_s$$

From the similarity of the triangles of the strain diagram, we get:

$$z_b = \frac{\epsilon_u}{\epsilon_c + \epsilon_u}\ d = \frac{0.003}{f_y/E_s + 0.003}\ d \qquad 2\text{-}115$$

The depth of the rectangular stress block can be determined from equation 2-113; thus

$$y_b = \frac{N_{ub} + (A_s - A_s')\ f_y}{0.85\ f_{cp}\ b} \qquad 2\text{-}116$$

Also, by definition of the rectangular stress block, we have:

$$y_b = 0.85\ z_b$$

Introducing this value in equation 2-116, and using z_b from equation 2-115, one can determine the ultimate load under balanced conditions, N_{ub}, hence

$$\frac{N_{ub} + (A_s - A_s') f_y}{0.85 f_{cp} b} = 0.85 \frac{0.003}{f_y/E_s + 0.003} d \qquad \text{or}$$

$$N_{ub} = 0.72 f_{cp} b d \frac{0.003}{f_y/E_s + 0.003} - (A_s - A_s') f_y \qquad 2\text{-}117$$

The eccentricity e_{sb} which results in balanced conditions can then be obtained from equation 2-114 by substituting N_{ub} for N_u and y_b for y and solving for e_s. Loads with eccentricities smaller than e_{sb} result in primary compression failures at ultimate values larger than N_{ub}; loads with eccentricities larger than e_{sb} result in primary tension failures at loads smaller than N_{ub}.

Hence, if $e_s < e_{sb}$, failure is initiated by compression of the concrete; i.e., when $\epsilon_c = \epsilon_u = 0.003$ and $\sigma_s < f_y$.

<u>Illustrative examples:</u>

(1) It is required to determine the ultimate eccentric load of the section shown in Fig. 2-46 for an eccentricity e = 50 cms assuming f_{cp} = 200 kg/cm^2 and f_y = 2400 kg/cm^2.

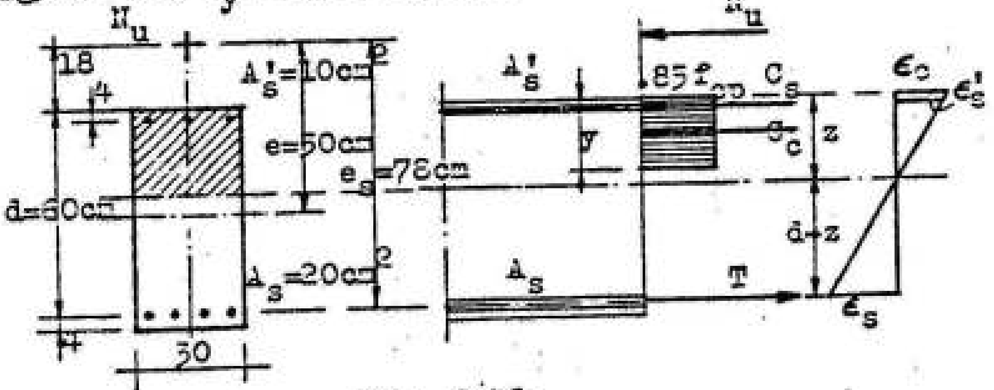


Fig. 2-46

Solution:

Assuming that failure will be initiated by yielding of the tension steel and $\sigma_s = \sigma_s' = f_y$ = 2400 kg/cm^2. Then

$$T = A_s f_y = 20 \times 2.4 = 48 \text{ ton}$$

$$C_s = A_s' f_y = 10 \times 2.4 = 24 \text{ ton}$$

$$C_c = 0.85 f_{cp} y b = 0.85 \times 0.2 \times 30 y = 5.1 y$$

Due to equilibrium between external and internal forces, we have

$$N_u = C_s + C_c - T$$ or

$$N_u = 24 + 5.1\ y - 48 = 5.1\ y - 24$$

i.e.

$$y = \frac{N_u + 24}{5.1}$$

The magnitude of y (and z) can be determined by taking moments about the point of application of N_u. That is,

$$T \times 78 = C_s \times 22 + C_c\ (y/2 + 18)$$ or

$$24 \times 78 = 24 \times 22 + 5.1\ y\ (y/2 + 18)$$ giving

$$y = 21.3 \quad \text{cms}$$

Therefore

$$z = y\ /\ 0.85 = 21.3\ /\ 0.85 = 25.1 \text{ cms}$$ and

$$N_u = 5.1 \times 21.3 - 24 = 85.63 \quad \text{tons}$$

The above calculations can only be correct if the strain in the compression steel ϵ'_s is bigger than its yield strain ϵ_y.

$$\epsilon_y = \frac{f_y}{E_s} = \frac{2.4}{2100} = \frac{1.14}{1000}$$

$$\epsilon'_s = \epsilon_c\ \frac{z - d'}{z} = \frac{3}{1000} \times \frac{25.1 - 4}{25.1} = \frac{2.52}{1000} > \frac{1.14}{1000}$$

Hence, at the instant at which the ultimate concrete strain is reached, the compression steel strain exceeds the yield strain, verifying the assumption that at ultimate load the compression steel is yielding.

(2) Assuming e in the previous example is equal to 20 cms, determine the ultimate load.

This problem can be calculated by successive approximations in the following manner:

First trial: Assume y = 38 cms i.e. z = 38 / 0.85 = 45 cms

The strain diagram gives:

$$\epsilon_s = \frac{\sigma_s}{E_s} = \epsilon_c\ \frac{d - z}{z}$$ so that

$$\sigma_s = 2100\ 000\ \frac{3}{1000} \times \frac{60 - 45}{45} = 2100 \quad \text{kg/cm}^2$$

Taking moments about the tension steel, we get:

N_u (20 + 32 - 4) = 0.85 x 0.200 x 38 x 30 (60 - 19) + 24 x 56 i.e.

$$N_u = 194.67 \text{ tons}$$

The summation of the axial forces gives:

$$194.67 = 0.85 \times 0.2 \times y \times 30 + 24 - 20 \times 2.1 \quad \text{or} \quad y = 41.5 \text{ cms}$$

Second trial: Assume y = 40.5 cms i.e. s = 40.5 / 0.85 = 47.5 cm

$$\sigma_s = 2100\,000 \times \frac{3}{1000} \times \frac{60 - 47.5}{47.5} = 1660 \text{ kg/cm}^2$$

and

N_u (20 + 32 - 4) = 0.85 x 0.200 x 40 x 30 (60 - 20.25) + 24 x 56

i.e. N_u = 198 tons

The summation of the axial forces gives:

198 = 5.1 y + 24 - 20 x 1.66 or y = 40.5 cms

which means that N_u = 198 tons is the right answer.

Calculating the ultimate load for balanced conditions from equation 2-117, we find

$$N_{ub} = 164 \text{ tons} > 85.6 \text{ tons and} < 198 \text{ tons}$$

The balanced height of the compression block is, according to equation 2-116, given by:

$$y_b = \frac{N_{ub} + (A_s - A_s') f_y}{0.85 f_{cp} b} = \frac{164 + (20 - 10) 2.4}{0.85 \times 0.2 \times 30} = 37 \text{ cms}$$

which is bigger than 25.1 cms of example 1 and smaller than 40.5 cms of example 2.

The eccentricity e_{sb} which results in balanced conditions can be obtained from equation 2-114 by substituting N_{ub} = 164 tons for 'N_u' and y_b = 37 cms for y and solving for e_s. Hence

$$N_{ub} e_{sb} = 0.85 f_{cp} y_b b (d - y_b/2) + A_s' f_y (d - d') \quad \text{or}$$

$$164 e_{sb} = 0.85 \times 0.2 \times 37 \times 30 (60 - \tfrac{37}{2}) + 10 \times 2.4 (60 - 4)$$

or e_{sb} = 56 cms and e = 26 cms

which means that, an ultimate load of 198 ton > N_{ub} = 164 ton acting at an eccentricity e = 20 cms < 26 cms will result in primary compression failure.

2.8- ECCENTRIC TENSION

a) Small eccentricity

If the normal tensile force N lies between the reinforcements A_{s1} and A_{s2}, (Fig. 2-47), the whole section will either be subject to tensile stresses only or tensile stresses and small compressive stresses.

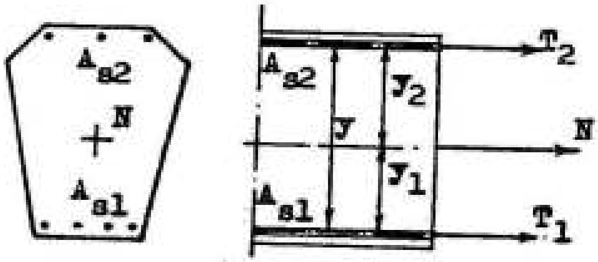


Fig. 2-47 Eccentric tension with small eccentricity

In both cases, the concrete section will be assumed as statically not acting and the steel alone must be sufficient to resist the tensile forces T_1 and T_2, where

$$T_1 = N\, y_2 / y \qquad \text{and} \qquad T_2 = N\, y_1 / y \qquad 2\text{-}118$$

In sections where no tension cracks are allowed, the maximum tensile stresses in the section, calculated according to the equations of Navier 2-99, must be smaller than the rupture strength of concrete.

b) Big eccentricity

Proceeding in the same manner as in case of eccentric compression, we get:

For a rectangular section in the cracked linear stage: (Fig. 2-48)

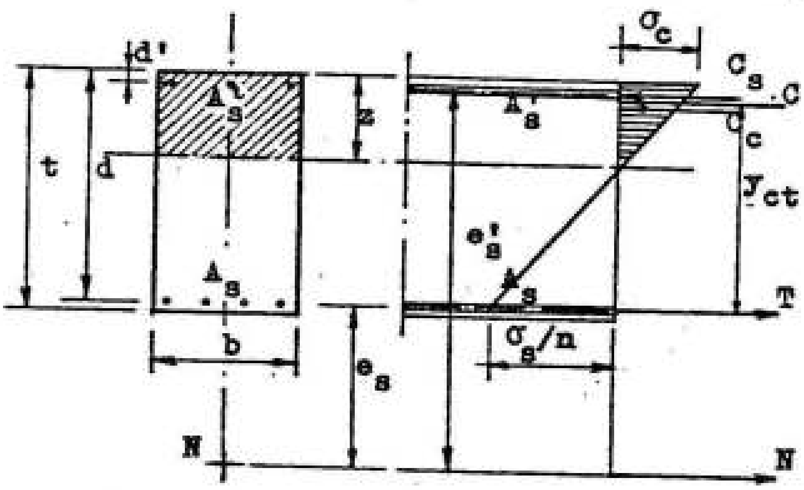


Fig. 2-48 Stresses in a rectangular section subject to eccentric tension with big eccentricity

we get the following relations:

Equation of neutral axis:

$$z^3 - z^2 .3\,(e_s + d) - z\,\frac{6\,n}{b}\,(A_s\,e_s + A_s'\,e_s') + \frac{6\,n}{b}\,(A_s\,d\,e_s + A_s'\,d'\,e_s') = 0 \qquad \text{..........2-119}$$

Stresses:

$$\sigma_c = \frac{M_s\,z}{\frac{b\,z^2}{2}\,(d - \frac{z}{3}) + n\,A_s'\,(z - d')\,(d - d')} \qquad 2\text{-}120$$

$$\sigma_s = n\,\sigma_c\,\frac{d - z}{z} \qquad 2\text{-}121$$

For a rectangular section at failure, we get the following relations. (Fig. 2-49)

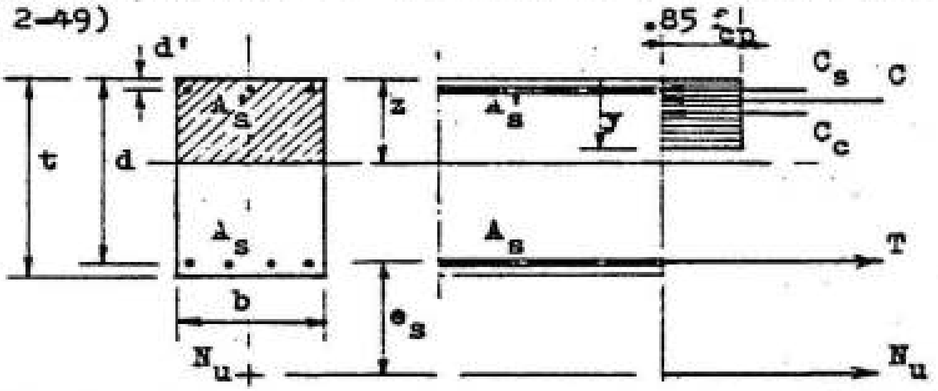


Fig. 2-49 Ultimate stresses in a rectangular section subject to eccentric tension with big eccentricity

Equations of equilibrium:

$$N_u = T - C_s - C_c$$

or

$$-N_u = 0.85\,f_{cp}\,y\,b + A_s'\,f_y - A_s\,\sigma_s \qquad 2\text{-}122$$

and

$$-N_u . e_s = 0.85\,f_{cp}\,y\,b\,(d - y/2) + A_s'\,f_y\,(d - d') \qquad 2\text{-}123$$

At balanced conditions, we have:

Distance of neutral axis

$$z_b = \frac{0.003}{f_y/E_s + 0.003}\,d \qquad 2\text{-}124$$

Height of compression block

$$y_b = \frac{(A_s - A_s')\,f_y - N_{ub}}{0.85\,f_{cp}\,b} \qquad 2\text{-}125$$

Balanced normal tensile force

$$N_{ub} = (A_s - A_s') f_y - 0.72 f_{cp} b d \frac{0.003}{f_y/E_s + 0.003} \qquad 2\text{-}126$$

2-9 Shear Stresses and Diagonal Tension in Sections Subject to Eccentric Forces

a) Elastic analysis for the case of big eccentricities (Fig. 2-50)

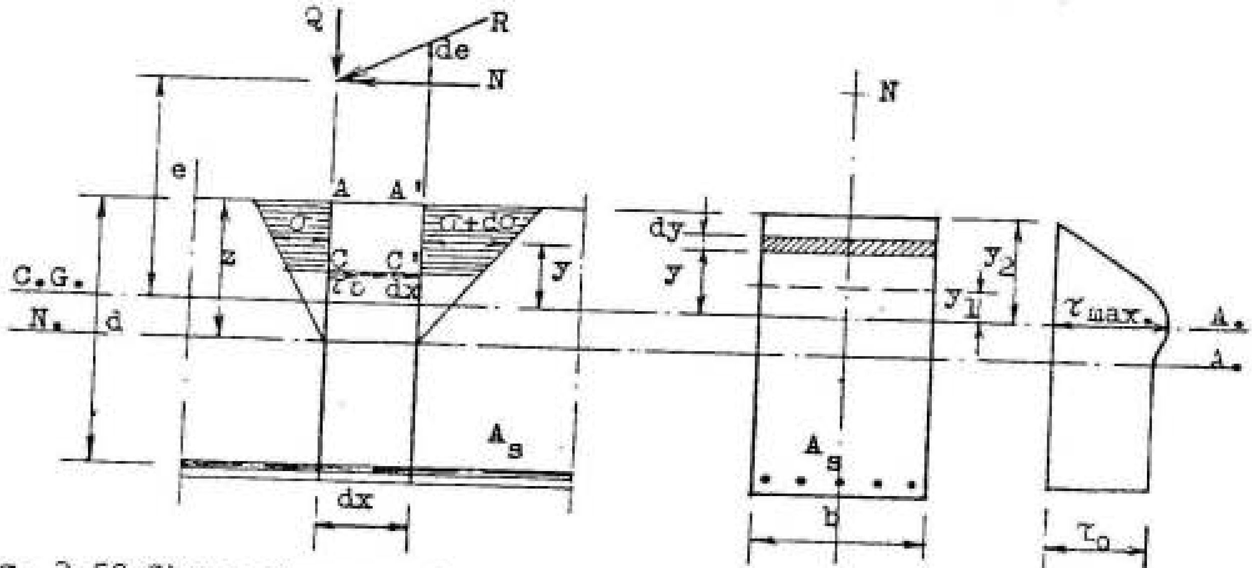


Fig. 2-50 Shear stresses in a section subject to eccentric force with big eccentricity

The normal stresses in any section can be given, according to the modified equation of Navier, from the relation:

$$\sigma = \frac{N}{A_v} + \frac{N\,e\,y}{I_v} \qquad 2\text{-}127$$

in which

A_v = the virtual area

I_v = the moment of inertia of the virtual area about the axis passing through the center of gravity of the section.

Assuming that the position of the neutral axis does not vary in two successive sections at a distance dx, then, A_v and I_v represent constant values, and

$$d\sigma = \frac{N\,de}{I_v}\,y$$

The sum of the shear stresses acting on a horizontal surface CC' is equal to the difference between the normal stresses on A'C' and AC, thus:

$$\tau\,b\,dx = \int_{y1}^{y2} b\,dy\,d\sigma$$

or

$$\tau\, b = \frac{N}{I_v}\cdot\frac{de}{dx}\int_{y1}^{y2} b\, y\, dy$$

but

$$\frac{N\, de}{dx} = \frac{dM}{dx} = Q$$

and

$$\int_{y1}^{y2} b\, y\, dy = S_v$$

in which

S_v = statical moment of the virtual area above CC' about the C.G.-axis.

Therefore

$$\tau = \frac{Q\, S_v}{I_v\, b} \qquad 2\text{-}128$$

This equation has the same form as equation 2-22, with S_v and I_v instead of S and I.

The value of y_2 is given by:

$$y_2 = \frac{n\, A_s\, d + b\, \frac{z^2}{2}}{n\, A_s + b\, z} \qquad 2\text{-}129$$

The value of S_v required to determine τ_o is computed from the statical moment of n times the area of the tension steel about the C.G.-axis. Thus

$$S_v = n\, A_s\, (d - y_2) \qquad 2\text{-}130$$

The moment of inertia of the virtual area about the C.G.-axis is given by:

$$I_v = \frac{b}{3}\left[\, y_2^3 + (z - y_2)^3 \right] + n\, A_s\, (d - y_2)^2 \qquad 2\text{-}131$$

The principal tensile stress σ_1 is given by the relation:

$$\sigma_1 = -\frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} + \tau^2} \qquad 2\text{-}132$$

Below the neutral axis, σ_1 is equal to τ_o because $\sigma = 0$. The shear stress τ_{max} at the center of gravity of the section is here of no importance because the corresponding principal stress is not maximum. The difference between σ_1 and τ_o is to be taken in consideration only in cases of small eccentricities and small tension reinforcements.

A comparison between the values of the principal diagonal tensile stresses σ_1 and the shear stresses for sections subject to eccentric forces with different big eccentricities is shown in Fig. 2-51.

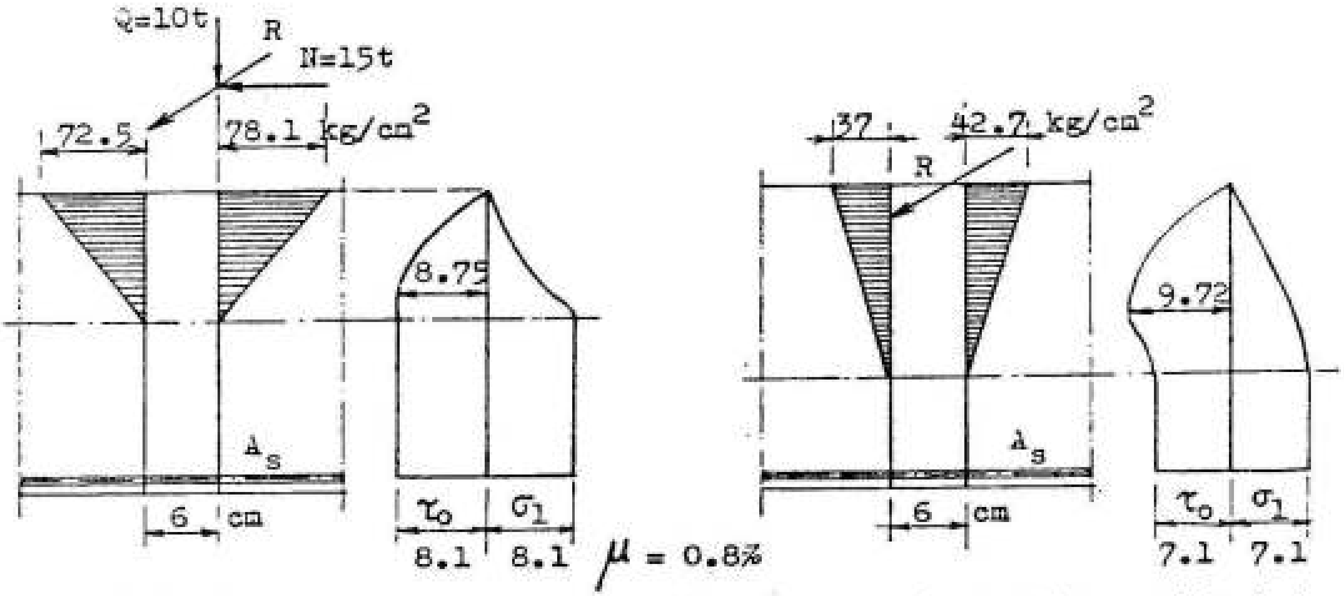


Fig. 2-51 Shear and principal tensile stresses in sections subject to eccentric forces with big eccentricity

Application to T-sections

If we assume that all the compressive stresses are resisted by the flange only, and that the center of compression lies in the middle of the flange, the shear stress can be simply determined in the following manner. (Fig. 2-52)

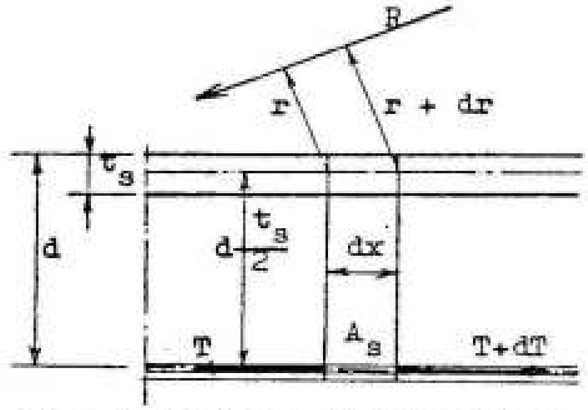


Fig. 2-52 Shear in T-sections subject to eccentric forces

Taking moments of external and internal forces about centers of flange in twc successive sections at a distance dx, we get:

$$R\ r = T\ (d - \frac{t_s}{2})$$

$$R\ (r + dr) = (T + dT)\ (d - \frac{t_s}{2})$$

Substracting the first relation from the second, one finds

$$R\ dr = dT\ (d - \frac{t_s}{2})$$

Therefore

$$dT = \frac{R\ dr}{d - \frac{t_1}{2}}$$

but

$$dT = \tau_o\ b\ dx$$

then

$$b\ \tau_o = \frac{dT}{dx} = \frac{R}{d - t_s/2} \cdot \frac{dr}{dx}$$

but

$$\frac{dr}{dx} = \frac{Q}{R}$$

therefore

$$\tau_o = \frac{Q}{b\,(d - t_s/2)} \qquad 2\text{-}133$$

b) Elastic analysis for the case of eccentric compression with small eccentricity (Fig. 2-53)

In this case, the shear stress is given by:

$$\tau = Q\,S_v \,/\, I_v\, b$$

The principal stress is

$$\sigma_1 = -\frac{\sigma}{2} + \sqrt{\frac{\sigma^2}{4} + \tau^2}$$

acting at an angle α where

$$\tan 2\alpha = -2\,\tau/\sigma$$

The principal tensile stresses σ_1 in a section subject to eccentric compression with small eccentricity are smaller than the shear stresses τ (Fig. 2-53) and generally need no special reinforcement.

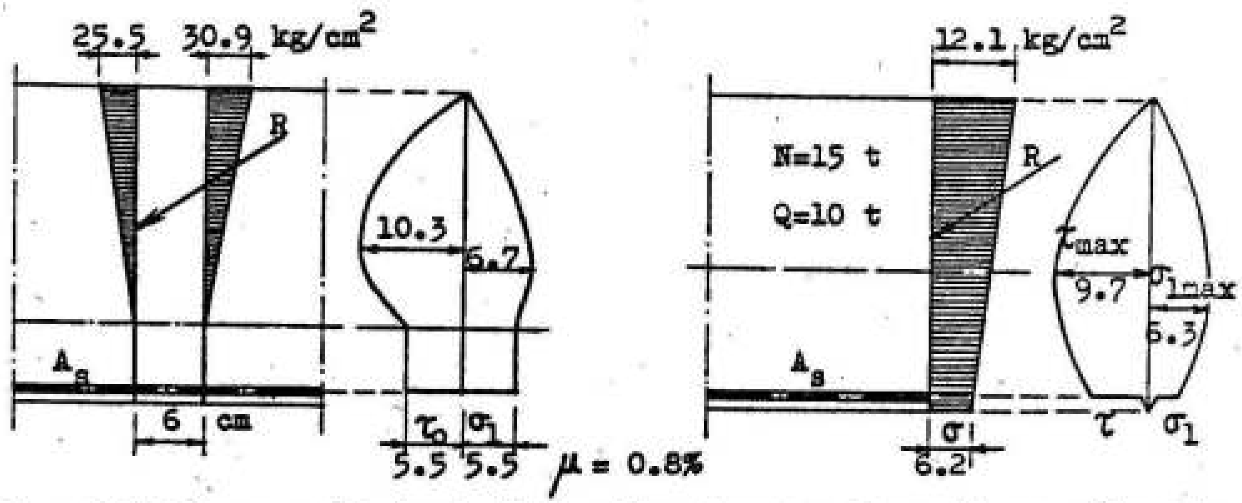


Fig. 2-53 Shear and principal tensile stresses in sections subject to eccentric compression with small eccentricity

The principal tensile stresses σ_1 in a section subject to eccentric tension with small eccentricity need not be checked because they are a bit bigger than the normal tensile stresses and nearly prallel to them.

c) Ultimate strength analysis for the case of big eccentricity

In this case, the ultimate principal stress σ_{1u} may be assumed equal to τ_{ou} and treated in the same way as in case of simple bending.

CHAPTER 3

DESIGN OF REINFORCED CONCRETE ELEMENTS

3.1 General Considerations

The UNESCO-reinforced concrete international manual states, under scope and object of the design, the following:

> "All reinforced concrete structures or structural elements should be designed and constructed that they are able, with appropriate safety, to withstand all the loads, superimposed loads and other actions liable to occur during construction and in use."
>
> "The object of the design calculations is to guarantee sufficient safety against the structure being 'unfit for service'.
>
> A structure is considered to have become 'unfit' when one or more of its members ceases to perform the function for which it was designed, owing to failure, buckling due to elastic, plastic or dynamic instability, excessive cracking, excessive elastic or plastic deformation, etc."
>
> "For residential buildings which are not exceptional in character and which comprise not more than five storeys (i.e., four upper floors) the building owner may authorise the designer to employ a simplified design method, e.g., a method based on the use of the modular ratio - working stress method with linear distribution of stresses -, provided that the overall safety of each of its component members are checked to ensure that, in all circumstances and for any combination of loads, superimposed loads and other actions, they are at least equal to the structural safety that can be obtained by rigorous application of the code of practice detailed in this manual." [By the use of

the limit states and ultimate strength design methods].

We gave in the previous chapter, the behavior of reinforced concrete under different load stages, showing the analysis:

1. under service loads using the elastic analysis in which both concrete and steel are assumed elastic. A design according to this method of analysis is generally called: " elastic design or working stress design",

2. at failure loads using the ultimate strength analysis in which the inelastic behavior of both concrete and steel is taken in cosideration. Modern designs based on this method of analysis are known under different names, the most common of which are: " the ultimate strength design" used in Germany and the U.S.A. and " the limit states design " used in the U.S.S.R. and recmmended by the C.E.B. (Comité Européen du Béton).

The notion of 'limit state' as defined by the UNESCO is as follows

> " To each of the conditions in which a structure becomes 'unfit for service' corresponds a particular state called a 'limit state'. These limit states are respectively:
>
> a) the ultimate limit state "failure";
> b) the limit state of instability "buckling";
> c) the limit state of cracking; and,
> d) the limit state of deformation; etc.

By working-stress design methods, allowable stresses are established as some fraction of the strength of concrete f_{cp} and the yield strength of the steel f_y. Members are proportioned so that these allowable stresses are not exceeded when service working loads are applied.

The ultimate -strength design methods, on the other hand, base the design on conditions just before failure. Members are proportioned so that the full strength of the cross-section is just utilized when the ultimate load is applied. The ultimate load is obtained by multiplying the actual dead load and the anticipated live load by separate load factors greater than unity. The concrete compressive stress distribution just before failure is quite different from the triangular distribution assumed in the working-stress design methods. [Compare Figs. 2-22 to 2-28 by Figs. 2-10 to 2-18].

It has been proved by tests that working-stress design methods do not give correct information regarding the actual factor of safety against failure of a reinforced concrete member.

The factor of safety is defined as the ratio between the load that would cause total collapse to that used as the service or working load, and it has been found that the value of this factor is far different from the ratio of ultimate strength to the so-called working stress. However, elastic analysis does give a realistic representation of conditions in a member at working load and therefore is useful in predicting service load behavior characteristics such as deflection and crack width. Furthermore, shrinkage and creep are inelastic strains; they take place under working loads and cannot be treated in an elastic manner. These two factors give the main reason to the known fact that the strains, and the corresponding stresses,under working loads, calculated according to the elastic theory, are very far from the real strains that are measured in a structural element under the same load.

Ultimate strength methods, on the other hand, are not affected by shrinkage and creep and permit an accurate evaluation of the strength capacities and the corresponding factor of safety against failure. However, strength analysis gives little information about conditions at service loads.

Accordingly, one may conclude that the elastic theory is still retained as a design method because of the long habit of the profession of thinking in terms of elastic stresses, and that the more realistic ultimate strength methods will very soon become dominant.

3.2 Safety Provisions

As stated before, the design calculations must guarantee sufficient safety against the structure being rendered 'unfit for service', i.e., it must be safe against failure and serviceable in use. Safety requires that the strength of the structure be adequate, with appropriate factor of safety, for all loads and other actions that are liable to occur during construction and in use. Serviceability requires that deflections be adequately small, that cracks, if any, be kept to tolerable limits, that vibrations be minimized.

An appropriate margin of safety is essential due to different sources of uncertainty and which may be listed as follows*:

1. Actual loads may differ from those assumed in the design.

---

* Winter and others " Design of concrete structures "

2. Actual loads may be distributed in a manner different from that assumed in the design.

3. The assumptions and simplifications inherent in any analysis may result in calculated load effects - moments, shears, etc.- different from those which a more rigorous analysis would furnish.

4. The actual structural behavior may differ from the assumed, owing to imperfect knowledge.

5. Actual member dimensions may differ from those specified by the designer.

6. Reinforcement may differ from that specified by the designer, and may not be in its proper position.

7. Actual material strength may be different from that specified by the designer.

In establishing the safety margin, consideration must be given to some other factors, namely:

1. The nature of failure, whether ductile or sudden; the latter should possibly be avoided, otherwise , a large factor of safety is essential.

2. Consequences of failure; the factor of safety should be bigger if it results in losses of life or big losses of property, and increases with the increase of the losses.

a) Safety provisions for working-stress design method

In the working-stress design method, the safety provisions are supposed to be satisfied if the maximum stresses due to the maximum possible internal forces under the effect of dead and live working-loads are smaller than the allowable values which may be assumed as follows:

Table 3.1 Allowable stresses in steel reinforcement

Kind of steel	Yield stress kg/cm^2	Allowable stress kg/cm^2
Normal mild	2200 - 2400 2800 $< f_y$	1400 $f_y/2$ & < 1600
Deformed high grade: normal Deformed high grade: cold treated	3600 4000 - 4800	2000 2200 - 2400

The maximum allowable concrete compressive stress in beams, slabs (with a thickness bigger than 20 cms) and elements subject to eccentric forces with big eccentricity shall be assumed $\leqslant$ 50% of the minimum guaranteed compressive strength of concrete.

This stress must be reduced gradually to 0.75 its value in 8 cms thick slabs and T-sections with a compression flange of 8 cms.

The allowable compressive stress in concrete may be increased to 60% of the minimum guaranteed strength, provided that this increase is approved by the responsible authorities.

For the allowable shear stresses: If the shear stress τ in a beam is:

$$\tau < \tau_1 = 0.5\sqrt{f_{cp}}$$

no special calculation is essential for the web reinforcement on condition that this is bigger than the minimum allowed values. If

$$\tau_1 < \tau < \tau_2 = 1.5\sqrt{f_{cp}}$$

web reinforcements are essential to resist $\tau_s = \tau - \tau_c$. In this relation, τ_s = the shear stress resisted by the web reinforcement, and τ_c = the shear stress resisted by the concrete $\leqslant 0.25\sqrt{f_{cp}}$. If

$$\tau_2 < \tau \leqslant \tau_3 = 1.8\sqrt{f_{cp}}$$

inclined stirrups and deformed bent bars are essential.

In Egypt, τ_c is assumed equal to zero; the values of the allowable compressive and shear stresses are as shown in table 3.2

Table 3.2 Allowable stresses in concrete

Kind of stress		Allowable stress in kg/cm^2							
Cube strength Prism ,,	C_{28} f_{cp}	120 105	160 135	180 150	200 165	225 185	250 200	275 220	300 240
Axial comp.	σ_{co}	30	45	50	55	60	65	70	75
Simple bending and eccentric forces with big eccentricity:									
Slabs t=8-10	σ_c	25-30	40-45	45-50	50-55	55-60	60-65	65-70	70-75
t=10-12	σ_c	30-35	45-50	50-55	55-60	60-65	65-70	70-75	75-80
t=12-20	σ_c	35-40	50-60	55-60	60-70	65-75	70-80	75-85	80-90
Beams & slabs t 20 cm	σ_c	40	60	65	70	75	80	85	90
Shear stresses:									
Slabs	τ_1	6	7	8	8	9	9	10	10
Other elements	τ_1	4	5	6	6	7	7	8	8
All elements	τ_2	14	16	17	18	19	20	21	22

b) Safety provisions for ultimate-strength design method

In order to compute the ultimate resistance and to have an appropriate factor of safety, one has to increase the dead and live loads by the so-called load factors which are chosen such that these ultimate loads have an acceptably small probability of ever being exceeded. At the same time, one has to reduce the internal resistance of the sections by the so-called 'capacity reduction factor' which depends on the nature and consequences of failure.

1. Load factors giving the required strength

Structures and structural members shall be designed to have strengths at all sections at least equal to the structural effects of the design loads and forces in such combinations as given in the following:

1. The required strength U provided to resist dead load G and live load P shall be at least equal to

$$U = 1.5\ G + 1.8\ P \qquad 3\text{-}1$$

2. In the design of a structure or member, if resistance to the structural effects of a specified wind load W must be included in the design, the following combination of G, P and W shall be investigated in determining the greatest required strength U

$$U = 0.75\ (1.5\ G + 1.8\ P + 1.8\ W) \qquad 3\text{-}2$$

Where the cases of P having its full value or being completely absent shall both be checked to determine the most severe condition

$$U = 0.9\ G + 1.3\ W \qquad 3\text{-}3$$

but in any case the strength of the member or structure shall not be less than required by equation 3-1.

3. If resistance to specified earthquake loads or forces E must be included in the design, the requirements of article 2 shall apply except that E shall be substituted for W.

4. If lateral earth pressure EP must be included in the design, the strength U shall be at least equal to

$$U = 1.5\ G + 1.8\ P + 1.8\ EP \qquad 3\text{-}2\ a$$

Where G or P reduce the effect of EP, the ultimate strength U shall be at least equal to

$$U = 0.9\ G + 1.8\ EP \qquad 3\text{-}3\ a$$

5. For lateral pressure from liquids LP, the provisions for art. 4 shall apply, except that 1.5 LP shall be substituted for 1.8 EP, i.e.

$$U = 1.5\,G + 1.8\,P + 1.5\,LP \qquad 3\text{-}2\ b$$

or

$$U = 0.9\,G + 1.5\,LP \qquad 3\text{-}3\ b$$

The vertical pressure of liquids shall be considered as dead load, with due regard to variation in liquid depth.

6. Dynamic effects if any, shall be included with the dynamic live load P_d by multiplying it by $(1 + \alpha)$, where

$$\alpha = 0.4 /(1 + l/5) + 0.6 /(1 + G/P) \qquad 3\text{-}4$$

where l denotes the free (unrestrained) length of the element considered in meters.

P denotes the total superimposed live loads and dynamic loads.

This empirical amplification coefficient takes account of the unfavourable effect of transient dynamic phenomena (as occur in bridges, overhead crane track girders, etc.). Permanent functioning conditions and cyclic phenomena (e.g. rotating machinery with reciprocating action as well as impact coefficient)are not allowed for in this coefficient and should be the subject of special investigation in each case.

2. Capacity reduction factors

The strength of a member or cross section in terms of load, moment, shear, or stress shall be taken as the strength calculated in accordance with the requirements and assumptions given in chapter 2, including a 'capacity reduction factor Ω'. The following values for Ω shall be used:

Bending , with or without axial tension 0.90

Axial compression or axial compression combined with bending

a. Spirally reinforced members 0.75

b. Other members 0.70

c. The values given in a and b may be increased linearly to 0.90 as the axial ultimate load in the compression member under consideration N_u decreases from $0.1 f_{cp} A_c$ to zero for sections with symmetrical reinforcement and $(t-d'-d'')/t$ not less than 0.70.

d. The values given in a and b may be increased linearly to 0.90 as N_u decreases from $0.1 f_{cp} A_c$ or N_b whichever is smaller, to zero for sections with small axial compression not satisfying c.

Shear and torsion 0.85

Bearing on concrete 0.70

Bending in plain concrete or in concrete with reinforcement less than the minimum value 0.65

c) Safety provisions for limit-states design method

1. Load factors

For the ultimate limit state and limit state of instability, the loads are to be increased by the load factors given under b)/1.

The limit states of cracking and deformation are to be determined under the effect of the specified dead and live loads under working conditions.

2. Capacity reduction factors

For ultimate limit state, the capacity reduction factor γ is chosen for concrete $\gamma_c = 1.6$ and for steel $\gamma_s = 1.15$, so that

$$\text{design stress of concrete} = \frac{.85 \text{ conc. strength } f_{cp}}{\gamma_c} = \bar{\sigma}_c$$

$$\text{design stress of steel} = \frac{\text{yield stress } f_y}{\gamma_s} = \bar{\sigma}_s$$

In elements subject to axial compression, or eccentric compression with small eccentricity, an extra reduction of 25% of the concrete strength is specified because of the possible sudden failure of such elements.

3. Limit state of instability

It has been found that the analysis for the limit state of instability can be transformed into the usual analysis of the ultimate limit state of eccentric compression by conventionally introducing a complementary eccentricity to the direct force causing an 'additional bending moment' that has to be added to the initial loading system on which the design of the section for the ultimate limit state is based.

4. Limit state of cracking

Cracking is a phenomenon specifically associated with reinforced concrete, for the structural elements loaded in tension or in bending are normally cracked in their usual conditions of service. However, for the sake of the durability of the structure it is necessary to impose certain limits upon the cracking or, to be more precise, certain maximum values of the crack widths are not allowed to be exceeded. These maximum widths in each case define the limit state of cracking of the member or structure concerned, having due regard to environmental and service conditions thereof.

The analysis of the limit state of cracking in bending and in tension should, in normal cases, be confined to checking that the rules of good construction have been applied in the design and distribution of the reinforcement, whose quantity has first been determined by means of the analysis of the ultimate limit state(failure); or any other approved method. These practical rules for the design and distribution of the reinforcement define in each case, for a given total section, the maximum diameter of the reinforcing bars as a function of the percentage, the elastic limit and the bond properties of the steel, and of the tensile strength of the concrete.

To take account of their varying conditions of environment and service, structures are divided into three classes, according to the possible consequences of cracking with regard to the behavior and durability of the structures:

Class 1: Structural members which must ensure watertightness or are exposed to aggressive actions. These are members in which cracking is very harmful, either because they have to be watertight (e.g., walls of tanks, shipping locks or dry docks) or because they are exposed to a particularly aggressive medium. In this class of structural members, the maximum width of cracks must not exceed 0.1 mm. i.e.,

$$w_1 \text{ max.} \ngtr 0.1 \text{ mm}$$

Class 2. Unprotected ordinary structural members. In these members cracking of the tensile zones is harmful either because they are exposed to the effects of the weather (as is the case with outdoor structures such as bridges and other civil engineering works) or because they are exposed to a humid and aggressive atmosphere (as in the case of certain industrial structures, factory roofs, or workshop buildings in which considerable quantities of water vapour are liable to be produced). This class may also be taken to include members which have to support very fragile claddings or facings which would suffer harmful consequences from excessive cracking and deformation. In this class, the maximum width of cracks must not exceed 0.2 mm. i.e.,

$$w_1 \text{ max.} \ngtr 0.2 \text{ mm}$$

Class 3. Protected ordinary structural members. In these members cracking is not harmful and does not have any seriously adverse effects upon the preservation of the reinforcing steel nor upon the durability of the structure. Interior structural members of buildings in normal atmosphere are, for example included in this class. The maxim- crack width, in this case, must not exceed 0.3 mm. i.e.,

$$w_1 \text{ max.} \ngtr 0.3 \text{ mm}$$

In order to limit the width of cracks to the specified values, one has:

1) to use dense concrete,

2) to have sufficient spacing and cover for the reinforcing bars; so that the width of the member is bigger than four times the sum of the diameters of the main reinforcing bars arranged in one row (if these are plane bars) or twice the sum (if they are deformed bars providing improved bond behavior).

3) to use several bars at moderate spacing than larger bars of equivalent area. The objective is therefore one of distributing the reinforcement in the concrete tension zone.

Although cracking cannot be expected to be eliminated it is generally more desirable to have many fine hair cracks than a few wide cracks; this is generally the case when deformed bars are used as tension reinforcement.

Control of cracking may be particularly important when tension reinforcement percentage is bigger than half the maximum allowed value according to U.S.D. i.e. $\mu > \frac{1}{2} \times 0.75\, \mu_b = 0.375\, \mu_b$.

Extensive laboratory studies have shown that crack width is proportional to steel stress, thickness of concrete cover and the embedment area of each individual reinforcing bar i.e. the area of concrete serrounding each bar in the zone of maximum tension.

The ACI-provisions of 1977-Code for crack width are based on the Gergely - Lutz expression* for crack width as follows:

$$w_1 \max = C\, \beta_h\, \sigma_s \cdot \sqrt[3]{d_c\, A} \qquad 3\text{-}5$$

where, refering to Fig. 3-1

w_1 = crack width at tension face of beam in mm,

$\beta_h = h_2 / h_1$, the ratio of the distances to the working stress N-A from the extreme tension fiber and from the centroid of the main tension steel,

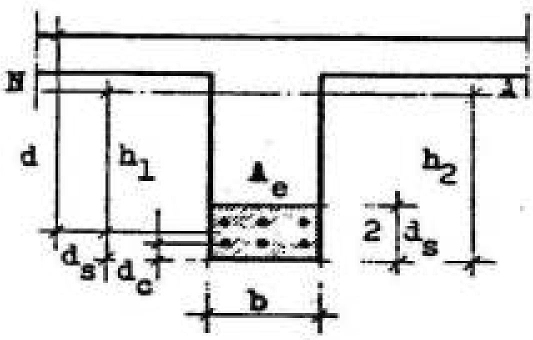


Fig. 3-1

σ_s = service load stress in the steel in kg / mm^2,

d_c = concrete cover measured from the extreme tension fiber to the

* Maximum crack width in reinforced concrete flexural members. American Concrete Institute 1968.

center of the bar located closest thereto in mm,

$A = A_e / n$ = embedmant area = effective tension area of concrete serrounding the main tension reinforcing bars and having the same centroid as the reinforcement, divided by the number of bars in mm^2,

n = number of bars, for different sizes use $n = A_s$ / area for largest bar,

C = an experimental constant = 11×10^{-5} mm^2 / kg.

This equation has been converted into a form in which a simple calculation could be made to arrive at reasonable reinforcing details as indicated by laboratory tests, without acually emphasizing crack width.

Dividing equ. 3-5 by $C \beta_h$ the quantity Z, whose value is taken as a measure for crack width, is defined.

$$\underline{Z_{limit}} = \sigma_s \sqrt[3]{d_c A} = \underline{\frac{w_i \max}{C \beta_h}} \qquad 3\text{-}6$$

For simplicity β_h may be taken at an approximate value of 1.2 in beams. In order to have numerical values for the quantity Z_{limit} according to equation 3-6 in the ACI-code, crack widths have been limited to (0.41 and 0.33 mm) for interior and exterior exposure, respectively. Substitution of these values in equation 3-6 gives:

For w_i max = 0.41 mm $\quad Z_{limit} = \dfrac{0.41}{1.2 \times 11 \times 10^{5}} = 3100$ kg/mm

" " = 0.33 mm $\quad Z_{limit}$ = = 2500 "

" " = 0.30 mm $\quad Z_{limit}$ = = 2270 "

" " = 0.20 mm $\quad Z_{limit}$ = = 1500 "

" " = 0.10 $\quad Z_{limit}$ = = 760 "

The expressions 3-5 and 3-6 have been found to apply to one-way slabs and wide beams; in which case, the average value for $\beta_h = h_2/h_1$ is about 1.35 for floor slabs rather than 1.2 which applies for beams.

The limiting values of Z will correspondingly be as follows:

w_i max	0.41	0.33	0.30	0.20	0.10	mm
Z_{limit}	2760	2220	2000	1330	665	kg/mm

Examples

1) A beam 250 x 800 mm is reinforced by 8 ϕ 25 mm ($A_s \cong 4000$ mm^2) ar-

ranged as shown in Fig. 3-2. The width of crack w_1 max. must be smaller than 0.20 mm. The stress in steel σ_s = 20 kg/mm^2.

Examine the crack control situation.

For w_1 max = 0.20 mm, Z_{limit} 1500 kg/mm

$$Z_{limit} = \sigma_s \sqrt[3]{d_c\ A}$$

d_c = 25 + 12.5 = 37.5 mm

A_e = 250 x 125 = 33750 mm^2 n = 8

A = 33750 / 8 = 3850 mm^2

$$Z_{limit} = 20\sqrt[3]{37.5 \times 3850} = 1000 \quad \text{kg/mm}$$

Crack width $w_1 = 0.20 \times \frac{1000}{1500} = 0.133$ mm

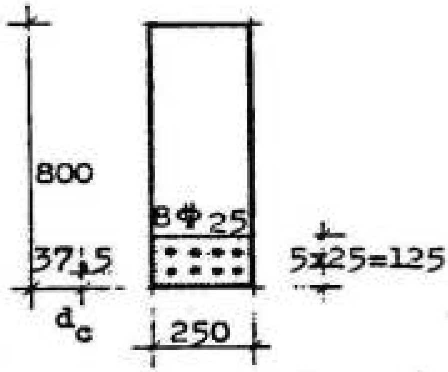


Fig. 3-2

2) The lowest section of the wall of a water tank is 400 mm thick and reinforced by ø 16 mm (A_s = 200 mm^2) spaced @ 140 mm on the water side, the cover is 30 mm and the stress in steel σ_s = 12 kg/mm^2. Determine the expected crack width.

d_c = 30 mm   A = 60 x 140 = 8400 mm^2

$$Z_{limit} = 12 \cdot \sqrt[3]{30 \times 8400} = 755 \quad \text{kg/mm}$$

Crack width $w_1 = 0.20 \times \frac{755}{1330} = 0.113$ mm

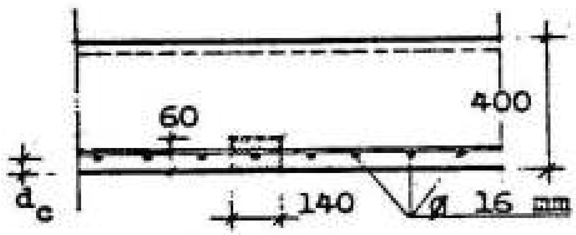


Fig. 3-3

5. Limit state of deformation

Deformations are a phenomenon specifically associated with reinforced concrete, since structural members loaded in bending, compression or tension are normally subjected to deformation under their usual conditions of service. However, for the sake of serviceability and durability of structures, it is necessary that certain deformation limits -or, more precisely, certain maximum values of the deflections- should not be exceeded. In each case, having due regard to the nature and the extent of possible damage due to deformation, these maximum values define the limit state of deformation of the member or structure considered.

In order to control the deflections, reinforced concrete members subject to bending shall be designed to have adequate stiffness to to limit deflections or any deformations which may adversely affect the strength or serviceability of the structure at service loads.

The minimum thickness given in the following table shall apply for members of one-way construction not supporting or attached to partitions or other construction likely to be damaged by large deflections unless the computation of deflection indicates that lesser thickness may be used without adverse effects.

Table 3.3 Minimum thickness of beams or one-way slabs unless deflections are computed

Member	Minimum thickness t			
	simply supported	one end continuous	both ends continuous	cantilever
Solid one-way slabs	l/20	l/24	l/28	l/10
Beams or ribbed one-way slabs	l/16	l/18.5	l/21	l/8

Deflections which occur immediately upon application of the load, the so-called instantaneous deflections, can be calculated by methods based on the elastic behavior of flexural members.

Due to shrinkage and creep, these instantaneous deflections continue to increase for some time after load application, at a decreasing rate, and the longtime deflections may exceed the instantaneous deflections by a large amount. For this reason, methods for estimating both the instantaneous and longtime deflections are essential.

When calculating the elastic instantaneous deflections, the main problem is the determination of the appropriate flexural rigidity EI for a member consisting of two materials with properties and behavior as widely different as steel and concrete.

If the maximum moment in a flexural member is sufficiently small so that the tension stress in the concrete is smaller than the flexural tensile strength, no tension cracks will occur and the full uncracked section is acting. In this stage, the moment of inertia is to be included in the calculations for the full concrete section plus n times the area of the steel reinforcement ($n = E_s/E_c$), and E is the modulus of elasticity of concrete E_c.

When significant changes of the effective cross section of a beam along the span length are involved, they should be included in calculating the deflections. In such cases, the best approach is to account for the variable flectural rgidity EI .

Many methods have been suggested for calculating the value of EI[1] proposing a single adjusted value for EI by one of the following methods: Fig. 3-4

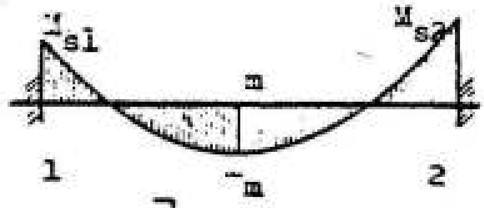


Fig. 3-4

1) Midspan value alone: $EI = (EI)_m$

This is the simplest method; the results are are within ± 20%.

2) Simple average:

$$EI = \frac{1}{2}\left[\frac{1}{2}(EI_1 + EI_2) + EI_m\right]$$

for a beam continuous at both ends 1 and 2; and

$$EI = \frac{1}{2}(EI_{1,2} + EI_m)$$

for a beam continuous at one end 1 or 2 only.

3) Weighted average:

$$EI = 0.7\ EI_m + 0.15\ (EI_1 + EI_2)$$

for a beam continuous at both ends 1 and 2; and

$$EI = 0.85\ EI_m + 0.15\ EI_{1,2}$$

for a beam continuous at one end 1 or 2 only.
For a single heavy concentrated load, only the midspan EI_m should be used.

Recently[2], the effect of load level and degree of cracking have been included in the calculation of I as follows:

$$I = \left(\frac{M_{cr}}{M_{max}}\right)^3 I_g + \left[1-\left(\frac{M_{cr}}{M_{max}}\right)^3\right] I_{cr} \leqslant I_g$$

where:

M_{cr} = cracking moment = $f_t\ I_g / y_t$

M_{max} = maximum service load moment acting at the condition under which deflection is computed,

I_g = gross moment of inertia (neglecting reinforcement),

I_{cr} = cracked transformed section moment of inertia,

f_t = unit tensile stress capacity in flexure. ACI-code gives:

$$f_t = 1.99\sqrt{f_{cp}}$$

y_t = distance from the N.A to extreme fiber of concrete in

1- Refer to ACI Jurnal Proceedings 70. December 1973.

2- Dan E. Branson "Instantaneous and time dependent deflection of reinforced concrete beams under working loads". Part 1, Report 7. Alabama Highway research report.

tension.

The modulus of elasticity of concrete E_c is assumed according to the ACI-Code equal to

$$E_c = 15100 \sqrt{f_{cp}}$$

For an excellant discussion of the deflection of concrete, refer to: "Reinforced Concrete Design" by Chu-Kia Wang and Charles G. Salmon. 3rd edition . Published by Harper and Row. New York and London.

As stated before, the deflections continue to increase for some time after load application due to shrinkage and creep. These additional long-time deflections may be obtained by multiplying the immediate deflection caused by the sustained part of the load by 2.0 when A'_s = 0; 1.2 when $A'_s = 0.5\ A_s$; and 0.8 when $A'_s = A_s$.

The effect of creep, shrinkage and compression steel on deflection has been more accurately evaluated by Branson* in the following manner:

$$\delta_{cr+sh} = k_s\ T\ (\delta_i)_d$$

where

δ_{cr+sh} = deflection due to creep and shrinkage

k_s = coefficient depending on ratio of comp. steel $\mu' = \frac{A'_s}{b\ d}$

$$k_s = 1/(1 + 50\ \mu')$$

T = time-dependent coefficient (creep + shrinkage), which may be taken from the following table,

$(\delta_i)_d$ = instantanious deflection due to dead load

Time-dependent Coefficient T Inkluding Both Creep and Shrinkage Effects for concrete members of common types, sizes and composition according to Branson*

Conc. stregth f_{cp} at 28 days	Average relative humidity, age when loaded								
	100 %			70 %			50 %		
	< 7d	14d	> 28d	< 7d	14d	> 28d	< 7d	14d	> 28d
170–280 kg/cm²	2.0	1.5	1.0	3.0	2.0	1.5	4.0	3.0	2.0
> 280 "	1.5	1.0	0.7	2.5	1.8	1.2	3.5	2.5	1.5

* Dan E. Branson:

1- "Design proceeduresfor computing deflections", ACI Journal. Proceedings, 65, September 1968.

2- " Compression steel effect on long-time deflections", ACI Jurnal. Proceedings, 68, August 1971.

It has to be noticed that the total working live load will act only infrequently on a member, and the sustained load deflection should be calculated for dead load plus some fraction of the working live load. The fractional part of the live load to be considered depends on the type of occupancy. For example, in c alculating long-time deflections for a beam carrying the floor of a residential appartment, one might reasonably consider about 20% of the live load to be sustained. Long-time deflections of a storage warehouse, may probably be based on 100% of design live load.

Branson suggests that the following percentages of the values in the previous table be used for sustained loads that are maintained for the periods indicated:

25% for 1 month or less
50% for 3 months
75% for 1 year
100% for 5 years or more

The maximum limits for immediate deflection due to live load computed as ab ove are:

1. For roofs which do not support plastered ceilings l/180
2. For roofs which support plastered ceilings or for floors which do not support partitions l/360
3. For a floor or roof construction intended to support or to be attached to partitions or other construction likely to be damaged by large deflections of the floor, the allowable limit for the sum of the immediate deflection due to live live load and the additional deflection due to shrinkage and creep under all sustained loads shall not exceed l/360

In normal cases, the UNESCO gives, for the control of deflections, the following ratio of span to effective depth $(l/d)_{max}$ as a factor of the maximum allowed deflection $(\delta/l)_{max}$ as follows:

$$\left(\frac{l}{d}\right)_{max} = \frac{4.5 \times 10^7}{f_y} \cdot \frac{1 - 2\mu_m}{1 + 2\psi} \left(\frac{\delta}{l}\right)_{max} \qquad 3\text{-}7$$

where

μ_m = mechanical ratio of main tension reinforcement $= \frac{A_s}{b\,d} \cdot \frac{\sigma_s}{\sigma_c} < 0.25$

σ_s and σ_c are the maximum allowed stresses in steel and concrete,

ψ = the ratio of the sustained load to the total load on the member considered. According to UNESCO:

ψ = 1/5 for service floors of buildings for public use,
,, = 1/4 ,, ,, ,, ,, ,, ,, private use, and
,, = 1/3 for roof structures of all buildings.

3.3 Working-stress Design Versus Modern Design Methods

Following are some reasons that called for introduction of modern design methods:

(1) Experimental investigations showed that analysis according to the elastic theory does not predict actual behavior especially at high stresses (stages II_n and III).

(2) Although the effects of shrinkage and creep influence the stress distribution significantly, their incorporation into the analysis requires knowledge of the complete history of events.

(3) By definition, a linearly elastic material does not exihibit any creep deformation. That is, the incorporation presents a deviation from straight-line theory of stress and strain.

(4) The ultimate strength of a structural member is more or less an invariant quantity that depends mainly on the geometry and reinforcement of a section and the mechanical properties of its materials.

(5) According to the method of elastic design, the idea of safety is ambiguous or at best simplistic as compared to the definite approach in the modern methods.

(6) The elastic theory does not allow for prediction of the ductility of a structural member. Consideration of ductility, however, is of a vital importance in the field of design for most dynamic effects.

(7) The method of elastic design leans heavily on the beam theory that neglects local effects of cracking. As a result, formulas for shear and bond yield misleading values.

(8) Effective distribution of reinforcement cannot be achieved without consideration of a more detailed analysis even for behavior in stage II . Modern methods recognise crack patterns in their recommendations for arrangement of reinforcement; this could not have been easily predicted from the elastic theory.

(9) It is true that the analysis of a section in stage II_n is more involved than that in stage II_l normally adopted in the elastic design. However, analysis according to stage III, which is used in modern theories, is usually simpler than that according to stage II_n.

The elastic theory has provided the means for designing strctures that are, so far, safe. However, lack of correlation with test results lead to introduction of many empirical formulas, which could ultimately lead to loss of insight.

3.4 Systematic Steps of Design

The following shows how to proceed systematically in the design of a reinforced concrete structure.

1. General lay-out of the structure

Assume that it is required to cover an area of ~15 x 20 ms without intermediate supports by a reinforced concrete flat roof having 5 ms minimum clear height if the soil at the site is sandy.

The roof being flat (chosen horizontal) and subject mainly to vertical dead and live loads, bending moments and shearing forces will be

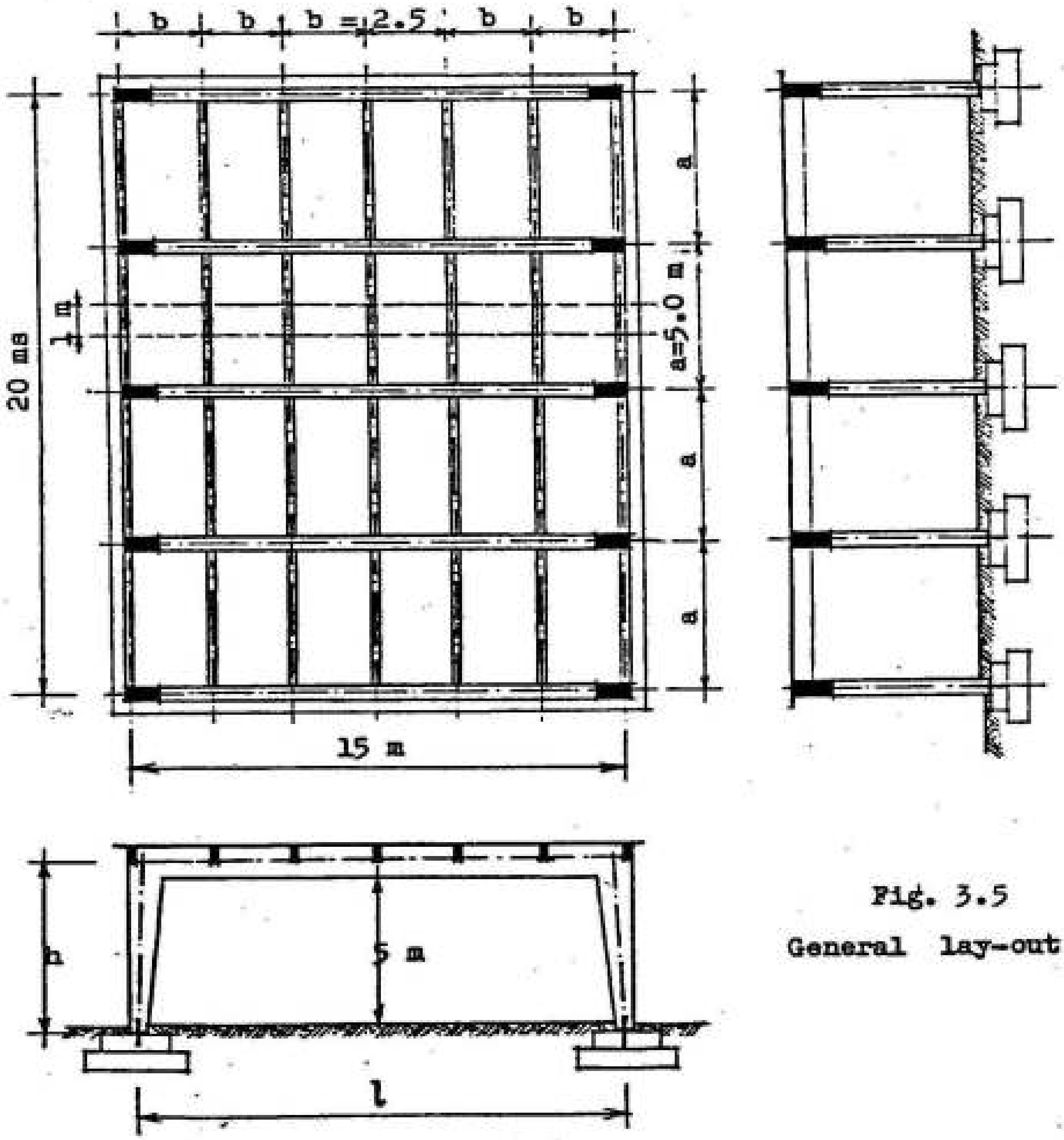


Fig. 3.5
General lay-out

created in the supporting elements of the roof. The bending moments in the main girders, spanning one of the two main dimensions of the area are proportional to the square of their span; hence it is more economic to choose the main girders spanning the smaller dimension of the area to be covered (i.e., in our case, 15 ms). In order to get a good distribution of the bending moments on the main girders and because the soil is sufficiently firm -sandy- a two hinged frame,-15 ms span and 5 ms clear height (distance between finished floor level -F.F.L.- and bottom of main girder) is chosen. The frames are to be arranged at distances giving reasonable loads and dimensions for the main frames and the other elements supported on them, the slabs and secondary beams, if any. A distance of 4 to 6 ms between the center lines of the frames is generally convenient (in our example, this distance is chosen 5 ms).

The frames are covered by a reinforced concrete slab whose own weight is 25 to 40 % of the loads acting on the frame and hence it is essential to choose its supporting elements such that its thickness is minimum (between 8 and 10 cms). In order to get this result, secondary beams are arranged at convenient distances, 2 to 5 ms, (in our example, 2.5 ms).

In this manner, the main supporting elements of our structure are: Fig. 3.5.

a) The slabs, supported in their shorter direction on the secondary beams, with a span b = 2.5 ms; in their longer direction, they are supported on the frames, with a span a = 5 ms. Such slabs, where $a/b > 2$, are called one-way slabs resisting all the loads acting on them in the shorter direction only. If $a/b < 2$, the slabs are called two-way slabs resisting the loads acting on them in both directions a and b.

b) The secondary beams: These are continuous beams of four equal spans each 5 ms, supporting, in addition to their own weight, the slab loads and transmitting them as concentrated to the main frames.

c) The main two-hinged frames having the following main dimensions: Fig. 3.6)

Span l = distance C.L to C.L hinges,

Height h = distance from C.L hinge to C.L of main girder.

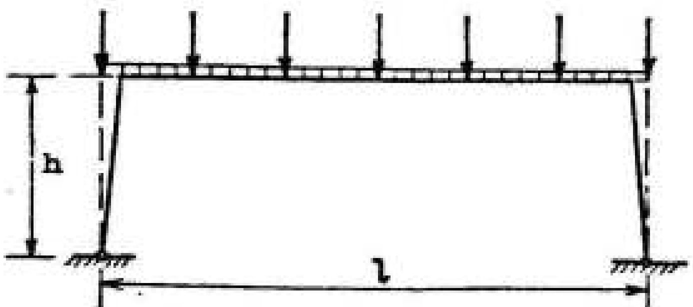


Fig. 3.6 Main frames

They carry mainly, in addition to their own weight, the reactions of the longitudinal secondary beams.

d) The foundations of the structure which must rest on a sufficiently firm stratum of the soil. If this stratum is near the ground surface, as in the case of our structure, surface foundations (e.g. isolated footings) are used. If it is deep in the ground, one of the systems of deep foundations (e.g. pile foundations) is recommended.

Having chosen the different supporting elements of the structure, a general lay-out (scale 1/100 or 1/200) giving the main concrete dimensions as that shown in Fig. 3.5 is to be drawn.

The design of each of the supporting elements must contain the following items:

The statical system, main dimensions and spans,
The loads and actions acting on the element,
The internal forces: max. values or absolute diagrams,
Dimensioning of critical sections and longitudinal steel,
Diagonal tension and web reinforcements, and
Detailing.

2. The statical system, main dimensions and spans

In order to be able to determine the internal forces of an element in a satisfactory manner, its statical system, main dimensions, spans, and eventually its flexural or torsional rigidity must be known in advance.

The span length, l , of members that are not built integrally with their supports shall be considered the clear span plus the depth of the slab or beam but shall not exeed the distance between centers of supports.

In analysis of continuous beams or frames, center to center distances shall be used in the determination of the internal forces. Moments at faces of supports may be used for design of beams and girders.

Solid or ribbed slabs with clear spans of not more than 3 ms that are built integrally with their supports may be considered as continuous slabs on knife edge supports with spans equal to the clear spans of the slab and the width of beams otherwise neglected.

3. The loads and actions

In order to satisfy the requirements given in 3.1, all structures shall be designed for all dead and live loads coming upon them.

The dead and superimposed permanent loads are to be calculated from the volume and the density of the relevant materials under the conditions of use. The following tables give the weight per cubic meter of some materials that may be required for design purposes.

Table 3.4 Weight per cubic meter and angle of repose of materials

Material	Weight kg/m^3	∢ of repose		Material	Weight kg/m^3
1- <u>Building materials</u>				2- <u>Storing mat. (contd)</u>	
Cement	1700	20		Benzine	600
Expanded clay	600	35		Bear and wine	1000
Gravel and sand	1800	30		Glycerine	1250
Gypsum (powder)	1500	25		Milk	1000
Homra (powder)	1500	20		Oils	1000
Iron slag	1800	40		Petroleum	800
Lime (stony)	700	45		Tar	1100
Hyd. lime (powder)	700	25		Paper (rolls)	1500
Limestone (powder)	1600	25		Paper (sheets) ...	1100
Pumice stone	900	35		Books and files ..	850
2- <u>Storing materials</u>				Books and files on shelves	600
Coal	1000	35		Cloth (folded) ...	1100
Coal (powder)	700	25		Clothes	300
Coke's coal	650	35		Fish (in boxes) ..	800
Corn, seeds & beans	800	25		Leather (rolled)	900
Flour	600	25		Drinks (in boxes)	850
Flour (in sacks) ...	500	—		Tobaco (packs) ...	500
Sugar	950	35		Fodder	400
Sugar (in sacks) ...	1600	—		Cork	300
Potatoes............	700	30		Glass (plates) ...	2500

The weight of the concrete and mortar, masonry, plaster, metals, timber and flooring are continued in the following table.

Table 3.4 Weight per cubic meter of materials (contd.)

Material	Weight kg/m^3
3- <u>Concrete and mortar</u>	
Gravel and sand, broken stone or slag concrete — not rfd	2300
Gravel and sand, broken stone or slag concrete — rfd	2500
Pumice or expanded clay conc. — not rfd	1400
Pumice or expanded clay conc. — rfd	1600
Cement or cement and homra mortar	2100
Lime and cement or lime and homra mortar	2000
Gypsum mortar witht. sand	1200
4- <u>Masonry</u>	
With Basalt	3000
,, Granite	2800
,, Marble or hard lime stone	2700
,, Ordinary lime stone	2400
,, Ordinary red bricks	1800
,, Hollow bricks	1200-1500
,, Lime stone bricks — solid	2000
,, Lime stone bricks — Hollow	1200
Increase in weight/m^3 masonry	
If weight 1500	200
If weight 1500-1700	100
If weight 1700	——

Material	Weight kg/m^3
5- <u>Metals</u>	
Lead	11400
Copper and Nickel	8900
Bronze and Brass	8500
Steel	7850
Cast iron	7250
Tin	7400
Zinc	7200
Magnesium	1850
6- <u>Timber</u>	
Pitch pine and Oak	800-900
Ordinary timber	600-700
7- <u>Plaster</u>	kg/m^2
Ordinary plaster	40
8- <u>Flooring</u> weight/cm	kg/m^2
Cement or concrete tiles	24
Asphalt tiles	22
Vinyl floors	13
Rubber	12
Slag (ground)	10
Timber	8
Cork	2
Foam plastic	0.5

Live loads: These comprise:

a) static superimposed loads, which vary only in a gradual manner, e.g., furniture, stored object of materials, etc.;

b) superimposed loads due to persons occupying or moving about in the rooms of the buildings.

To simplify the design calculations, these superimposed loads are assumed to be uniformly distributed. Their usual values are given in table 3.4. They include the dynamic coefficient for taking account of moving loads due to persons walking about.

c) dynamic superimposed live loads which produce a dynamic effect in the structure in consequence of movements and variation of the forces involved: persons, machinery, mobile equipment such as overhead travelling cranes, etc.

d) climatic superimposed loads due to the effects of wind, snow and earthquake actions.

a + b) Variable superimposed working live loads

The values for variable superimposed working live loads as indicated in table 3.5, are applicable to the design of ordinary residential buildings, office buildings or farm buildings, but not to civil engineering structures e.g. bridges, railway stations, etc.

It is recommended that the following values in which the dynamic coefficient is included, should be adopted in the calculations.

Table 3.5 Values of live loads in buildings

	Live load kg/m^2
a) Flat roofs:	
Inaccessible inclined roofs	50
Inaccessible horizontal roofs	100
Accessible roofs in private buildings	200
Accessible roofs in public buildings	500
b) Dwellings	200
c) Offices, classrooms, staircases and entrance halls of private buildings	300
d) Public rooms retail shops, restaurants, assembly rooms with fixed individual seatings and balconies of private buildings	400
e) Cinema theatres, dancing halls, amphitheatres, libraries, file rooms, bookshops, courts and balconies of public buildings	500
f) Assembly rooms without fixed seatings, passenger platforms	600
g) Baggage rooms, store-rooms, garages	750

Table 3.5 Values of live loads in buildings (contd.)

	Live load kg/m^2
h) Warehouses	1000
i) Freight platforms and houses, cotton loading platforms	1500

In case of inaccessible roofs, the live load carried by main girders covering an area more than 50 m^2 can be multiplied by 50 : the covered area. (e.g., for a main girder carrying an area of 80 m^2, the live load can be assumed as 50 : 80 = 62.5 % of the specified live load. This ratio should not be less than 50%.

Progressive reduction of live loads in multi-storey buildings

In a case where points of support carry the loads of several floors on which the maximum live loads are unlikely to be acting simultaneously (resdential buildings, offices, etc.), appropriately reduced values -as defined below- can permissibly be adopted in the design calculations for the load bearing members.

Let P_o be the live load upon the roof and let P_1, P_2, P_3 .... be the respective live loads upon the floors numbered 1, 2, 3, ..., starting from the top of the building; then the live loads for the successive floors are reduced in the following proportions:

total live load for the roof (no reduction)	P_o
,, ,, ,, ,, ,, top floor (no reduction)	P_1
,, ,, ,, ,, ,, floor directly below	0.90 P_2
,, ,, ,, ,, ,, next floor below this	0.80 P_3

and so on, applying a reduction of 10% per floor until a value of 0.5P is reached, which is thereafter retained for all the following floors below.

In residential buildings the reduction of the live loads should not be applied to any shop or office floors that may be present; for these the full live load should be adopted for designing the points of support.

In principle, no reduction of the live loads is allowed for the floors of warehouses, shops, schools or workshops.

Equivalent uniform load of lightweight partitions

To take account of the possibilities of changing the positions of lightweight demountable partitions, their effect can be allowed for by adding 75 to 125 kg/m^2 to the uniformly distributed live load if the weight of the partition varies between 100 and 150 kg/m^2.

If the live load on the floor under consideration is ≥ 500 kg/m^2, the load need not be increased for such partitions.

Horizontal forces on parapets

Balcony parapets, railings, handrails and their fixings should be designed to withstand a horizontal force applied at the top, this force being 80 kg/m for private buildings and 150 kg/m for those accessible to the public.

The static and elastic stability should be checked for this force by an amplification coefficient equal to 5/3.

c) Dynamic effect of moving live loads

Where moving live loads produce dynamic effects, and are supported by or communicated to a framework, allowance shall be made for such dynamic effects, including impact, by multiplying the live load value by $1 + \alpha$, where

$$\alpha = \frac{0.4}{1 + l/5} + \frac{0.6}{1 + G/P}$$

in which

l = free length of member carrying the dynamic load
G = total dead loads on member under consideration
P = total live and dynamic loads on member.

The Egyptian Code specifies that "unless otherwise specified, the following allowances (percent) shall be deemed to cover all forces set up by vibration, shock, kinetic action and impact":

Turbines, elevators 100%
Stationary vibrating machines 50%
Electric overhead cranes 25%
Hand-operated cranes 10%

d) Wind loads

The simplified rules given hereafter are applicable only to ordinary residential buildings, warehouses or factories comprising parallelepipedal blocks rectangular on plan, length a and breadth b, and of normal height h, consisting of substantially identical storeys.

For a structure connected by stiff walls and floors, no wind effects need to be considered except for specially tall buildings.

The effect of wind may be assumed as a uniform static pressure distributed over the area on the windward side equal to w_i which can be calculated from the relation:

$$w_i = 1.3\, k_r\, k_h\, w_d$$

in which

k_r = region coefficient as regards degree of exposure to wind,
k_h = height (relative to ground level) coefficient and
w_d = dynamic pressure of wind due to its velocity.

As regards k_r, one distinguishes three different regions as shown in table 3.6.

Table 3.6 Coefficient k_r in different regions

R e g i o n		k_r
I	Exposed to storms as sea shores, islands & top of hills	1.3
II	Moderately exposed to wind	1.0
III	Protected from storms by hills or standing structures	0.8

The height coefficient k_h for the parts of the structure higher than ground level by more than 10 ms can be determined from the relation:

$$k_h = 2.5\ (1 - \frac{42}{h + 60})$$

in which h is the height of the structure from ground level. For the lowest 10 ms assume $k_h = 1$.

The dynamic pressure w_d in kg/m^2 is a function of the wind velocity v in ms/sec and is given by:

$$w_d = v^2 / 16$$

Table 3.7 gives the wind velocities v and the corresponding dynamic pressure w_d.

Table 3.7 The dynamic pressure w_d for different wind velocities

Velocity of wind v								
	km / hour	50	75	100	125	150	175	200
	m / second	13.9	20.8	27.8	43.7	41.6	48.6	55.6
Dynamic pressure w_d	kg/m	12	27	48	75	108	148	193

Table 3.8 gives the classification of the regions according to the wind intensities

Region	Wind intensity	Wind velocity		Dynamic pressure w_d kg/m^2
		km / hour	m / second	
I	strong	150	41.6	108
II	moderate	125	34.7	75
III	weak	100	27.8	48

In one storey structures, where $h > 0.25\ b$, such as stores and factories, having inclined or curved roofs with α denoting the angle of the plane slope (in degrees), or of the tangent to the roof curve at its springing, the values of wind pressure relative to its dynamic pressure can be taken from table 3.9 in which, the positive sign is for normal pressure and the negative sign is for suction.

Table 3.9 Wind pressure as a factor of the slope angle of the roof

slope angle α in degrees	Values of the wind pressure relative to the dynamic pressure as a factor of the slope angle "α"	
	Windward side	Leeward side
20°	-0.7	-0.6
$20 - 30^\circ$	$-(2.1 - 0.07\alpha)$	-0.6
$30 - 60^\circ$	$+(0.9 - 0.03\alpha)$	-0.6
$> 60^\circ$	$+0.9$	-0.6

The Egyptian code specifies that: 'The wind load depends on the height and on the shape of exposed areas and it shall generally be calculated from the formula

$$w = c\ w_1$$

where w_1 is the pressure of wind per m^2 acting horizontally and it depends on the height of the building from the ground level. (Table 10)

Table 3.10 Wind pressure according to height (Egyptian Code)

Height of building m	0-8	8-20	20-100	100-200	200
Wind pressure w kg/m^2	50	75	100	125	150

These values could be reduced to one half, if sufficient data justify such a procedure.

c is a factor determined by aerodynamical tests according to the shape and size of the structure. If α is the angle of inclination of the roof to the horizontal, then:

$c = 1.2 \sin\alpha - 0.4$ for windward surface in case of sheds ,
$c = 1.6 \sin\alpha - 0.4$,, ,, ,, ,, ,, ,, towers,
$c = -0.4$,, leeward exposed surface.

For roofs repeated one after the other (as in saw-tooth structures) the first area exposed to wind shall receive the full wind pressure while the following areas shall carry only half the wind value.

The wind pressure on surfaces of revolution may be assumed 2/3 of the previous values.

Tall buildings (when $h > 2.5a$) should be specially investigated.'

4. Internal forces

All members of frames or continuous construction shall be designed to resist at all sections the maximum effects of the design loads as determined by the theory of elastic frames.

In the design of continuous construction, consideration may be limited to combinations of:

1) Design dead load on all spans with full design live load on two adjacent spans; (giving maximum connecting moment over support between the adjacent spans) and

2) Design dead load on all spans with full design live load on alternate spans; (giving maximum field moments).

In building frame analysis the live load may be considered to be applied only to the floor or roof under consideration, and the far ends of the columns may be assumed as fixed.

In normal buildings of usual types of construction, spans and storey heights, approximate methods for determining bending moments and shearing forces, given later, may be used.

Except where approximate values for bending moments are used, the nagative moments calculated by elastic theory at the supports of continuous flexural members for any assumed loading arrangement may each be increased or decreased by not more than

$$20\left(1-\frac{\mu-\mu'}{\mu_b}\right) \text{ per cent} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 3.8$$

These modified negative moments shall be used for calculations of the moments at sections within the spans. Such an adjustment shall be made only when the section, at which the moment is reduced is so designed that μ or $\mu - \mu'$ is equal to or less than $0.5\,\mu_b$ where μ_b is given by equations 2-69.

In case of design according to working stress methods, this modification in the connecting moment M may be limited to $\pm\,0.15$ M.

5. Dimensioning

Having determined the absolute maximum diagrams of the internal forces (thrust, bending moment, shearing force ... etc) acting on the element, the required concrete dimensions and steel reinforcement for specified known materials' qualities are then computed according to the method adopted for the design and based on the analysis given in chapter II.

It is allowed to make the design according to any of the methods stated at the beginning of this chapter, namely:

1) The working-stress design method

or one of the following two methods

2a) The ultimate-strength design method

2b) The limit-states design method.

The application of the methods 1 and 2a will be shown in the following simple example.

It is required to determine the concrete dimensions and main tension reinforcement of a simple beam 8 ms span, if it is subjected to uniformly distributed dead load g of 1.0 t/m' and live load p of 1.5 t/m'. The concrete strength is $f_{cp} = 200$ kg/cm^2 and the steel used is normal mild steel with $f_y = 2400$ kg/cm^2.

Max. B.M. due to dead load g:	$M_g = g\,l^2/8 = 1 \times 8^2/8$	$= 8$ mt
,, ,, ,, ,, live load p:	$M_p = p\,l^2/8 = 1.5 \times 8^2/8$	$= 12$ mt
,, S.F. ,, ,, dead load g:	$Q_g = g\,l/2 = 1 \times 8 / 2$	$= 4$ t
,, ,, ,, ,, live load p:	$Q_p = p\,l/2 = 1.5 \times 8 /2$	$= 6$ t

It has to be noted that the dimensions of the concrete sections as well as the area of the main tension reinforcement are generally governed by the normal stresses caused by the bending moments.

In simple bending, the equilibrium requires that:

$$C = T \qquad \text{and} \qquad C\,y_{ct} = T\,y_{ct} = M$$

i.e., we have two conditions of equilibrium and hence two unknowns only can be determined. All other values required for the design must either be known or assumed.

1) <u>Dimensioning according to the working-stress design method</u>

The most critical section is at mid-span as it is subject to the maximum moment of 20 mt under service loads.

The dimensions and reinforcement of the section must be so chosen that the maximum stress in concrete σ_c and in steel σ_s for the available materials' qualities under the effect of the maximum moment of 20 mt are not exceeded when the design is made according to stage II_1.

For concrete with $f_{cp} = 200$ kg/cm^2, the maximum allowable compressive stress in bending, according to table 3.2, is given by $\sigma_c = 80$ kg/cm^2

For mild steel with $f_y = 2400$ kg/cm^2, the maximum allowable stress is, according to table 3.1 , given by $\sigma_s = 1400$ kg/cm^2

Hence, for $M = 20$ mt , $\sigma_c = 80$ kg/cm^2 and $\sigma_s = 1400$ kg/cm^2 it is required to determine the concrete dimensions b and t, and the tension steel A_s.

In order to have two unknowns only, assume $b = 30$ cms.

a) Determination of position of neutral axis

Refering to equation 2-25 and Fig. 3.7

$$\frac{\sigma_c}{\sigma_s / n} = \frac{z}{d - z} \quad \text{where} \quad n = 15$$

Assuming $\frac{\sigma_s}{\sigma_c} = r$ and $z = \zeta d$,

we get: (equation 2-26)

$$\zeta = \frac{n}{r + n} \quad \ldots\ldots\ldots\ldots \quad 3\text{-}9$$

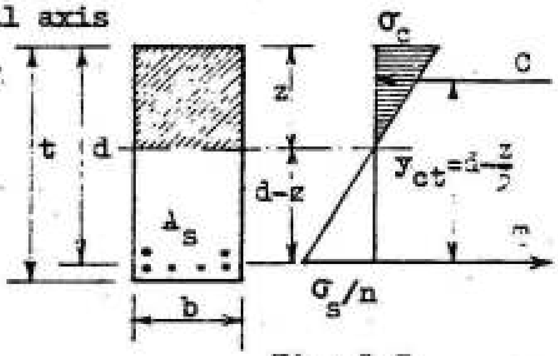


Fig. 3.7

b) The depth is determined from the condition of equilibrium:

$$C\, y_{ct} = M \qquad \text{or}$$

$$\sigma_c \frac{z}{2} b \left(d - \frac{z}{3}\right) = M \quad \text{or} \quad \sigma_c\, b \frac{d^2}{2} \zeta \left(1 - \frac{\zeta}{3}\right) = M \qquad \text{or}$$

$$\underline{d = c\sqrt{\frac{M}{\sigma_c\, b}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 3\text{-}10$$

where

$$c = \sqrt{\frac{2}{\zeta\left(1 - \frac{\zeta}{3}\right)}}$$

c) The area of the tension steel is determined from the condition:

$$T\, y_{ct} = M \qquad \text{or}$$

$$A_s\, \sigma_s \left(d - \frac{z}{3}\right) = M \quad \text{or} \quad A_s\, \sigma_s\, d\left(1 - \frac{\zeta}{3}\right) = M \qquad \text{or}$$

$$\underline{A_s = \frac{M}{\sigma_s\, \eta\, d}} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 3\text{-}11$$

where

$$\eta = 1 - \frac{\zeta}{3}$$

It has to be noted that c and η are dimensionless figures.

In the given example, we have

$$r = \frac{1400}{80} = 17.5, \quad \zeta = \frac{15}{15 + 17.5} = 0.462, \quad c = \sqrt{\frac{2}{0.462\left(1 - \frac{0.462}{3}\right)}}$$

i.e. $c = 2.26$ and $\eta = 1 - \frac{0.462}{3} = 0.846$

so that $d = 2.26\sqrt{\frac{20000}{80 \times .30}} = 65$ cms choose $t = 70$ cms and $$A_s = \frac{20000}{1400 \times 0.846 \times 0.65} = 26 \ \text{cm}^2$$

2a) Dimensioning according to ultimate-strength design method

The ultimate moment is given by:

$M_u = 1.5\ M_g + 1.8\ M_p$ or

$M_u = 1.5 \times 8 + 1.8 \times 12 = 33.6$ mt

The section is to be designed such that Ω. the ultimate resistance of the section is equal to M_u.

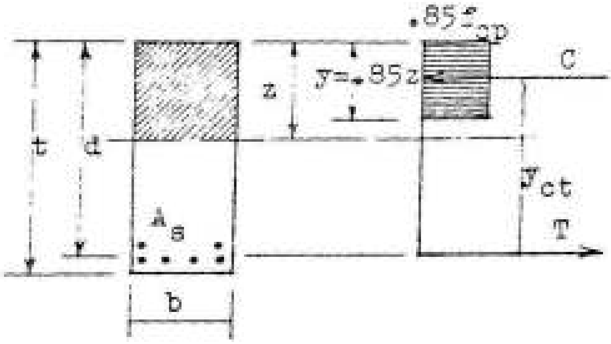


Fig. 3.8

a) The neutral axis is determined from the relation C = T or

$$0.85\ f_{cp}\ y\ b = A_s\ f_y$$

but $y = 0.85\ z$, $z = \zeta\ d$, $A_s = \mu\ b\ d$, and $f_y / f_{cp} = \varrho$ then

$$\zeta = \mu \varrho / 0.72 \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{3-12}$$

The balanced ratio of tension steel is given by equation 2-69a. Hence

$$\mu_b = \frac{0.72}{\varrho} \cdot \frac{6300}{6300 + f_y} = \frac{4536}{\varrho\ (6300 + f_y)} \quad \ldots\ldots\ldots\ldots \text{3-13}$$

In order to ensure ductile failure, it is recommended to choose a tensile steel ratio

$$\mu_{max} = 0.75\ \mu_b \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{3-14}$$

This ratio corresponds to the minimum depth of a section with tension steel only.

If the ratio of the tension steel μ is chosen $\leqslant 0.18/\varrho$, then the corresponding depth is generally convenient and is so big that the deflection of the beam need not be checked. Hence

$$\mu_{conv.} = 0.18/\varrho \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{3-15}$$

b) The depth can be determined from the relation: Fig. 3.8

$$M_u = \Omega\ C\ y_{ct}$$ or

$$M_u = \Omega\ C\ (d - y/2) = \Omega\ 0.85\ f_{cp}\ y\ b\ (d - y/2)$$

but $y = 0.85\ z = 0.85\ \zeta\ d$ and $\Omega = 0.9$ so that

$$M_u = 0.9 \times 0.85\ f_{cp} \times 0.85\ d^2\ b\ \zeta\ (1 - 0.425\ \zeta)$$ or

$$M_u = 0.648\ f_{cp}\ b\ d^2\ \zeta\ (1 - 0.425\ \zeta)$$ or

$$M_u = m_c\, f_{cp}\, b\, d^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 3\text{-}16$$

where

$$m_c = 0.648\, \zeta\, (1 - 0.425\, \zeta)$$

Therefore

$$d = c \sqrt{\frac{M_u}{f_{cp}\, b}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 3\text{-}17$$

where

$$c = \sqrt{\frac{1}{m_c}} = \sqrt{\frac{1}{0.648\, \zeta\, (1 - 0.425\, \zeta)}}$$

c) The area of steel can be determined from the relation:

$$M_u = \Omega\, T\, y_{ct} \qquad \text{or}$$

$$M_u = \Omega\, T\, (d - y/2) = 0.9\, A_s\, f_y\, (1 - 0.425\zeta)\, d \qquad \text{or}$$

$$M_u = A_s\, f_y\, \eta\, d \qquad \text{and}$$

$$A_s = \frac{M_u}{f_y\, \eta\, d} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 3\text{-}18$$

where

$$\eta = 0.9\, (1 - 0.425\zeta)$$

Applying these relations to the given example, we get:

$\varsigma = 2400 / 200 = 12$ and $\mu_b = \frac{4536}{12\,(5300 + 2400)} = 0.0434$

For minimum depth, we have:

$\mu = 0.75 \times 0.0434 = 0.0325$ & $\zeta = 0.0325 \times 12/0.72 = 0.542$,

corresponding $m_c = 0.648 \times 0.542\,(1 - 0.425 \times 0.542) = 0.27$

therefore $c = \sqrt{1/0.27} = 1.93$ chosen 2.00 and

$d_{min.} = 2.00 \sqrt{\frac{33600}{200 \times 0.3}} = 47.0$ cms choose $t = 53$ cms

We have further $\eta = 0.9\,(1 - 0.425 \times 0.542) = 0.693$

so that $A_s = \frac{33600}{2400 \times 0.693 \times 0.47} = 43\ \text{cm}^2$

For a convenient depth, choose $\mu = 0.18 / 12 = 0.015$

The corresponding $\zeta = 0.25$, $m_c = 0.145$, $c = 2.65$ and $\eta = 0.8$

Therefore

$d_{conv.} = 2.65 \sqrt{\frac{33600}{200 \times 0.3}} = 62.5$ cms choose $t = 68$ cms and

the corresponding area of steel is

$$A_s = \frac{33600}{2400 \times 0.80 \times 0.63} = 27.8\ \text{cm}^2$$

The design of other forms of sections under different kinds of internal forces, resistance of diagonal tension and detailing will be shown in the following chapters.

PART II

BEAMS, SLABS AND COLUMNS

CHAPTER 4

REINFORCED CONCRETE BEAMS

4.1 Statical System, Main Dimensions and Spans

Depending on the conditions at the supports, we recognize:

a) Simple beams, simple cantilevers and beams with overhanging cantilevers. Fig. 4-1 a.

b) Partially or totally fixed beams. Fig. 4-1 b.

c) Continuous beams or continuous frames. Fig. 4-1 c.

The framing action depends on the conditions at the supports, free or rigidly connected, the stiffness of the supports relative to that of the beam, conditions of loading .... etc.

Continuous beams supported on girders may be calculated according to the theory of continuous beams on free rigid supports.

d) Cantilever beams. Fig. 4-1 d.

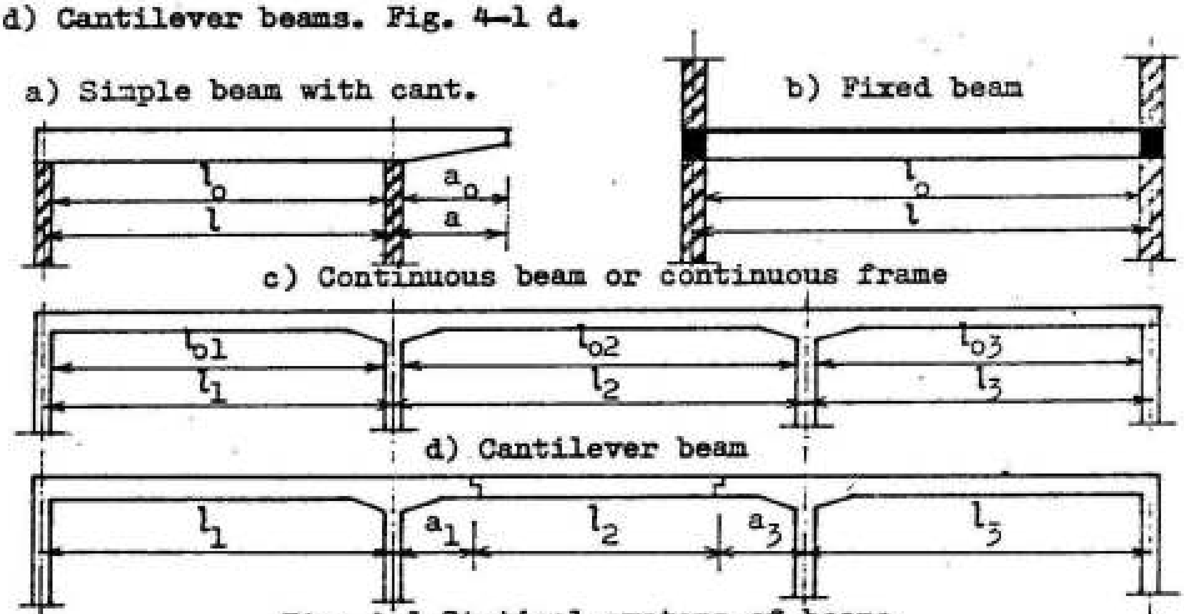


Fig. 4-1 Statical systems of beams

4.2 Loads

The beams carry their own weight, the slab loads and any other direct loads that may affect the design of the beam.

Slab load

If the beams carry one-way slabs (e.g. the secondary beams shown in

Fig. 3-3) then the slab load on any of the intermediate supports = load breadth (in the given example b) multiplied by the slab load p_s (dead or live).

If the beams carry two-way slabs (a/b of slab < 2), the load is resisted in the two directions a and b. The slab load transmitted to each beam can be calculated from the contributing areas shown in Fig. 4-2.

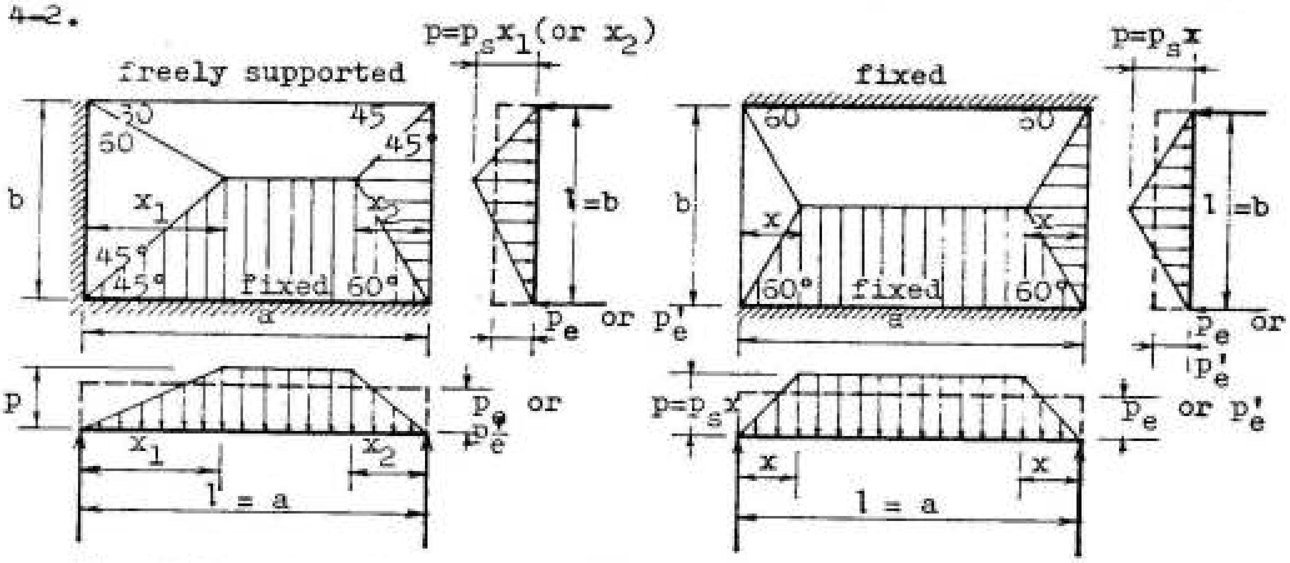


Fig. 4-2 Load on beams supporting two-way slabs with varying edge conditions

The lines defining the different areas bisect the corner between two edges having similar supporting conditions and are inclined 60° to the fixed edge if the other edge at the same corner is simply supported.

For two-way slabs with similar edge conditions, the slab load is, therefore, distributed on the edge beams as shown in Fig. 4-3.

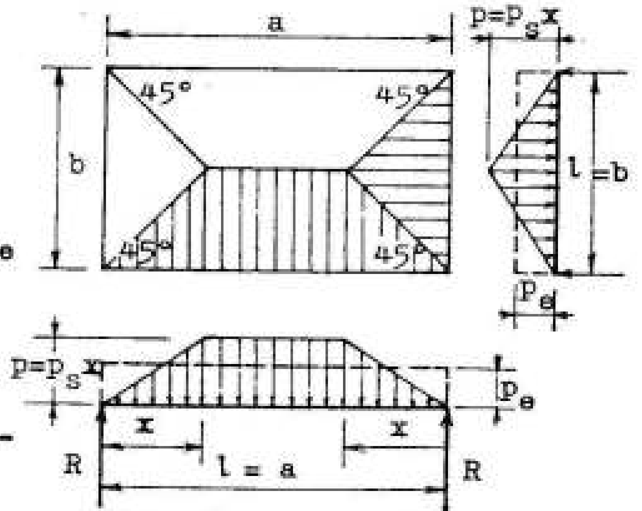


Fig. 4-3 Load on beams supporting two-way slabs with similar edge conditions

Such distribution means that the loads on beams supporting two-way slabs are either triangular or trapezoidal.

When calculating the bending moments, shearing forces or reactions for such loads, it is allowed to replace them by uniform loads covering the whole span and giving the same maximum value of the internal force under consideration.

In order to show the way in which this is done, let us first consider the bending moment and shearing force diagrams of a simple beam, span l, subject to a total load P acting symmetrically and distributed in the different ways shown in Fig. 4-4. It can be easily seen that the maximum bending moment increases with the concentration of the load towards the middle of the span while the maximum shearing force is the same in the four cases. Fig. 4-4.

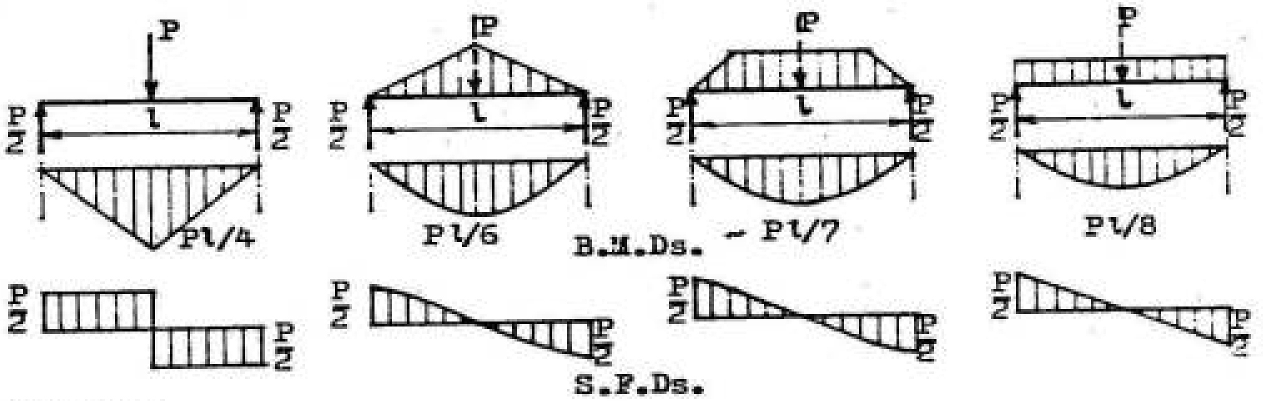


Fig. 4-4

An equivalent uniform load p_e replacing a triangular or a trapezoidal load with maximum load intensity p at the middle to give the same maximum moment in a simple beam symmetrically loaded, is given by the following relations: Fig. 4-3.

For a triangular load distribution:

$$p_e l^2 / 8 = p l^2 / 12 \qquad \text{or} \qquad p_e = 0.667\, p$$

For a trapezoidal load:

The reaction $R = \frac{p\,x}{2} + p\,(\frac{l}{2} - x) = \frac{p}{2}\,(l - x)$

The bending moment at the middle section

$$M = \frac{p}{2}\,(l - x)\,\frac{l}{2} - \frac{p\,x}{2}\,(\frac{l}{2} - \frac{2}{3}\,x) - \frac{p}{2}\,(\frac{l}{2} - x)^2 = \frac{p_e\, l^2}{8}$$

giving

$$p_e = p\left[1 - \frac{1}{3}\,(\frac{2\,x}{l})^2\right] = p \cdot (1 - \frac{1}{3\,s^2}) \qquad \text{or} \qquad p_e = \alpha\, p$$

where

$$s = l/2\,x \qquad \text{and} \qquad \alpha = (1 - \frac{1}{3\,s^2})$$

α is the coefficient of equivalent load for bending moments in beams with maximum load intensity in the middle.

In normal reinforced concrete structures, two-way slabs are generally assumed partially fixed and with similar supporting conditions on all four sides, so that the lines defining the different areas bisect the corners of the slab. In this case, the load distribution will be as shown in Fig. 4-3 and 2x equals the slab breadth b. Hence $l/2x = a/b$ = ratio of longer side to shorter side of slab under consideration.

For a normal square slab, the load on the beams will be triangular and $s = l/2x = a/b = 1$ gives $\alpha = 0.667$.

The reactions and maximum shearing forces in the four cases investigated in Fig. 4-4 are the same. Hence, the equivalent uniform load p'_e used for calculating the shearing force and reaction of beams supporting triangular or trapezoidal loads can be assumed equal to the average load, i.e.

equivalent load for shear and reaction = p'_e =

$$\frac{\text{load area x intensity of load}}{\text{loaded length}} = \beta p \qquad \text{or}$$

$$p'_e = \beta\, p$$

where β is the coefficient of equivalent load for shearing forces and reactions of beams.

For a triangular load $\beta = 0.50$

Values of α and β are given in table 4-1 and Fig. 4-5. (Sheet 9).

Table 4-1 Coefficients of equivalent loads on beams

$s=\frac{l}{2x}$	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
α	0.667	0.725	0.769	0.803	0.829	0.852	0.870	0.885	0.897	0.908	0.917
β	0.500	0.545	0.583	0.615	0.643	0.667	0.688	0.706	0.722	0.737	0.750

These coefficients may also be used for determining the bending moments, shearing forces and reactions of continuous beams.

For cantilevers, consider the actual load both for bending moments and shearing forces.

Wall loads

Only wall loads bounded by 60° lines from supports cause bending moments and shearing forces in the beam. Fig. 4-6.

The triangular or trapezoidal wall-load can be replaced by an equivalent uniform load giving the same maximum internal force. The coefficients α and β

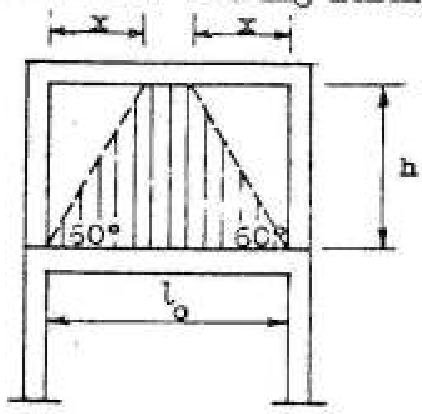


Fig. 4-6 Wall load

S H E E T   9

COEFFICIENTS GIVING EQUIVALENT AND AVERAGE UNIFORM LOADS ON BEAMS

SUPPORTING TWO-WAY SLABS

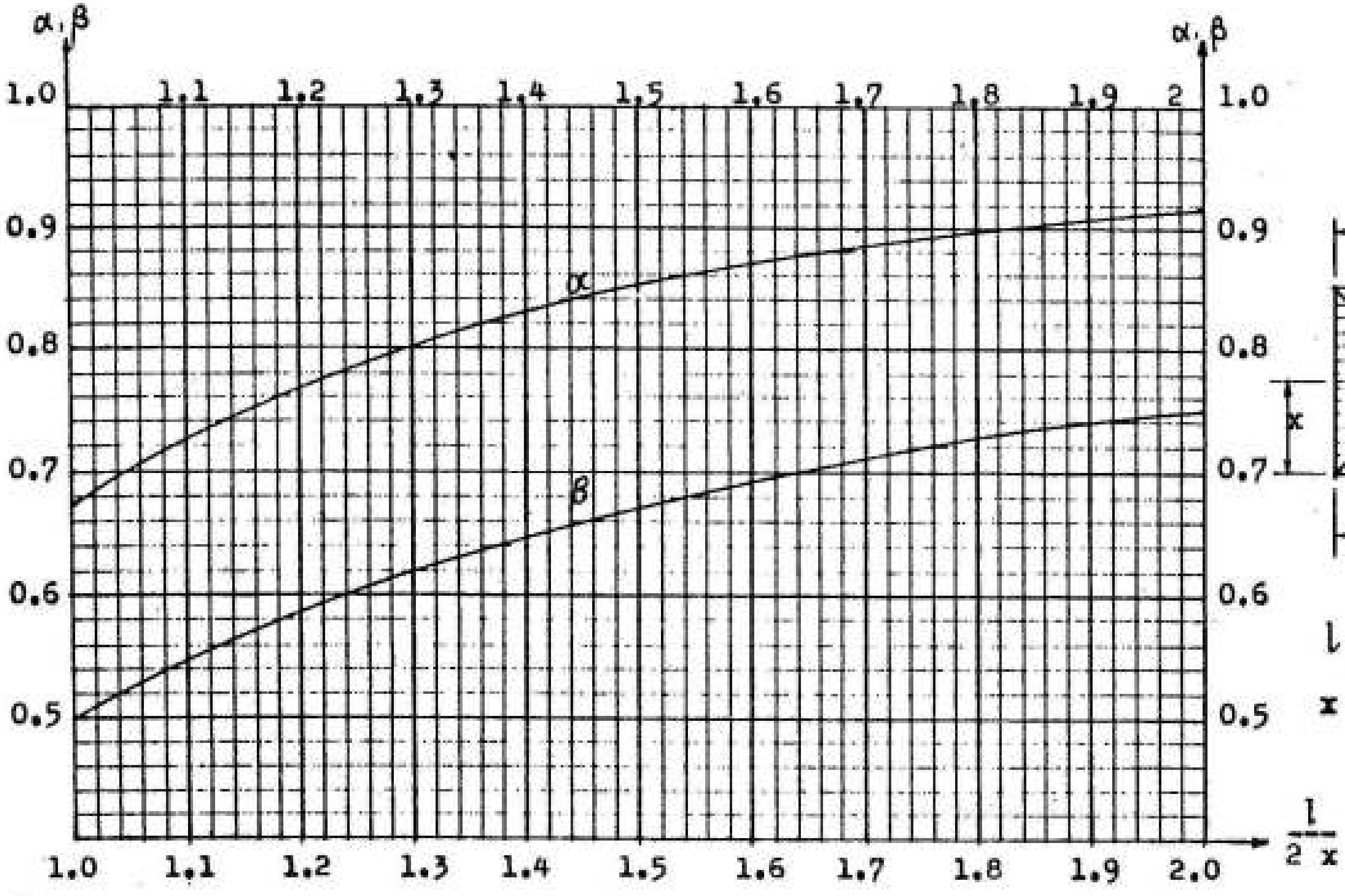
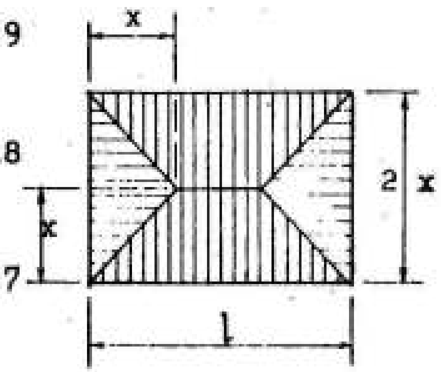


Fig. 4-5



l = span of beam

x = max. load breadth

Equivalent uniform load for calculating B.Ms. in beams = $\alpha w x$

Average uniform load for calculating S.Fs. in beams = $\beta w x$

depend on the ratio $s = l_o/2x$, where l_o is the clear span and $x = \frac{h}{\sqrt{3}}$ and can be determined from table 4-1 and Fig. 4-5.

When calculating the reactions of a wall load, its full weight is to be considered.

Illustrative examples

1) It is required to find the loads on the secondary beams and on one of the main intermediate frames of the roof structure shown in Fig. 3-5. Assuming

Slab loads: dead load $g_s = 400$ kg/m^2, live load $p_s = 200$ kg/m^2
Own weight of secondary beams = 200 kg/m'
Own weight of main frame .... = 1000 kg/m'
Slabs 5 x 2.50 ms may be assumed one-way slabs.

Loads on an intermediate secondary beam:

Dead load = own weight + slab dead load			
	200 + 2.5 x 400	or	g = 1200 kg/m'
Live load	2.5 x 200	or	p = 500 kg/m'
		total	w = 1700 kg/m'

Loads on an external secondary beam:

Dead load	200 + 1.25 x 400	or	g'= 700 kg/m'
Live load	1.25 x 200	or	p'= 250 kg/m'
		total	w'= 950 kg/m'

Loads on an intermediate main frame:

Uniformly distributed own weight			= 1000 kg/m'
Concentrated loads from intermediate secondary beams			
	1700 x 5	or	P = 8500 kgs
Concentrated loads from external secondary beams			
	950 x 5	or	P'= 4750 kgs

2) It is required to determine the loads on the secondary beams and intermediate main girder of the roof shown in Fig. 4-7. Assume

Slab loads: dead load $g_s = 400$ kg/m^2, live load $p_s = 200$ kg/m^2
Own weight of secondary beams = 200 kg/m'
Own weight of main girder ... = 500 kg/m'

Slabs with a = 5 ms, b = 3 ms and $s = a/b = 5/3 = 1.67 < 2$ are 2-way
Table 4-1 gives for s = 1.67 $\alpha = 0.88$ and $\beta = 0.70$

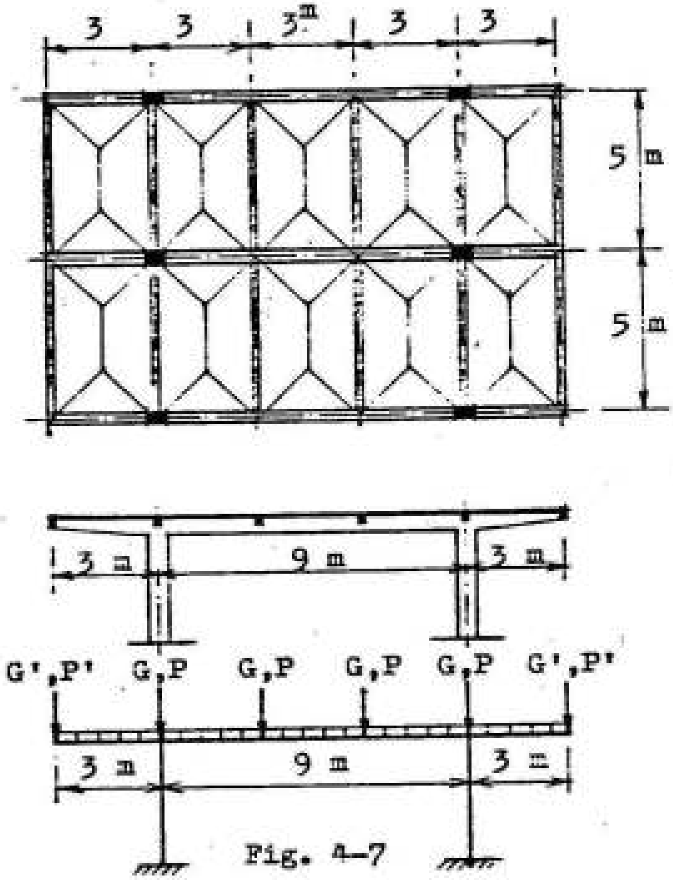


Fig. 4-7

Loads on an intermediate secondary beam:

For calculation of bending moments:

Dead load = own weight + slab dead load			
	200 + 0.88 x 3 x 400	or	g_e = 1256 kg/m'
Live load	0.88 x 3 x 200	or	p_e = 528 kg/m'
		total	w_e = 1784 kg/m'

For calculation of shearing forces and reactions:

Dead load	200 + 0.70 x 3 x 400	or	g_e' = 1040 kg/m'
Live load	0.70 x 3 x 200	or	p_e' = 420 kg/m'
		total	w_e' = 1460 kg/m'

Loads on an external beam:

For calculation of bending moments:

Dead load	200 + 0.88 x 1.5 x 400	or	g_e = 728 kg/m'
Live load	0.88 x 1.5 x 200	or	p_e = 264 kg/m'
		total	w_e = 992 kg/m'

For calculation of shearing forces and reactions:

Dead load	200 + 0.7 x 1.5 x 400	or	$g_e' = 620$ kg/m'
Live load	0.7 x 1.5 x 200	or	$p_e' = 210$ kg/m'
		total	$w_e' = 830$ kg/m'

Loads on intermediate main girder:

The slab load is transmitted to the main girder in two parts: The first part is transmitted directly and is composed of five triangular loads and the second part is transmitted indirectly as concentrated through the secondary beams and represents the reactions of the trapezoidal part of the slab load.

The slab load directly transmitted to the middle span of the main girder is composed of three triangular loads, i.e., the loads are not accumulating towards the middle of the span. Hence, the average load is to be used for computing both the bending moment and shearing force in the span of the main girder. Also for the cantilever arms of the main girder, the average load gives the same maximum bending moment and shearing force as the actual triangular load.

Hence, for calculating bending moments, shearing forces and reactions, we have:

Uniformly distributed dead load = 500 + 0.5 x 3 x 400 or $g = 1100$ kg/m'

Uniformly distributed live load = 0.5 x 3 x 200 or $p = 300$ kg/m'

Concentrated loads from intermediate secondary beams:

Dead load = 1.15* x 1040 x 5 or $G = 5980$ kgs

Live load = 1.15* x 420 x 5 or $P = 2415$ kgs

Concentrated loads from external secondary beams:

Dead load = 1.15* x 620 x 5 or $G' = 3565$ kgs

Live load = 1.15* x 210 x 5 or $P' = 1208$ kgs

3) It is required to calculate the dead and live loads on the beam c d e shown in Fig. 4-8. Assume:

Slab loads: dead load $g_s = 450$ kg/m^2, live load $p_s = 300$ kg/m^2

Own weight of beams a b and e f = 150 kg/m'

Own weight of beam c d e ...... = 300 kg/m'

---

* This figure is the continuity factor. (Refer to internal forces in 4-3)

Wall load on beam a b = 250 kg/m^2

Wall load on beam c d e = 500 kg/m^2

Height of walls = 3.0 ms

Reaction of beam a b at b:

For a/b = 5/4 = 1.25 β = 0.6

Dead loads = own weight + slab dead load

or [150 + (1 + 0.6 x 2) 450] 2.5

i.e. G = 2850 kgs

Live loads = L.L from slab + wall load

or [(1 + 0.6 x 2) 300 + 3 x 250] 2.5

i.e. P = 3525 kgs

Reaction of edge beam at e:

G'= (150 + 1 x 450) 5 or G' = 3000 kgs

P'= 1 x 300 x 5 or P' = 1500 kgs

Uniform loads on cantilever arm:

Dead load = own weight or g = 300 kg/m

Live load = wall load

= 500 x 3 or p = 1500 kg/m

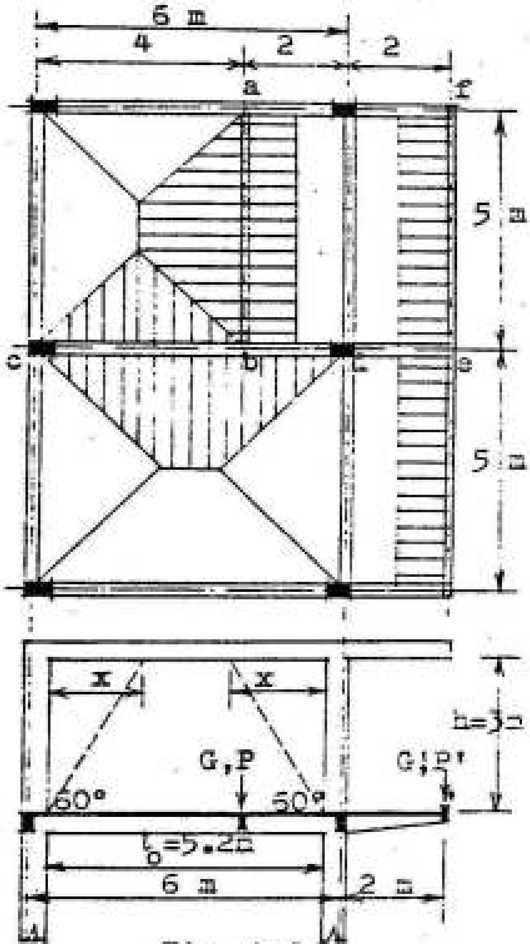


Fig. 4-8

Uniform loads on span c d: for a/b = 6/5 = 1.2 α = 0.769 & β = 0.583

The slab triangular load on c b being not maximum at the middle of the span, it will be considered with its average value both for bending moment and shearing force.

Loads for bending moment:

Dead load on c d: 300 + 0.769 x 2.5 x 450 or g_1 = 1165 kg/m'

Additional dead load on c b: .5 x 2 x 450 or g_2 = 450 kg/m'

Live loads on c d:

The wall load on c d will be assumed as live load. Only the part of the wall inside the dotted lines inclined 60° to the horizontal cause bending moments and shearing forces. Fig. 4-8. It will be replaced by a uniform load giving the same maximum internal forces and treated as trapezoidal load (table 4-1 and Fig. 4-5).

For $x = h/\sqrt{3} = 3/\sqrt{3} = 1.72$ ms $s = l_o / 2x = 5.2/2x1.72 = 1.50$

Table 4-1 gives $\alpha = 0.852$ & $\beta = 0.667$ so that

Live load on c d = 0.769 x 2.5 x 300 + 0.852 x 3 x 500 or

p_1 = 1857 kg/m'

Additional live load on c b = 0.5 x 2 x 300 or p_2 = 300 kg/m'

Loads for shearing force:

Dead load on c d = 300 + 0.583 x 2.5 x 450 or g_1' = 955 kg/m'

Additional dead load on c b = 0.5 x 2 x 450 or g_2' = 450 kg/m'

Live load on c d =0.583 x 2.5 x 300 + 0.667 x 3 x 500 or

p_1' = 1437 kg/m'

Additional live load on c b = 0.5 x 2 x 300 or p_2' = 300 kg/m'

When calculating the reaction of the wall load, its full weight is to be considered.

4.3 Internal Forces

In addition to the items given in 3.4 under 4), the following can be allowed.

1) The absolute internal forces (bending moments, shearing forces,.. etc.) and reactions can be determined in a continuous beam according to the known methods in theory of structures, e.g. the method of three moments, the moment distribution method, ... etc. assuming that the beam is supported on free rigid supports; i.e., the rigidity between the beam and the supporting elements as well as the settlement of the supports is neglected.

2) Connecting moments may be reduced due to the breadth of the supporting element according to a parabolic curve as shown in Fig. 4-9.

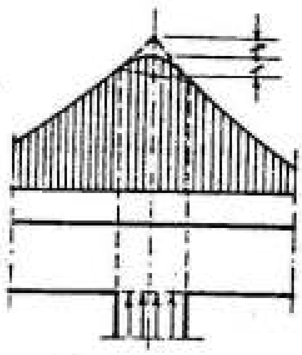


Fig. 4-9

3) The positive bending moments considered in the design of intermediate panels of continuous beams should not be smaller than the positive moments of the same span if it were totally fixed at the supports. This is because a positive bending moment smaller than that of total fixation means that the angles of rotation at the supports are negative (smaller than zero) which is not liable to take place in reinforced concrete structures due to the rigidity of the supports.

4) Negative field moments due to heavy live loads to be considered in the design may be taken with 2/3 calculated values due to the ri-

gidity of the supporting girders and columns.

5) The connecting moments of continuous beams as calculated by the theory of structures may be further reduced by a redistribution of the bending moments according to (4), article 3-4.

A redistribution is not allowed in the following cases:

i) If the bending moments are determined from an approximate or empirical method.

ii) If the bending moment is statically determinate.

iii) If condition 3.8 given in (4), article 3-4 is not satisfied (especially in case of design according to the ultimate strength method).

6) Internal forces and stresses in beams and cantilevers can be determined according to the known methods of simple plane structures if the dimensions of their cross section, breadth and depth, are small relative to their span.

If the depth is bigger than the span in simple cantilevers, 0.50 the span in simple beams or 0.40 the span in continuous beams, they have to be treated as "deep cantilevers or deep beams" and can be solved according to the recognized methods of the theory of elasticity.

Illustrative examples

1-It is required to determine the absolute maximum bending moment, shearing force - diagrams and reactions for the continuous beam shown in Fig. 4-10 due to the given dead and live loads. Assume that the beam is of constant moment of inertia.

Dead loads:
$g = 1$ t/m , $G = 3$ t
Live loads:
$p = 1.5$ t/m , $P = 6$ t

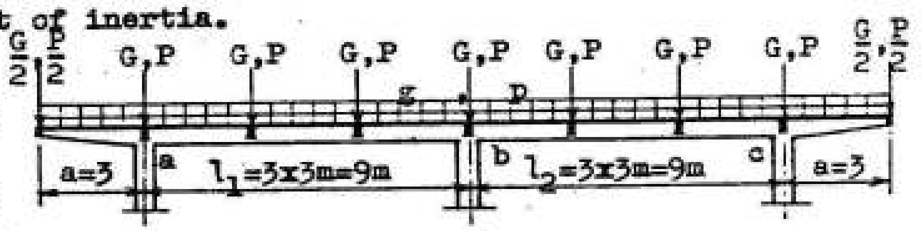


Fig. 4-10

Solution

The absolute maximum bending moment, shearing force-diagrams and reactions in such a continuous beam can be determined by superposition of the following cases:

i) Dead loads on all the beam,

ii) Live load on any of the cantilevers, and

iii) Live load on any of the spans a b or b c.

The equation of three moments will be used for determining the statically indeterminate connecting moment M_b. Accordingly

$$M_a\, l_1 + 2\, M_b\, (l_1 + l_2) + M_c\, l_2 = -\,6\, r_b$$

in which

$l_1 = l_2 = 9$ ms and

r_b = the elastic reaction at b so that

$$9\ M_a + 36\ M_b + 9\ M_c = -\ 6\ r_b$$

The shearing force in any statically indeterminate span will be calculated from the relation

$$Q = Q_o + \frac{M_r - M_l}{l}$$

in which

Q_o = the shearing force of the simple span,
M_r and M_l are the connecting moments at the right and left supports of the span l under consideration.

The reaction R of any support will be determined from the relation

$$R = Q_r - Q_l$$

in which

Q_r and Q_l are the shearing forces to the right and to the left of the support under consideration.

i) Case 1: Dead loads G and g on beam and cantilevers.

The connecting moment M_b:

$$M_a = M_c = -\ (\frac{G}{2}\ a + g\ \frac{a^2}{2}) = -\ (\frac{3 \times 3}{2} + \frac{1 \times 3^2}{2}) \qquad = \quad -\ 9 \text{ mt}$$

$$r_b = 2\ (\frac{G\ l^2}{9} + \frac{g\ l^3}{24}) = 2\ (\frac{3 \times 9^2}{9} + \frac{1 \times 9^3}{24}) \qquad = 114.75 \text{ m}^2\text{t}$$

So that

$$-\ 9 \times 9 + 36\ M_b - 9 \times 9 = -\ 6 \times 114.75 \qquad \text{or} \qquad M_b = -\ 14.6 \text{ mt}$$

The shearing forces:

$$Q_a \text{ left} = -\ (\frac{G}{2} + g\ a) = -(\frac{3}{2} + 3 \times 1) \qquad = -\ 4.5 \text{ t}$$

$$Q_a \text{ right} = (G + \frac{g\ l}{2}) + \frac{M_b - M_a}{l}$$

$$= 3 + \frac{1 \times 9}{2} + \frac{-\ 14.6 + 9}{9} \qquad = +\ 6.9 \text{ t}$$

$$Q_b \text{ left} = -\ 3 - \frac{1 \times 9}{2} + \frac{-\ 14.6 + 9}{9} \qquad = -\ 8.1 \text{ t}$$

The reactions not including the concentrated load over the support:

R_a = 6.9 + 4.5 = 11.4 tons = R_c

R_b = 2 x 8.1 = 16.2 tons

ii) <u>Case 2</u> : Live loads $\frac{P}{2}$ and p on left cantilever.

The connecting moment M_b:

$$9 M_a + 36 M_b + 0 = 0 \qquad \text{or} \qquad M_b = -\frac{M_a}{4}$$

in which

$$M_a = -\left(\frac{P}{2} a + \frac{p a^2}{2}\right) = -\left(\frac{6}{2} \times 3 + \frac{1.5 \times 3^2}{2}\right) = -15.75 \text{ mt}$$

So that

$$M_b = +3.94 \text{ mt}$$

The shearing forces:

$$Q_a \text{ left} = -\frac{6}{2} - 1.5 \times 3 = -7.50 \text{ t}$$

$$Q_{a-b} = \frac{+3.94 + 15.75}{9} = +2.19 \text{ t}$$

$$Q_{b-c} = \frac{0 - 3.94}{9} = -0.44 \text{ t}$$

The reactions:

$$R_a = 2.19 + 7.50 = +9.69 \text{ t}$$

$$R_b = -0.44 - 2.19 = -2.63 \text{ t} \qquad \text{and} \qquad R_c = +0.44 \text{ t}$$

iii) <u>Case 3</u> : Live loads P and p on a b.

The connecting moment M_b:

$$36 M_b = -6 \left(\frac{1.5 \times 9^3}{24} + \frac{6 \times 9^2}{9}\right) \qquad \text{or} \qquad M_b = -16.59 \text{ mt}$$

The shearing forces:

$$Q_a = (6 + 1.5 \times 4.5) - \frac{16.59}{9} = 12.75 - 1.84 = 10.91 \text{ t}$$

$$Q_b \text{ left} = -12.75 - 1.84 = -14.59 \text{ t} \qquad \text{and} \qquad Q_{b-c} = 1.84 \text{ t}$$

The reactions not including the direct concentrated load over the support:

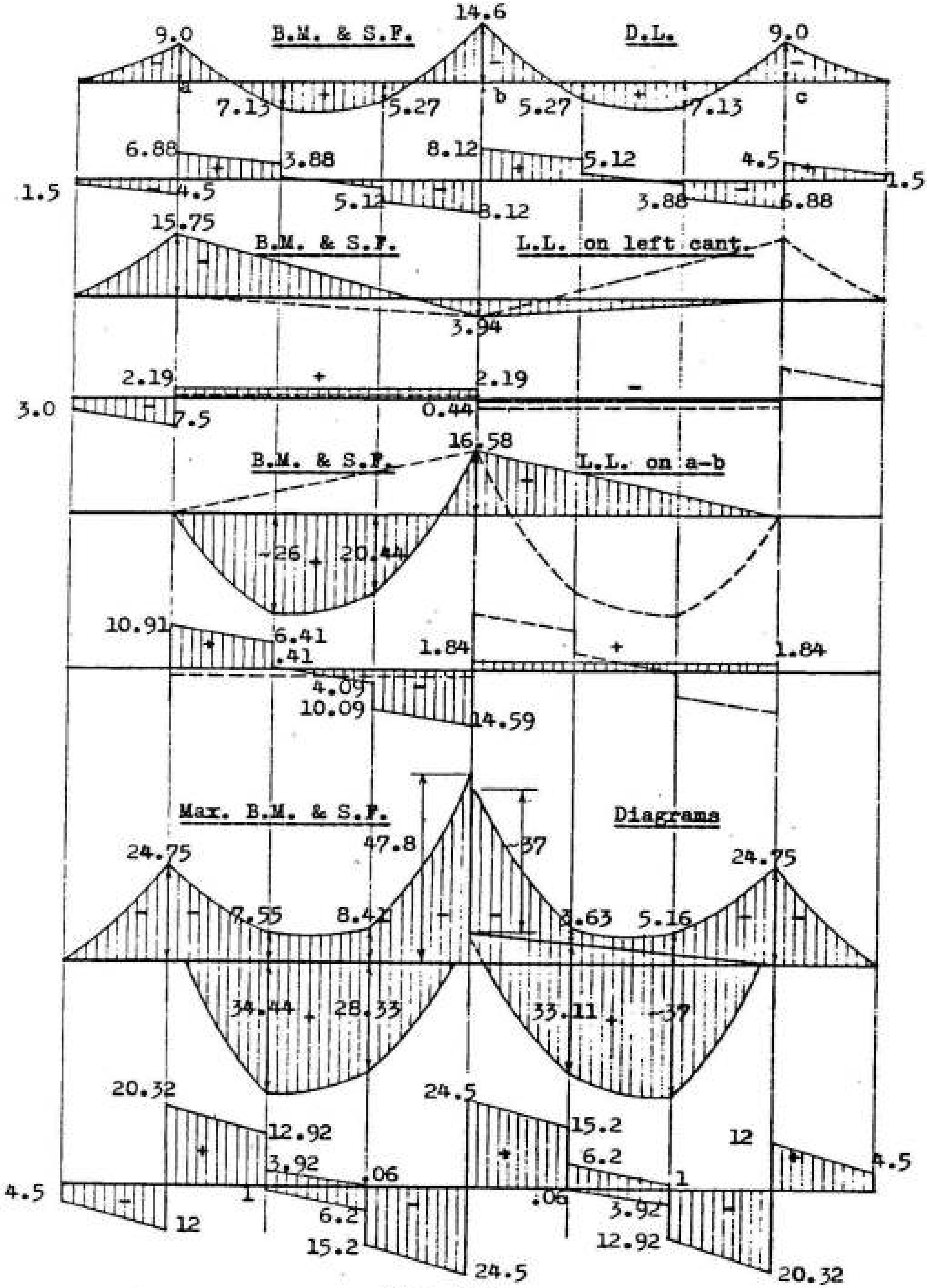
$$R_a = Q_a = 10.91 \text{ t} \qquad R_b = 1.84 + 14.59 = 16.43 \text{ t} \qquad R_c = -1.84 \text{ t}$$

The absolute maximum bending moment and shearing force can then be determined by superposition as shown in Fig. 4-11.

The maximum reactions are:

$$R_{a\,max} = R_{c\,max} = 11.38 + 9.69 + 0.44 + 10.91 = 32.42 \text{ tons}$$

$$R_{b\,max} = 16.24 + 2 \times 16.43 = 49.10 \text{ tons}$$


B.M. & S.F.
D.L.
9.0
14.6
9.0
a
b
c
7.13
5.27
5.27
7.13
6.88
3.88
8.12
5.12
4.5
1.5
4.5
5.12
8.12
3.88
6.88
1.5
B.M. & S.F.
L.L. on left cant.
15.75
3.94
2.19
2.19
3.0
7.5
0.44
B.M. & S.F.
L.L. on a-b
16.58
26
20.44
10.91
6.41
.41
1.84
1.84
4.09
10.09
14.59
Max. B.M. & S.F.
Diagrams
47.8
37
24.75
24.75
7.55
8.41
3.63
5.16
34.44
28.33
33.11
37
20.32
24.5
12.92
15.2
12
3.92
6.2
.06
1
4.5
4.5
12
6.2
15.2
24.5
.06
3.92
12.92
20.32
Fig. 4-11

The maximum calculated connecting moment of 47.8 mt can be reduced to ≃ 0.85 x 47.8 ≃ 40 mt by redistribution of the bending moments, i.e., by shifting the base line of the absolute bending moment diagram to the shown heavy line. Accordingly, the maximum field moment will be:

$$34.44 + \frac{0.15}{3} \times 47.8 \simeq 37 \text{ mt}$$

A further reduction of the maximum connecting moment can be achieved by assuming the bending moment over the intermediate support in the form of a parabolic curve.

Accordingly, this beam may be designed for the maximum moments:

$\underline{M_a = 24.75 \text{ mt}}$ $\underline{M_m \simeq 37 \text{ mt}}$ $\underline{M_b \simeq 37 \text{ mt}}$

Example 2

Assuming that the beam shown in Fig. 4-12 is of constant moment of inertia, it is required to determine the maximum bending moment and shearing force diagrams due to a dead load g and a live load p.

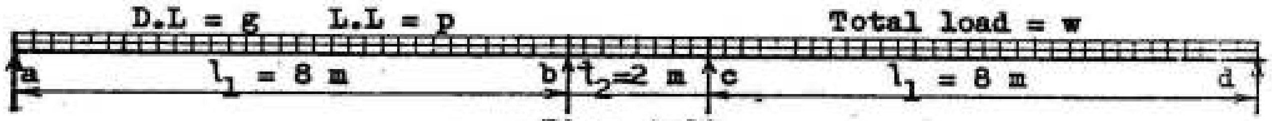
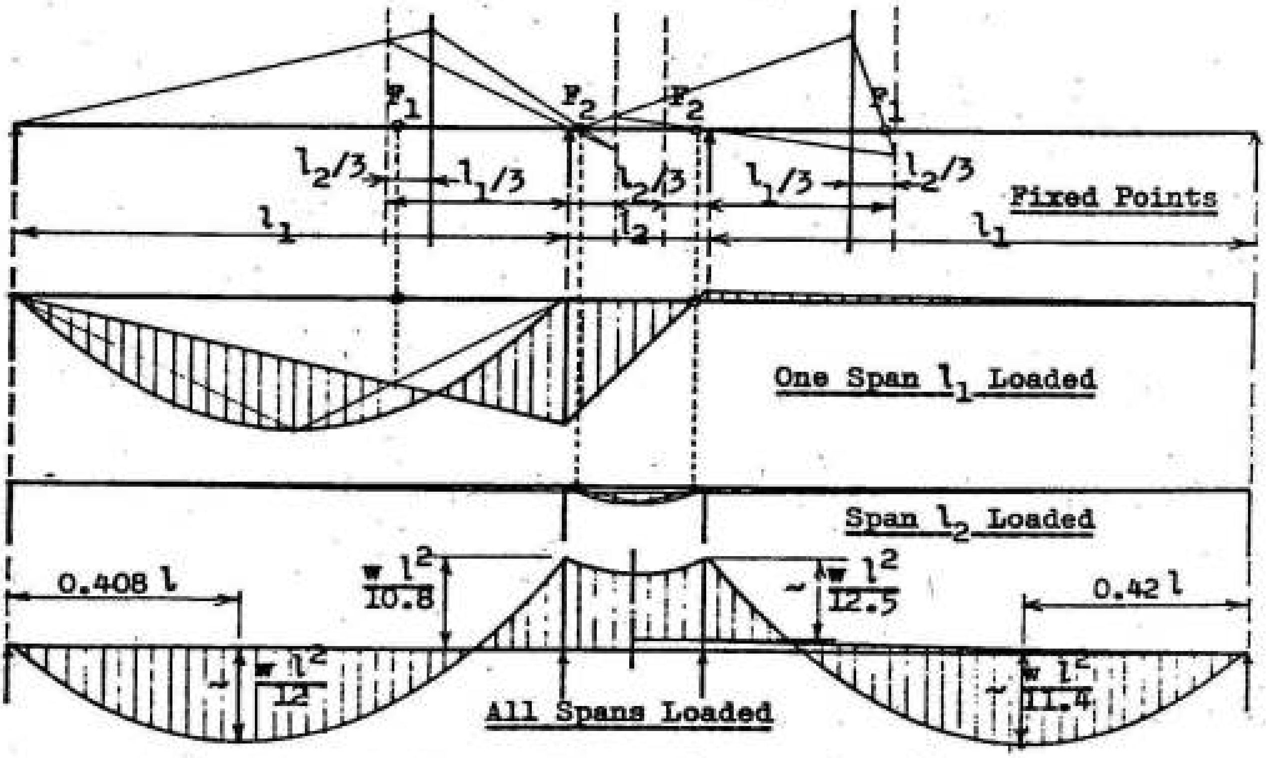


Fig. 4-12

Before determining the bending moments, it may be of advantage to determine the fixed points of the beam. Fig. 4-13.



Calculated B.M.D. Fig. 4-13 Re-distributed B.M.D.

From the shown construction, it can be easily seen that the fixed points F_1 are very near to the third lines which mean that the shorter span causes fixation to the longer one; whereas the fixed points F_2 are near the intermediate supports which mean that the shorter span behaves more or less as a simple beam.

The bending moments created from loading the shorter span or those trasmitted through it to other spans is relatively small, so that it is sufficient in such cases to solve the beam for one case of loading with all spans loaded. In this case, we get:

$$2 M_b (l_1 + l_2) + M_c l_2 = - 6 \left(\frac{w l_1^3}{24} + \frac{w l_2^3}{24}\right)$$

Due to symmetry, $M_b = M_c$. Assuming further that $l_2 = \alpha l_1$, then:

$$2 M_b (l_1 + \alpha l_1) + \alpha M_b l_1 = - 6 \left(\frac{w l_1^3}{24} + \frac{\alpha^3 w l_1^3}{24}\right)$$

giving

$$M_b = - \frac{w l_1^2}{4} \cdot \frac{1 + \alpha^3}{2 + 3\alpha} = - \frac{w l_1^2}{k}$$

in which

$$k = \frac{4 (2 + 3\alpha)}{1 + \alpha^3}$$

Assuming that $\bar{M}_b$ is the bending moment at b for the case of total fixation, we get:

α	0	0.1	0.15	0.2	0.25	0.3
k	8	9.19	9.77	10.32	10.83	11.30
$M / \bar{M}$	1	.871	.819	.775	.739	.708

Accordingly, the maximum connecting moment in the given example (α = 0.25) is given by

$$M_b \simeq - \frac{w l_1^2}{10.8}$$

and the maximum field moment by:

$$M_m \simeq + \frac{w l_1^2}{12}$$

at a distance equal to 0.408 l from exterior support.

It is however allowed to reduce the maximum connecting moment by - 15% of its value, so that it is allowed to assume:

$$M_b \simeq - 0.85 \frac{w l_1^2}{10.8} = - \frac{w l_1^2}{12.5}$$

in which case

$$M_m \simeq + \frac{w l_1^2}{11.4}$$

at a distance equal to 0.42 l from exterior support. Fig. 4-13.

If the beam were continuous with a short span on both sides and proceeding in the same way as in the previous case, we get for a uniform load: (Fig. 4-14)

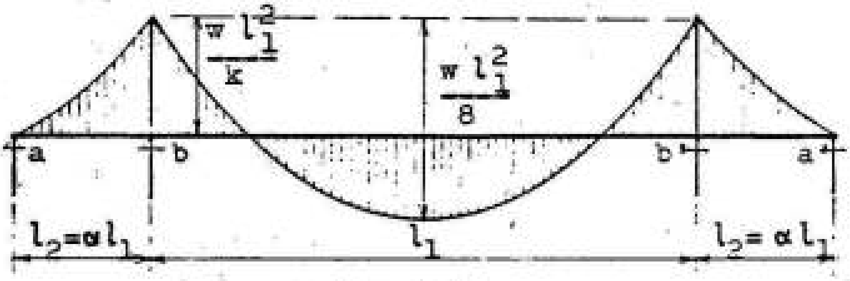


Fig. 4-14

$$2 M_b (l_1 + l_2) + M_b l_1 = - 6 \left(\frac{w l_1^3}{24} + \frac{w l_2^3}{24}\right)$$

or

$$2 M_b l_1 (1 + \alpha) + M_b l_1 = - \frac{w l_1^3}{4} (1 + \alpha^3)$$

or

$$M_b = - \frac{w l_1^2}{4} \cdot \frac{1 + \alpha^3}{3 + 2\alpha} = - \frac{w l_1^2}{k}$$

in which

$$k = \frac{4 (3 + 2\alpha)}{1 + \alpha^3}$$

Accordingly, we have:

α	0	0.10	0.15	0.20	0.25	0.30
k	12	12.79	13.16	13.49	13.79	14.02
$M / \bar{M}$	1	0.938	0.912	0.889	0.871	0.856

A moment re-distribution up to 15% is allowed as has been shown in the previous case.

The bending moments, shearing forces and reactions in a continuous beam with two, three, four and five equal spans subject to uniform dead load g and live load p are shown in tables 4-2 A. Beams subject to symmetrical concentrated dead loads G and live loads P are given in tables 4-2 B.

If the number of equal spans is bigger than five, assume the internal forces of the two exterior spans equal to those of the five - span beam, and the internal forces of all interior spans equal to those of the middle span of the five-span beam.

With the help of these tables, it is possible to draw the absolute maximum bending moment and shearing force diagrams of a continuous beam of constant moment of inertia with equal spans.

Table 4-2. Values of bending moments and shearing forces in continuous beams of constant moment of inertia for 2, 3, 4 and 5 spans

A. Uniform loads

1. Two equal spans

$\frac{x}{l}$	Bending moment			$\frac{x}{l}$	Shearing force		
	D.L. g	L.L. p			D.L. g	L.L. p	
	M_g	max. $+M_p$	max. $-M_p$		Q_g	max. $+Q_p$	max. $-Q_p$
0	0	0	0	0	+ 0.3750	0.4375	0.0625
0.2	+ 0.0550	0.0675	0.0125	0.1	+ 0.2750	0.3437	0.0687
0.4	+ 0.0700	0.0950	0.0250	0.2	+ 0.1750	0.2624	0.0874
0.6	+ 0.0450	0.0825	0.0375	0.3	+ 0.0750	0.1932	0.1182
0.8	− 0.0200	0.0300	0.0500	0.5	− 0.1250	0.0898	0.2148
.85	− 0.0425	0.0152	0.0577	0.7	− 0.3250	0.0287	0.3537
.90	− 0.0675	0.0061	0.0736	0.8	− 0.4250	0.0119	0.4369
.95	− 0.0950	0.0014	0.0964	0.9	− 0.5250	0.0027	0.5277
1.0	− 0.1250	0	0.1250	1.0	− 0.6250	0	0.6250
	$g\,l^2$	$p\,l^2$	$p\,l^2$		$g\,l$	$p\,l$	$p\,l$

max. $R_a = 0.375\,g\,l + 0.4375\,p\,l$ max. $R_b = 1.250\,(g + p)\,l$

2. Three equal spans

	$\frac{x}{l}$	M_g	max. $+M_p$	max. $-M_p$	$\frac{x}{l}$	Q_g	max. $+Q_p$	max. $-Q_p$
First Span	0.2	+ 0.0600	0.0700	0.0100	0	+ 0.40	0.4500	0.0500
	0.4	+ 0.0800	0.1000	0.0200	0.1	+ 0.30	0.3560	0.0563
	0.6	+ 0.0600	0.0900	0.0300	0.2	+ 0.20	0.2752	0.0752
	0.8	0	0.0402	0.0402	0.3	+ 0.10	0.2065	0.1065
	.85	− 0.0212	0.0277	0.0490	0.5	− 0.10	0.1042	0.2042
	.90	− 0.0450	0.0204	0.0645	0.7	− 0.30	0.0443	0.3443
	.95	− 0.0712	0.0171	0.0883	0.8	− 0.40	0.0280	0.4280
	1.0	− 0.1000	0.0167	0.1167	0.9	− 0.50	0.0193	0.5191
Second span	.05	− 0.0762	0.0141	0.0903	1.0	− 0.60	0.0167	0.6167
	.10	− 0.0550	0.0151	0.0701	0	+ 0.50	0.5833	0.0833
	.15	− 0.0362	0.0205	0.0568	0.1	+ 0.40	0.4870	0.0870
	.20	− 0.0200	0.0300	0.0500	0.2	+ 0.30	0.3991	0.0991
	.30	+.005 | .022 x	0.0550	0.0500	0.3	+ 0.20	0.3210	0.1210
	.50	+.025 | .042 x	0.0750	0.0500	0.5	0	0.1979	0.1979
		$g\,l^2$	$p\,l^2$	$p\,l^2$		$g\,l$	$p\,l$	$p\,l$

max. $R_a = 0.40\,g\,l + 0.45\,p\,l$ max. $R_b = 1.10\,g\,l + 1.20\,p\,l$

x Min. value of field moment (case of total fixation)

4-2 A Values of B.Ms. & S.Fs. in cont. beams due to uniform loads

3. Four equal spans

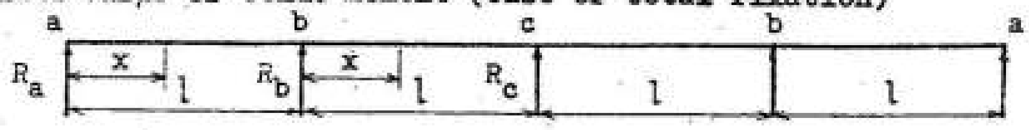
Span	x/l	Bending moment D.L. g M_g	Bending moment L.L. p max. $+M_p$	Bending moment L.L. p max $-M_p$	x/l	Shearing force D.L. g Q_g	Shearing force L.L. p max. $+Q_p$	Shearing force L.L. p max. $-Q_p$
First span	0	0	0	0	0	+ 0.3929	0.4464	0.0535
	0.2	+ 0.0586	0.0693	0.0107	0.1	+ 0.2929	0.3528	0.0599
	0.4	+ 0.0771	0.0986	0.0214	0.2	+ 0.1929	0.2717	0.0788
	0.6	+ 0.0557	0.0879	0.0321	0.3	+.0.0929	0.2029	0.1101
	0.8	− 0.0057	0.0374	0.0431	0.5	− 0.1071	0.1007	0.2079
	.85	− 0.0273	0.0248	0.0522	0.7	− 0.3071	0.0410	0.3481
	.90	− 0.0514	0.0163	0.0677	0.8	− 0.4071	0.0247	0.4319
	.95	− 0.0780	0.0139	0.0920	0.9	− 0.5071	0.0160	0.5231
	1.0	− 0.1071	0.0134	0.1205	1.0	− 0.6071	0.0134	0.6205
Second span	.05	− 0.0816	0.0116	0.0932	0	+ 0.5357	0.6027	0.0670
	.10	− 0.0586	0.0145	0.0721	0.1	+ 0.4357	0.5064	0.0707
	.15	− 0.0380	0.0198	0.0578	0.2	+ 0.3357	0.4187	0.0830
	.20	− 0.0200	0.0300	0.0500	0.3	+ 0.2357	0.3410	0.1053
	.40	+.027 | .037*	0.0736	0.0464	0.5	+ 0.0357	0.2150	0.1833
	.50	+.036 | .042*	0.0804	0.0446	0.7	− 0.1643	0.1435	0.3078
	.60	+.034 | .037*	0.0771	0.0429	0.8	− 0.2643	0.1222	0.3855
	.80	+ 0.0014	0.0417	0.0403	0.9	− 0.3643	0.1106	0.4743
	.85	− 0.0130	0.0345	0.0475	1.0	− 0.4643	0.1071	0.5714
	.90	− 0.0300	0.0310	0.0610		g l	p l	p l
	.95	− 0.0495	0.0317	0.0812				
	1.0	−.071 | .083*	0.0357	0.1071				
		$g\,l^2$	$p\,l^2$	$p\,l^2$				

max R_a = 0.393 g l + 0.446 p l

max R_b = 1.143 g l + 1.223 p l

max R_c = 0.929 g l + 1.143 p l

* Minimum value of bend. moment (case of total fixation)



4-2 A Values of B.Ms. & S.Fs. in cont. beams due to uniform loads

4. Five equal spans

Bending moment

Span	$\frac{x}{l}$	D. L. g M_g	L.L. p max. $+M_p$	L.L. p max. $-M_p$
First span	0	0	0	0
	0.2	+ 0.0589	0.0695	0.0105
	0.4	+ 0.0779	0.0989	0.0211
	0.6	+ 0.0568	0.0884	0.0316
	0.8	− 0.0042	0.0381	0.0423
	0.9	− 0.0497	0.0183	0.0680
	1.0	− 0.1053	0.0144	0.1196
Second span	0.1	− 0.0576	0.0140	0.0717
	0.2	− 0.0200	0.0300	0.0500
	0.4	+.025 / .037*	0.0726	0.0474
	0.5	+.033 / .042*	0.0789	0.0461
	0.6	+.031 / .037*	0.0753	0.0447
	0.8	− 0.0042	0.0389	0.0432
	0.9	− 0.0366	0.0280	0.0646
	1.0	−.079 / .083*	0.0323	0.1112
Third span	0.1	− 0.0339	0.0293	0.0633
	0.2	+ 0.0011	0.0416	0.0405
	0.4	+ 0.0411	0.0805	0.0395
	0.5	+ 0.0461	0.0855	0.0395
		$g\,l^2$	$p\,l^2$	$p\,l^2$

Shearing force

$\frac{x}{l}$	D.L. g Q_g	L.L. p max. $+Q_p$	L.L. p max. $-Q_p$
0	+ 0.3947	0.4474	0.0526
0.1	+ 0.2947	0.3537	0.0590
0.2	+ 0.1947	0.2726	0.0779
0.3	+ 0.0947	0.2039	0.1091
0.5	− 0.1053	0.1017	0.2067
0.7	− 0.3053	0.0419	0.3472
0.8	− 0.4053	0.0257	0.4309
0.9	− 0.5053	0.0169	0.5222
1.0	− 0.6053	0.0144	0.6196
0	+ 0.5263	0.5981	0.0718
0.1	+ 0.4263	0.5018	0.0755
0.2	+ 0.3263	0.4141	0.0878
0.3	+ 0.2263	0.3364	0.1101
0.5	+ 0.0263	0.2146	0.1882
0.7	− 0.1737	0.1391	0.3128
0.8	− 0.2737	0.1179	0.3916
0.9	− 0.3737	0.1063	0.4800
1.0	− 0.4737	0.1029	0.5766
0	+ 0.5000	0.5909	0.0909
0.1	+ 0.4000	0.4944	0.0944
0.2	+ 0.3000	0.4063	0.1063
0.3	+ 0.2000	0.3279	0.1279
0.5	0	0.2045	0.2045
	$g\,l$	$p\,l$	$p\,l$

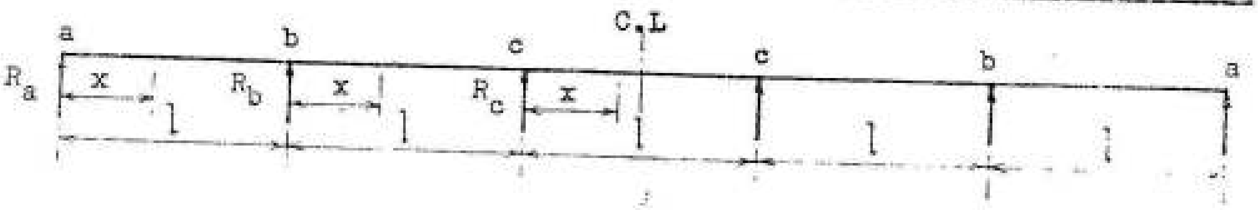
max. R_a = 0.395 g l + 0.447 p l

max. R_b = 1.132 g l + 1.218 p l

max. R_c = 0.974 g l + 1.168 p l

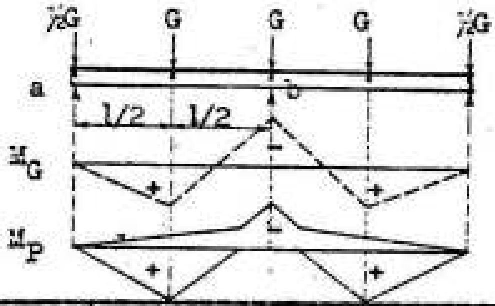
Note:

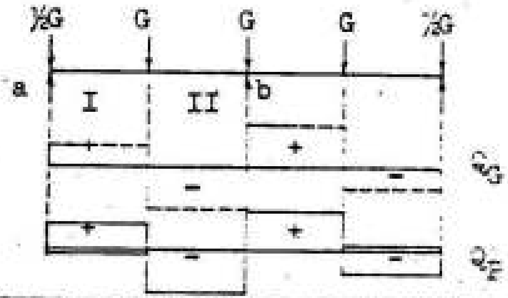
* Minimum value of bend. moment. (case of total fixation)



B. Concentrated loads

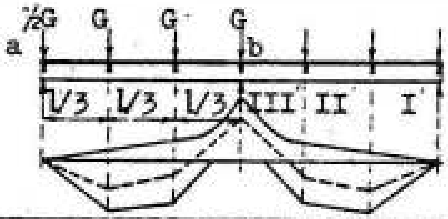
1-a. Two equal spans with concentrated loads at middle of spans





$\frac{x}{l}$	Bending moment D.L. G M_G	L.L. P max. $+M_P$	L.L. P max. $-M_P$	Part	Shearing force D.L. G Q_G	L.L. P max. $+Q_P$	L.L. P max. $-Q_P$
0.50	+ 0.1562	0.2032	0.0468	I	+ 0.3125	0.4062	0.0938
.842	- 0.0789	0	0.0789	II	- 0.6875	0	0.6875
0.90	- 0.1188	0	0.1188		G	P	P
1.00	- 0.1875	0	0.1875	max. R_a = 0.8125 G + 0.9062 P			
	G l	P l	P l	max. R_b = 2.3750 G + 2.3750 P			

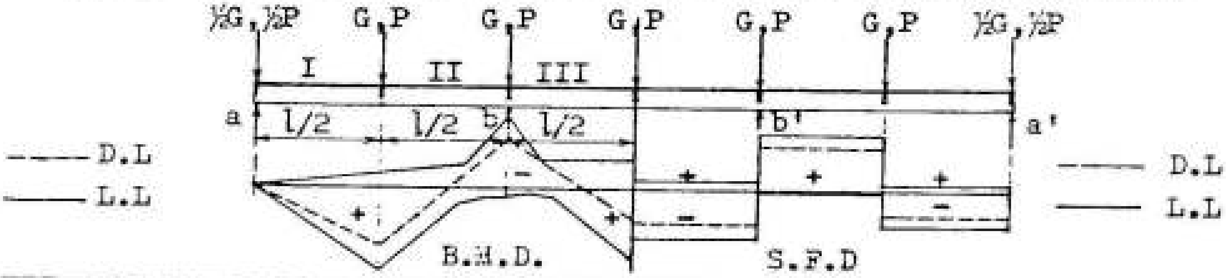
1-b. Two equal spans with concentrated loads at third points of spans



$\frac{x}{l}$	Bending moment D.L. G M_G	L.L. P max. $+M_P$	L.L. P max. $-M_P$	Part	Shearing force D.L. G Q_G	L.L. P max. $+Q_P$	L.L. P max. $-Q_P$
.333	+ 0.2222	0.2778	0.0555	I	+ 0.6667	0.8331	0.1667
.667	+ 0.1111	0.2223	0.1110	II	- 0.3333	0.1650	0.4980
.800	- 0.0667	0.0667	0.1332	III	- 1.3333	0	1.3333
.850	- 0.1333	0.0171	0.1503		G	P	P
.900	- 0.2000	0	0.2000				
.950	- 0.2664	0	0.2667				
1.00	- 0.3333	0	0.3333	max R_a = 1.1667 G + 1.3331 P			
	G l	P l	P l	max R_b = 3.6667 G + 3.6667 P			

4-2 B. Values of B.Ms. & S.Fs. in cont. beams due to concent. loads

2-a. Three equal spans with concent. loads at middle of spans



Span	$\frac{x}{l}$	Bending moment D.L. G M_G	Bending moment L.L. P max. $+M_P$	Bending moment L.L. P max. $-M_P$	Part	Shearing force D.L. G Q_G	Shearing force L.L. P max. $+Q_P$	Shearing force L.L. P max. $-Q_P$
1st span	0.5	+ 0.1750	0.2124	0.0374	I	+ 0.3500	0.4250	0.0750
	0.8	− 0.0200	0.0400	0.0600	II	− 0.6500	0.0250	0.6750
	0.9	− 0.0850	0.0224	0.1076	III	+ 0.5000	0.6250	0.1250
	1.0	− 0.1500	0.0250	0.1750		G	P	P
2nd span	0.1	− 0.1000	0.0124	0.1124				
	0.2	− 0.0500	0.0250	0.0750				
	0.5	+.100 | .125*	0.1750	0.0750		max. R_a = 0.850 G + 0.925 P		
		G	P	P		max. R_b = 2.150 G + 2.300 P		

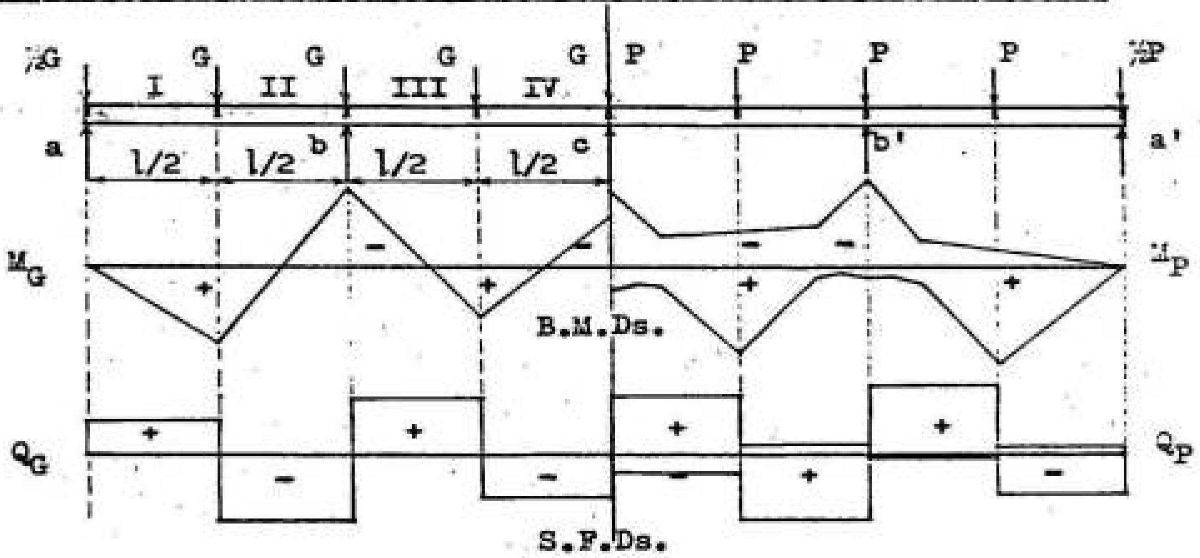
2-b. Three equal spans with concent. loads at third points of spans

span	$\frac{x}{l}$	Bending moment D.L. G M_G	Bending moment L.L. P max. $+M_P$	Bending moment L.L. P max. $-M_P$	Part	Shearing force D.L. G Q_G	Shearing force L.L. P max. $+Q_P$	Shearing force L.L. P max. $-Q_P$
1st span	0.33	+ 0.2444	0.2889	0.0444	I	+ 0.7333	0.8667	0.1332
	0.67	+ 0.1556	0.2442	0.0888	II	− 0.2667	0.2040	0.4707
	0.80	− 0.0133	0.0933	0.1065	III	− 1.2667	0.0444	1.3110
	0.90	− 0.1400	0.0399	0.1800	IV	+ 1.0000	1.2222	0.2222
	1.00	− 0.2667	0.0444	0.3111	V	0	0.4539	0.4539
2nd span	0.10	− 0.1667	0.0222	0.1890		G	P	P
	0.20	− 0.0667	0.0667	0.1335				
	0.33	+.067 | .111*	0.2000	0.1335				
	0.50	+.067 | .111*	0.2000	0.1335		max. R_a = 1.233 G + 1.367 P		
		G	P	P		max. R_b = 3.267 G + 3.533 P		

* Min. value of field moment (case of total fixation)

4-2 B. Values of B.Ms. & S.Fs. in cont. beams due to concent. loads

3-a. Four equal spans with concent. loads at middle of spans



Span	$\frac{x}{l}$	Bending moment D.L. G M_G		L.L. P max. $+M_P$	L.L. P max $-M_P$
First span	0.5	+ 0.1696		0.2098	0.0402
	0.8	− 0.0286		0.0358	0.0644
	0.9	− 0.0946		0.0180	0.1126
	1.0	− 0.1607		0.0200	0.1808
Second span	0.1	− 0.1053		0.0100	0.1154
	0.2	−.050	.025*	0.0250	0.0750
	0.5	+.116	.125*	0.1830	0.0670
	0.8	−.018	.025*	0.0424	0.0602
	0.9	−.062	.075*	0.0376	0.1000
	1.0	−.107	.125*	0.0536	0.1608
		G		P	P

Part	Shearing force D.L. G Q_G	L.L. P max. $+Q_P$	L.L. P max. $-Q_P$
I	+ 0.3393	0.4198	0.0804
II	− 0.6607	0.0200	0.6808
III	+ 0.5536	0.6540	0.1004
IV	− 0.4464	0.1608	0.6072
	G	P	P

max. R_a = 0.8393 G + 0.9198 P

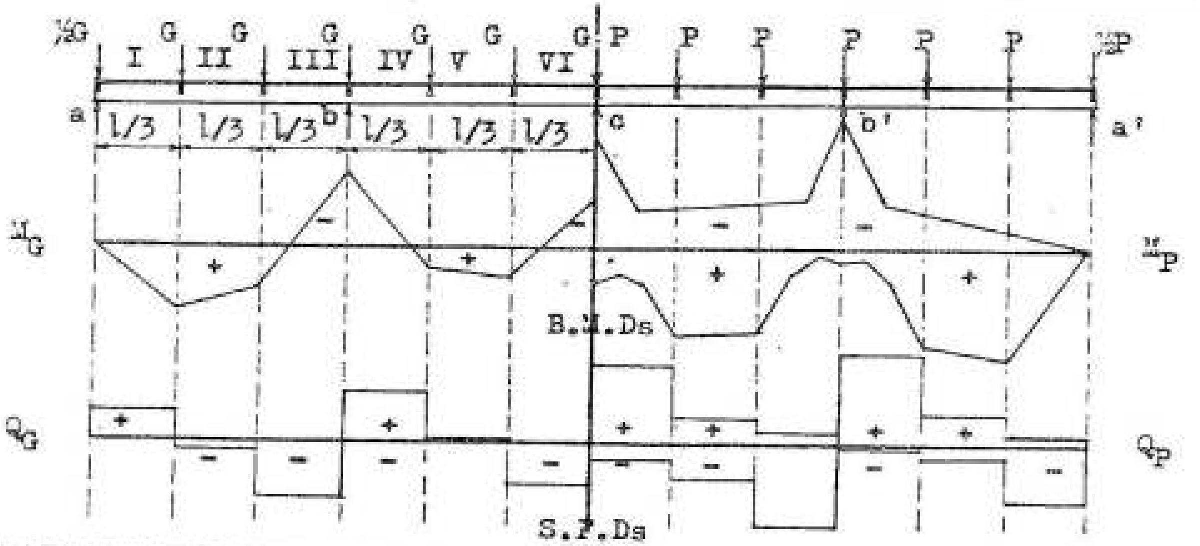
max. R_b = 2.2143 G + 2.3348 P

max. R_c = ·1.8928 G + 2.2144 P

* Minimum value of bending moment; (case of total fixation)

4-2 B. Values of B.Ms. & S.Fs. in cont. beams due to concent. loads

3-b. Four equal spans with concent. loads at third points of spans



Span	$\frac{x}{l}$	Bending moment D.L. G M_G		L.L. P max. $+M_P$	L.L. P max. $-M_P$
First span	.333	+ 0.2381		0.2859	0.4477
	.667	+ 0.1429		0.2382	0.0954
	.800	− 0.0285		0.0858	0.1143
	.900	− 0.1571		0.0321	0.1893
	1.00	− 0.2856		0.0357	0.3213
Second span	.100	− 0.1761		0.0177	0.1938
	.200	−.067	.022*	0.0669	0.1332
	.333	+.080	.111*	0.2064	0.1269
	.667	+ 0.1114		0.2223	0.1110
	.800	−.009	.022*	0.0978	0.1071
	.900	−.100	.122*	0.0669	0.1665
	1.00	−.190	.222*	0.0954	0.2856
		G		P	P

Part	Shearing force D.L. G Q_G	L.L. P max. $+Q_P$	L.L. P max. $-Q_P$
I	+ 0.1744	0.8571	0.1428
II	− 0.2856	0.1950	0.4806
III	− 1.2856	0.0357	1.3212
IV	+ 1.0954	1.2738	0.1785
V	+ 0.0954	0.5106	0.4152
VI	− 0.9046	0.2859	1.1904
	G	P	P

max. R_a = 1.2144 G + 1.3571 P

max. R_b = 3.3810 G + 3.5950 P

max. R_c = 2.8092 G + 3.3808 P

* Minimum values of bending moment; (case of total fixation)

Illustrative Example

As an example for the application of tables 4-2, the method for determining the absolute maximum bending moment, shearing force-diagrams and reactions of the beam shown in Fig. 4-10 will be illustrated: (Fig. 4-15).

1) Determine the dead load, positive live load and negative live load-bending moment and shearing force diagrams as well as reactions due to the uniform dead load g of 1 ton per meter and the live load p of 1.5 ton per meter using table 4-2. A. case 1. (Fig. 4-15, 1-a and b). Accordingly, we have:

If $g\,l^2 = 1 \times 9^2 = 81$ mt and $p\,l^2 = 1.5 \times 9^2 = 121.5$ mt

then, we get, for example:

for $x/l = 0.4$	for $x/l = 1$
$M_g = + 0.07 \times 81 = 5.67$ mt	$- 0.125 \times 81 = 10.125$ mt
$+ M_p = 0.095 \times 121.5 = 11.55$,,	0
$- M_p = 0.025 \times 121.5 = 3.04$,,	$0.125 \times 121.5 = 15.20$,,

and for $g\,l = 1 \times 9 = 9$ t and $p\,l = 1.5 \times 9 = 13.50$ t,

then, we get:

for $x/l = 0$	for $x/l = 1$
$Q_g = + 0.375 \times 9 = 3.375$ t	$- 0.625 \times 9 = 5.625$ t
$+ Q_p = 0.4375 \times 13.5 = 5.906$ t	0
$- Q_p = 0.0625 \times 13.5 = 0.844$ t	$0.625 \times 13.5 = 8.438$ t

The reactions for this case are:

$$\max R_a = 0.375\,g\,l + 0.4375\,p\,l = 0.375 \times 9 + 0.4375 \times 13.5 = 9.28 \text{ t}$$

$$\max R_b = 1.25\,(g + p)\,l = 1.25 \times (1 + 1.5) \times 9 = 28.125 \text{ t}$$

2) The dead load, positive live load and negative live load-bending moment and shearing force diagrams as well as reactions due to the concentrated dead loads G = 3 ton and live loads P = 6 ton can be determined in the same way using table 4-2. B. case 2-b. (Fig. 4-15, 2-a and b).

3) The bending moments, shearing forces and reactions due to cantilever loads are to be treated from the first principles, as has been shown in example 1 (Fig. 4-10). In this case (Fig. 4-15, 3-a and b), we have:

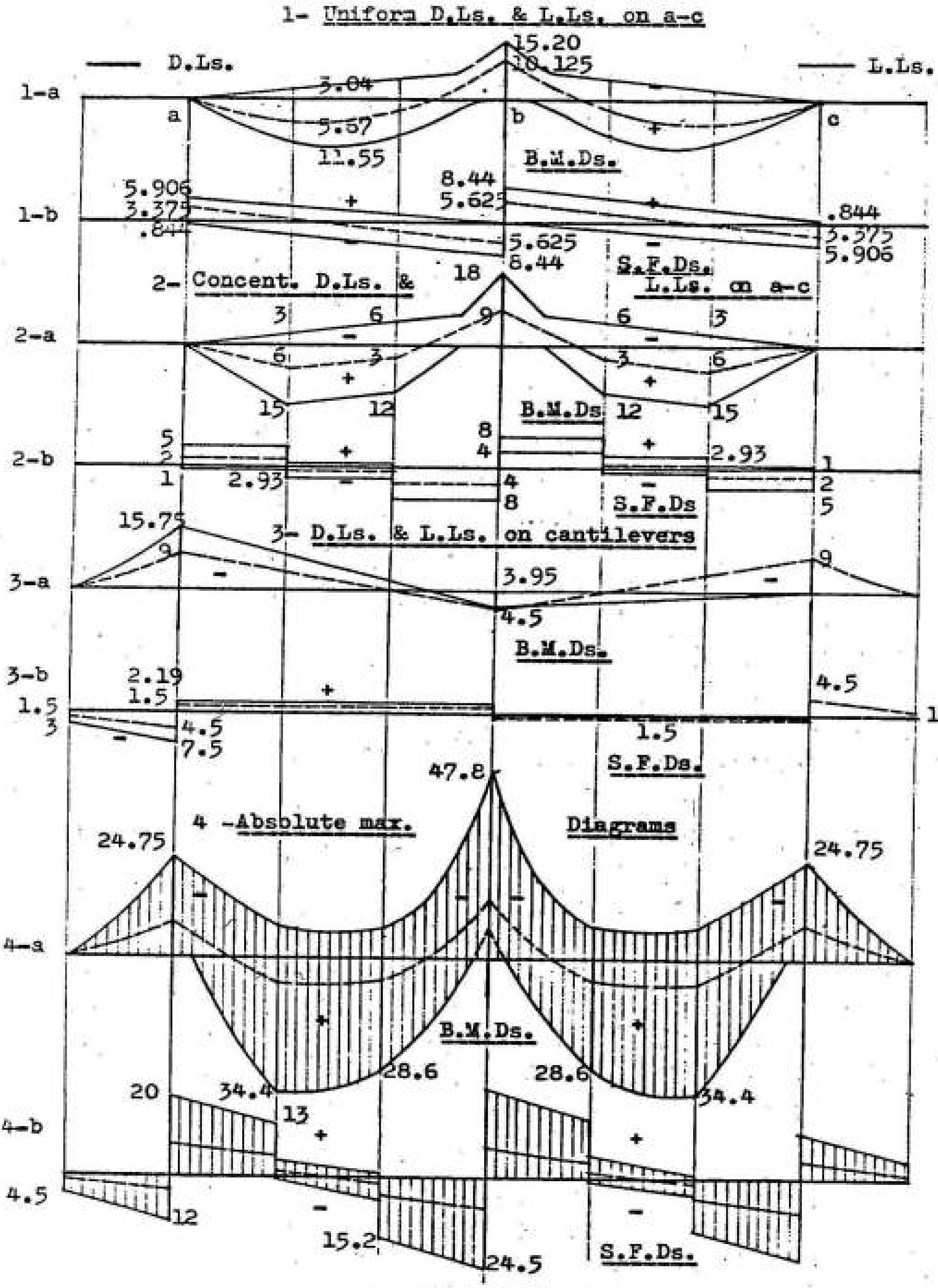

1- Uniform D.Ls. & L.Ls. on a-c
D.Ls.
L.Ls.
1-a
a
b
c
15.20
10.125
3.04
5.67
11.55
B.M.Ds.
1-b
5.906
3.375
.844
8.44
5.625
S.F.Ds.
2- Concent. D.Ls. & L.Ls. on a-c
18
2-a
B.M.Ds
2-b
2.93
S.F.Ds
3- D.Ls. & L.Ls. on cantilevers
15.75
3-a
3.95
4.5
B.M.Ds.
3-b
2.19
1.5
7.5
S.F.Ds.
47.8
4 -Absolute max. Diagrams
24.75
4-a
B.M.Ds.
28.6
34.4
20
13
4-b
4.5
12
15.2
24.5
S.F.Ds.

Fig. 4-15

For dead loads on both cantilevers:

$$M_a = -\left(\frac{g\,a^2}{2} + \frac{G\,a}{2}\right) = -\left(\frac{1 \times 3^2}{2} + \frac{3 \times 3}{2}\right) = -\,9 \text{ mt}$$

$$M_b = -\,M_a / 2 = +4.5 \text{ mt}$$

For the left cantilever loaded:

$$M_a = -\left(\frac{p\,a^2}{2} + \frac{P\,a}{2}\right) = -\left(\frac{1.5 \times 3^2}{2} + \frac{6 \times 3}{2}\right) = -\,15.75 \text{ mt}$$

$$M_b = -\,M_a / 4 = +\,3.94 \text{ mt}$$

The maximum bending moment and shearing force diagrams are determined by drawing the dead load diagrams and superposing to them, once the positive live load diagrams and once the negative live load diagrams. (Fig. 4-15, 4-a and b).

The results are nearly the same as those calculated in example 1. (Fig. 4-11).

The maximum field and connecting moments in continuous beams of equal spans l and constant moment of inertia subject to uniform dead loads g and live loads p are given in table 4-3 for different ratios of g / w in the form:

$$M = w\,l^2 / K$$

in which

$w = g + p$ = total uniform load per meter run

The Egyptian code of practice for the design and construction of reinforced concrete in buildings gives the following items:

The bending moments in continuous beams may be calculated according to the theory of beams on knife edge rigid supports. In case of equal spans or unequal spans with a maximum difference of 20% and uniform loads on different spans, the following values for bending moments may be assumed:

Beams of one span: maximum positive bending moment $M = +\,w\,l^2 / 8$

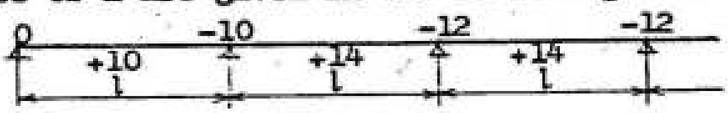
Two spans: maximum positive bending moment $M = +\,w\,l^2 / 10$

Negative bending moment at interm. support $M = -\,w\,l^2 / 8$

More than two spans: maximum bending moment $M = w\,l^2 / k$

The values of k are given in the following:

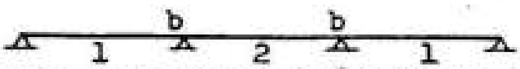
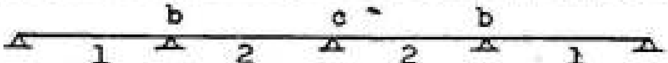
k =



$$M = \frac{w\,l^2}{k}$$

Table 4-3 Bending moments in continuous beams of equal spans and constant moment of inertia due to uniform dead loads g and live loads p

$$M = \frac{w\,l^2}{K} \qquad w = g + p$$

g/w	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1. Two equal spans 											
K_1	10.5	10.8	11.1	11.4	11.7	12.1	12.5	12.9	13.3	13.8	14.2
$-K_b$	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0	8.0
2. Three equal spans 											
K_1	10.0	10.2	10.4	10.6	10.9	11.1	11.4	11.6	11.9	12.2	12.5
K_2	13.3	14.3	15.4	16.7	18.2	20.0	22.2	25.0	28.6	33.3	40.0
$-K_b$	8.6	8.7	8.9	9.0	9.1	9.2	9.4	9.5	9.7	9.8	10.0
3. Four equal spans 											
K_1	10.2	10.4	10.6	10.9	11.1	11.4	11.7	12.0	12.3	12.6	12.9
K_2	12.4	13.2	14.0	14.9	16.0	17.2	18.7	20.4	22.4	24.9	28.0
$-K_b$	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9.0	9.1	9.2	9.3
$-K_c$	9.3	9.7	10.0	10.4	10.8	11.2	11.7	12.2	12.7	13.3	14.0
4. Five equal spans 											
K_1	10.1	10.3	10.6	10.8	11.0	11.3	11.6	11.9	12.2	12.5	12.8
K_2	12.7	13.5	14.3	15.3	16.5	17.9	19.5	21.4	23.8	26.6	30.4
K_3	11.7	12.3	12.9	13.6	14.3	15.2	16.2	17.3	18.5	20.0	21.7
$-K_b$	8.3	8.5	8.6	8.7	8.8	8.9	9.0	9.1	9.2	9.4	9.5
$-K_c$	9.0	9.3	9.6	9.9	10.2	10.5	10.9	11.3	11.7	12.2	12.7
5. ∞ equal spans 											
K_m	12.0	12.6	13.3	14.1	15.0	16.0	17.2	18.4	20.0	21.8	24.0
$-K_c$	8.8	9.1	9.3	9.5	9.9	10.1	10.5	10.8	11.2	11.6	12.0
g/w	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

Shear in end spans at first interior support $Q = 1.15\ w\ l/2$

Shear at all other supports $Q = w\ l/2$

In case of equal spans and heavy live loads, the negative field moments in intermediate spans may be calculated from the relation:

$$M_{min} = \frac{l^2}{24}\left(g - \frac{2}{3}p\right)$$

If the breadth of the girder, wall or column supporting the beam is bigger than 1/5 span and the beam is monolithically cast and reinforced with the support, it may be considered as totally fixed at its ends, provided that in case of walls sufficient weight should exist to ensure this fixation.

Moments due to fixation of exterior spans of continuous beams to supporting columns is given in chapter 7.

4.4 Dimensioning

The assumptions and conditions of equilibrium are given in chapter 2; the basic methods of dimensioning and their application to rectangular sections with tension reinforcements only using both the elastic method and the ultimate strength method have been shown in chapter 3.

The main steps of dimensioning depend on whether we are using the elastic method or the ultimate strength method.

In dimensioning of sections subject to simple bending (or eccentric forces with big eccentricity) both methods are allowed, and hence, will be illustrated. It will be tried however to use the same form of equations for both methods of design and for all popular types of sections; and, to formulate these formulae in such a way that the method applied can be immediately distinguished. In addition, it has been tried to use dimensionless coefficients only.

1) T and ⅂-sections

If a slab is monolithically cast with a beam and it lies in its compression zone, it is allowed to assume that a part of the slab acts with the beam and they form together a T or an ⅂-section as shown in Fig. 4-16.

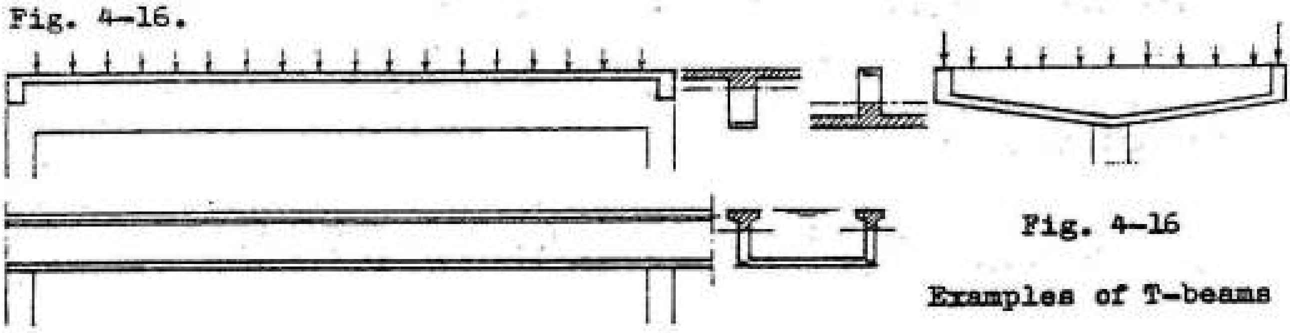


Fig. 4-16

Examples of T-beams

The effective breadth of T and ⅂- beams

The effective width of a T or an ⅂-section is the width of the compression zone that effectively participates in the flexural load capacity of the member.

The effective width of the compression flange of a T-section depends on a large number of parameters, including: the support conditions of the beam considered (freely supported or continuous at the supports); the method of load application (distributed loads or locally concentrated loads); the ratio of the length of the beam (between free supports or between points of zero bending moment) to the width of the rib (or web) and to the distance between consecutive ribs; the ratio of the thickness of of the flange (or slab) to the depth of the beam; and the presence of haunches at the junction of flange and rib.

In the Egyptian Code of Practice, the effective width of T and ⅂-sections is assumed not to exceed the least of the following values:

a) In case of T-sections: Fig. 4-17

$B = 12\ t_s + b + 2\ b_s$ or

B = 1/3 distance between points of zero bending moment in the span. i.e. $l/3$ in simple beams and @ $l/4.5$ in continuous beams; or

B = width of the smaller adjacent field of slab (calculated from center line of beam and in direction normal to it).

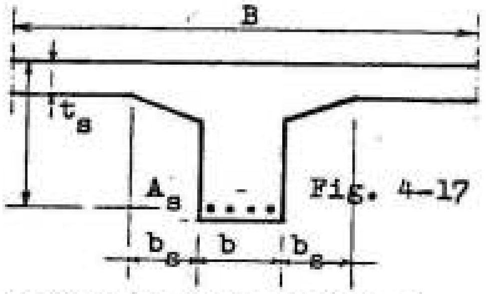


Fig. 4-17

b) In case of ⅂-sections that are not liable to rotate: Fig. 4-18

$B' = 4.5\ t_s + b + b_s$ or

B'= 1/6 distance between points of zero bending moment in the span. i.e. $l/6$ in simple beams and @ $l/9$ in continuous beams. or

B'= ½ width of adjacent field.

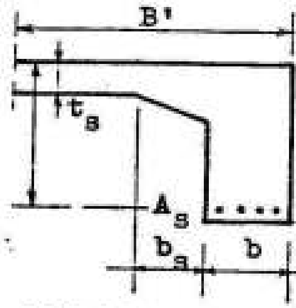


Fig. 4-18

The following limitations are also specified in the Code:

- Minimum thickness of flange to be considered in the design is 8 cm.
- The top reinforcement in flange transverse to rib should ensure the monolithic action between flange and web; it should extend the full width of the flange and must not be less than 0.3% of the cross sectional area of the flange. The maximum spacing between the bars of this reinforcement is 20 cms.

- Stirrups in the web should extend to top of the flange to ensure the monolithic action between flange and web.
- Isolated beams in which the T-form are used only for purpose of providing additional compression area should have a flange thickness not less than ½ the width of web and a total flange width not more than four times web thickness.

2) Dimensioning of sections by the elastic method

For structures to be designed with reference to this method, the allowable stresses in steel and concrete under the service loads and actions should not exceed values given in chapter 3 (tables 3-1 and 3-2).

In the following, we give the method of dimensioning of the popular forms of sections generally used by the engineer in most of the designs, namely: the rectangular, the T, the ⌉ and the triangular sections.

1. Rectangular sections with tension reinforcement only

Given: M, b, σ_c and σ_s. Required: d and A_s. Fig 3-7.

It has been proved (refer to equations 3-9 to 3-11) that:

$$d = c\sqrt{\frac{M}{\sigma_c\, b}} \qquad \text{and} \qquad A_s = \frac{M}{\sigma_s\, \eta\, d} \qquad 4\text{-}1$$

in which

$$c = \sqrt{\frac{2}{\zeta\,(1 - \frac{\zeta}{3})}} \qquad \eta = \frac{y_{ct}}{d} = 1 - \frac{\zeta}{3} \qquad \zeta = \frac{z}{d} = \frac{n}{r + n}$$

in these relations, we have: $n = E_s/E_c$ & $r = \sigma_s/\sigma_c$

The ratio of the tension steel $\mu = A_s/b\,d$ can be determined from the relation:

$$C = T \qquad \text{or} \qquad \sigma_c\,\frac{b\,z}{2} = A_s\,\sigma_s$$

Substituting $z = \zeta\, d$ and $A_s = \mu\, b\, d$, we get:

$$\sigma_c\, b\, \zeta\,\frac{d}{2} = \mu\, b\, d\, \sigma_s \qquad \text{but} \qquad \sigma_s/\sigma_c = r \qquad \text{then}$$

$$\mu = \frac{\zeta}{2\,r} \qquad 4\text{-}2$$

The values of ζ, η, c and μ for n = 15 and different values of $r = \sigma_s/\sigma_c$ are shown in table 4-4. (Sheet 10)

The values of ζ, η, and c do not change for any other value of n when r is replaced by $r_n = 15\, r / n$.

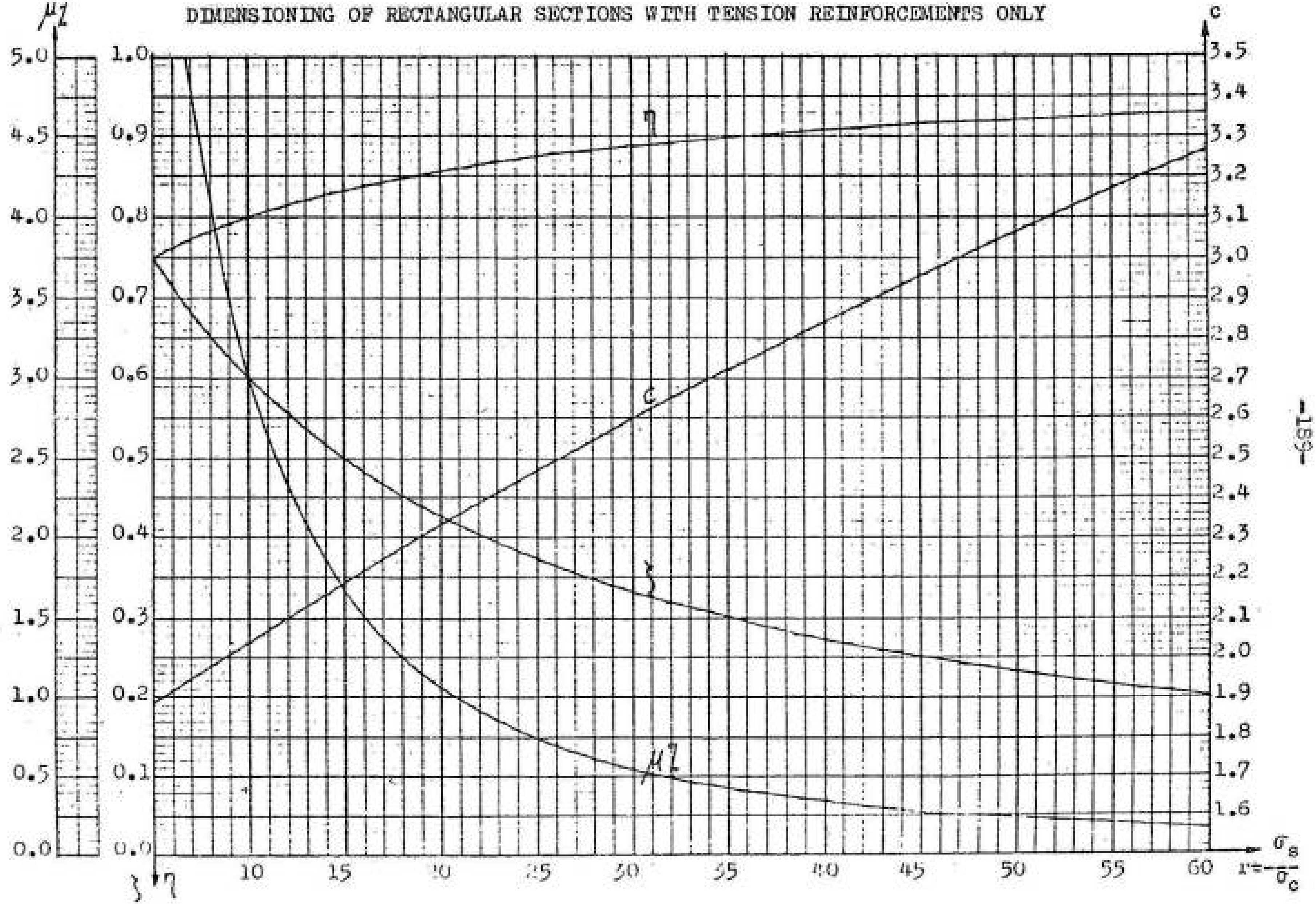
Table 4-4 Dimensioning of Rectangular Sections with Tension Reinforcements only. n = 15

$$d = c\sqrt{\frac{M}{\sigma_c\, b}} \qquad A_s = \frac{M}{\sigma_s\, \eta\, d}$$

$r = \frac{\sigma_s}{\sigma_c}$	$\zeta = \frac{z}{d}$	$\eta = \frac{y_{ct}}{d}$	c	$\mu = \frac{A_s}{bd}$	$r = \frac{\sigma_s}{\sigma_c}$	$\zeta = \frac{z}{d}$	$\eta = \frac{y_{ct}}{d}$	c	$\mu = \frac{A_s}{bd}$
0	1.000	0.667	1.73		30	0.333	0.889	2.60	0.00556
1	0.938	688	1.76	0.46875	31	326	891	2.63	526
2	882	706	1.79	22060	32	319	894	2.65	498
3	833	722	1.83	13889	33	313	896	2.68	473
4	789	737	1.86	09868	34	306	898	2.70	450
5	0.750	0.750	1.89	0.07500	35	0.300	0.900	2.72	0.00429
6	714	762	1.92	05952	36	294	902	2.75	409
7	682	773	1.95	04870	37	289	904	2.77	390
8	652	783	1.98	04076	38	283	906	2.79	372
9	625	792	2.01	03472	39	278	907	2.81	356
10	0.500	0.800	2.04	0.03000	40	0.273	0.909	2.84	0.00341
11	577	808	2.07	2622	41	268	911	2.87	327
12	556	815	2.10	2315	42	263	912	2.89	313
13	536	821	2.13	2060	43	258	914	2.91	300
14	517	828	2.16	1847	44	254	915	2.93	289
15	0.500	0.833	2.19	0.01667	45	0.250	0.917	2.96	0.00278
16	484	839	2.22	1512	46	246	918	2.98	267
17	469	844	2.25	1379	47	242	920	3.00	257
18	455	849	2.28	1262	48	238	921	3.02	248
19	441	853	2.31	1161	49	234	922	3.04	239
20	0.429	0.857	2.33	0.01072	50	0.231	0.923	3.06	0.00231
21	417	861	2.36	992	51	227	924	3.09	223
22	405	865	2.39	922	52	224	925	3.11	215
23	395	868	2.42	858	53	220	927	3.13	208
24	385	872	2.44	801	54	217	928	3.15	201
25	0.375	0.875	2.47	0.00750	55	0.214	0.929	3.17	0.00194
26	366	878	2.50	704	56	211	930	3.19	188
27	357	881	2.52	662	57	208	931	3.21	182
28	349	884	2.55	623	58	205	932	3.23	177
29	341	886	2.57	588	59	202	933	3.25	172
30	0.333	0.889	2.60	0.00556	60	0.200	0.933	3.27	0.00167

SHEET 10 SIMPLE BENDING

DIMENSIONING OF RECTANGULAR SECTIONS WITH TENSION REINFORCEMENTS ONLY



Illustrative examples

1) Given: $M = 16$ mt, $b = 0.3$ m, concrete C200 with $\sigma_c = 70$ kg/cm^2 and normal mild steel with $\sigma_s = 1400$ kg/cm^2. Required d and A_s.

Solution

$r = \sigma_s/\sigma_c = 1400/70 = 20$, table 4-4 gives $c = 2.33$ and $\eta = 0.857$

Hence, using t and m units, we get:

$$d = 2.33\sqrt{\frac{16}{700 \times 0.3}} = 0.645 \text{ ms} \qquad \text{chosen depth} \quad t = 70 \text{ cm}$$

$$A_s = \frac{16}{1.4 \times 0.857 \times 0.645} = 20.6 \text{ cm}^2 \qquad \text{chosen } 4\,\varnothing\,22 + 2\,\varnothing\,19$$

Area of $4\,\varnothing\,22 + 2\,\varnothing\,19 = 15.2 + 5.7 = 20.9$ cm^2

2) Assuming that $n = 10$, then for the section given in example 1, $r = 20$ will be replaced by $r_n = \frac{15}{10} \times 20 = 30$. In which case:

$$c = 2.6 \qquad \text{and} \qquad \eta = 0.889$$

i.e. $$d = 2.6\sqrt{\frac{16}{700 \times 0.3}} = 0.715 \text{ ms} \qquad \text{chosen} \quad t = 76 \text{ cm}$$

$$A_s = \frac{16}{1.4 \times 0.889 \times 0.715} = 18 \text{ cm}^2 \qquad \text{chosen} \quad 5\,\varnothing\,22$$

The depth and area of tension steel can however be determined from the relations:

$$\underline{d = k_1\sqrt{\frac{M}{b}}} \qquad \text{and} \qquad \underline{A_s = \frac{M}{k_2\, d}} \qquad 4\text{-}3$$

in which

$$k_1 = \frac{c}{\sqrt{\sigma_c}} \quad \frac{\text{cm}}{\sqrt{\text{kg}}} \qquad \text{and} \qquad k_2 = \eta\,\sigma_s \quad \text{kg/cm}^2$$

In these relations k_1 and k_2 are not dimensionless and in order to get d in cms and A_s in cms^2, M/b in the equation of d and M/d in the equation of A_s must be in kgs.(Table 4-5 and sheet 11)

After the introduction of the ultimate load theory , and the allowance of both, the elastic and the ultimate load methods in the design, it seems to the author that the proposed form of equations:

$$d = c\sqrt{\frac{M}{\sigma_c\, b}} \qquad \text{and} \qquad A_s = \frac{M}{\sigma_s\,\eta\, d}$$

is more convenient, because from the first look to these equations, one can distinguish the method and the stresses used in the design.

Beams of given depth

If the depth is given and the area of tension steel only is required, one has to distinguish the following cases:

a) The depth is bigger than the balanced depth in which the full value of the allowable stresses is utilized. In this case, it is sufficient to assume $\eta = 0.9$ and to determine the corresponding area of steel.

Example 3

Assume that the effective depth specified for example 1 is 70 cms and the area of the required corresponding tension steel is required.

The 70 cms is bigger than the required balanced depth of 64.5 cms; if we assume $\eta = 0.9$, then:

$A_s = \frac{16}{1.4 \times 0.9 \times 0.7} = 18.1 \ cm^2$ chosen 5 Ø 22 area 19 cm^2.

The actual stresses in the section may be determined from table 1 page 62 as follows:

$\mu = A_s / b\ d = 19 / 30 \times 70 = 0.00905$ table 1 gives

$c_1 = 5.736$ and $c_2 = 127.6$ therefore

$$\sigma_c = 5.736 \times \frac{16 \times 10^5}{30 \times 70^2} = 62.6 \ kg/cm^2$$

$$\sigma_s = 127.6 \times \frac{16 \times 10^5}{30 \times 70^2} = 1392 \ kg/cm^2$$

b) If the specified depth is smaller than the balanced, one can design the section using tension reinforcements only from the first principles as follows:

$$M = C\ y_{ct} = \frac{b\ z}{2} \sigma_c \left(d - \frac{z}{3}\right)$$

giving

$$z = 1.5\ d \left(1 - \sqrt{1 - \frac{8\ M}{3\ b\ d^2\ \sigma_c}}\right) \qquad 4\text{-}4$$

Because

$$C = T \quad \text{or} \quad \frac{b\ z}{2} \sigma_c = A_s\ \sigma_s$$

and

$$\sigma_s = \frac{n\ (d - z)\ \sigma_c}{z} \qquad 4\text{-}5$$

then

$$A_s = \frac{b\ z^2}{2\ n\ (\ d - z)} \qquad 4\text{-}6$$

Example 4

Table 4-5 Dimensioning of Rectangular Sections with Tension Reinforcements only. n = 15

$$d = k_1\sqrt{\frac{M}{b}} \qquad A_s = \frac{M}{k_2\, d}$$

Values of k_1 and k_2 for kg and cm units

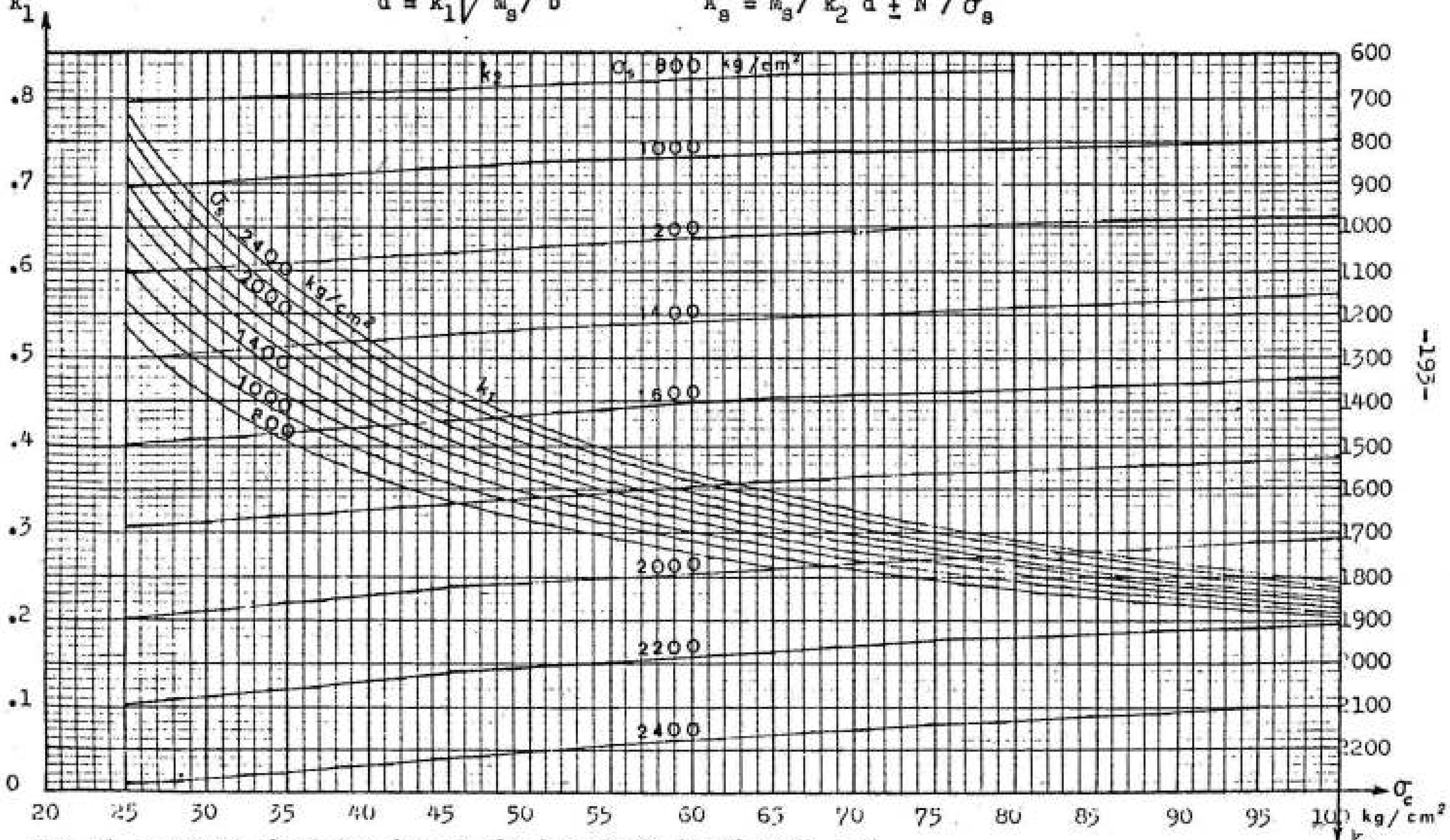
σ_c	σ_s	800	1000	1200	1400	1600	1800	2000	2200	2400
25	k_1	.538	.568	.604	.636	.676	.702	.731	.760	.785
	k_2	715	909	1105	1301	1499	1697	1894	2092	2292
30	k_1	.459	.490	.518	.547	.575	.598	.621	.645	.656
	k_2	704	897	1091	1286	1483	1679	1877	2075	2274
35	k_1	.408	.433	.457	.479	.502	.521	.542	.562	.590
	k_2	594	885	1078	1273	1469	1665	1862	2060	2256
40	k_1	.369	.390	.411	.430	.449	.466	.484	.502	.520
	k_2	686	875	1068	1260	1455	1651	1846	2044	2240
45	k_1	.339	.357	.375	.392	.408	.423	.438	.454	.472
	k_2	678	866	1056	1248	1441	1636	1832	2028	2226
50	k_1	.314	.330	.345	.361	.375	.389	.401	.415	.426
	k_2	671	857	1046	1237	1430	1628	1818	2014	2210
55	k_1	.296	.308	.321	.334	.347	.359	.371	.383	.394
	k_2	665	849	1037	1227	1419	1611	1806	2000	2194
60	k_1	.277	.289	.301	.313	.324	.335	.347	.360	.367
	k_2	659	842	1028	1217	1408	1600	1793	1987	2184
65	k_1	.262	.273	.284	.294	.305	.317	.324	.340	.344
	k_2	654	835	1021	1208	1398	1589	1782	1975	2170
70	k_1		.259	.269	.279	.288	.296	.306	.315	.325
	k_2		829	1013	1200	1389	1578	1771	1963	2158
75	k_1		.247	.256	.265	.274	.282	.289	.298	.306
	k_2		823	1006	1192	1379	1570	1761	1952	2144
80	k_1		.237	.245	.253	.261	.268	.276	.283	.290
	k_2		818	1000	1185	1371	1561	1750	1941	2132
85	k_1		.227	.235	.242	.250	.255	.263	.274	.277
	k_2		813	994	1178	1363	1552	1740	1931	2122
90	k_1		.219	.226	.233	.240	.246	.250	.258	.265
	k_2		809	988	1171	1357	1543	1731	1920	2110
95	k_1		.211	.217	.224	.230	.236	.243	.248	.252
	k_2		804	983	1165	1349	1535	1722	1912	2102
100	k_1		.204	.210	.216	.222	.229	.233	.239	.245
	k_2		800	978	1159	1342	1528	1715	1904	2004

SHEET 11

SIMPLE BENDING AND ECCENTRIC FORCES WITH BIG ECCENTRICITY (ELASTIC DESIGN)

Dimensioning of Rectangular Sections with Tension Reinforcements Only

$d = k_1 \sqrt{M_s / b}$ $A_s = M_s / k_2 d \pm N / \sigma_s$



N.B. M_s = moment about tension steel; for simple bending: $M_s = M$

Assume that the specified effective depth for example 1 is 60 cm only. Then

Applying equation 4-4, we get:

$$z = 1.5 \times 60 \left(1 - \sqrt{1 - \frac{8 \times 16 \times 10^5}{3 \times 30 \times 60^2 \times 70}}\right) = 30.8 \quad \text{cms}$$

Equation 4-5 gives:

$$\sigma_s = \frac{15\ (60 - 30.8)\ 70}{30.8} = 995 \quad kg/cm^2$$

Equation 4-6 gives:

$$A_s = \frac{30 \times 30.8}{2 \times 15(60 - 30.8)} = 32.5 \quad cm^2$$

It is clear that, in this case, the stress in the tension steel will be low and its amount will correspondingly be high.

Such a section of limited depth may also be designed using double (tension and compression) reinforcement as follows.

2. <u>Rectangular sections with double reinforcements</u>

Given σ_c, σ_s, M, b and d (smaller than the balanced depth). Required A_s and A'_s.

This problem can be solved by three methods. Fig. 4-19.

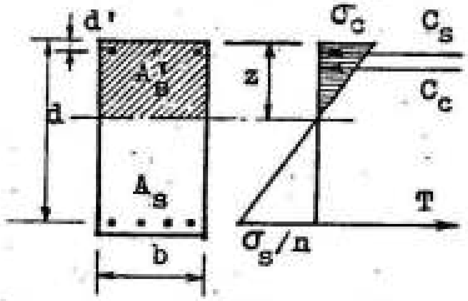


Fig. 4-19

<u>First method</u>

By use of the fundamental principles. Knowing σ_c and σ_s, the position of the neutral axis can be determined from the relation:

$$z = \frac{n\,\sigma_c}{\sigma_s + n\,\sigma_c}\,d \qquad 4\text{-}7$$

Taking moments about the center of gravity of the compression reinforcement, we get:

$$T\ (d - d') - C_c\ \left(\frac{z}{3} - d'\right) = M \qquad \text{or}$$

$$A_s\,\sigma_s\ (d - d') - \sigma_c\,\frac{z\,b}{2}\left(\frac{z}{3} - d'\right) = M \qquad 4\text{-}8$$

from which, one can determine A_s.

Taking moments about the center of gravity of the tension reinforcement, we get:

$$C_c\ \left(d - \frac{z}{3}\right) + C_s\ (d - d') = M \qquad \text{or}$$

$$\sigma_c\,\frac{z\,b}{2}\left(d - \frac{z}{3}\right) + A'_s\,\sigma'_s\ (d - d') = M \qquad 4\text{-}9$$

knowing further that $\sigma_s' = \frac{n\,\sigma_c\,(z - d')}{z}$ one can determine A_s'.

Second method

The moment M_1 and the area of steel A_{s1} of the balanced section (section with tension reinforcements only designed for the maximum allowed values of σ_c and σ_s) can be determined from the relations:

$$d = c\sqrt{\frac{M_1}{\sigma_c\, b}} \qquad \text{or} \qquad M_1 = \frac{\sigma_c\, b\, d^2}{c^2} \qquad 4\text{-}11$$

and

$$A_{s1} = \mu\, b\, d \qquad 4\text{-}12$$

c and μ are to be taken from table 4-4 for the given allowed value of $r = \sigma_s/\sigma_c$.

M_1 must be smaller than M, otherwise, the section needs no compression reinforcement, and
$M_2 = M - M_1$ will be resisted by compression reinforcements A_s' and additional tension reinforcements A_{s2}; where

$$A_s' = \frac{M_2}{\sigma_s'\,(d - d')} \qquad 4\text{-}13$$

in which

$$\sigma_s' = \frac{n\,\sigma_c\,(z - d')}{z}$$

The magnitude of z can be determined from equation 4-7.

One has further:

$$A_{s2} = \frac{M_2}{\sigma_s\,(d - d')} \qquad 4\text{-}14$$

and

$$A_s = A_{s1} + A_{s2} \qquad 4\text{-}15$$

Third method: Use of curves.

It has been proved (equation 4-7) that:

$$z = \zeta d = \frac{n\,\sigma_c}{\sigma_s + n\,\sigma_c}\, d, \qquad \text{with } r = \frac{\sigma_s}{\sigma_c}, \qquad \zeta = \frac{z}{d} = \frac{n}{r + n}.$$

Assuming $y_{ct} = \eta d$, $d' = \beta d$, $A_s = \mu b\, d$, $A_s' = \mu' b\, d$ and $\alpha = \mu'/\mu$, it has been found in equation 2-39 that:

$$\sigma_c = c_1 \frac{M}{b\, d^2}$$

which gives

$$d = c\sqrt{\frac{M}{\sigma_c\, b}} \qquad 4\text{-}16$$

where $c = \sqrt{c_1} = \sqrt{\dfrac{1}{\frac{\zeta}{2}(1 - \frac{\zeta}{3}) + n\alpha\mu\frac{\zeta - \beta}{\zeta}(1 - \beta)}}$

We have further:

$$C = T \qquad \text{or} \qquad \sigma_c \frac{b\,z}{2} + A_s^{!}\,\sigma_s^{!} = A_s\,\sigma_s$$

Substituting $\sigma_s^{!} = \dfrac{n\,\sigma_c\,(z - d')}{z}$ and $\dfrac{\sigma_s}{\sigma_c} = r$, we get

$$\sigma_c \frac{b\,z}{2} + n\,A_s^{!}\,\sigma_c\,\frac{z - d'}{z} = A_s\,\sigma_s \qquad \text{or}$$

$$\frac{b\,\zeta\,d}{2} + n\,\alpha\,\mu\,b\,d\,\frac{\zeta - \beta}{\zeta} = \mu\,b\,d\,r \qquad \text{or}$$

$$\mu = \frac{\frac{1}{2}\zeta}{r - n\,\alpha\,\frac{\zeta - \beta}{\zeta}} \qquad 4\text{-}17$$

We have further $$A_s^{!} = \alpha\,A_s \qquad 4\text{-}18$$

The values of c and μ are given in sheet 12 assuming n = 15 and $\beta = d'/d = 0.08$.

The curves shown on sheet 12 can be used for the dimensioning as well as the determination of stresses in rectangular sections.

Example 5

Given M = 16 mt, b = 30 cm, d = 60 cm, $\sigma_c = 70$ kg/cm^2 and σ_s = 1400 kg/cm^2. Required A_s and $A_s^{!}$.

First method

The position of the neutral axis can be determined from equation 4-7. Hence

$$z = \frac{15 \times 70}{1400 + 15 \times 70} \times 60 = 25.71 \quad \text{cms}$$

Taking moments about center of gravity of compression steel, we get:

$$A_s \times 1400\,(60 - 4) - 70 \times \frac{25.71 \times 30}{2}\left(\frac{25.71}{3} - 4\right) = 16 \times 10^5$$

giving $$A_s = 22.0 \quad \text{cm}^2$$

Further, we have:

$$\sigma_s^{!} = 15 \times 70 \times \frac{25.71 - 4}{25.71} = 887 \quad \text{kg/cm}^2$$

Taking moments about the center of gravity of the tension steel, we get:

$$70 \times \frac{25.71 \times 30}{2}\left(60 - \frac{25.71}{3}\right) + A_s^{!} \times 887\,(60 - 4) = 16 \times 10^5$$

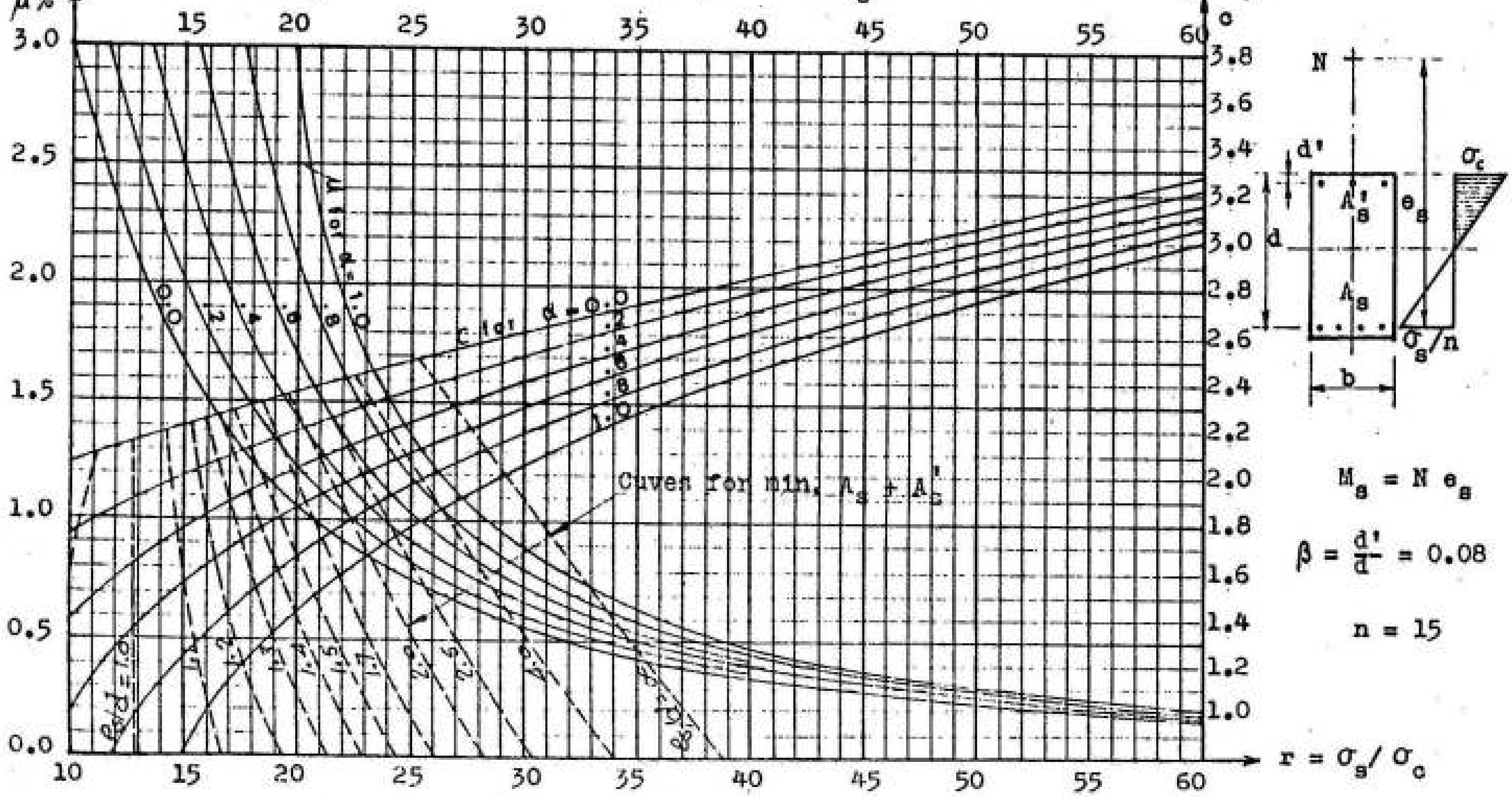
giving $$A_s^{!} = 4.35 \quad \text{cm}^2$$

SHEET 12

SIMPLE BENDING AND ECCENTRIC FORCES WITH BIG ECCENTRICITY (ELASTIC DESIGN)

Dimensioning of Rectangular Sections with Double Reinforcements

$d = c\sqrt{\frac{M_s}{\sigma_c b}}$ $\qquad A_s = \mu b d \pm \frac{N}{\sigma_s}$ $\qquad A'_s = \alpha\mu b d$



N.B. M_s = moment about tension steel; for simple bending $M_s = M$

Second method

For a balanced section with $\sigma_c = 70$ kg/cm^2 and $\sigma_s = 1400$ kg/cm^2, $r = 1400/70 = 20$ and table 4-4 gives: $\zeta = 0.429$, $c = 2.33$ and $\mu = 0.01072$. So that:

$$0.6 = 2.33 \sqrt{\frac{M_1}{700 \times 0.3}} \quad \text{or} \quad M_1 = 13.8 \text{ mt}$$

and

$$A_{s1} = 0.01072 \times 30 \times 60 = 19.3 \text{ cm}^2$$

$$M_2 = 16 - 13.8 = 2.2 \text{ mt}$$

$$A_{s2} = \frac{2.2}{1.4 \times 0.56} = 2.8 \text{ cm}^2$$

$$A_s = 19.3 + 2.8 = 22.1 \text{ cm}^2$$

$$z = 0.429 \times 60 = 25.74 \text{ cm}$$

$$\sigma_s' = 15 \times 70 \times \frac{25.74 - 4}{25.74} = 886 \text{ kg/cm}^2$$

$$A_s' = \frac{2.2}{886 \times 0.56} = 4.44 \text{ cm}^2$$

Third method

$$0.6 = c \sqrt{\frac{16}{700 \times 0.3}} \quad \text{or} \quad c = 2.18$$

For $r = 1400/70 = 20$, and $c = 2.18$, sheet 12 gives: $\alpha = 0.2$ & $\mu = 1.23\%$

Therefore

$$A_s = \frac{1.23}{100} \times 30 \times 60 = 22.1 \text{ cm}^2$$

$$A_s' = 0.2 \times 22.1 = 4.42 \text{ cm}^2$$

The three methods give nearly the same result.

Example 6

Given $M = 16$ mt, $b = 30$ cm, $d = 60$ cm, $A_s = 20$ cm^2 and $A_s' = 4$ cm^2. Required: σ_c and σ_s.

$$\mu = 20/30 \times 60 = 0.0111 \quad \text{and} \quad \alpha = 4/20 = 0.2$$

Sheet 12 gives: $\sigma_s/\sigma_c = 21.2$ & $c = 2.20$, so that

$$0.6 = 2.2 \sqrt{\frac{16}{\sigma_c \times 0.3}} \quad \text{or} \quad \sigma_c = 715 \text{ t/m}^2 = 71.5 \text{ kg/cm}^2$$

therefore

$$\sigma_s = 21.2 \times 71.5 = 1520 \text{ kg/cm}^2$$

which means that the stresses in both concrete and steel are high.

3. T-sections with tension reinforcements

The effective breadth B to be considered in the design of T-sections is given in 4.4, 1).

Working stresses in a T-section

T-sections subjected to simple bending are to be designed for working values of σ_c smaller than those allowed in rectangular sections. The reasons can be summarized in the following:

i) Stress distribution along the breadth B of a T-beam is not constant, but bigger at the center line of the web and smaller at the edges of the effective breadth B. The average value is smaller than the maximum. Fig. 4-20. The magnitude depends on the assumed breadth B.

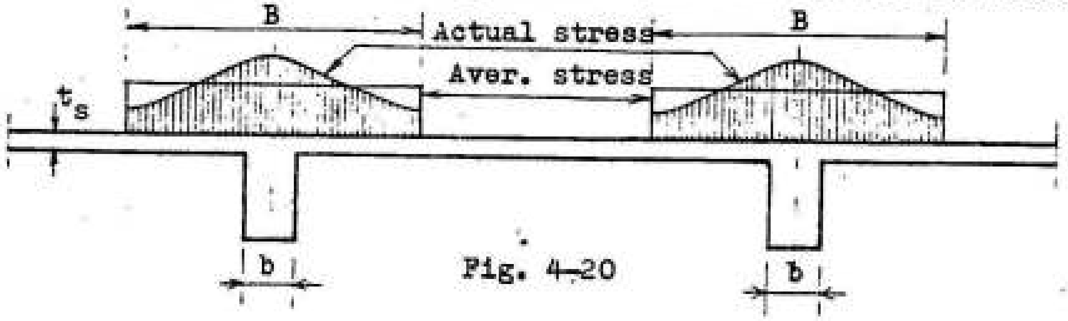


Fig. 4-20

ii) If an element at the top fiber of the flange of a T-beam is considered, it is found that it is subject to compressive stresses parallel to the axis of the beam (due to bending moments in the beam) and tensile stresses normal to it (due to bending moments in the slab). These tensile stresses tend to reduce the strength of the element and respectively the allowable stresses. (Fig. 4-21).

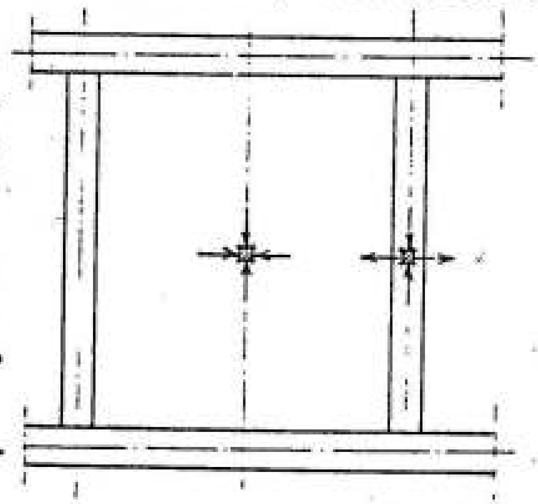


Fig. 4-21

iii) It is more economic to design a T-section with a reduced stress (smaller than half the allowable stress in a rectangular sec.) than with higher values. Because, the higher the value of σ_c, the smaller is the depth and the higher is the amount of the tension steel.

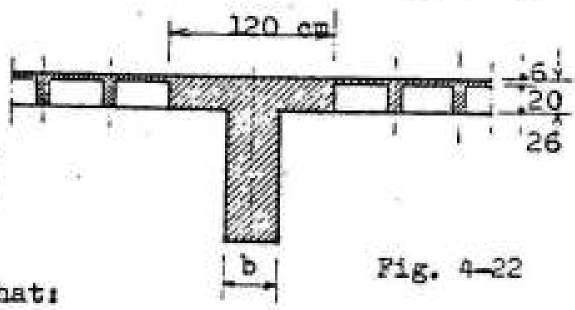


Fig. 4-22

Because of these reasons, it is specified in the Egyptian code that:

The economic design of T-sections may be attained by using working stresses not exceeding <u>half</u> the values of σ_c allowed in rectangular sections given in table 3-2. At any rate, the working stress should in no case exceed 0.75 σ_c when the full effective width of the flange is considered in the calculations.

If for structural purposes, the breadth B is chosen less than the specified value, the design stress can be relatively increased.

In a beam supporting a hollow or a ribbed slab, (Fig. 4-22), the solid part of the slab which may be assumed as the effective breadth of the T-section is generally much smaller than the specified breadth B of the T-section, as

$$B = 12\ t_s + b = 12 \times 26 + 40 = 352 \quad cm \gg 120 \quad cm$$

The allowable stress may be increased to 0.80 - 1.0 σ_c of the rectangular section and the design may still be within the economic limits.

<u>Dimensioning of T-sections</u>

The design of a T-section depends on whether the neutral axis is inside or outside the flange of the section.

The position of the neutral axis with respect to the compression flange of the section may be estimated in the following manner:

$$z = \zeta\ d = \zeta \cdot c \sqrt{\frac{M}{\sigma_c\ B}} \qquad \text{but} \qquad c = \sqrt{\frac{2}{\zeta\,(1 - \zeta/3)}}$$

Therefore

$$z = \zeta \sqrt{\frac{2}{\zeta\,(1 - \zeta/3)}}\ \sqrt{\frac{M}{\sigma_c\ B}} \qquad \text{but} \qquad \zeta = \frac{n}{r + n}$$

then

$$z = \sqrt{\frac{6\ n}{3\ r + 2\ n}} \cdot \sqrt{\frac{M}{\sigma_c\ B}}$$

For n = 15, we get:

$$z = \sqrt{\frac{30}{\sigma_c\ (r + 10)}} \cdot \sqrt{\frac{M}{B}} \qquad \text{or}$$

$$\underline{z = k \sqrt{\frac{M}{B}}} \qquad 4\text{-}19$$

in which

$$k = \sqrt{\frac{30}{\sigma_c\ (r + 10)}}$$

Using normal mild steel with $\sigma_s = 1400\ kg/cm^2$ and assuming σ_c = 25 to 35 kg/cm^2, then r = 56 - 40, and

$$\underline{z = 0.13 \sqrt{\frac{M}{B}}} \qquad 4\text{-}19a$$

Using high grade steel with $\sigma_s = 2000$ kg/cm^2 and $\sigma_c = 30$ to 40 kg per cm^2 then r = 67 - 50, and

$$z = 0.112 \sqrt{\frac{M}{B}} \qquad \text{4-19b}$$

In order to determine the depth of a T-section, one has first to calculate z (equations 4-19) defining the position of the neutral axis.

In case the neutral axis lies inside the flange ($z < t_s$), then the equations of dimensioning rectangular sections can be used, one has only to replace b by B. i.e.

$$d = c \sqrt{\frac{M}{\sigma_c\, B}} \quad \text{and} \quad A_s = \frac{M}{\sigma_s\, \eta\, d} = \mu\, B\, d \qquad \text{4-20}$$

In case the neutral axis lies outside the flange, ($z > t_s$), the compression zone having the form of a T-section can be replaced by a rectangular section of breadth $B_\lambda = \lambda\, B$ and depth z. (Fig. 4-23).

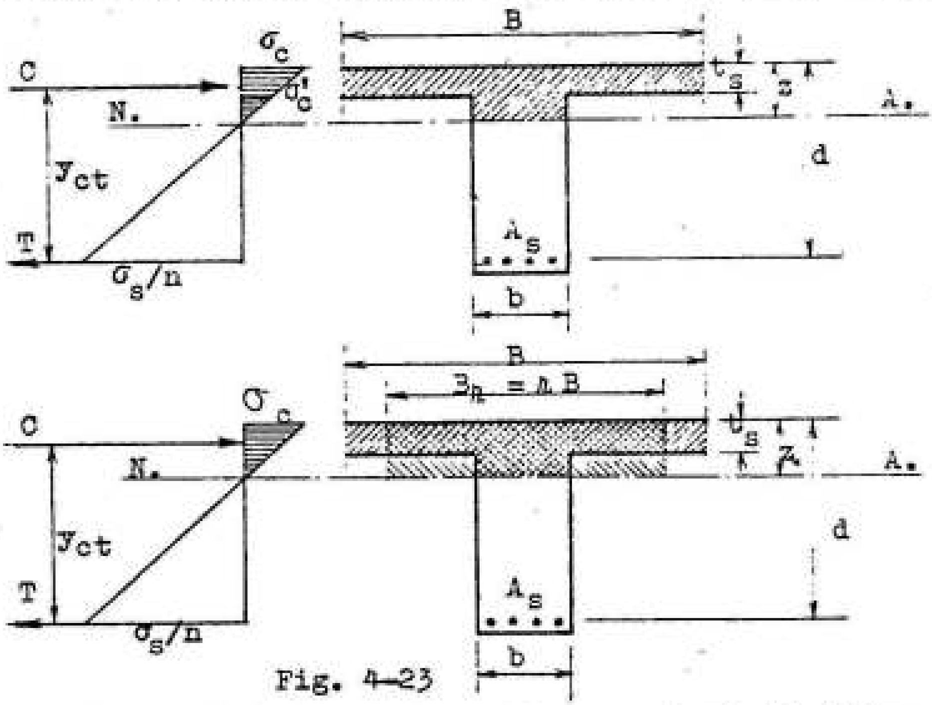


Fig. 4-23

The external moment being equal to the moment of the internal stresses, we get:

For a rectangular compression zone, B_λ z, we have:

$$M = A_s\, \sigma_s\, (d - \tfrac{z}{3}) = \tfrac{1}{2}\, \sigma_c\, z\, (d - \tfrac{z}{3})\, B_\lambda \qquad \text{(a)}$$

For the T-compression zone:

$$M = A_s\, \sigma_s\, y_{ct} = \left[\tfrac{1}{2}\, \sigma_c\, B\, z - \tfrac{1}{2}\, \sigma_c'\, (z - t_s)\, (B - b)\right] y_{ct}$$

but $\quad \sigma_c' = \sigma_c \dfrac{z - t_s}{z} \quad$ therefore

$$M = \tfrac{1}{2}\,\sigma_c \left[z\,B - (z - t_s)^2\,(B - b)\,\frac{1}{z} \right] y_{ct} \qquad \text{(b)}$$

In order to get the same value for A_s, σ_c, σ_s and respectively z, we should further have:

$$y_{ct} = d - \frac{z}{3}$$

Substituting this value in equation (b) and equalizing it to the relation given in equation (a), we get:

$$z\,B_\lambda = z\,B - (z - t_s)^2\,(B - b)\,/\,z$$

Introducing further $B_\lambda = \lambda\,B$ then

$$\lambda = 1 - \left(1 - \frac{t_s}{z}\right)^2 \left(1 - \frac{b}{B}\right) \qquad 4\text{-}21$$

The values of λ are given in sheet 13.

Example

Given a T-section with t_s = 9 cm, b = 30 cm subject to M = 16 mt. Required d and A_s if concrete C200 and normal mild steel are used.

Solution

According to tables 3-1 and 3-2: $\sigma_s = 1400$ and $\sigma_c = 70$ kg/cm^2.

Assume the working stress for concrete in the T-sec. = 35 kg/cm^2

The effective breadth $B = 12\,t_s + b = 12 \times 9 + 30 = 138$ cms

but $z = 0.13\sqrt{\frac{M}{B}} = 0.13\sqrt{\frac{16000}{1.38}} = 14$ cm $> t_s$, then:

One has to use sheet 13 giving the effective reduced breadth of the T-section. Hence

For $\frac{t_s}{z} = \frac{9}{14} = 0.64$ and $\frac{B}{b} = \frac{138}{30} = 4.6$, we get $\lambda = 0.9$, so that

$$B_\lambda = 0.9 \times 138 = 124 \text{ cm}$$

For $\sigma_c = 35$, $\sigma_s = 1400$ kg/cm^2, $r = \frac{1400}{35} = 40$ and table 4-4 gives: $c = 2.84$ and $\eta = 0.909$ therefore

$$d = 2.84\sqrt{\frac{16}{350 \times 1.24}} = 0.545 \text{ m}$$

and

$$A_s = \frac{16}{1.4 \times 0.909 \times 0.545} = 23 \text{ cm}^2$$

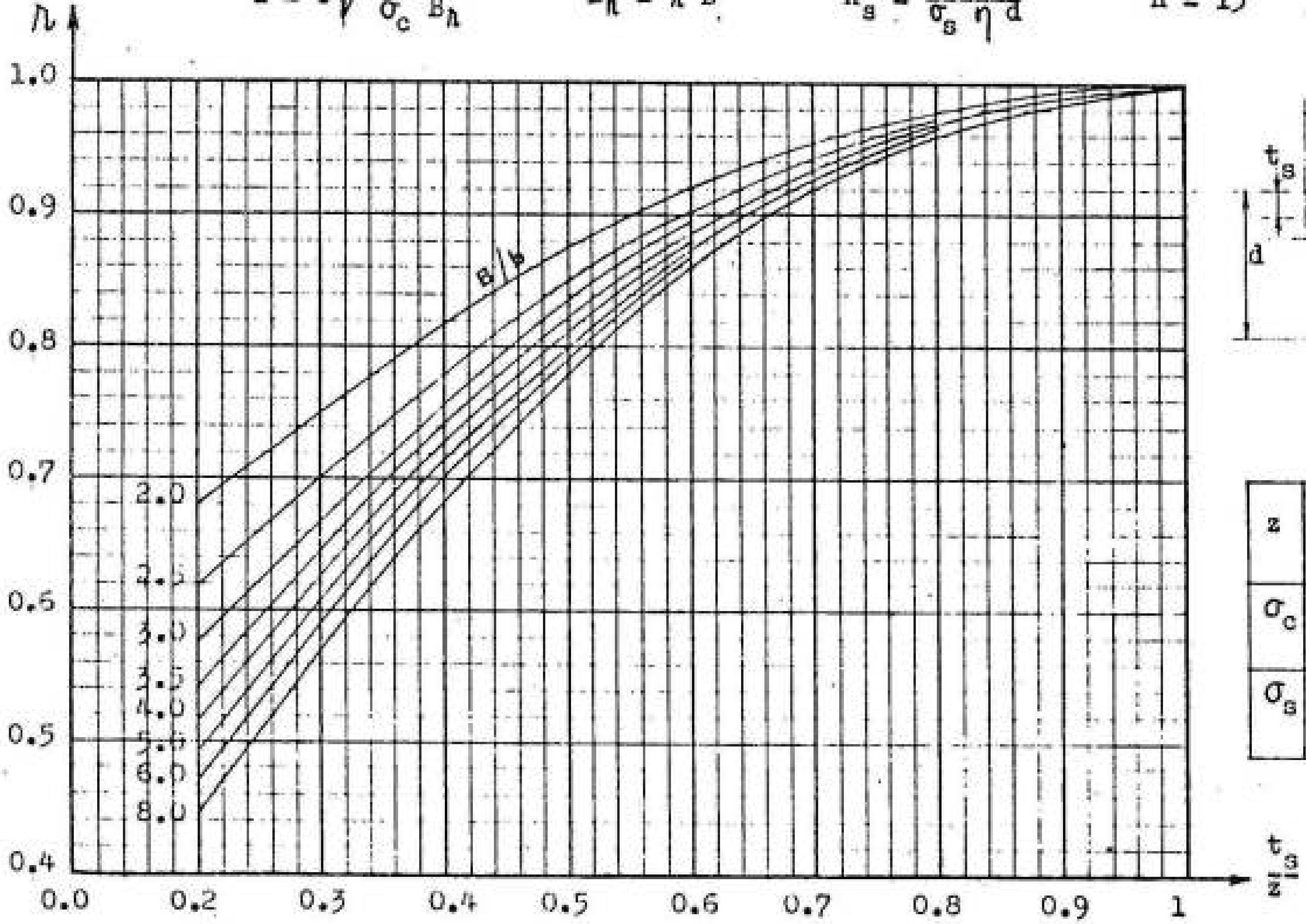
Although the concrete stress is low, the depth is relatively small and the area of steel is relatively big. A bigger depth of 58 to 60 cms corresponding to a smaller working concrete stress may be more convenient.

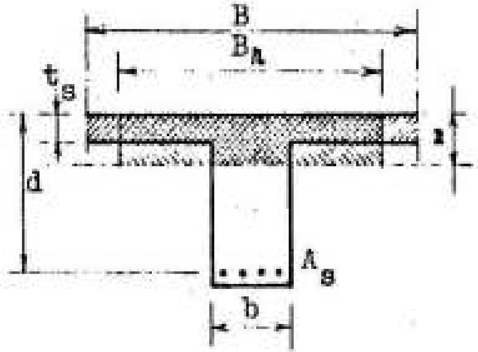
SHEET 13

SIMPLE BENDING (ELASTIC DESIGN)

Reduced Breadth of T-Sections

$d = c\sqrt{\frac{M}{\sigma_c B_h}}$ $B_h = h\,B$ $A_s = \frac{M}{\sigma_s \eta d}$ $n = 15$





z	$0.13\sqrt{\frac{M}{B}}$	$0.112\sqrt{\frac{M}{B}}$
σ_c	25 - 35 kg/cm^2	30 - 40 kg/cm^2
σ_s	1400 kg/cm^2	2000 kg/cm^2

4. T-sections with double reinforcements

It has been stated before that a T-section is generally more economic if it is designed for a reduced allowable concrete stress smaller than half the stress allowed in a rectangular section.

It is however possible to reduce the depth of a T-section by designing the section for the limit of the allowable concrete compressive stress which is limited to 3/4 the stress allowed in a rectangular section; in which case, the design will be uneconomic.

It is therefore not recommended to reduce the depth of a T-section so that the use of compression steel seems to be essential.

A T-section with compression reinforcement can however be designed in the same way as rectangular sections, in which case, the design will be extremely uneconomic and the deflection has generally to be checked.

5. T-sections with tension reinforcements

Referring to Fig. 2-16, the assumptions, and basic equations 2-49 and 2-50, one can proceed as follows:

The position of the neutral axis can be determined from the relation:

$$\frac{\sigma_c}{\sigma_s/n} = \frac{z}{d - \frac{3}{4} z}$$

Assuming $\sigma_s / \sigma_c = r$ and $z = \zeta d$, then

$$z = \frac{n}{r + \frac{3}{4} n} d = \zeta d \quad \text{where} \quad \zeta = \frac{n}{r + \frac{3}{4} n}$$

We have further:

$$C\, y_{ct} = \frac{B\, z}{2} \frac{\sigma_c}{3} (d - \frac{z}{4}) = M \qquad \text{or}$$

$$\frac{B\, \zeta\, d}{2} \cdot \frac{\sigma_c}{3} (d - \frac{\zeta\, d}{4}) = M \qquad \text{or}$$

$$d = \sqrt{\frac{6}{\zeta (1 - \frac{\zeta}{4})}} \cdot \sqrt{\frac{M}{\sigma_c\, B}} \qquad \text{or}$$

$$d = c \sqrt{\frac{M}{\sigma_c\, B}} \qquad \text{4-22a}$$

where

$$c = \sqrt{\frac{6}{\zeta (1 - \frac{\zeta}{4})}}$$

Assuming $B = 2\, b$ then

$$d = \bar{c} \sqrt{\frac{M}{\sigma_c\, b}} \qquad \text{4-22}$$

where $$\bar{c} = \sqrt{\frac{3}{\zeta\,(1 - \frac{\zeta}{4})}}$$ and

$$A_s = \frac{M}{\sigma_s\, y_{ct}} = \frac{M}{\sigma_s\,(1 - \zeta\, d/4)} = \frac{M}{\sigma_s\, d\,(1 - \zeta/4)}$$ or

$$A_s = \frac{M}{\sigma_s\, \eta\, d} \qquad 4\text{-}23$$

where

$$\eta = (1 - \zeta/4)$$

The values of ζ, $\bar{c}$ and η for n = 15 and different values of $r = \frac{\sigma_s}{\sigma_c}$ are given in table 4-6.

Table 4-6 Dimensioning of ⅂-sections with tension reinforcements

r	10	15	20	25	30	35	40	45	50	55	60
ζ	0.706	0.571	0.480	0.414	0.364	0.324	0.293	0.267	0.245	0.226	0.211
η	0.823	0.857	0.880	0.896	0.909	0.919	0.927	0.933	0.939	0.943	0.946
$\bar{c}$	2.273	2.476	2.665	2.844	3.011	3.174	3.324	3.470	3.611	3.752	3.875

For sections with tension reinforcements only, if an ⅂-section is compared with a rectangular section having the same breadth b, it is found that the depth of the ⅂-section is @ 15% more than that of the rectangular section.

According to the Egyptian Code of Practice, an ⅂-section, that is not liable to rotate, is to be calculated as a symmetrical T-section of flange breadth B'; where B' is the smallest of:

$4.5\ t_s + b + b_s$,
1/6 distance between points of zero bending moments,
1/2 width of adjacent field.

Example

Design the ⅂-section shown in Fig. 4-24 to carry a bending moment of 8000 kgm. Concrete quality is C200 and normal mild steel.

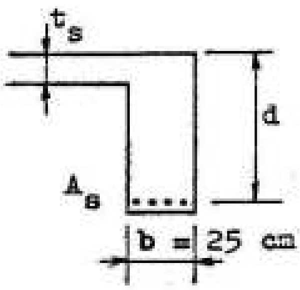


Fig. 4-24

Solution

Allowable stresses according to tables 3.1 and 3.2 are: $\sigma_s = 1400\ kg/cm^2$, $\sigma_c = 70\ kg/cm^2$ i.e.

$r = 1400 / 70 = 20$

Assuming a section with tension reinforcements only, the section can be designed by one of the following three different methods: a) using table 4-6, b) as rectangular and c) according to Egypt. code.

a) Design according to table 4-6

For $r = 20$, table 4-6 gives: $\eta = 0.88$ and $\bar{c} = 2.665$ so that

$$d = \bar{c}\sqrt{\frac{M}{\sigma_c\ b}} = 2.665\sqrt{\frac{8000\ 00}{70 \times 25}} = 57 \text{ cm} \qquad \text{and}$$

$$A_s = \frac{M}{\sigma_s\ \eta\ d} = \frac{8\ 000\ 00}{1400 \times 0.88 \times 57} = 11.4 \text{ cm}^2$$

b) Design as a rectangular section

For $r = 20$, table 4-4 gives: $\eta = 0.857$ and $c = 2.33$ so that

$$d = 1.15 \times 2.33\sqrt{\frac{M}{\sigma_c\ b}} = 2.68\sqrt{\frac{8000\ 00}{70 \times 25}} = 57.2 \text{ cm} \qquad \text{and}$$

$$A_s = \frac{M}{\sigma_s\ \eta\ d} = \frac{8\ 000\ 00}{1400 \times 0.857 \times 57.2} = 11.7 \text{ cm}^2$$

c) Design according to Egyptian Code

$B' = 4.5\ t_s + b = 4.5 \times 10 + 25 = 70$ cm

Assuming further $\sigma_c = \frac{1}{2} \times 70 = 35$ kg/cm^2, then

$r = 1400/35 = 40$ and table 4-4 gives: $\eta = 0.909$ and $c = 2.84$

Therefore:

$$d = c\sqrt{\frac{M}{\sigma_c\ B'}} = 2.84\sqrt{\frac{8000\ 00}{35 \times 70}} = 51 \text{ cm} \qquad \text{and}$$

$$A_s = \frac{8000\ 00}{1400 \times 0.909 \times 51} = 12.4 \text{ cm}^2$$

Assuming $d = 57$ cm, then σ_c will be smaller than 35 kg/cm^2 and one may assume: $y_{ct} = d - t_s/2$ in which case, we have $y_{ct} = 57 - 10/2 = 52$ cm and

$$A_s = \frac{M}{\sigma_s\ y_{ct}} = \frac{8000\ 00}{1400 \times 52} = 11.0 \text{ cm}^2$$

6. ⅂-sections with double reinforcements

The dimensioning of ⅂-sections with double reinforcements can be done basically by using the second method explained in rectangular sections.

7. Triangular sections with tension reinforcements only

For given values of σ_c and σ_s and referring to Fig. 2-17, one has:

$$\frac{\sigma_c}{\sigma_s/n} = \frac{z}{d - z}$$

Assuming that $\sigma_s/\sigma_c = r$ and $z / d = \zeta$, we get:

$$\zeta = \frac{z}{d} = \frac{n}{r + n}$$

and

$$y_{ct} = \eta\, d = d - \frac{z}{2} \qquad \text{refer to 2-52}$$

or

$$\eta = 1 - \frac{\zeta}{2}$$

Assuming further that the total depth of the section t is 1.08 its effective depth d or t = 1.08 d and solving equation 2-54 for d, we get

$$M = \sigma_c \frac{t\, z^2\, (d - ½\, z)}{6\, t} = \sigma_c \frac{b\, \zeta^2\, d^3\, (1 - ½\zeta)}{6 \times 1.08\, d}$$

or

$$d^2 = \frac{6.48\, M}{\sigma_c\, b\, \zeta^2\, (1 - ½\zeta)}$$

or

$$\underline{d = c \sqrt{\frac{M}{\sigma_c\, b}}} \qquad 4\text{-}24$$

where

$$c = \frac{1}{\zeta}\sqrt{\frac{6.48}{1 - ½\zeta}}$$

Assuming $b = \beta\, t = 1.08 \beta\, d$ then

$$d^2 = \frac{6.48\, M}{1.08\, \sigma_c\, \beta\, d\, \zeta^2\, (1 - ½\zeta)} \quad \text{or} \quad d^3 = \frac{6\, M}{\sigma_c\, \beta\, \zeta^2\, (1 - ½\,\zeta)} \quad \text{or}$$

$$\underline{d = c' \sqrt[3]{\frac{M}{\sigma_c\, \beta}}} \qquad 4\text{-}24\text{ a}$$

where

$$c' = \sqrt[3]{\frac{6}{\zeta^2\, (1 - ½\zeta)}}$$

In both cases:

$$\underline{A_s = \frac{M}{\sigma_s\, \eta\, d}} \qquad 4\text{-}25$$

The values of c, c' and η for n = 15 and different values of $r = \sigma_s/\sigma_c$ are given in table 4-7.

Table 4-7. Dimensioning of triangular sections with tension steel only

r	10	15	20	25	30	35	40	45	50	55	60
ζ	0.606	0.500	0.429	0.375	0.333	0.300	0.273	0.250	0.231	0.214	0.200
η	0.700	0.750	0.785	0.812	0.833	0.850	0.863	0.875	0.884	0.893	0.900
c	5.071	5.879	6.697	7.533	8.376	9.258	10.04	10.88	11.72	12.59	13.42
c'	2.877	3.174	3.463	3.746	4.020	4.281	4.535	4.787	5.025	5.274	5.503

8. Triangular sections with double reinforcements

In this case also, one can proceed in the same way as in the second method shown in rectangular sections.

Examples

1) It is required to design a triangular section subject to M = 5000 kgm if concrete C200 and normal mild steel are used. The breadth of the section b is double its depth t.

Solution

According to tables 3.1 and 3.2, the allowable stresses are: σ_s = 1400 kg/cm^2 and σ_c = 70 kg/cm^2 which mean that r = 20. For this value, table 4-7 gives: $\eta = 0.785$ and $c' = 3.463$.

Therefore, for $\beta = 2$, we get:

$$d = c'\sqrt[3]{\frac{M}{\sigma_c \beta}} = 3.463\sqrt[3]{\frac{5000\ 00}{70 \times 2}} = 53 \text{ cm}$$

so that t = 57 cm and b = 114 cm

$$A_s = \frac{M}{\sigma_s \eta d} = \frac{5000\ 00}{1400 \times 0.785 \times 53} = 8.6 \text{ cm}^2 \quad 7\ \text{Ø}\ 13$$

2) If t is chosen 50 cms only, determine A_s and A_s'.

Solution

Using basically the second method explained in rectangular sections, one can proceed as follows: Fig. 4-25.

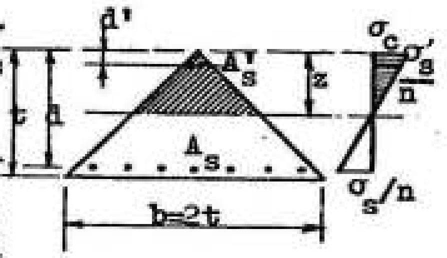


Fig. 4-25

The maximum moment that can be resisted by a triangular section, 50 cms deep and 100 cms wide, if tension reinforcements only were used for σ_c = 70 kg/cm^2 and σ_s = 1400 kg/cm^2 (i.e. r = 20), can be determined from relation 4-24a knowing that d = 46 cm, c' = 3.463 and $\beta = 2$.

$$46 = 3.463\sqrt[3]{\frac{M_1}{70 \times 2}} \quad \text{or} \quad M_1 = 3250\ 00 \text{ kg cm}$$

The corresponding reinforcement is given by:

$$A_{s1} = \frac{M_1}{\sigma_s \eta d} = \frac{3250\ 00}{1400 \times 0.785 \times 46} = 6.4 \text{ cm}^2$$

Table 4-7 gives further $\zeta = 0.429$, so that $z = \zeta d = 0.429 \times 46$ or z = 19.7 cms. Assuming that the compression steel lies at d' = 4 cms from the upper surface, the moment $M_2 = M - M_1$ will be resisted by additional tension steel A_{s2} acting at a stress σ_s = 1400 kg/cm^2, and compression steel acting at a stress σ_s', where

$$\sigma_s' = n\ \sigma_c\ \frac{z - d'}{z} = 15 \times 70 \times \frac{19.7 - 4}{19.7} = 838 \quad kg/cm^2$$

Therefore

$$A_{s2} = \frac{M_2}{(d - d')\ \sigma_s} = \frac{(5000 - 3250)\ 100}{(46 - 4)\ 1400} = 3.0 \quad cms^2$$

$$A_s' = \frac{M_2}{(d - d')\ \sigma_s'} = \frac{(5000 - 3250)\ 100}{(46 - 4)\ \ 838} = 5.0 \quad cms^2$$

and

$$A_s = A_{s1} + A_{s2} = 6.4 + 3.0 = 9.4 \quad cms^2$$

$$A_s' \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ = \underline{5.0 \quad cms^2}$$

$$\text{total} \quad = 14.4 \quad cms^2$$

3) Dimensioning of sections by the ultimate strength method

Recent codes of practice give all necessary provisions so that the intended safety is ensured. As stated before, safety is normally associated with adequate warning which, in turn, requires a desi... for a ductile failure.

For sections to be designed according to this method and referring to equations 3-12 to 3-18, it has been shown that:

1. The ultimate resistance of the section must be bigger than or equal to the ultimate moment M_u which may act on the section. M_u is to be determined according to provisions given by relations 3-1 to 3-4. For the general case of dead loads g and live loads p, we have:

$$M_u = 1.5\ M_g + 1.8\ M_p$$

2. The ultimate compression on the section C is resisted by the concrete compression zone of height z. The stress distribution due to C is assumed rectangular with a maximum ordinate = 0.85 f_{cp} and a height $y = 0.85\ z$ where z is the height from the neutral axis to the edge of the compression zone.

3. The total ultimate tension T is resisted by the tension steel only, and for ductile failure $\sigma_s = f_y$.

4. In order to have ductile failure, the ratio of the tension steel μ must be smaller than the balanced ratio μ_b. (Refer to equations 2-69a and 3-13).

5. To ensure ductile failure, it is recommended to choose:

$$\mu_{max} < 0.75\ \mu_b$$

This ratio corresponds to the minimum depth of a section with tension reinforcements only.

6. The ultimate moment of resistance of the section $C\ y_{ct}$ or $T\ y_{ct}$ must be reduced by the capacity reduction factor Ω given in article 3-2. For simple bending $\Omega = 0.9$.

7. In order to avoid sudden failure of the tension steel, its ratio is limited by a minimum value

$$\mu_{min} = 0.1/f_y \qquad 4\text{-}26$$

except in cases where the chosen amount of tension steel is 30% more than the calculated amount.

8. The deflection of a beam is to be checked if the steel ratio is bigger than:

$$0.18\ f_{cp}/f_y = 0.18/\rho \qquad \text{refer to 3-15}$$

This ratio gives generally a convenient design.

We give in the following, the design of some common forms of sections.

1. Rectangular sections with tension reinforcements only

Refer to Fig. 3-8

Given M_u, b, f_{cp} and f_y. Required d and A_s.

It has been proved that:

$$\zeta = \mu\rho / 0.72 \qquad \text{refer to 3-12}$$

$$M_u = \Omega C\ y_{ct} \qquad \text{or}$$

$$M_u = m_c\ f_{cp}\ b\ d^2 \qquad \text{refer to 3-16}$$

and

$$d = c\sqrt{\frac{M_u}{f_{cp}\ b}} \qquad \text{refer to 3-17}$$

in which

$$c = \sqrt{\frac{1}{m_c}} = \sqrt{\frac{1}{0.648\ \zeta\ (1 - 0.425\ \zeta)}}$$

and

$$M_u = \Omega T\ y_{ct} \qquad \text{or}$$

$$A_s = \frac{M_u}{f_y\ \eta\ d} \qquad \text{refer to 3-18}$$

in which

$$\eta = 0.9\ (1 - 0.425\ \zeta)$$

Table 4-8 gives the values of ζ, m_c, c and η.

Sheet 14 gives the values of c and η for different values of $\zeta = \mu\rho/0.72$.

Table 4-8. Dimensioning of rectangular sections with tension reinforcements only U S D

$\zeta = z/d$	μ_c	c	η	$\zeta = z/d$	μ_c	c	η
0.05	0.0317	5.62	0.88	0.20	0.1186	2.90	0.82
0.06	0.0379	5.14	0.88	0.22	0.1292	2.78	0.82
0.07	0.0440	4.77	0.87	0.24	0.1397	2.68	0.81
0.08	0.0501	4.47	0.87	0.26	0.150	2.58	0.80
0.09	0.0561	4.22	0.87	0.28	0.150	2.50	0.79
0.10	0.0621	4.01	0.86	0.30	0.170	2.43	0.785
0.11	0.0680	3.84	0.86	0.32	0.180	2.36	0.78
0.12	0.0738	3.68	0.85	0.34	0.188	2.30	0.77
0.13	0.0796	3.54	0.85	0.36	0.198	2.25	0.76
0.14	0.0853	3.42	0.85	0.38	0.205	2.20	0.75
0.15	0.0910	3.32	0.84	0.40	0.215	2.16	0.75
0.16	0.0966	3.22	0.84	0.45	0.236	2.06	0.73
0.17	0.1022	3.13	0.83	0.50	0.255	1.98	0.71
0.18	0.1077	3.05	0.83	0.55	0.273	1.91	0.69
0.19	0.1132	2.98	0.83	0.60	0.290	1.86	0.67

For given qualities of concrete and steel, the value of $\rho = f_y/f_{cp}$ is known, so that the magnitude of μ to be chosen can be determined.
However, for min. depth $\mu = 0.75\,\mu_b$
(for normal cases, these values are given in table 2, page 86)
for conv. depth $\mu = 0.18/\rho$

Having known μ, the value of ζ can be calculated fromrelation 3-12. The corresponding values of c and η, required for the design are then extracted fromtable 4-8. In normal cases one may assume: (4-27)

For a convenient depth $d = 2.65\sqrt{\dfrac{M_u}{f_{cp}\,b}}$ and $A_s = \dfrac{M_u}{0.80\,f_y\,d}$

For min. depth & max A_s $d = 2.0\sqrt{\dfrac{M_u}{f_{cp}\,b}}$ and $A_s = \dfrac{M_u}{0.71\,f_y\,d}$

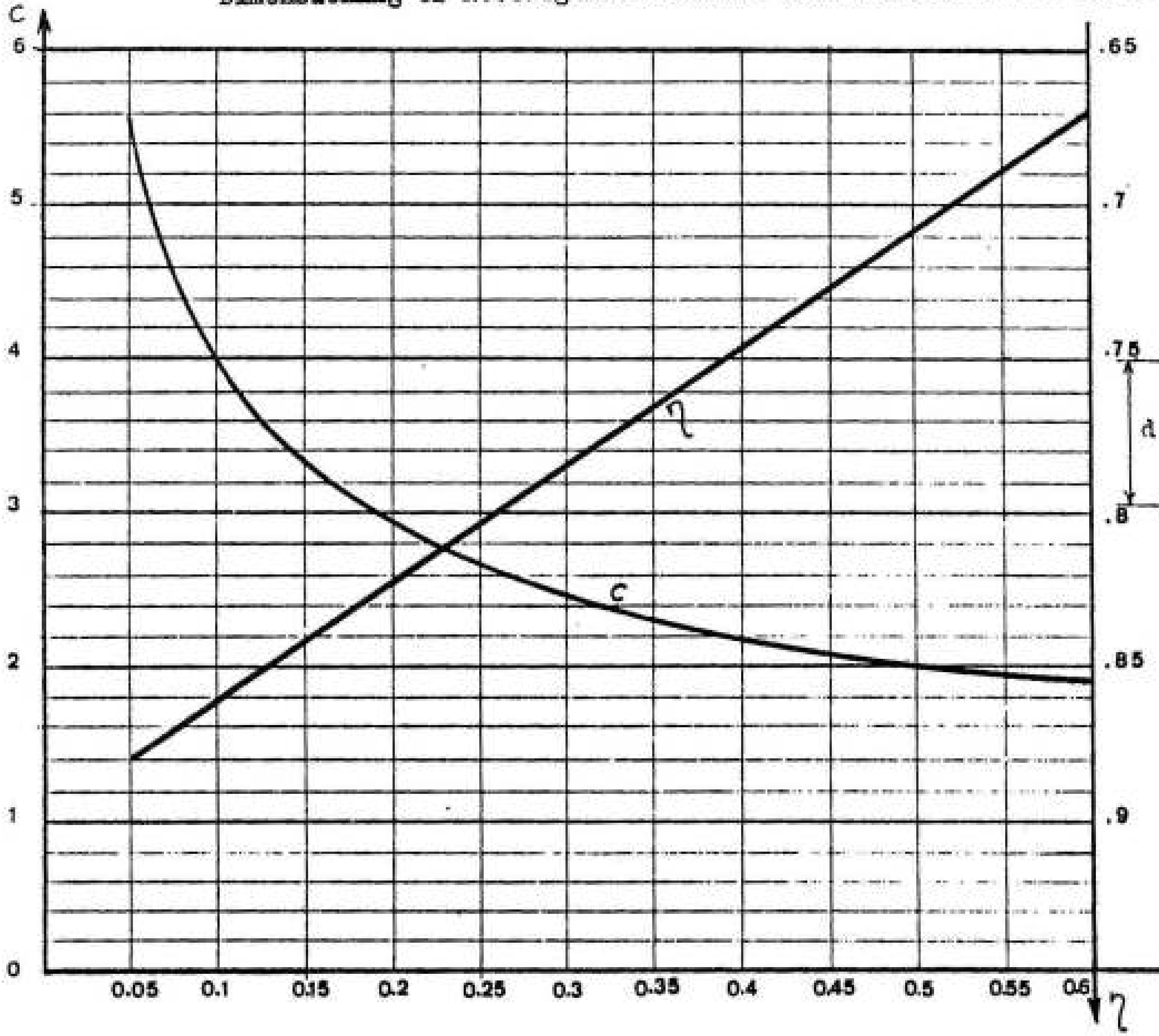
Hence, the ultimate moment at failure for a section subject to simple bending with tension reinforcements only (taking the specified reduction factor in consideration) is therefore given by:

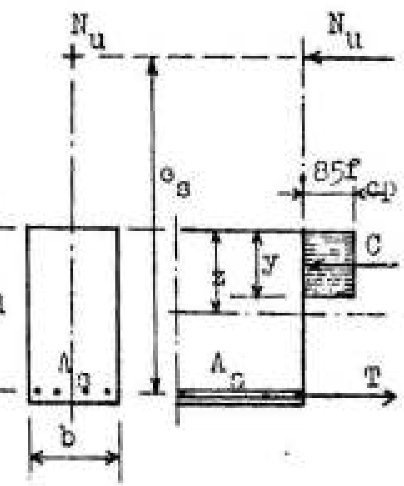
$$M_u = 0.25\,f_{cp}\,b\,d^2 \quad \text{..................a}$$
$$M_u = 0.71\,f_y\,A_s\,d \quad \text{..................b} \qquad 4\text{-}28$$

S H E E T 1 4

SIMPLE BENDING AND ECCENTRIC FORCES WITH BIG ECCENTRICITY (ULTIMATE STRENGTH DESIGN)

Dimensioning of Rectangular Sections with Tension Reinforcements Only





$$d = c\sqrt{\frac{M_{su}}{f_{cp}\,b}}$$

$$A_s = \frac{M_{su}}{f_y\,\eta\,d} \pm \frac{N_u}{f_y}$$

$$M_{su} = N_u\,e_s$$

2. Rectangular section with double reinforcement

If it is exceptionally required to design a section for a limited depth smaller than that corresponding to the balanced ratio of steel, i.e. (Fig. 4-26)

$$d < 2\sqrt{\frac{M_u}{f_{cp}\, b}} \qquad 4\text{-}29$$

compression reinforcement may be used.

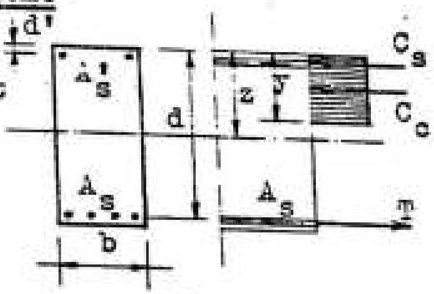


Fig. 4-26

The maximum ultimate moment that can be resisted by a rectangular section with tension reinforcement so that tension failure is ensured, is given by equation 4-28 a. Hence

$$M_{u1} = 0.25\, f_{cp}\, b\, d^2 \qquad 4\text{-}30$$

The corresponding area of tension steel ($0.75\mu_b$) is given by equation 4-28 b. Hence

$$A_{s1} = M_{u1} \,/\, 0.71\, f_y\, d \qquad 4\text{-}31$$

If $M_u > M_{u1}$, then $M_{u2} = M_u - M_{u1}$ is to be resisted by compression reinforcement A_s' and additional tension steel A_{s2}, so that

$$M_{u2} = \Omega\, A_s'\, f_y\, (d - d') = \Omega\, A_{s2}\, f_y\, (d - d')$$

or

$$A_{s2} = A_s' = M_{u2} \,/\, \Omega\, f_y\, (d - d') \qquad 4\text{-}32$$

in which d' = cover of compression reinforcement. The total tension reinforcement is therefore:

$$A_s = A_{s1} + A_{s2} \qquad 4\text{-}33$$

It is recommended to have:

1. $A_s' < \frac{1}{2} A_{s1}$

   i.e. $A_s < 1.5\, A_{s1}$

2. $y > 2\, d'$

   i.e. $y_{ct} \leqslant d - d'$

This condition insures that the stress in the compression steel at failure reaches the yield stress by specifying that its line of action lies nearer to the outside fiber of the section than the resultant compression C acting on the section.

3. It is highly desirable, for reasons given earlier, that failure takes place by tensile yielding rather than by crushing of the concrete. This can generally be ensured if condition 2-76 is satisfied. Hence

$$\mu - \mu' < 0.75\,\mu_b$$

Illustrative Examples

1. Using the ultimate strength method, it is required to determine the depth and area of tension steel of a rectangular section of breadth b = 30 cms subject to a dead load moment M_g = 14 mt and a live load moment M_p = 16 mt.

The materials used are concrete C200 and normal mild steel having a yield stress $f_y = 2300$ kg/cm^2.

Compare the results with those determined according to the working stress method.

Solution

a. Design according to the ultimate strength method

The ultimate moment is given by:

$$M_u = 1.5\ M_g + 1.8\ M_p = 1.5 \times 14 + 1.8 \times 16 = 21 + 28.8 = 50 \text{ mt}$$

Design compressive stress .... $f_{cp} = 165$ kg/cm^2

Tensile yield stress of steel .... $f_y = 2300$ kg/cm^2

A convenient depth and the corresponding tension steel may be chosen according to equation 4-27. Hence

$$d = 2.65\sqrt{\frac{50 \times 10^5}{165 \times 30}} = 84.4 \text{ cm} \quad \& \quad A_s = \frac{50 \times 10^5}{0.8 \times 2300 \times 84.4} = 32.4 \text{ cm}^2$$

The minimum depth and the corresponding steel are given by:

$$d = 2.0\sqrt{\frac{50 \times 10^5}{165 \times 30}} = 63.4 \text{ cm} \quad \& \quad A_s = \frac{50 \times 10^5}{0.71 \times 2300 \times 63.4} = 48.5 \text{ cm}^2$$

b. Design according to the working stress method

Tables 3.1 and 3.2 give:

Allowable concrete compressive stress $\sigma_c = 70$ kg/cm^2

Allowable steel tensile stress .... $\sigma_s = 1400$ kg/cm^2

So that $r = \sigma_s/\sigma_c = 1400 / 70 = 20$

Table 4-4 gives for r = 20, the values: $c = 2.33$ & $\eta = 0.857$

For $M = 14 + 16 = 30$ mt we get:

$$d = 2.33\sqrt{\frac{30 \times 10^5}{70 \times 30}} = 88 \text{ cm} \quad \& \quad A_s = \frac{30 \times 10^5}{1400 \times 0.857 \times 88} = 28.4 \text{ cm}^2$$

c. Assuming a depth of 88 cms and determining the steel by the USD

Equation 3-16 gives:

$$m_c = \frac{M_u}{f_{cp}\ b\ d^2} = \frac{50 \times 10^5}{165 \times 30 \times 88^2} = 0.130$$

According to table 4-8, the corresponding η is given by: $\eta = 0.82$.
So that,

$$A_s = \frac{M_u}{f_y \eta d} = \frac{50 \times 10^5}{2300 \times 0.82 \times 88} = 30.0 \quad cm^2$$

d. Check of stresses for the minimumdepth of 63.4 cms and the corresponding steel area of 48.5 cm² according to the WSD

For $M = 30$ mt , $b = 30$ cms , $d = 63.4$ cms and $A_s = 48.5$ cm²
we have:

$$z = \frac{n A_s}{b} (-1 + \sqrt{1 + \frac{2 d b}{n A_s}})$$

or

$$z = \frac{15 \times 48.5}{30} (-1 + \sqrt{1 + \frac{2 \times 63.4 \times 30}{15 \times 48.5}}) = 36.5 \text{ cm}$$

$$\sigma_c = \frac{2 M}{b z (d - z/3)} = \frac{2 \times 30 \times 10^5}{30 \times 36.3 (63.4 - 36.3/3)} = 107 \quad kg/cm^2$$

Such a high value is not allowed.

$$\sigma_s = \frac{M}{A_s (d - z/3)} = \frac{30 \times 10^5}{48.5 (63.4 - 36.3/3)} = 1205 \quad kg/cm^2$$

e. Required reinforcements A_s and A_s' according to the WSD

For $M = 30$ mt , $b = 30$ cms , $d = 63.4$ cms , $\sigma_c = 70$ kg/cm²
and $\sigma_s = 1400$ kg/cm², we have: $r = 1400/70 = 20$ and

$63.4 = c\sqrt{\frac{30 \times 10^5}{70 \times 30}}$ or $c = 1.675$ and

sheet 12 gives: $\alpha = 0.8$ & $\mu = 2.1$ % or

$$A_s = \frac{2.1}{100} \times 30 \times 63.4 = 40 \quad cm^2$$ and

$$A_s' = 0.8 \times 40 = 32 \quad cm^2$$

$$\text{total} = 72 \quad cm^2$$

against 48.5 cm² required by ultimate strength method.

It can be seen that for a depth of 63.4 cms = d_{min}, the section, calculated according to the working stress method, needs 32 cm² compression reinforcement. It has been proved by tests that the ultimate moment of the section is approximately the same as given by the ultimate strength method without all this compression reinforcement. This means that by using the USD-method there is a wide range for choosing smaller depths without the necessity of compression reinforcement.

2. It is required to design the beam given in the previous example if the maximum total depth allowed is 60 cms.

The theoretical depth is, in this case, limited to: $d = 55$ cms

The minimum depth without compression steel is given by:

$$d = 2\sqrt{M_u / f_{cp}\, b} = 2\sqrt{50 \times 10^5 / 165 \times 30} = 63.4 \text{ cms} > 55 \text{ cms}$$

The maximum moment M_{u1} that can be resisted by the given section, according to equation 4-28 a is given by:

$$M_{u1} = 0.25\, f_{cp}\, b\, d^2 = 0.25 \times 165 \times 30 \times 55^2 = 37.4 \times 10^5 \text{ kg cm}$$

The corresponding tension reinforcement, according to equation 4-28 b is given by:

$$A_{s1} = M_{u1} / 0.71\, f_y\, d = 37.4 \times 10^5 / 0.71 \times 2300 \times 55 = 41.6 \text{ cm}^2$$

The moment to be resisted by compression steel and additional tension steel is:

$$M_{u2} = M_u - M_{u1} = 50 \times 10^5 - 37.4 \times 10^5 = 12.6 \times 10^5 \text{ kg cm}$$

$$A_s' = A_{s2} = M_{u2} / \Omega\, f_y\, (d - d') = 12.6 \times 10^5 / 0.9 \times 2300 \times 50 = 12.2 \text{ cm}^2$$

The total tension reinforcement is therefore given by:

$$A_s = A_{s1} + A_{s2} = 41.6 + 12.2 = 53.8 \text{ cm}^2$$

$$\text{Total } A_s + A_s' = 53.8 + 12.2 = 66.0 \text{ cm}^2$$

Limitations:

1. $A_s \leqslant 1.5\, A_{s1}$ but $A_s = 53.8$ cm^2 and $A_{s1} = 41.6$ cm^2
   $1.5 \times 41.6 = 62.5 > 53.8$ i.e. condition 1 is satisfied.

2. $y_{ct} \leqslant d - d'$ but $d' = 5$ cm i.e. $y_{ct} \leqslant 55 - 5 = 50$ cm
   It is known that $M_u = A_s\, f_y\, y_{ct}$ so that
   $y_{ct} = M_u / A_s\, f_y = 50 \times 10^5 / 53.8 \times 2300 = 40.5$ cm < 50 cm
   which means that condition 2 is also satisfied.

3. To verify whether condition 3 is satisfied, especially because the depth is small and the percentage of tension steel is high, we have:

$$(\mu - \mu')\,\% = \frac{100\,(A_s - A_s')}{b\, d} = \frac{100\,(53.8 - 12.2)}{30 \times 55} = 2.52\%$$

μ_b can be calculated from equation 3-13; thus

$$\mu_b = \frac{4536}{\rho\,(6300 + f_y)} \quad \text{but} \quad \rho = \frac{f_y}{f_{cp}} = \frac{2300}{165} = 14$$

so that:

$$\mu_b = \frac{4536}{14\ (6300 + 2300)} = 3.77\ \%$$

$$0.75\,\mu_b = 0.75 \times 3.77\% = 2.83\% \ > \ 2.52\%$$

which means that condition 3 is satisfied and it is ensured that ductile failure will take place.

It is recommended that a special check of deflection be made if $\mu - \mu' > 0.18/\rho$. In this case,
$0.18/\rho = 0.18/14 = 0.0128$ is much smaller than $\mu - \mu' = 0.0252$. A check of deflection is therefore required.

3. T-sections with tension reinforcements only

In the T-beam design, the flange is usually determined by the slab design, and the web size -breadth and depth- is often determined by shear or requirements other than moment e.g. economy. The depth may however be chosen according to the methods given in the working stress design method, i.e.

$$d = c\sqrt{\frac{M}{\sigma_c\ B_R}} \simeq 0.5\sqrt{\frac{M}{B_R}}$$

so that the design for moment includes only the choice of the area of tension steel A_s, a check of brittle failure and some consideration of deflection.

A flange of T-section provides generally enough compression area to eliminate the possibility of brittle failure in compression. This is frequently confirmed by a neutral axis which falls high in the flange and turns a T-beam design into the design of a wide rectangular beam.

The ratio of the tension steel μ must be smaller than or equal to 0.75 the balanced ratio, i.e.

$$\mu < 0.75\mu_b$$

and, as C = T, the actual compression block is restricted to 0.75 of the balanced condition; this will generally lead to $y < t_s$. Thus most T-beams are only rectangular beams in flexural behavior. The methods shown in the analysis, Figures 2-26, 27 and 28 can be used in the design as will be shown in the following example.

Example

A floor system consists of a 10 cm concrete slab supported by continuous T-beams 9 ms span. Assuming that M_g = 15 mt and M_p = 25 mt, determine the tension steel at midspan if high grade steel with f_y = 3600 kg/cm² and concrete C200 are used. Assume t = 90 cms and b = 30 cms.

a. Check of the assumed depth

$$d = 0.5\sqrt{\frac{M}{B_\lambda}}$$ in which

$B_\lambda = \lambda\, B$, and B is the smaller of:

$B = 12\ t_s + b = 12 \times 10 + 30 = 150$ cm and $l/4.5 = 900/4.5 = 200$ cms

Therefore B = 150 cm

According to working stress design method, we have:

Position of neutral axis (equation 4-19 b) is given by:

$$z = 0.112\sqrt{\frac{M}{B}} = 0.112\sqrt{\frac{40\ 000}{1.5}} = 18.3 \text{ cms}$$ so that

according to sheet 13, we have: for $\frac{B}{b} = \frac{150}{30} = 5$ and $\frac{t_s}{z} = \frac{10}{18.3} = .55$

$\lambda = 0.84$, so that $B_\lambda = \lambda\, B = 0.84 \times 150 = 126$ cms and

$$d = 0.5\sqrt{\frac{40\ 000}{1.26}} = 89 \text{ cms}$$

Therefore, the assumed total depth t of 90 cms (d = 85 cms) is acceptable.

b. Area of tension steel according to ultimate strength theory

$$M_u = 1.5\ M_g + 1.8\ M_p = 1.5 \times 15 + 1.8 \times 25 = 67.50 \text{ mt}$$

If the stress block depth y were equal to the flange thickness of 10 cms, then $y_{ct} = d - \frac{t_s}{2} = 85 - 5 = 80$ cms and

$$A_s = \frac{M_u}{\Omega\ f_y\ y_{ct}} = \frac{67.5}{0.9 \times 3.6 \times 0.80} = 26.1 \text{ cm}^2$$

Therefore $\bar{\mu} = A_s/B\ d = 26.1/150 \times 85 = 0.00205$

Applying equation 2-78, we get:

$$z = \frac{\bar{\mu}\ f_y\ d}{0.72\ f_{cp}} = \frac{0.00205 \times 3600 \times 85}{0.72 \times 165} = 5.26 \text{ cms}$$

z being smaller than t_s, then the section behaves as a wide rectangular section with breadth B = 150 cm and effective depth d = 85 cm.

Using table 4-8, we have:

For $m_c = \frac{M_u}{f_{cp} B d^2} = \frac{67.5 \times 10^5}{165 \times 150 \times 85^2} = 0.0377$ we have $\eta = 0.88$

and $A_s = \frac{M_u}{f_y \eta d} = \frac{67.5 \times 10^5}{3600 \times .88 \times 85} = 25 \text{ cm}^2$

If it happens that $z > t_s$, a T-beam analysis is indicated and one can proceed in the following manner:

a) One determines the steel A_{s1} required to balance the compressive force in the overhanging portions of the flange (equation 2-79). Introducing the capacity reduction factor , we get:

$$A_{s1} = \frac{0.85 f_{cp} (B - b) t_s}{\Omega f_y} \qquad 4\text{-}34$$

The force $A_{s1} f_y = \frac{0.85 f_{cp}}{\Omega} (B - b) t_s$ acts at a lever arm = $(d - \frac{1}{2} t_s)$ to provide the resisting moment:

$$M_{u1} = \Omega A_{s1} f_y (d - \tfrac{1}{2} t_s) \qquad 4\text{-}35$$

b) The remaining moment $M_{u2} = M_u - M_{u1}$ will be resisted by the compression in the rectangular portion of the beam. The corresponding m_c is given by:

$$m_c = \frac{M_{u2}}{f_{cp} b d^2}$$

and the required area of steel $A_{s2} = A_s - A_{s1}$, can be calculated from the relation:

$$A_{s2} = \frac{M_{u2}}{f_y \eta d} \qquad 4\text{-}36$$

where η is to be extracted fromtable 4-8; and

c)
$$A_s = A_{s1} + A_{s2} \qquad 4\text{-}37$$

It has also to be checked whether ductile failure is ensured by satisfying equation 2-84. i.e.

$$\mu - \mu_1 < 0.75 \mu_b$$

A check of deflection has also to be made if

$$\mu - \mu_1 > 0.18/\rho$$

where $\mu = A_s / b\ d$ and $\mu_1 = A_{s1} / b\ d$

Assuming that the dimensions of the T-beam of the previous example are limited to: $b = 25$ cm, $t = 65$ cm, $d = 58$ cm and $t_s = 8$ cm then,

$$B = 12 \times 8 + 25 = 121 \text{ cm}$$

If we assume that y is equal to the flange thickness, then:

$y_{ct} = 58 - 4 = 54$ cm and $A_s = \dfrac{57.5}{0.9 \times 3.6 \times 0.54} = 38.6\ \text{cm}^2$, or

$\bar{\mu} = \dfrac{38.6}{121 \times 58} = 0.0055$ and $z = \dfrac{0.0055 \times 3600 \times 58}{0.72 \times 165} = 9.7$ cm

z being bigger than t_s, a T-beam analysis is indicated. According to equation 4-34, we have

$$A_{s1} = \frac{0.85\ f_{cp}\ (B - b)\ t_s}{\Omega\ f_y} = \frac{0.85 \times 165\ (121 - 25)\ 8}{0.9 \times 3600} = 33.33\ \text{cm}^2$$

According to equation 4-35, we have:

$M_{u1} = \Omega\ A_{s1}\ f_y\ (d - \tfrac{1}{2} t_s) = 0.9 \times \dfrac{100}{3} \times 3600\ (58 - 4) = 52.5 \times 10^5$ kg cm

so that $M_{u2} = M_u - M_{u1} = 67.5 - 52.5 = 15$ mt and

$$m_c = \frac{M_{u2}}{f_{cp}\ b\ d^2} = \frac{15 \times 10^5}{165 \times 25 \times 58^2} = 0.0175$$

The value of η according to table 4-8 is given by: $\eta = 0.7825$

so that $A_{s2} = \dfrac{M_{u2}}{f_y\ \eta\ d} = \dfrac{15 \times 10^5}{3600 \times 0.7825 \times 58} = 9.15\ \text{cm}^2$

The total amount of steel is therefore:

$$A_s = A_{s1} + A_{s2} = 33.33 + 9.15 = 42.48\ \text{cm}^2$$

Hence, we have:

$\mu = A_s/b\ d = 42.48/25 \times 58 = 0.0292$ and $\mu_1 = A_{s1}/b\ d = 0.023$

with $\rho = 3600/165 = 21.8$, the value of μ_b is given by:

$$\mu_b = \frac{4536}{21.8\ (6300 + 3600)} = 0.021$$

i.e. $\mu - \mu_1 = 0.0292 - 0.0230 = 0.0062$ and $0.75\mu_b = 0.157$, so that $\mu - \mu_1 < 0.75\,\mu_b$ i.e. ductile failure is ensured.

Further, we have: $0.18/\rho = 0.18/21.8 = 0.00825$

i.e. $\mu - \mu_1 < 0.18/\rho$, so that no check of deflection is required.

4. Sections of other forms with single and double reinforcements

Following the principles explained with respect to rectangular and T-sections, any other symmetrical form of section with single or double reinforcement may be treated.

As an example, a triangular section subject to simple bending will be treated for both cases.

4-a. Triangular sections with tension reinforcements

Given: M_u, b, f_{cp} and f_y.

Required: t and A_s. One may assume here: $t/d = b/b' = 1.08$. (Fig. 4-27)

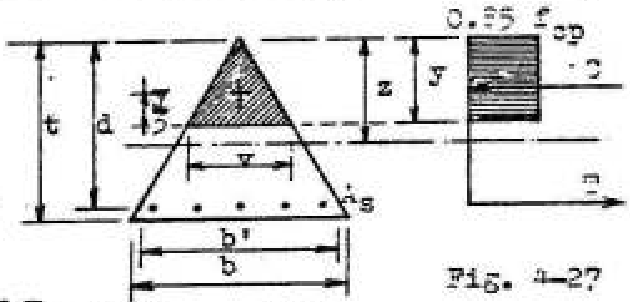


Fig. 4-27

Solution

We have: C = T or $0.85\ f_{cp} \frac{v\ y}{2} = A_s\ f_s$, but

$\frac{v}{b} = \frac{y}{t}$, $y = 0.85\ z$, $z = \zeta\ d$, $t = 1.08\ d$, $b = 1.08\ b'$, i.e.

$A_s = \mu/2\ b'\ d = 0.463\ \mu\ b\ d$ so that $v = 0.787\ b\ \zeta$,

Therefore $0.85\ f_{cp} \cdot \frac{0.787\ b\ \zeta}{2} \cdot 0.85\ \zeta\ d = 0.463\ \mu\ b\ d\ f_y$ and knowing that $f_y / f_{cp} = \rho$ then

$$0.614\ \zeta^2 = \mu\ \rho \quad \text{or} \quad \zeta = 1.276\sqrt{\mu\ \rho} \quad \ldots\ldots\ldots\ldots \quad 4\text{-}38$$

The area of tension steel can be determined from the relation:

$M_u = \Omega\ T\ (d - \frac{2\ y}{3}) = 0.9\ A_s\ f_y\ (1 - 0.567\ \zeta)\ d$ or

$M_u = A_s\ f_y\ \eta\ d$ giving

$$A_s = \frac{M_u}{f_y\ \eta\ d} \quad \ldots\ldots\ldots\ldots \quad 4\text{-}39$$

where

$$\eta = 0.9\ (1 - 0.567\ \zeta)$$

We have further

$M_u = \Omega\ C\ (d - \frac{2\ y}{3}) = 0.9\ \frac{v\ y}{2} \cdot 0.85\ f_{cp}\ (d - \frac{2\ y}{3})$ and

substituting $v = 0.787\ b\ \zeta$ & $y = 0.85\ \zeta\ d$, we get:

$M_u = 0.266\ \zeta^2\ b\ d^2\ f_{cp}\ (1 - 0.567\ \zeta) = m_c\ f_{cp}\ b\ d^2$ or

$$m_c = \frac{M_u}{f_{cp}\ b\ d^2} \quad \ldots\ldots\ldots\ldots \quad 4\text{-}40$$

in which $m_c = 0.266\,\zeta^2\,(1 - 0.567\,\zeta)$ and

$$d = c\sqrt{\frac{M_u}{f_{cp}\,b}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 4\text{-}41$$

where

$$c = \sqrt{\frac{1}{m_c}}$$

If $b/t = \beta$, and $t = 1.08\,d$ then $b = 1.08\,d\beta$. Substituting this value in the equation $M_u = m_c\,f_{cp}\,b\,d^2$, we get:

$M_u = m_c\,f_{cp}\,(1.08\,d\beta)\,d^2 = \bar{m}_c\,f_{cp}\,\beta\,d^3$ where $\bar{m}_c = 1.08\,m_c$

so that

$$d = \bar{c}\sqrt[3]{\frac{M_u}{f_{cp}\,\beta}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 4\text{-}42$$

in which

$$\bar{c} = \sqrt[3]{\frac{1}{1.08\,m_c}}$$

The values of ζ, η, m_c, c and $\bar{c}$ are given in table 4-9 assuming $t = 1.08$ d.

Table 4-9. Dimensioning of triangular sections with tension reinforcements only by USD-method

ζ	0.350	0.400	0.450	0.500	0.550	0.600	0.650	0.700	0.750	0.800	0.850
η	0.722	0.696	0.671	0.644	0.619	0.594	0.568	0.543	0.517	0.491	0.466
m_c	0.024	0.030	0.036	0.043	0.050	0.057	0.071	0.079	0.086	0.093	0.100
c	6.455	5.774	5.270	4.822	4.472	4.189	3.753	3.558	3.410	3.279	3.162
$\bar{c}$	3.380	3.137	2.952	2.782	2.645	2.531	2.354	2.271	2.208	2.152	2.100

For minimum depth, $\mu = 0.75\mu_b$. Knowing f_y and f_{cp}, ρ and μ_b can be determined from equation 3-13; so that μ for minimum depth is known. The corresponding value of ζ can then be calculated from equation 4-38. For this value of 'ζ', c, $\bar{c}$, and η required for the minimum depth and the corresponding area of tension steel can be extracted from table 4-9. They can generally be given by:

$$d = 3.4\sqrt{\frac{M_u}{f_{cp}\,b}} \quad \text{or} \quad d = 2.2\sqrt[3]{\frac{M_u}{f_{cp}\,\beta}} \qquad 4\text{-}43$$

and

$$A_s = \frac{M_u}{0.51\,f_y\,\eta\,d} \qquad 4\text{-}44$$

4.b Triangular sections with double reinforcements

If it is exceptionally required to design a triangular section for a limited depth smaller than that corresponding to equation 4-43, compression reinforcements, using the same methods applied before, may be used as will be shown in the following example:

Example

A triangular section with b = 60 cm is subjected to M_u = 12 mt. If f_y = 3600 kg/cm^2 and f_{cp} = 200 kg/cm^2. Determine:

1. The minimum total depth t and the maximum area of tension steel A_s.
2. If the maximumdepth allowed is ½ b = 30 cm, determine A_s and A'_s - if required-.

Solution

1. Equations 4-43 and 4-44 give:

$$\text{min. } d = 3.4\sqrt{\frac{M_u}{f_{cp}\ b}} = 3.4\sqrt{\frac{12}{2000 \times 0.6}} = 0.34 \text{ m} \qquad \text{i.e.}$$

$$\underline{\text{min. } t = 37 \text{ cm}}$$

$$\text{max. } A_s = \frac{M_u}{0.51\ f_y\ d} = \frac{12}{.51 \times 3.6 \times 0.34} = 19.2 \text{ cm}^2$$

2. Max. t = 30 cm (37 cm determined in 1.), then, compression reinforcement is required.

$$d = 27 = 3.4\sqrt{\frac{M_{u1}}{2000 \times 0.6}} \qquad \text{or} \qquad M_{u1} = 7.60 \text{ mt}$$

$$A_{s1} = \frac{7.6}{0.51 \times 3.6 \times 0.27} = 15.4 \text{ cm}^2$$

$$M_{u2} = M_u - M_{u1} = 12.0 - 7.6 = 4.4 \text{ mt}$$

$$A'_s = A_{s2} = \frac{M_{u2}}{0.9\ f_y\ (d - d')} = \frac{4.4}{0.9 \times 3.6 \times 0.24} = 5.7 \text{ cm}^2$$

$$A_s = A_{s1} + A_{s2} = 15.40 + 5.70 = 21.1 \text{ cm}^2$$

So that, total amount of steel in the section is given by:

$$\text{Total } A_s = 26.80 \text{ cm}^2$$

This ratio being very high, a special check of deflection is essential.

4.5 Shearing Forces and Diagonal Tension

General considerations

The dimensions of a beam are generally determined to meet its flexural requirements; in addition, it must be safe against premature fail ure due to diagonal tension resulting mainly from the shearing forces. The analysis of beams subject to shear and diagonal tension has been shown in 2.6.

Beams may be designed with cross sections sufficiently large that the concrete can resist all the diagonal tension; however, it is generally more economic to use the smaller dimensions required for bending and to provide the beam with additional steel called 'web reinforcement' to carry the excess shear greater than that which can be carried by concrete alone.

Web reinforcement to resist diagonal tension may take one of the following forms: Fig. 4-28.

1. Bent bars; angle of bent with axis of beam lies between 30 and 60 degrees (normally 45 degrees).

2. Stirrups normal to the main longitudinal reinforcement of the beam;

3. Inclined stirrups; angle of inclination with the axis of the beam $\gg$ 30°.

The stirrups given under 2 and 3 may be closed or open with two or more branches.

4. Any combination of 1, 2 and 3.

The stirrups must generally be properly anchored in the compression zone of the beam.

One of the main functions of stirrups is to make the compression and tension zones of a beam act integrally and therefore they have to extend over the whole span of the beam. They are generally chosen 6 to 8 mm diameter (eventually 5 mm diameter for small beams and 10 mm diameter for exceptionally heavy beams), and equally spaced, 5 to

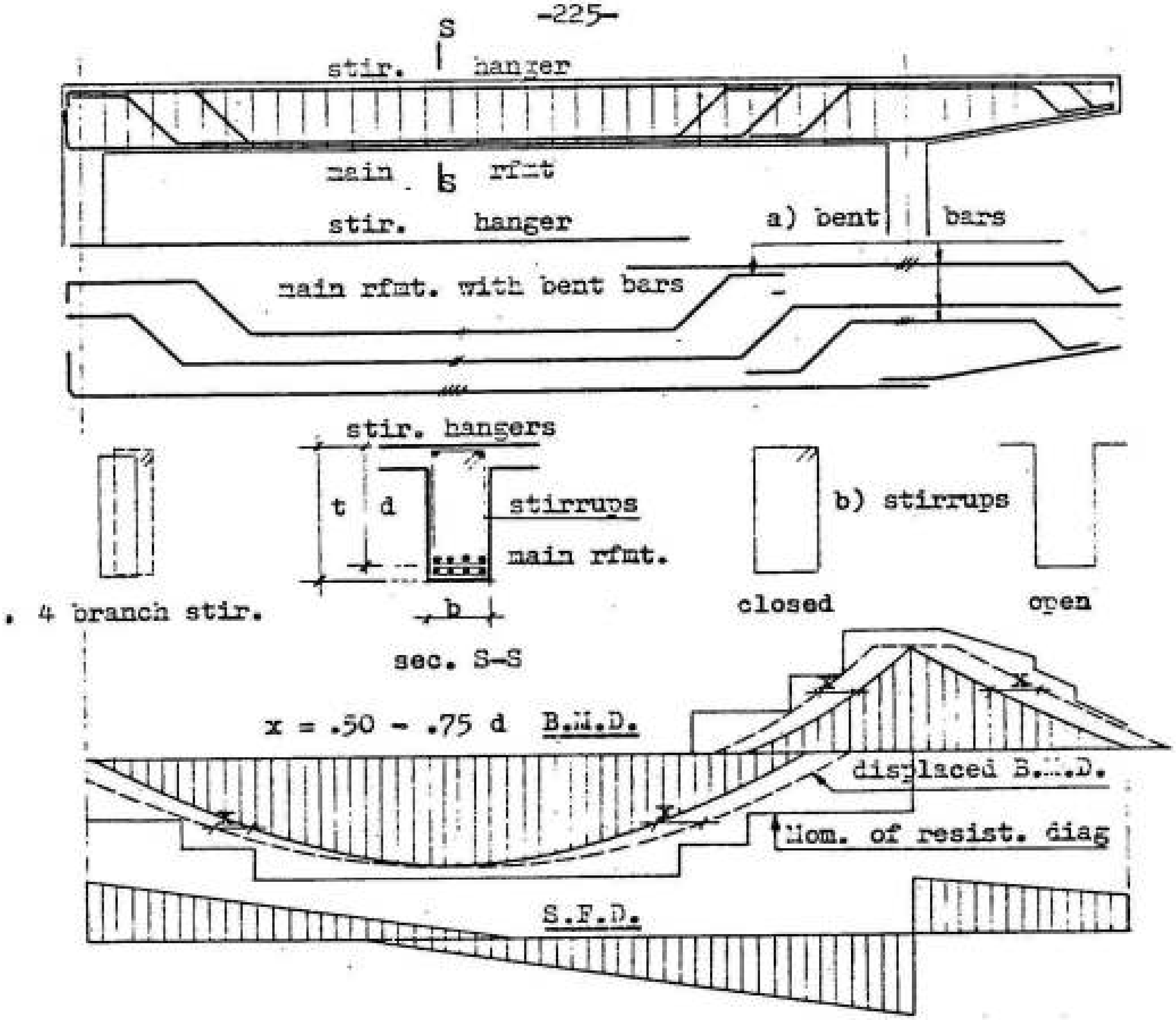


Fig. 4-28

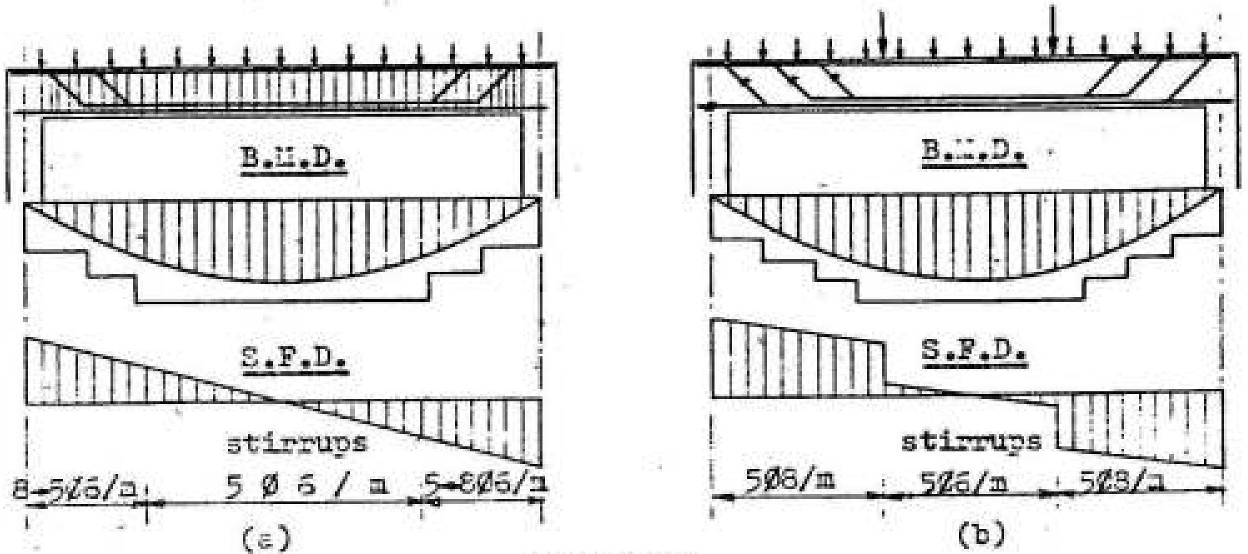


Fig. 4-29

7 per meter. In special cases, it is allowed to change the spacing , and eventually the diameter along the axis of the beam, wider spacing and smaller diameter at positions of low shear stresses and closer spacing with/or bigger diameter at positions of high shear stresses as shown in Fig. 4-29.

Bent bars are generally a part of the tension longitudinal reinforcement of the beam. The reinforcement which is no longer needed for flexural tension is bent at positions of high shear stresses as shown in Figs. 4-28a and 4-29.

Determination of the amount of web reinforcement

a) Using the working-stress- design method

One can determine the amount of web reinforcement required in a beam in the following manner: Fig. 4-30.

1- For the shearing forces Q acting on the beam, determine the diagonal tensile stresses at the different points of the beam from relation 2-85 (or eventually 2-87) namely:

$$\tau = \frac{Q}{y_{ct}\ b} \qquad \text{evt.} \qquad \tau = \frac{Q_{red}}{y_{ct}\ b}$$

The maximum shearing force Q, and therefore the maximum calculated shear stress τ, generally occur immediately adjacent to a support. Numerous tests have shown, however, that the first diagonal crack occurs not directly at the support, but at some distance from it of the order of the depth of the beam. This is so because at and adjacent to the supports the additional local stresses caused by reactions counteract crack formation. For this reason, it is generally specified that the maximum shear to be considered is that at a distance x from the face of the support, where x = 0.5 d to d and d = the effective depth of the beam.

It is further allowed that, in case a concentrated load P acts at a distance a from a support where a $<$ 2 d, the diagonal tensile stresses due to P are reduced by the ratio a / 2 d.

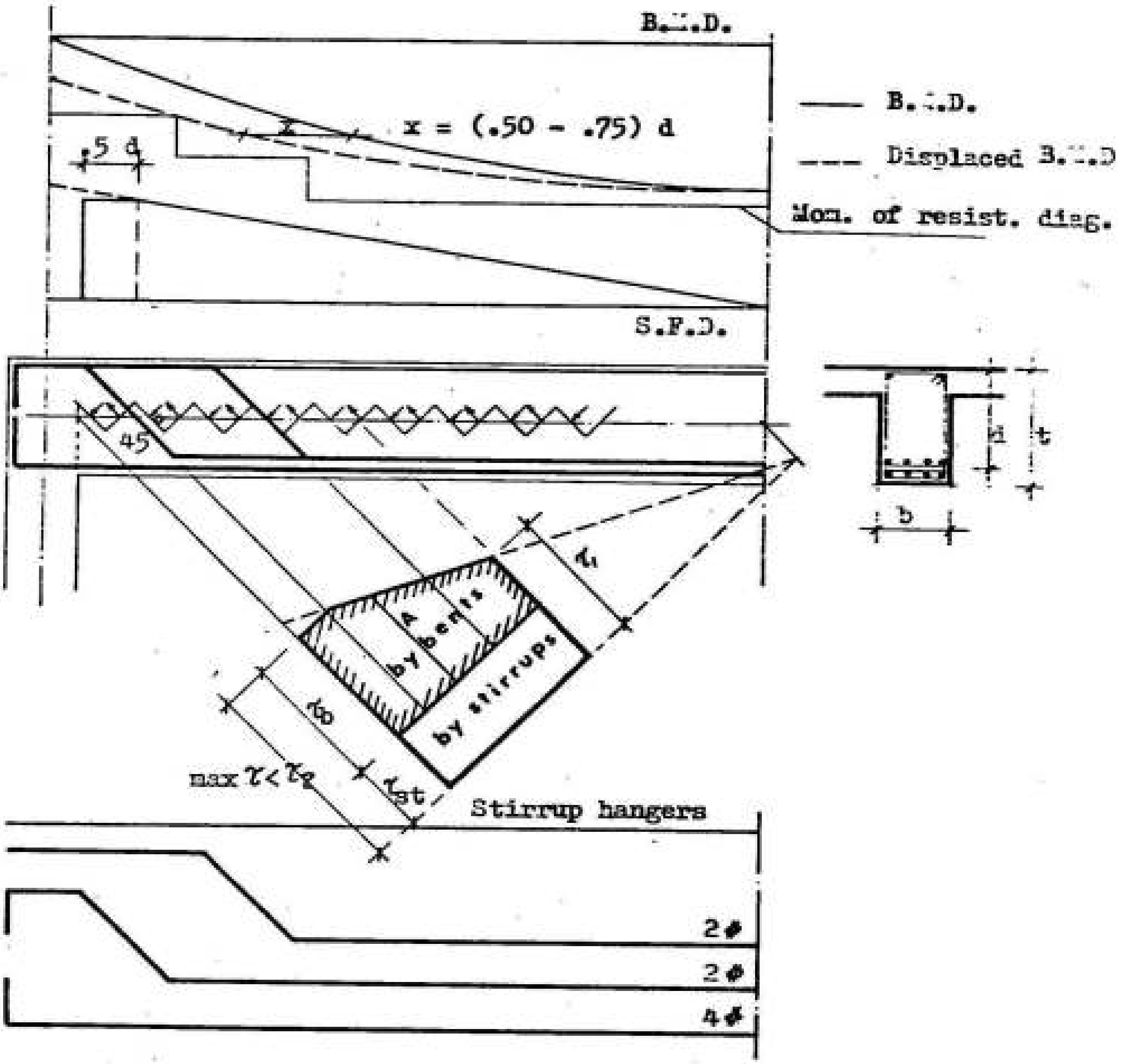


Fig. 4-30 Determination of web reinforcement

2- Draw the maximum diagonal tension acting on the beam (in the zone between the neutral axis and the tension steel of any section) in the direction of the principal tensile stresses as has been shown in Fig. 2-32. This direction can however be known immediately if the bending moment diagram of the beam is drawn on its tension side; because in this case, the diagonal tension diagram takes the direction of the side of the bending moment diagram at the support under consideration and is inclined 45° with the axis of the beam. It is generally represented on the center line of the beam as shown in Fig.4-30.

In zones where the shear stresses τ are smaller than τ_1 specified in table 3-2, no special calculation is essential for the web reinforcement on condition that this is bigger than the minimum allowed values given by:

$\min A_{st}$ = 0.15 % of the cross section of the web or

$3.5 / f_y$ where f_y is the yield stress of the stirrups.

Hence, for normal mild steel with $f_y = 2300$ kg/cm^2, $\min A_{st}$ = 0.15 %.

Accordingly, for a beam 25 x 75 cms, the minimum area of stirrups to be used per meter is:

$\min A_{st} = \frac{0.15}{100} \times 25 \times 75 = 2.8$ cm^2 i.e. two branch stirrups 5 Ø 6 mm/m giving an area of 2.8 cm^2,

and for a beam 50 x 160 cms, the minimum area of stirrups is given by:

$\min A_{st} = \frac{0.15}{100} \times 50 \times 160 = 12$ cm^2 i.e. four branch stirrups 6 Ø 8 mm/m giving an area of 12.0 cm^2.

If the resistance of concrete to shear stresses is neglected (as is generally the case in Egypt), the part of the diagonal tension diagram between τ_1 and $\tau_{max} \leqslant \tau_2$, shown heavy in Fig. 4-30, is to be resisted by web reinforcements.

If a part $\tau_c \leqslant 0.25\sqrt{f_{cp}}$ is allowed to be resisted by the compression zone of the beam, it has first to be deducted from the diagonal tension diagram, and the remaining part, shown heavy in Fig. 4-31, is to be resisted by web reinforcement.

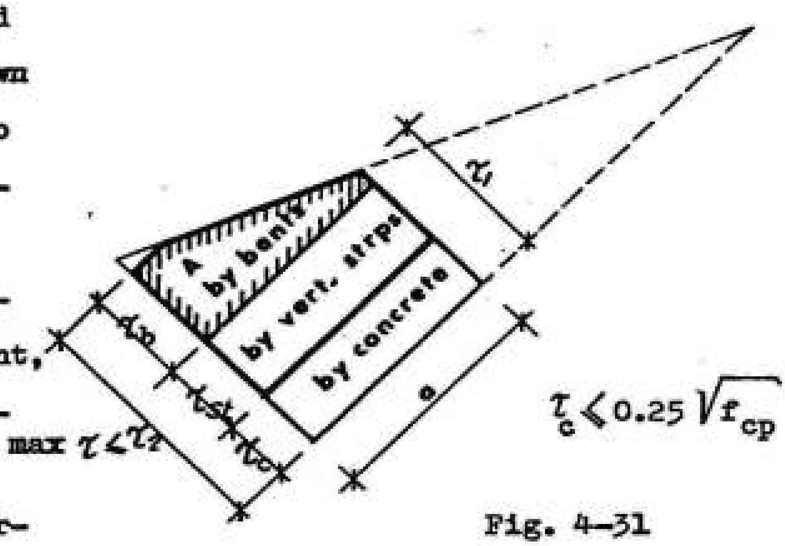


Fig. 4-31

3- In order to determine the web reinforcement, it is recommended to proceed as follows:

Choose first the stirrups and determine τ_{st} to that can be resisted by them from relation 2-95 (eventually 2-96) giving for vertical stirrups:

$$\tau_{st} = \frac{A_{st}\,\sigma_s}{b\ s}$$

where A_{st} = area of cross section of the stirrups in the section of the beam under consideration, and

s = spacing between the stirrups.

If, for example, the stirrups in a beam 25 x 75 cm are chosen 5 ϕ 8 mm / m with two branches and of normal mild steel, then

the allowable stress in steel .... σ_s = 1400 kg/cm^2

the breadth ob the beam .... b = 25 cms

the spacing of the stirrups .... s = 20 cms

the area of two branches ϕ 8 mm .... A_{st} = 1.00 cm^2

then

$$\tau_{st} = \frac{1.00 \times 1400}{25 \times 20} = 2.8 \quad kg/cm^2$$

It is however recommended to choose the stirrups such that τ_{st} is 0.25 to 0.40 τ.

The area of the main steel to be bent is equal to the hatched area A multiplied by the breadth b of the web divided by the allowable stress of the bent bars σ_s.

Assuming that the average value of the shear stress to be resisted by the bent bars τ_b = 8.00 kg/cm^2, the length a = 150 cms, the breadth of the beam b = 25 cms and the bent bars are of high grade steel with σ_s = 2000 kg/cm^2, then, the area of steel to be bent A_{sb} is given by:

$$A_{sb} = \frac{8.00 \times 150 \times 25}{2000} = 15.00 \quad cm^2$$

It has to be noted that the area A is to be exactly calculated.

b- <u>Using the ultimate-strength design method</u>

One can proceed here as in the previous case using the ultimate values of the loads and stresses together with the corresponding capacity reduction factors as follows:

The diagonal tension is to be calculated for the ultimate shearing force Q_u, where

$$Q_u = 1.5\, Q_g + 1.8\, Q_p$$

The ultimate diagonal tensile stress τ_u is given by the relation:

$$\tau_u = \frac{Q_u}{y_{ct}\, b}$$

In beams of variable depth, Q_u is to be replaced by $Q_{u\ red}$ where

$$Q_{u\ red} = Q_u - \frac{M_u \tan\alpha}{y_{ct}}$$ where

α is the inclination of one surface of the beam with respect to the other (Fig. 2-33).

One can assume further that:

$\tau_{1u} \leqslant 0.8\sqrt{f_{cp}}$ $\tau_{2u} \leqslant 2.5\sqrt{f_{cp}}$ $\tau_{cu} = 0.4\sqrt{f_{cp}}$

Assuming that the capacity reduction factor for shear is 0.85, then

and $$\tau_{st\ u} = \frac{0.85\, A_{st}\, f_y}{b\ s}$$

$$A_{sb\ u} = \frac{A.b}{0.85\, f_y}$$

Example

It is required to determine the concrete dimensions and the steel reinforcement required for a double cantilever shed (Fig. 4-32) subject to a concentrated load W = 12 ton (of which G = 8 tons and P = 4 tons) acting at the free ends. The materials used are: C200 for concrete, high grade steel for longitudinal tension reinforcement and normal mild steel for compression reinforcement (if any) and stirrups.

Assume the breadth of the cantilevers = 30 cms and their maximum depths at the free and fixed ends = 50 and 80 cms respectively

Solution

Average own weight = $\frac{0.4 + 0.7}{2} \times 0.3 \times 2500 \simeq 400$ kg/m

The bending moment and shearing force diagrams are shown in Fig. 4-32.

The solution will be made using both the working stress and the ultimate strength design methods, in order to be able to compare the results.

Dimensioning

a- Using the working-stress-design method

The bending moment in the cantilevers cause tension in the upper fiber where the slab lies, so that they behave as rectangular.

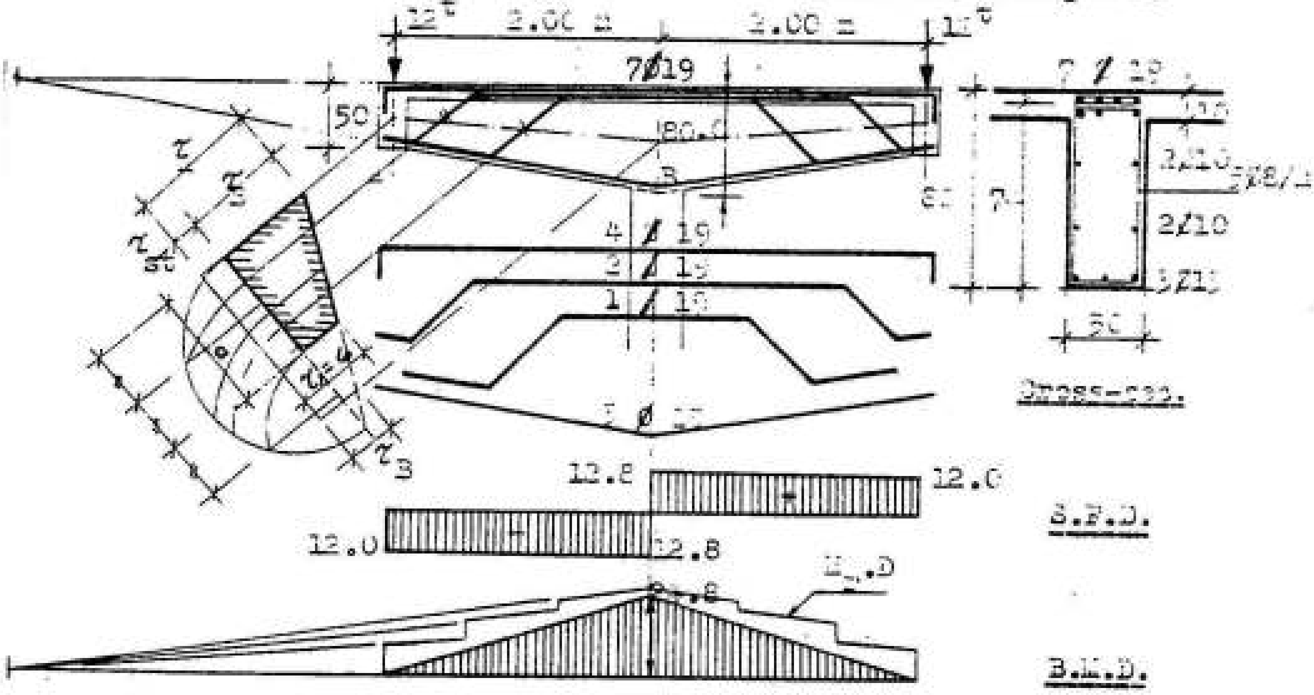


Fig. 4-32

The maximum value of the bending moment of 24.8 mt takes place at the sections of fixation. For this section, we have:

$$d = c\sqrt{\frac{M}{\sigma_c\, b}}$$

in which

d = 74 cms , b = 30 cms and according to table 3-2, we have:

$$\sigma_c = 70 \quad kg/cm^2 \quad \text{for concrete}$$

$$\sigma_s = 2000 \quad kg/cm^2 \quad \text{for high grade steel}$$

so that $r = 2000 / 70 = 28.6$ and

$$74 = c\sqrt{\frac{24.8 \times 10^5}{70 \times 30}} \quad \text{giving} \quad c = 2.15$$

According to sheet 12 $\alpha = 0.7$ and $\mu = 0.85$ % so that

$$A_s = \frac{0.85}{100} \times 30 \times 74 = 18.8 \quad cm^2 \quad 7 \phi 19 \quad (19.9\ cm^2)$$

$$A_s' = 0.7 \times 18.9 = 13.2 \quad cm^2 \quad 5 \phi 19 \quad (14.2\ cm^2)$$

The ultimate-strength-design method will show whether this compression reinforcement is really required.

b- Using the ultimate-strength-design method

For concrete C200, $f_{cp} = 165$ kg/cm^2; for high gr. steel $f_y = 3600$ kg/cm^2

$M_u = 1.5\ M_g + 1.8\ M_p = 1.5 \times 16.8 + 1.8 \times 8 = 39.6$ mt

For a maximum depth of 74 cms, we have:

$$m_s = \frac{M_u}{f_{cp}\ b\ d^2} = \frac{39.6 \times 10^5}{165 \times 30 \times 74^2} = 0.146$$, table 4-8 gives $\eta = 0.805$,

so that $$A_s = \frac{M_u}{f_y\ \eta\ d} = \frac{39.6 \times 10^5}{3600 \times 0.805 \times 74} = 18.5 \text{ cm}^2 \quad 7 \# 19$$

The value of η being bigger than 0.71, then we have ductile failure and there is no need for compression reinforcement. In spite of that, it is recommended to put 3 Ø 13 in the compression side of the cantilever.

Diagonal tension

a- Using the working-stress-design method

The diagonal tensile stress τ_A at the free end A is given by:

$$\tau_A = \frac{Q_A}{0.87\ b\ d} = \frac{12000}{0.87 \times 30 \times 45} = 10.2 \text{ kg/cm}^2$$ and

The diagonal tensile stress τ_B at the fixed end B is given by:

$$\tau_B = \frac{Q_{B\ red}}{0.87\ b\ d} \quad \text{where} \quad Q_{B\ red} = Q_B - \frac{M_B}{0.87\ d} \tan\alpha \quad \& \quad \tan\alpha = \frac{30}{200}$$

Therefore $$Q_{B\ red} = 12.8 - \frac{24.80}{0.87 \times 0.74} \times \frac{30}{200} = 7.00 \text{ tons}$$

So that $$\tau_B = \frac{7000}{0.87 \times 30 \times 74} = 3.64 \text{ kg/cm}^2$$

Assuming stirrups two branches 5 Ø 8 per meter with $A_{st} = 2 \times .5 = 1$ cm^2, then,

$$\tau_{st} = \frac{A_{st}\ \sigma_s}{b\ \ s} = \frac{1.0 \times 1400}{30 \times 20} = 2.33 \text{ kg/cm}^2$$

The area of steel to be bent A_{sb} is therfore given by:

$$A_{sb} = \frac{A\ .\ a\ .\ b}{\sigma_s} = \left(\frac{10.2 + 6}{2} - 2.33\right) \frac{90 \times 30}{2000} = 7.8 \text{ cm}^2 \quad \text{chosen } 3 \# 19$$

b- Using the ultimate-strength-design method

$Q_{A\ u} = 1.5\ Q_g + 1.8\ Q_p = 1.5 \times 8 + 1.8 \times 4 = 19.2$ tons

$Q_{B\ u} = \quad = 1.5 \times 8.8 + 1.8 \times 4 = 20.4$ tons and $$Q_{u\ red} = Q_u - \frac{M_u}{y_{ct}\ b} \tan\alpha$$

where $\tan\alpha = \frac{80-50}{200} = \frac{30}{200}$ and $M_u = 39.6$ mt so that:

$$Q_{u\ red} = 20.4 - \frac{39.6}{0.87 \times 0.74} \times \frac{30}{200} = 11.2 \text{ t,}$$ therefore

$$\tau_{A\ u} = \frac{19\ 200}{0.87 \times 30 \times 45} = 17.3 \text{ kg/cm}^2$$ and

$$\tau_{B\ u} = \frac{11\ 200}{0.87 \times 30 \times 74} = 5.8 \text{ kg/cm}^2$$

We have further:

$$\tau_{1\ u} = 0.8\sqrt{f_{cp}} = 0.8\sqrt{155} = 10.0 \text{ kg/cm}^2$$ and

$$\tau_{st\ u} = \frac{\Omega A_{st}\ f_y}{b\ \ s} = \frac{0.85 \times 1.0 \times 2300}{30 \times 20} = 3.25 \text{ kg/cm}^2$$

The distance a for this case is also equal to 90 cms, Fig. 4-33, so that:

$$A_{sb} = \left(\frac{17.3+10}{2} - 3.25\right)\frac{90 \times 30}{.85 \times 3600}$$

$= 8.75$ cm^2 chosen 3 # 19

The results are almost the same as in case of working-stress-design method.

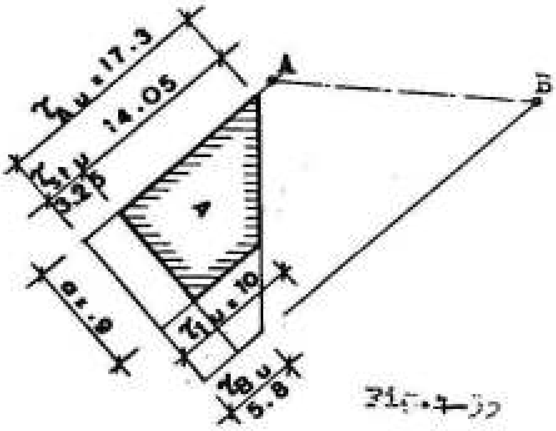


Fig. 4-33

The bent bars are to be distributed over the length a and placed at the center of gravity of the corresponding area of the diagonal tension diagram, as shown in Fig. 4-32.

Special considerations

a- Bent bars in relatively deep beams

In deep beams, where the depth is bigger than about 1/7 the span, the bents may be inclined to the horizontal by an angle varying between 45° and 60° as shown in Fig. 4-34. In such a case, only the component of A_{sb} along the diagonal is to be equal to the diagonal tension; i.e. the area of steel to be bent

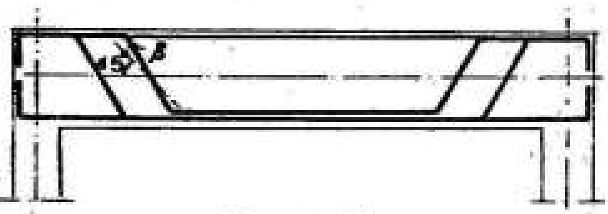


Fig. 4-34

should be increased by the ratio $1/\cos\beta$, where β is the angle between the bent bar and the diagonal making 45° with the axis of the beam. Fig. 4-34.

b- Beams with compression reinforcements

In beams with compression reinforcements, the stirrups must be of the closed box type; they shall not be less than 6 mm in diameter spaced not farther apart than 12 bar diameter nor 40 stirrup diameter.

c- Maximum spacing of bent bars

The maximum spacing of bent bars should not exceed the theoretical depth d of the beam if τ is bigger than 10 kg/cm^2; for smaller values of τ, the maximum spacing may be increased to 1.5 d.

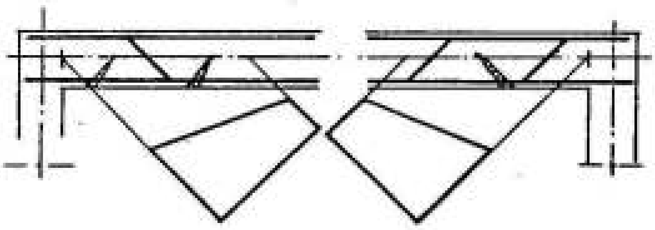


Fig. 4-35

If the spacing of bent bars is chosen more than 2d, a crack due to diagonal tension may be developed between the bents. Fig. 4-35.

A convenient arrangement of bent bars in simple cases is shown in figures 4-28, 29, 30 and 32.

d- Haunches

The requirements of bending moments or shearing forces can, in many cases, be satisfied by increasing the depth or breadth of the beam towards the support by arranging vertical or horizontal haunches as shown in Fig. 4-36 a and b. It has to be noted that a vertical haunch is more effective in resisting bending moments than a horizontal haunch; because the bending moment is proportional to b d^2.

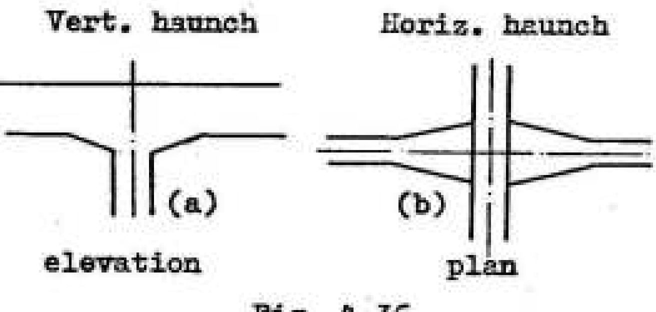


Fig. 4-36

The slope of the effective vertical haunch must not be more than 3:1, otherwise, the principal normal compressive stress σ_1 prallel to the outer surface of the haunch is excessive relative to the horiz. normal stress σ_c.

According to Fig. 4-37, we have:

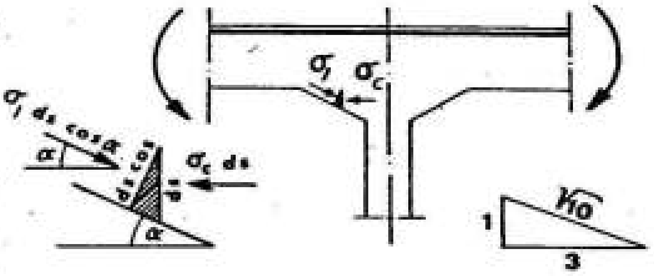


Fig. 4-37

$(\sigma_1 \text{ ds } \cos\alpha) \cos\alpha = \sigma_c \text{ ds}$

so that $\sigma_1 = \sigma_c / \cos\alpha^2$ 4-45

For a slope 1:1 $\cos\alpha^2 = 1/2$ and $\sigma_1 = 2\,\sigma_c$ **not allowed!**

For a slope 3:1 $\cos\alpha^2 = 9/10$ and $\sigma_1 = 1.11\,\sigma_c$ accepted.

4.6 Bond and Anchorage

General considerations

For concrete and steel to work together in a beam, it is necessary that stresses be transferred between the two materials. The term bond is used to describe the means by which slip between concrete and steel is prevented or minimized. Wherever the tensile or compressive stresses in a bar change, bond stresses must act along the surface of the bar to produce the change.

On smooth or plain bars, bond strength is largely dependent on the natural adhesion between the bar and concrete; but even after adhesion is broken, friction between the materials continues to provide a considerable bond resistance. Friction resistance is low for smooth bars' surfaces and is higher for rough surfaces.

Deformed bars give a higher bond resistance by providing an interlock between the steel and concrete.

In this case, the bond strength is primarily dependent on the bearing (compressive) strength of the concrete against the lugs as sketched in Fig. 4-38a and the shear strength of the concrete between the lugs. Adhesion and friction become minor elements in this case. The pressure on the concrete necessarily has an outward component producing ring tension and leads to splitting on weak planes along the bar. This longitudinal splitting of the concrete covering the bar is usually the limiting strength factor when the cover is not sufficiently thick. Fig. 4-38.

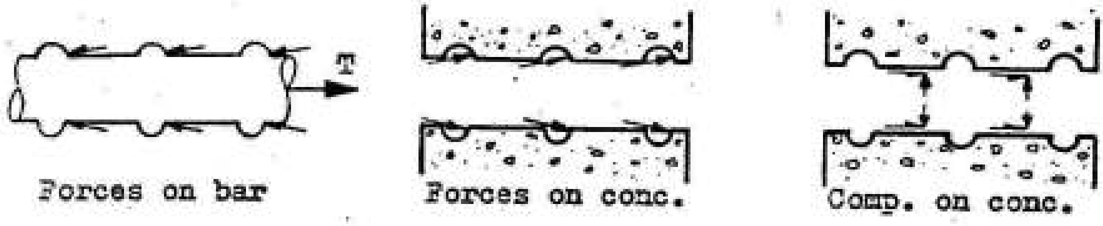


Fig. 4-38

Flexural bond and anchorage bond

It has been proved by tests that bond stress in a beam is neither uniform nor gradually varying from point to point. However, there exists two rather approximate methods for measuring bond; they are called flexural bond and anchorage or development bond.

Flexural bond is the average bond stress over the entire length of a bar which is necessary to change the bar stress , say, from T to T + dT. This change in stress is a function of dM and hence of shear.

Anchorage or development bond is the average bond stress over the entire length of bar within which the bar tension is being changed from T to zero (anchorage length) or from zero to T (development length). Such bond is a function of bar tension and bar length and is not determined by the external shear. It may be considered that anchorage or development bond determinations are more reliable and meaningful than flexural bond calculations.

a- Flexural bond

Consider a short length dx of the beam in Fig. 4-39 subject to a constant shear Q and moments M_1 and $M_2 = M_1 + dM_1$. These moments will produce the bending stresses shown in Fig. 4-39.

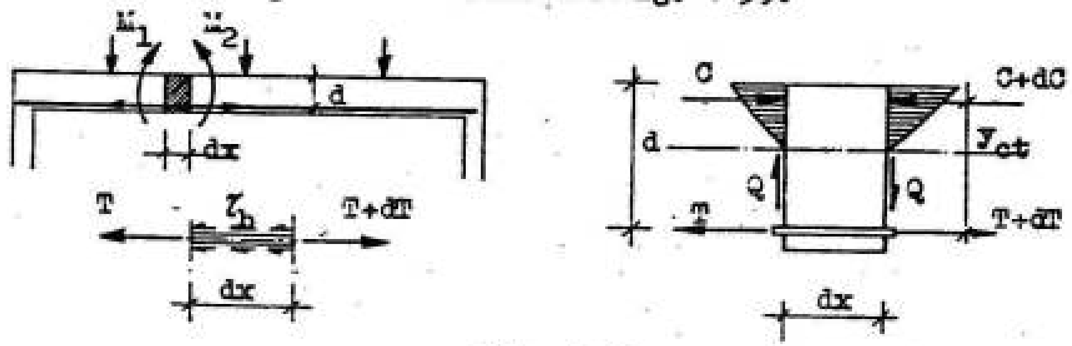


Fig. 4-39

The change in the bending moment dM produces a change in the bar force $dT = dM/y_{ct}$. Since the bar or bars must be in equilibrium, this change in bar force is resisted at the contact surface between concrete and steel by an equal and opposite force produced by the bond between the steel and concrete. If the average unit bond stress is called τ_b and the perimeter of bar or bars is ΣO , then

$$dT = \tau_b \, \Sigma O \, dx = dM/y_{ct}$$

so that

$$\tau_b = \frac{dM}{dx} \cdot \frac{1}{\Sigma O \, y_{ct}} = \frac{Q}{\Sigma O \, y_{ct}} = \frac{Q}{0.87 \, d \, \Sigma O}$$

This bond stress is caused by the change dM in bending moment and, for this reason, is known as flexural bond.

The allowable values of τ_b are as follows:

For deformed tension bars $\tau_b = 3.2\sqrt{f_{cp}} / \phi \leqslant 35$ kg/cm^2

,, ,, comp. ,, $\tau_b = 1.6\sqrt{f_{cp}} \leqslant 28$,,

Allowed values for plain bars = ½ deformed bars $\leqslant 10$ kg/cm^2

The ultimate values according to the ultimate-strength-design method are:

For deformed tension bars $\tau_b = 6.4\sqrt{f_{cp}} / \phi \leqslant 56$ kg/cm^2

,, ,, comp. ,, $\tau_b = 3.2\sqrt{f_{cp}} \leqslant 56$,,

Ultimate values for plain bars = ½ deformed bars $\leqslant 17$ kg/cm^2.

From the above values, it is clear that the bond between deformed bars and concrete is double as much as that of plain bars and hence its use is recommended.

When tension cracks are developed at the central part of a beam, the bond between the steel and concrete is broken at the positions of the cracks. Under heavy loads, the width and number of tension cracks is increased and the beam may collapse as the bar is pulled through the concrete. To prevent this occurance, end anchorage is essential, Fig. 4-40. If the anchorage is adequate, such a beam will not collapse even if the bond is broken over the entire length between anchorages. This is so because the beam behaves, under these conditions, as an arch with a tie as shown in Fig. 4-40, in which the shaded uncracked conc -

rete represents the arch and the tension reinforcements the tie. In this case, the force in the tension steel, over the entire unbonded length, is constant and equals $T = M_{max} / y_{ct}$. In consequence, the to-

Fig. 4-40

tal steel elongation, in such beams, is larger than in those in which bond is preserved, resulting in larger deflections and larger widths of cracks.

In this manner, one can easily imagine that the cutting off of the longitudinal tension bars weakens the tie and reduces the bearing capacity of the beam. The shearing forces can however be resisted by the inclined compressive forces of the arch.

It is consequently recommended to use for the tension reinforcements deformed bars giving high bond resistance and, to introduce at least one third of such reinforcements to the supports and to anchor them well beyond the center line of the supports.

b- <u>Anchorage or development bond</u>

It should be differentiated between the flexural bond mentioned above and the anchorage or development bond due to the direct pull of a bar from a concrete block.

Consider a bar of diameter Ø is embedded in concrete for a length l, Fig. 4-41.

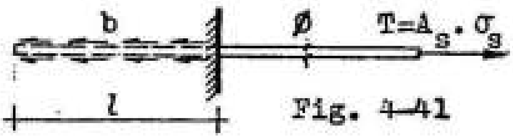


Fig. 4-41

In order to develop a tensile stress σ_s without the bond stress τ_b being exceeded, the length l should fulfil the following equation:

$$A_s \sigma_s = \tau_b \, l \, \Theta$$

where Θ is the circumference of a bar of diameter Ø. So that

$$\tfrac{1}{4} \pi \phi^2 \sigma_s = \tau_b \, l \, \pi \phi$$

or
$$l = \frac{\sigma_s}{4\,\tau_b}\,\phi$$

Using the ultimate-strength-design method and introducing the capacity reduction factor Ω, we get:

$$l = \frac{f_y}{4\,\Omega\,\tau_{bu}}\,\phi$$

For $\Omega = 0.85$, we get finally:

$$l = \frac{f_y}{3.4\,\tau_{bu}}\,\phi \qquad 4\text{-}46$$

Calculating the anchorage length l for a concrete quality C200 with $f_{cp} = 165\ kg/cm^2$ and different kinds of steel using both the working-stress and the ultimate-strength design methods, we get the relations shown in the following table:

Type of steel	Anchorage length l			
	W . S . D.		U . S . D .	
Deformed	$\sigma_s = 1400$	$8.5\ \phi^2$	$f_y = 2400$	$8.6\ \phi^2$
Plain	$\sigma_s = 1400$	$17.0\ \phi^2$	$f_y = 2400$	$17.2\ \phi^2$
Deformed	$\sigma_s = 2000$	$12.2\ \phi^2$	$f_y = 3600$	$12.9\ \phi^2$

Using the slightly bigger values of the ultimate-strength-design method (U.S.D.), the required anchorage length for different diameters is as shown in table 4-10.

Table 4-10 Anchorage length of reinforcing bars

Type of bars	Anchorage length l in cms for diameters ..... in mm						
	13	16	19	22	25	28	32
Def. normal	30	30	31	41	54	67	88
plain normal	30	44	62	83	107	135	176
def. high gr.	30	33	47	62	80	101	131

Anchorage length for compression bars is to be chosen bigger than half that for tension bars and not less than 30 cms.

Conclusions:

It has to be noted that

a) The results according to both the W.S.D. and the U.S.D. methods do not differ significantly; the use of any one is sufficient.

b) If the anchorage length at any section is sufficient, the magnitude of the flexural bond stress along the beam τ_b (or τ_{bu}) is of only secondary importance since the integrity of the member is assured even if minor local bond failures occur.

c) The actual local intensity of the bond stress along the embedded length of a bar is not constant. It varies according to the crack pattern, the distance from the section of maximum steel stress, and on other factors as can be seen from the schematic distribution of the bond stresses in a pullout test at low and high forces T.* Fig. 4-42.

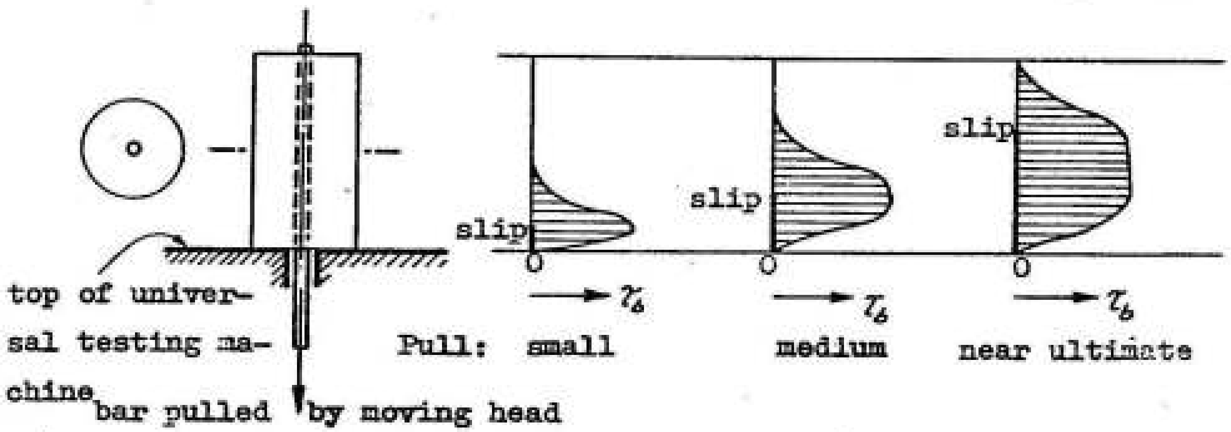


Fig. 4-42: Pullout test and bond stress distribution

d) In a pullout test, failure will usually occur (1) by longitudinal slipping of the concrete in the case of deformed bars, or (2) by pulling the bar through the concrete in the case of smooth bars, or (3) by breaking the bar, if the embedment is long enough.

e) In case the desired stress in a bar cannot be developed by bond alone, it is necessary to provide mechanical anchorage at the end of the bar, usually by means of a 90° bend or a 180° hook as shown in Fig. 4-

* Ferguson: Reinforced Concrete Fundamentals. Published by John Wiley and Sons, Inc. New York. London. Sydney.

43 a and b.

f) If the bar is bent at 45° (event. 60°) to get its anchorage length in the compression zone of the beam , its anchorage length may be reduced to 2/3 l given by previous equations, provided that r > 8 ∅ for normal mild steel and > 12 ∅ for high grade steel.

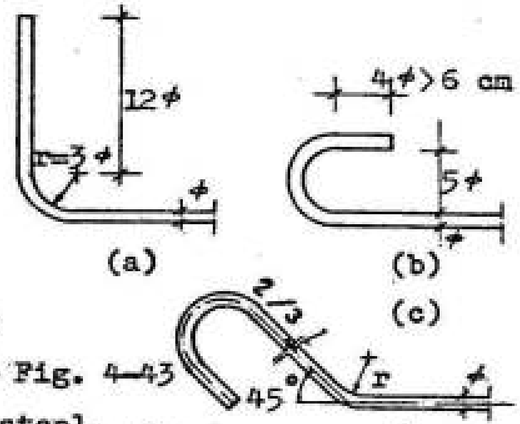


Fig. 4-43

g) The anchorage for stirrups can be chosen as shown in Fig. 4-44.

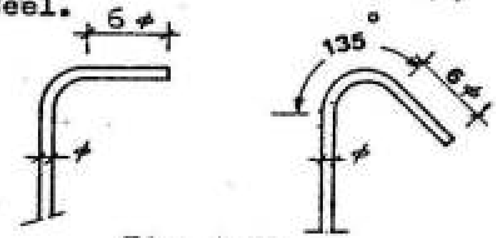


Fig. 4-44

Splices

The required length of splice for bars acting in tension is the same as the anchorage length . Splices at points of maximum stress are not desirable but sometimes cannot be avoided. Strength seems very little affected by whether the spliced bars are in contact or separated, so that any of the arrangements shown in Fig. 4-45 may be used. Splitting develops more rapidly when

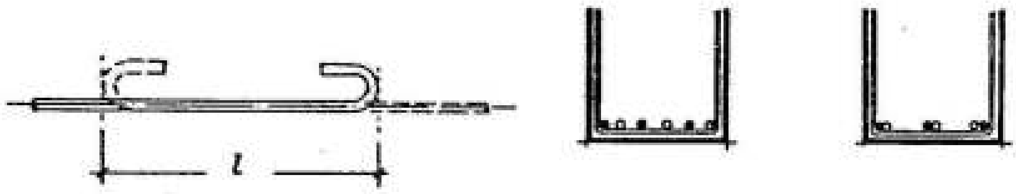


Fig. 4-45

several splices are spaced close together laterally and for this reason, it is recommonded to stagger the splices so that not more than 30 to 40 % of the bars (and generally not more than 2 to 3 bars) are spliced at the same position.

4-7 Constructional Details

1) Breadth of beams.

Breadth b of the rib of beams is usually chosen equal to the breadth of wall underneath. For free choice, b can be taken 0.50 to 0.40 t for small beams up to 50 cms depth, 0.40 to 0.30 t for medium beams

50 to 100 cms depth, and 0.35 to 0.25 t for beams deeper than 100 cms. As mentioned before, the breadth of the beam is not so effective as its depth ($M \propto b\ d^2$), thus it is recommended to choose the minimum breadth sufficient to place the steel and pour the concrete adequately keeping the minimum distances shown in Fig. 4-51.

In order to avoid the formation of longitudinal cracks in the tension zone of a beam, its width b is to be chosen such that it is bigger than four times the sum of the diameters of the main reinforcing bars arranged in one row, if these are plain bars, or, twice the sum if they are deformed bars providing improved bond behavior.

2) Depth of beams

The depth and longitudinal reinforcement are to be first determined to satisfy the requirements of the bending moment. For this chosen depth, the stirrups and diagonal tension reinforcement are then calculated in the manner shown in 4-5.

Governing for the depth of a beam is generally its field moment. For sections at the support, one may either add a haunch if required and possible or use compression steel (smaller than 0.40 tension steel) which is generally the field straight bars of the beam extended to the support.

If for a beam of approximately equal spans a constant depth is required, one may increase the field moments (for sections acting as T) and decrease the connecting moments (for sections acting as rectangular) by redistribution of bending moments explained in articles 3-4 and 4-3.

A further reduction of the connecting moment is possible by rounding the bending moment over the support, (refer to Fig. 4-9). A limited depth for sections at the support will be convenient for the whole beam. (Fig. 4-46).

When choosing the beams of a floor in a building, it is recommended to limit the number of the variable dimensions to a minimum on condition that the economy is not impaired; e.g. all beams in a floor to be 12 or

25 cms wide and, 40, 50 or 60 cms deep.

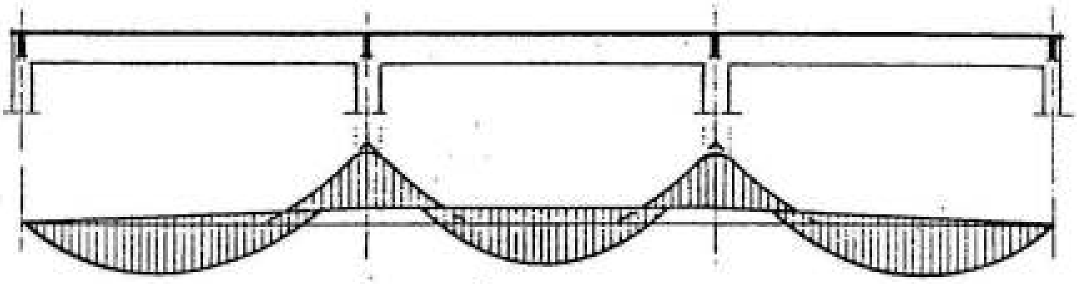


Fig. 4-46

It is however easier and may be more economic to standardize the design than to spare a few number of concrete cubic meters.

3) Main reinforcements

The main reinforcement of a beam is to be choosen with due regard to its cross-section. The diameter chosen ranges between 13 mm - 16 mm in small beams, 16 mm - 19 mm in medium beams and 22 mm - 25 mm in heavy beams. In exceptional cases, one may use 10 mm-diameter reinforcements in very small beams; and bigger than 25 mms - diameter in very heavy beams. For diameters bigger than 16 mms, the use of deformed normal or high grade reinforcements is more convenient and recommended.

Reinforcements are generally arranged in one or two horizontal layers; only in particular cases, in three or more layers.

The convenient number of main bars in small beams is 4 (eventually 2 or 3 in very small beams), in medium beams, is 5 to 8 bars, more than 8 bars may be used in heavy beams of big dimensions.

Wherever at any section of a flexural member, tension reinforcement is required, its amount should not be less than: $9/f_y$ or

A_s min $\geqslant$ 0.4 % if normal mild steel is used,

or A_s min $\geqslant$ 0.25% if deformed high grade steel is used

referred to the cross sectional area of the web unless the area of reinforcement provided at every section, positive or negative, is at least one third greater than that required by analysis.

It is advisable, moreover, not to use more than two successive diameters of tension reinforcements, positive or negative, in one span of a beam.

4) Stirrup hangers and skin reinforcement

The area of the stirrup hangers in a beam may be chosen $\geqslant 0.20\ A_s$.

If the depth of the web exceeds 65 cm, one should provide longitudinal distribution reinforcement, called skin reinforcement; its geometrical percentage, referred to the web section, should be at least = 0.05 % at each of the two faces. Furthermore, the individual bars of this reinforcement should be $\geqslant$ 6 mm diameter and are not spaced farther than 20 cms apart. (Fig. 4-47).

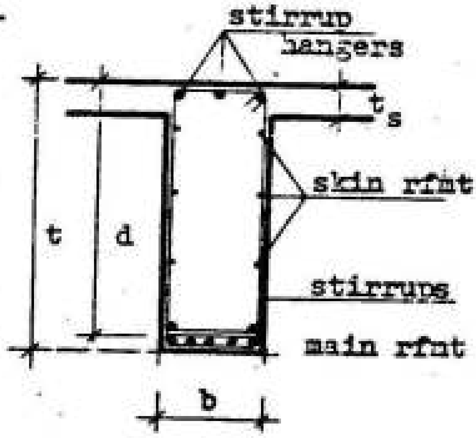


Fig. 4-47

5) Area of cross-section of steel reinforcements

The area of cross-section of steel reinforcing bars use in Egypt is given in table 4-11.

Table 4-11 Area of cross-section of reinforcing bars

Ø	wt kg/m	Area of cross-section in cm^2 for									
		1	2	3	4	5	6	7	8	9	10
6	.222	.283	.566	.848	1.13	1.41	1.70	1.98	2.25	2.54	2.83
8	.395	.503	1.01	1.51	2.01	2.51	3.02	3.52	4.02	4.52	5.03
10	.617	.785	1.57	2.36	3.14	3.93	4.71	5.50	6.28	7.07	7.85
13	1.04	1.33	2.66	3.98	5.31	6.64	7.96	9.29	10.6	11.9	13.3
16	1.58	2.01	4.02	6.03	8.04	10.1	12.1	14.1	16.1	18.1	20.1
19	2.23	2.84	5.67	8.50	11.3	14.2	17.0	19.9	22.7	25.5	28.4
22	2.98	3.80	7.60	11.4	15.2	19.0	22.8	26.6	30.4	34.2	38.0
25	3.85	4.91	9.82	14.7	19.6	24.5	29.5	34.4	39.3	44.2	49.1
28	4.83	6.16	12.3	18.5	24.6	30.8	37.0	43.1	49.3	55.4	61.6
32	6.31	8.04	16.1	24.1	32.2	40.2	48.3	56.3	64.3	72.4	80.4

The following diameters may also be used: 5, 7, 12, 14, 18, 20, 24, 26, 30, and 34 mm.

6) Convenient choice of reinforcements

The number and diameter of bars must increase gradually according to their area and with due regard to the dimensions of the beam. Table

4-12 gives a convenient choice of bars for the shown areas A_s.

Table 4-12 Convenient choice of reinforcements

A_s in cm^2	convenient rfmt		A_s in cm^2	convenient rfmt		A_s in cm^2	convenient rfmt
5.31	4Ø13		12.10	6Ø16		30.40	8Ø22
6.64	5Ø13		12.52	3Ø19 + 2Ø16		34.20	9Ø22
6.68	2Ø16 + 2Ø13		14.20	5Ø19		24.50	5Ø25
8.04	4Ø16		17.00	6Ø19		29.50	6Ø25
9.69	2Ø16 + 2Ø19		19.00	5Ø22		34.40	7Ø25
10.10	5Ø16		19.90	7Ø19		39.30	8Ø25
11.30	4Ø19		22.80	6Ø22		44.20	9Ø25
11.70	2Ø19 + 3Ø16		26.60	7Ø22		49.10	10Ø25

7) <u>The moment of resistance and the convenient arrangement of the main longitudinal reinforcing bars</u>

The checking of the anchorage conditions at the ends of longitudinal reinforcing bars should be based on the bending moment diagram , which should be suitably displaced to take account of the increase of tensile stresses in the longitudinal reinforcement: a) when diagonal cracks are formed, (refer to Fig. 2-35 and explanation given under 3, page 97); and b) when vertical cracks are developed along the span (refer to Fig. 4-40).

It can, however, be easily seen from this figure (4-40) that when cracks are formed, the tension in steel is increased towards the supports of a simple beam and may reach M_{max} / y_{ct}.

The displaced diagram (Fig. 4-48), which serves as a basis for designing the longitudinal reinforcement, is obtained by shifting the enveloping curve of the bending moments parallel to the center line of the member in the most unfavourable direction by an amount x where

$$x = (0.5\text{-}0.75)d \qquad 4\text{-}47.$$

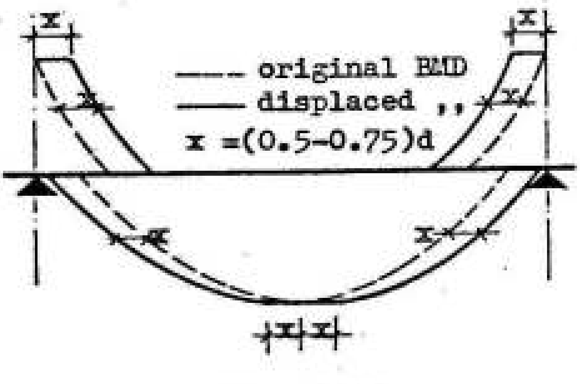


Fig. 4-48

a) Simple beams

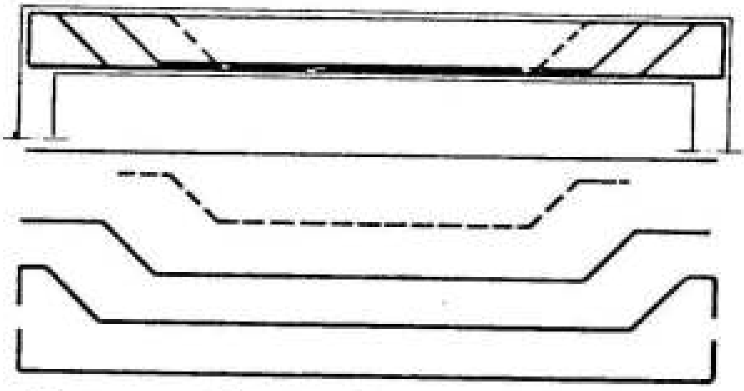
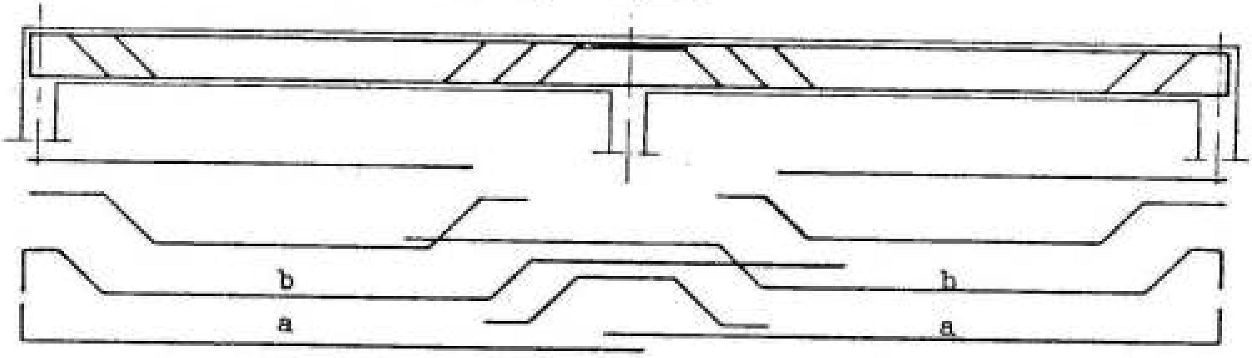
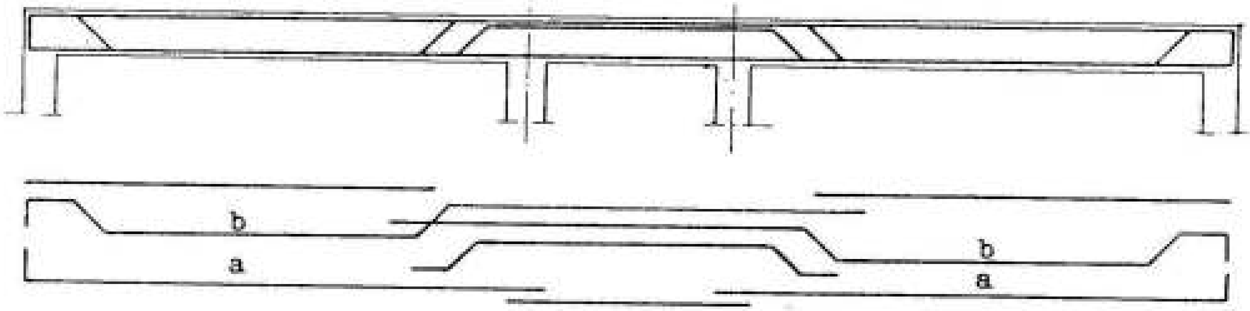


Fig. 4-49

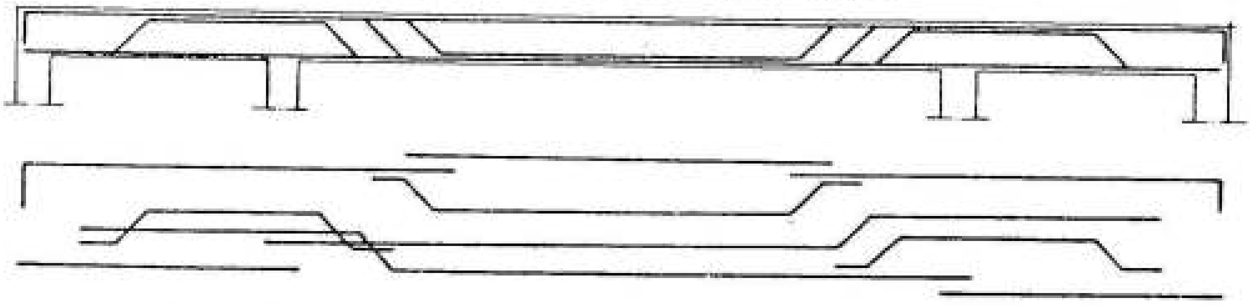
b) Continuous Beams Two equal spans



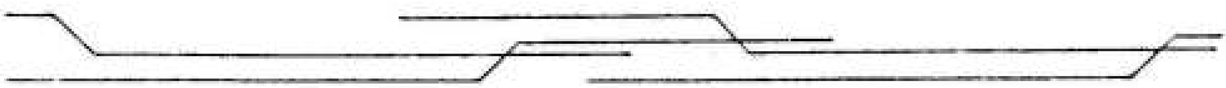
Short span between long spans



Long span between two short spans



The basic bars a + b may be arranged in the form:



The anchorage length l should be taken outside the displaced diagram.

The moment of resistance M_r of tension reinforcing bars A_s acting at a stress σ_s in a section of effective depth d is given by:

$$M_r = A_s \sigma_s y_{ct} = A_s \sigma_s \eta d = A_s k_2 d$$

The values of k_2 are given in table 4-5 and sheet 11 and the value of η may be assumed in this equation equal to 0.87, so that

$$\underline{M_r = 0.87 A_s \sigma_s d} \qquad 4\text{-}48$$

The chosen reinforcements must satisfy the requirements of the displaced diagram, Fig. 4-48 and the diagonal tension diagram Fig. 4-30.

The basic forms of reinforcements in a beam are as shown in Fig. 4-49.

Figures 4-28, 4-29, 4-30 and 4-32 show examples of convenient arrangements of reinforcements in simple beams, simple cantilevers and beams with overhanging cantilevers.

It has to be noted that swimming bent bars to resist diagonal tension only as those shown in Fig. 4-50 a , are to be avoided.

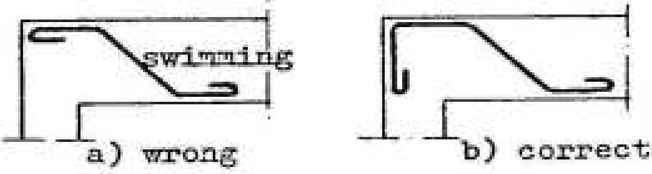


Fig. 4-50

Because of possible settlements of supports on medium and weak soil, and to be able to cover the displaced bending moment diagram over the supports, it is advisable to increase the top reinforcements at the supports of continuous beams by 15 to 20 % according to the possible settlement.

8) Spacing of bars

The spacing of reinforcing bars- i.e. the distances between adjacent bars within a section- should be sufficient to enable the concreting to be done in an entirely satisfactory manner. In particular, the bars in a section should be spaced that the freshly mixed concrete can be properly placed without risk of segregation and that the concrete surrounding the reinforcement can be efficiently vibrated.

The values shown in Fig. 4-51 correspond to normal concreting cast in situ. Subject to special justification, they may be reduced in the case of factory-made precast members or in the case of temporary structures.

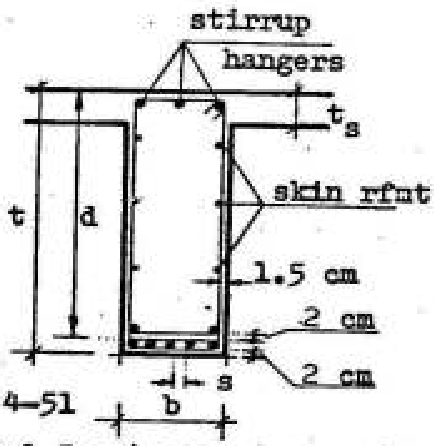


Fig. 4-51

$s > Ø > 2.5$ cm $>$ max. aggr. size + 0.5 cm

Clear distance between beam bars in any one layer shall not be less than the diameter of the bars, or 2.5 cm, or maximum size of aggregate plus 5 mm, whichever is the greater.

The concrete cover to the reinforcement- i.e., the distance from the bar to the walls of the formwork or to the free surface of the concrete- should be sufficient to enable concreting to be done in an entirely satisfactory manner, so that, in particular, all risk of segregation is obviated and the concrete can be compacted to the density that is essential to providing suitable protection of the reinforcement against corrosive agents.

The following values of cover shall be adopted as minimum figures for beams: 2.0 cm or bar diameter, if larger, for main bars and 1.5 cm for stirrups in indoor structures. In outdoor structures (if not plastered) : 2.5 to 3.0 cm for main bars and 1.5 to 2.0 cm for stirrups.

In foundations or elements subject to water or liquids a minimum cover of 3.0 cm is essential. In elements subject to attack from chemical agents, the cover shall be determined according to circumstances and may be bigger than 4.0 cm.

On the other hand, if the cover, in exceptional cases, does exceed 4.0 cm, the designer should provide an additional reinforcing network (e.g. expanded metal) within the thickness of the concrete cover so that no hair cracks appear in it.

The UNESCO has however allowed that in any particular vertical line in the cross-section the designer may provide two bars which are in contact with each other. On the other hand, he should not provide more than two bars in contact with one another in one and the same vertical line, unless he makes special arrangements to enable the freshly mixed concrete to fill all cavities perfectly.

In any particular horizontal layer the designer should not provide two bars which are in contact with each other, unless there is sufficient space on each side of each group of two bars to insert a vibrator. On no account should more than two bars be placed in contact with one another in one and the same horizontal layer.

On these conditions the above requirements are applicable to groups of bars in contact, provided that each group is replaced by a single fictitious bar with the same centroid as the group and with a cross-sectional area equal to the total cross-sectional area of the bars in the group.

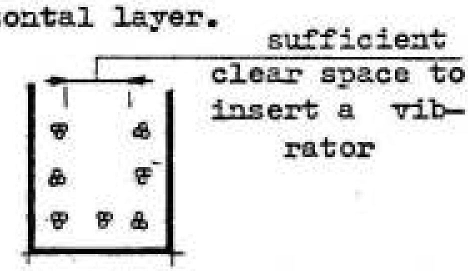


Fig. 4-52

To facilitate the placing of the concrete it may sometimes be advantageous to form groups of three bars as shown in Fig. 4-52. In this way very good embedment of the steel and good quality of the concrete can be ensured.

9) <u>Welding of reinforcing bars</u>

Welding is allowed only if the welding operations are carried out by a properly qualified welder and are under constant and strict supervision.

The method of welding should not cause any impairment of the mechanical properties of the steel.

If cold-treated reinforcing bars are welded, they generally loose their high quality properties and become mild steel having the same properties as those of the steel from which they originated. Welding of cold-treated reinforcing bars is permissible only on condition that tests are performed to verify that the method of welding employed in no way impairs the mechanical properties of the bars (and especially the elastic limit and the ultimate strength). The manufacturers of such reinforcing steel often supply practical information with regad to this.

Welded splices should be confined to straight portions of reinforcing bars. Besides, they should be staggered in the longitudinal direction by a length equal to at least twenty times the bar diameter (20 Ø) in the case of plain round bars and at least ten times the bar diameter 10 Ø in the case of deformed bars.

Finally, in any one particular cross-section the transmission of tensile forces by welded splices in bars should be effected by not more than one-third of the total cross-sectinal area of the reinforcement.

Weld splices are best made by means of lap joints produced by electric arc welding in longitudinal weld heads. They are to be so arranged such that the axis of the welded bar is not changed. The length of the longitudinal weld heads should not exceed five times the diameter of the bar. Fig. 4-53.

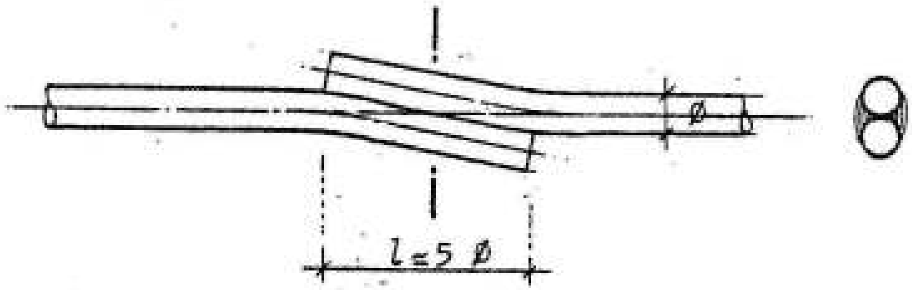


Fig. 4-53

10) Buckling of beams

According to the Egyptian code, the laterally unsupported length of rectangular beams should not be more than 30 times the breadth of the web otherwise stresses in concrete should be reduced due to the possible buckling liable to take place in the compressed part as given in table 4-13.

Table 4-13 Stress-reduction-factors of slender beams

Slenderness ratio l/b	30	40	50	60
Reduction factor	1.00	0.75	0.50	0.25

CHAPTER 5

REINFORCED CONCRETE SLABS

5.1 Types of Slabs

A slab may be supported on two opposite sides only as shown in Fig. 5-1, in which case, the structural action of the slab is essentially one-way; the loads being carried by the slab in the direction perpendicular to the supporting beams.

Such a slab may be solid, Fig. 5-1, ribbed or hollow, Fig. 1-4.

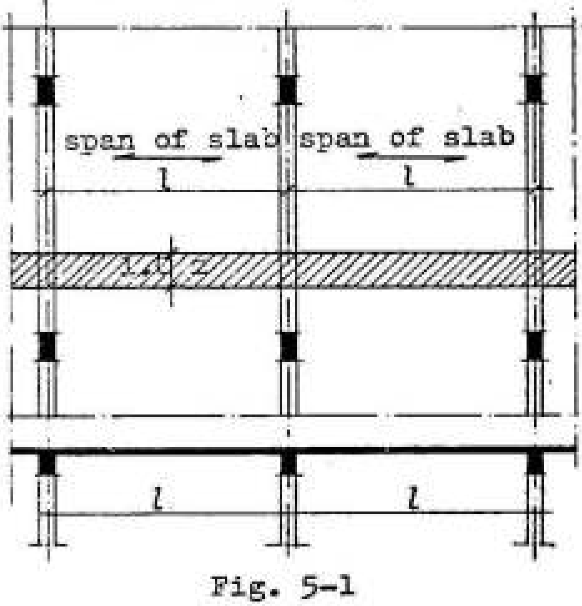


Fig. 5-1

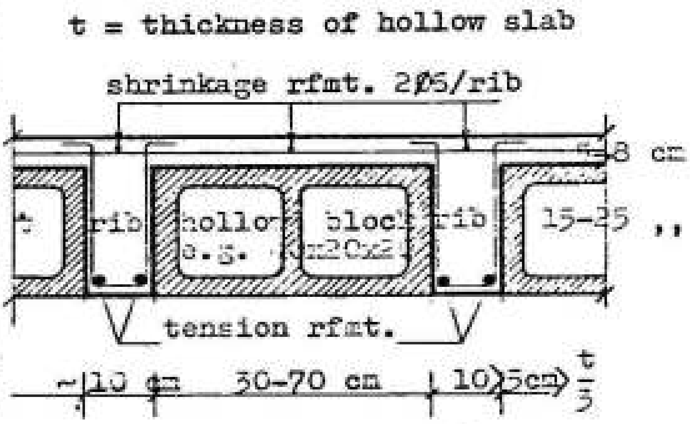


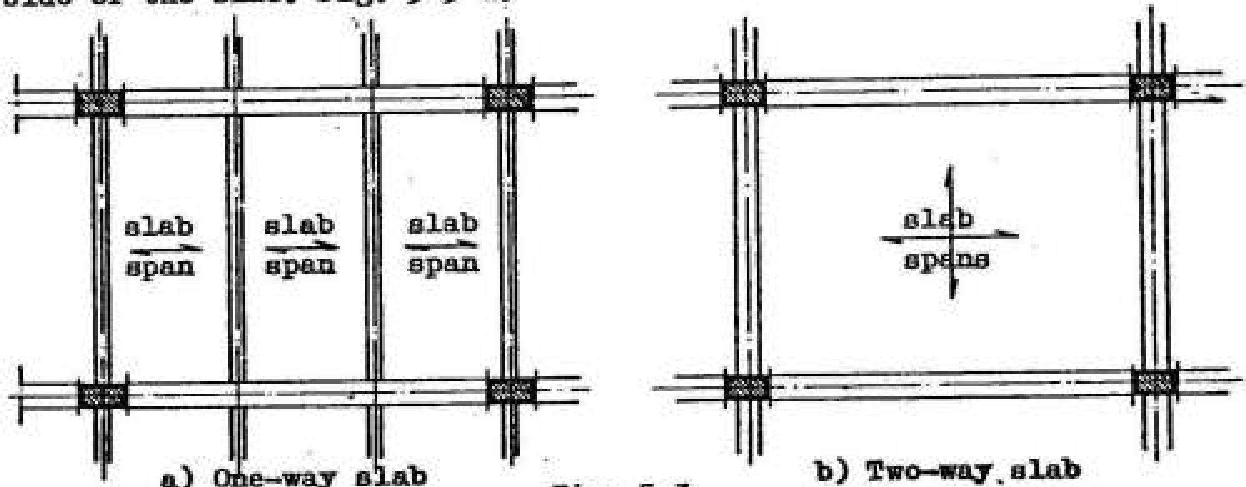
Fig. 5-2

In ribbed slabs, the concrete in tension below the neutral axis at the field of the slab, being generally neglected in the design, is removed and replaced by closely spaced ribs (distance between center lines is 40 to 80 cms and normally 50 cms) perpendicular to the supporting beams to carry the tension reinforcement. The concrete above the neutral axis -generally 5 to 8 cms and called compression slab- and the tension steel in the ribs form the compression and tension zones of the slab.

If it is required to have a flat surface for the bottom of the slab, hollow blocks, generally statically not acting, are arranged between the ribs, as shown in Fig. 5-2, which shows the cross-section of a hollow block slab perpendicular to the ribs and parallel to the supporting beams.

Slabs may be supported on all four sides, as in Fig. 5-3, so that two way-action is obtained and the load is transmitted to the supporting beams in two directions.

In cases where the slab is supported on all four sides and the ratio of length to width of one slab panel is larger than 2, most of the load is carried in the short direction, and one-way action is obtained in effect, even though supports are provided at the ends of the longer side of the slab. Fig. 5-3 a.



a) One-way slab Fig. 5-3 b) Two-way slab

If the ratio of length to width is smaller than 2, then the slab is a two-way. Fig 5-3 b.

As has been previously shown in Fig. 1-3, it is possible to support a floor slab directly on columns without the aid of beams or girders in the form of flat-slabs. The thickened portion of the slab, termed the drop panel, at the columns and the enlarged column head, called column capital are to be noted, both of them serve the double purpose of reducing shear stresses in the slab near the column supports and provide greater effective depth for negative bending moments.

It is however possible to construct a flat slab without drop panels and column capitals even in cases of irregular column arrangements of apartment, office ... etc. floors carrying walls and light live loads.

Both two-way slabs and flat slabs can be of the ribbed or the hollow-block type as shown, for example, in Fig. 1-5.

5.2 <u>One-way slabs</u>

A one-way slab, Fig. 5-1, may be considered as composed of a series of strips -each one meter wide- cut out at right angles to the supporting beams. Each of such strips behaves essentially as a rect. beam 1 m wide and t ms deep. The width is comparatively big relative to the depth. It can however be analyzed by the methods dealing with rectangular beams. The bending moments are generally calculated for a unit width: in this case, the load per unit length on the imaginary beam is the load per unit area on the slab. Since all the loads are transmitted to the supporting beams, the main reinforcing steel should be placed at right angles to these beams. A small percentage of steel, called distributers, should be arranged in the other direction for the following reasons: 1) to connect the main reinforcements and fix them in position 2) to distribute any concentrated loads that may act on the slab, 3) to resist any bending moment that may act normal to the main reinforcement in case of one-way slabs with supports in the longer direction, and 4) to resist shrinkage and temperature stresses.

<u>Spans</u>

For freely supported and fixed slabs .... $l = 1.05 l_o$

For continuous slabs l = distance CL to CL supports ... $l < 1.05 l_o$

However, the span need not exceed the clear span + the depth of the slab. Fig. 5-4.

Slabs of clear spans of not more than 3 ms that are integrally built with sufficiently rigid supports may be designed as continuous beams on knife edge supports with spans equal to the clear spans of the slab and the breadth of the supports otherwise neglected.

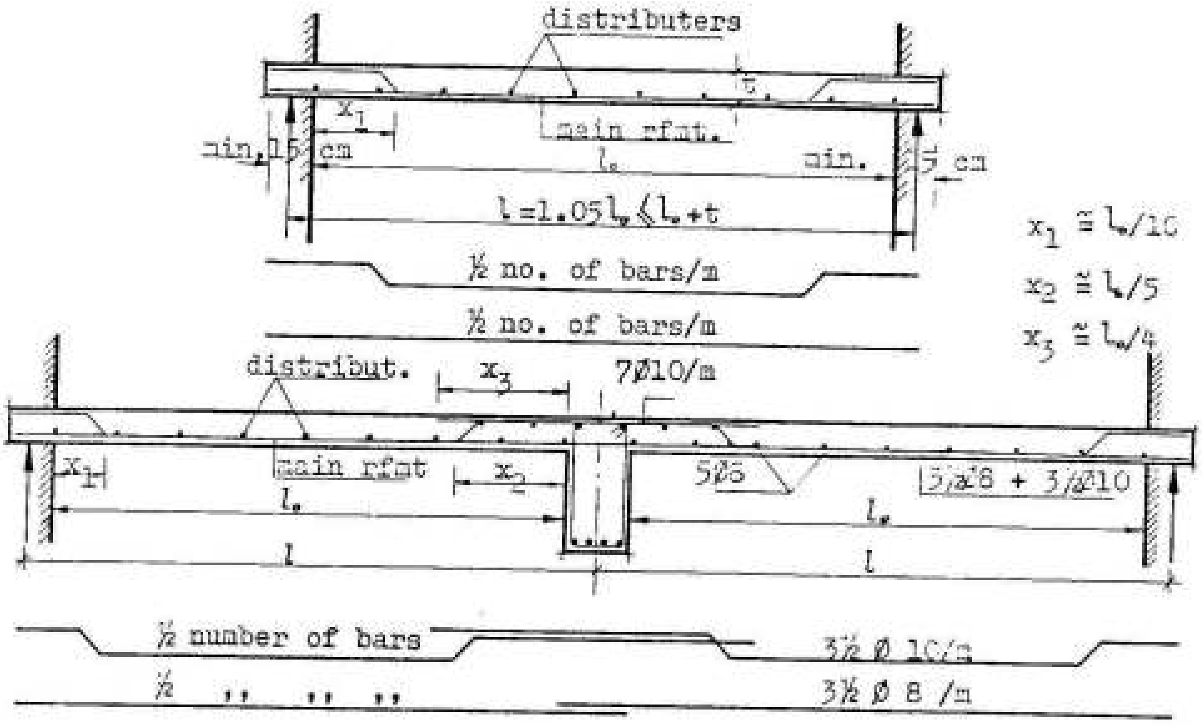


Fig. 5-4

Loads

Loads on different elements of a structure (especially slab loads) are to be determined with the utmost care; e.g. the dead load of a roof slab 10 cms thick covered with 2 cms thick cement tiles, Fig. 5-5, is to be determined in the following manner:

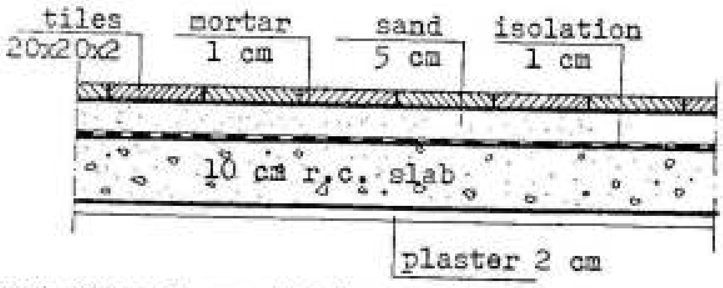


Fig. 5-5

Cement tiles	2 cms thick	....	0.02 x 2200 =	44 kg/m^2	
Mortar	1 cm ,,	....	0.01 x 2100 =	21 ,,	
Sand	5 cms ,,	....	0.05 x 1800 =	90 ,,	
Isolation	1 cm ,,	....	=	5 ,,	
Plaster	2 cms ,,	....	0.02 x 1400 =	28 ,,	188 kg/m^2
R.C. slab	10 cms ,,	....	0.10 x 2500 =		250 ,,
		Total dead load	=		438 ,,

The weight of normally used floorings is as follows:

Weight of timber floor with slag or light filling .... 60-80 kg/m^2
,, ,, ,, ,, ,, ordinary ,, ...100-120 ,,
,, ,, cement tiles-floor on sand .... 180-200 ,,

Live loads in buildings are given in table 3-5.

Minimum thickness of one-way slabs

Concrete compression will seldom cotrol the flexural design of one way slabs, particularly if U.S.D. methods are used. Flexural designs will ordinarily consist in selecting a slab depth which will permit the use of an economically low steel ratio and which will not allow unsightly or damaging deflections. The recommended minimum thicknesses of one-way slabs are:

Simply supported slabs	l/20,	Egypt. code specifies	l/35
One end continuous ,,	l/24	,,	---
Both ends continuous	l/28	,,	l/44
Cantilevers	l/10	,,	l/15

However, the smaller thicknesses may be used if the calculation of deflections, according to 3.2 (limit state of deformation), proves that no adverse effects will take place.

Bending moments and shearing forces

In order to show the behavior of one-way slabs supported on four sides where length over breadth of slab panel is bigger than 2, it is desirable to illustrate the physical action of such slabs.

A long narrow slab supported on four sides will take under load a troughlike shape except near the ends.

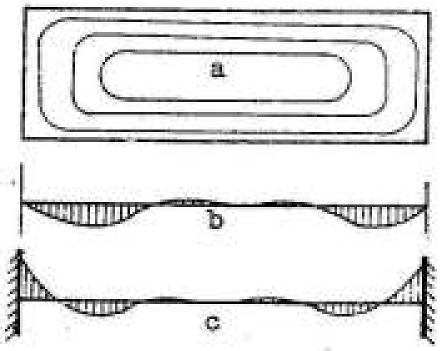


Fig. 5-6

The contour lines of a deflected simply supported long slab are shown in Fig. 5-6 a ; its bending moment in the longitudinal direction is shown in Fig. 5-6 b.

For fixed edges, there must be a transition zone around the edges in which the slope gradually turns downwards from the horizontal edge tangents; the bending moment of a fixed slab in the longitudinal direction is shown in Fig. 5-6 c.

It is clear that in long spans the longitudinal bending moments occur not at the center but at the sides of the slab. The moment over its mid-span length is small or zero and the short center strips act almost as one-way slabs.

Mathematical analysis shows that the positive bending moment of a simple slab or the negative bending moment of a fixed slab in the longitudinal direction are nearly the same regardless of the long span length. They are almost the same as they would be for a square panel having the short span dimensions.

In normal buildings (such as appartment buildings, offices, warehouses ... etc.) the thickness of the slab is generally 10 to 15 cms and the reinforcing bars are in most cases 8 to 10 mm diameter. The thickness of the slab and the diameter of the reinforcing bars being small, it is generally advisable not to depend on a full development of the connecting moment of continuous slabs. Accordingly, the calculation of continuous slabs according to the theory of continuous beams as discussed in 3.4 and 4.3, may only be allowed if special care is taken to ensure the right position of the top steel resisting the connecting moments during concreting. It is more likely to assume partial fixation on continuous sides of a slab and at the outside supports built into bricks or masonry walls or connected to relatively heavy beams.

The Egyptian code specifies the following for one-way slabs:

1. Connecting moments may be reduced according to a parabolic curve as shown in Fig. 4-9.
2. The positive bending moments considered for the design of intermediate panels of continuous slabs should not be smaller than the positive moments of the same span if it were totally fixed at the supports.
3. Negative bending moments at the supports of slabs built into brick or masonry walls in a manner that they develop partial end restraint

should not be less than $M = w\,l^2 / 16$. The positive moment in this field is however to be considered as if the moment at the end is zero.

4. Negative field moments due to live load are to be considered half calculated values due to the torsional resistance of the supporting beams, in case the slabs are cast monolithically at the same time with the beams.

5. A slab should be considered to have fixed ends when these ends are sufficiently secured to other parts of the construction having such rigidity as to prevent any rotation at the ends under all variations of loading.

6. In case of equal spans, or unequal spans with a maximum difference of 20% and uniform loads, the following values of the bending moments may be assumed:

Max. bending moment in slabs of one span .... $M = + w\,l^2 / 8$

In case of two continuous spans .... $M+ = w\,l^2 / 10$

$M- = w\,l^2 / 8$

In case of more than two spans:

Center and support of end spans .... $M = \pm w\,l^2 / 10$

Center and support of interior spans .... $M = \pm w\,l^2 / 12$

In case of heavy live loads, the negative field moment in intermediate spans is given by .... $\min. M = (g-p/2)\,l^2/24$

The shear stresses in slabs are generally low, they need to be checked in exceptional cases of heavy loads.

Dimensioning

The dimensioning of slabs in normal cases is generally made according to the W.S.D method and the allowable stresses in case of standard concrete C160-180 and normal mild steel st37 may be chosen according to table 5-0 which gives in addition to σ_c and σ_s the values of k_1 and k_2*.

---

* The design coefficients k_1 and k_2 are given in table 4-5 and sheet 11

Table 5-0. Values of the allowable stresses σ_c and σ_s and the dimensioning coefficients of slabs k_1 and k_2

thickness t cms	σ_c	σ_s	k_1	k_2	design equations
8	40	1400	0.430	1250	
10	45	,,	0.392	1248	$d = k_1 \sqrt{M}$
12	50	,,	0.361	1237	d in cm, M in kgm
15	55	,,	0.334	1227	
20	60	,,	0.313	1217	$A_s = M / k_2 d$
> 20	65	,,	0.294	1208	A_s in cm^2, $\frac{M}{d}$ in kg

The stresses are chosen higher for bigger thicknesses of slabs for the following reasons:

1. Section of slab, as far as concrete is concerned, is more uniform,
2. Thinner slabs, as far as moment of resistance is concerned, are more sensitive against possible variations in thickness or laying out of reinforcements.

Example

It is required to design a one-way simple floor slab 3 ms span if the floor cover and plaster are 150 kg/m^2 and the live load is 250 kg per m^2.

Assume thickness of slab 12 cms i.e. weight = 0.12 x 250 = 300 kg/m^2

Load w = 150 + 300 + 250 = 700 kg/m , $M = \frac{w\,l^2}{8} = \frac{700 \times 3^2}{8} = 785$ kg m

For σ_c = 50 kg/cm^2 & σ_s = 1400 kg/cm^2, k_1 = 0.361 and k_2 = 1237,

then $d = 0.361 \sqrt{785} = 10$ cms chosen t = 12 cms i.e. (d = 10.5 cms)

and $A_s = \frac{785}{1237 \times 0.105} = 6.05$ cm^2 ,, 8 ø 10 mm/m (6.28 cm^2)

Distributers = 0.2 x 6.05 = 1.2 cm^2 5 ø 6 mm/m (1.41 cm^2)

The details are similar to Fig. 5-4.

Reinforcement

The main reinforcements of a slab should satisfy the requirements of the internal forces . One has either to choose the number and diameter of bars required per meter, e.g. 6 ø 10 mm/m or to choose the diameter and spacing between the bars, e.g. Ø 10 mm @ 15 cms.

In order to have a regular arrangement of reinforcements over the intermediate supports of a continuous slab, it is advisable to choose the same number of bars in the different spans of a slab and to bend half these bars as shown in Fig. 5-4.

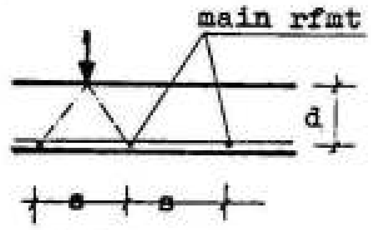
The Egyptian code specifies:

1. The reinforcement is to be arranged so as to cover all the tension zones "according to the displaced bending moment diagram" and extend beyond the end of these zones to develop the necessary anchorage length.

2. Min.slab reinforcement in the main direction is to be 0.25% of the required cross sectional area of slab, and not less than 0.15% of the actual section.

3. In continuous slabs of equal or nearly equal spans under normal loading conditions: at least one half of the main reinforcement is to be bent at a point 1/5 of the clear span from the face of the support and this top reinforcement is to be extended in neighbouring panels to 1/5 of these panels plus anchorage length or 1/4th the span, if the arrangement of bars is not made according to the bending moment diagram.

4. Maximum spacing of main reinforcement in middle of spans is to be 1.5 total thickness of slab but not more than 20 cms, "in order to ensure the homogeneity of the tension zone of the slab and to avoid local cracks due to concentrated loads as shown in Fig. 5-7". However, for slabs smaller than 10 cms thick 6 bars per meter may be allowed.



$s \leq 1.5\ d \leq 20$ cms

Fig. 5-7

5. Straight reinforcement extended to the supports should not be less than 1/3 of the total amount of positive reinforcement used in the middle of the span.

6. In case of uniformly distributed load, distributing bars normal to main reinforcement should not be less than 0.2 of the main reinforcement and not less than 4 Ø 6 mm per meter.
7. Minimum diameter of slab reinforcement is usually 6 mm for straight bars and 8 mm for bent bars. Bars of smaller diameters may be used for less important work and in prefabrication.

The indication of slab reinforcements on plans can be illustrated by one of the manners shown in Fig. 5-8. For small scales and simple standard cases, indication I is generally sufficient; for bigger scales > 1/50, indication II and sometimes sections, e.g. Fig. 5-4 may be necessary.

Supports

The breadth of the support of a slab should not be less than the thickness of the slab and not smaller than 8 cms. A half brick wall should not normally be used as a bearing wall.

5.3 Two-way Slabs

Performance of two-way slabs

Most rectangular reinforced concrete slabs are supported on all four sides by beams, girders or walls. Fig. 5-3. If the ratio of length to breadth of any panel is < 2, the slab bends under load into a dished surface which means that any point of the slab is curved in both principal directions; i.e. the loads are transmitted to the supporting beams in both directionsand consequently bending moments are created in the same directions. To resist these moments, the slab must be reinforced, in its tension zones, in both directions (refer e.g. to Figs. 5-4 and 5-11). The part of the load transmitted to each of the supporting beams will be as shown in 4.2.

Grashoff approximate calculation

The easiest way to visualize the performance of a two-way simple slab, subject to uniform load w/m^2, is to think of it as consisting

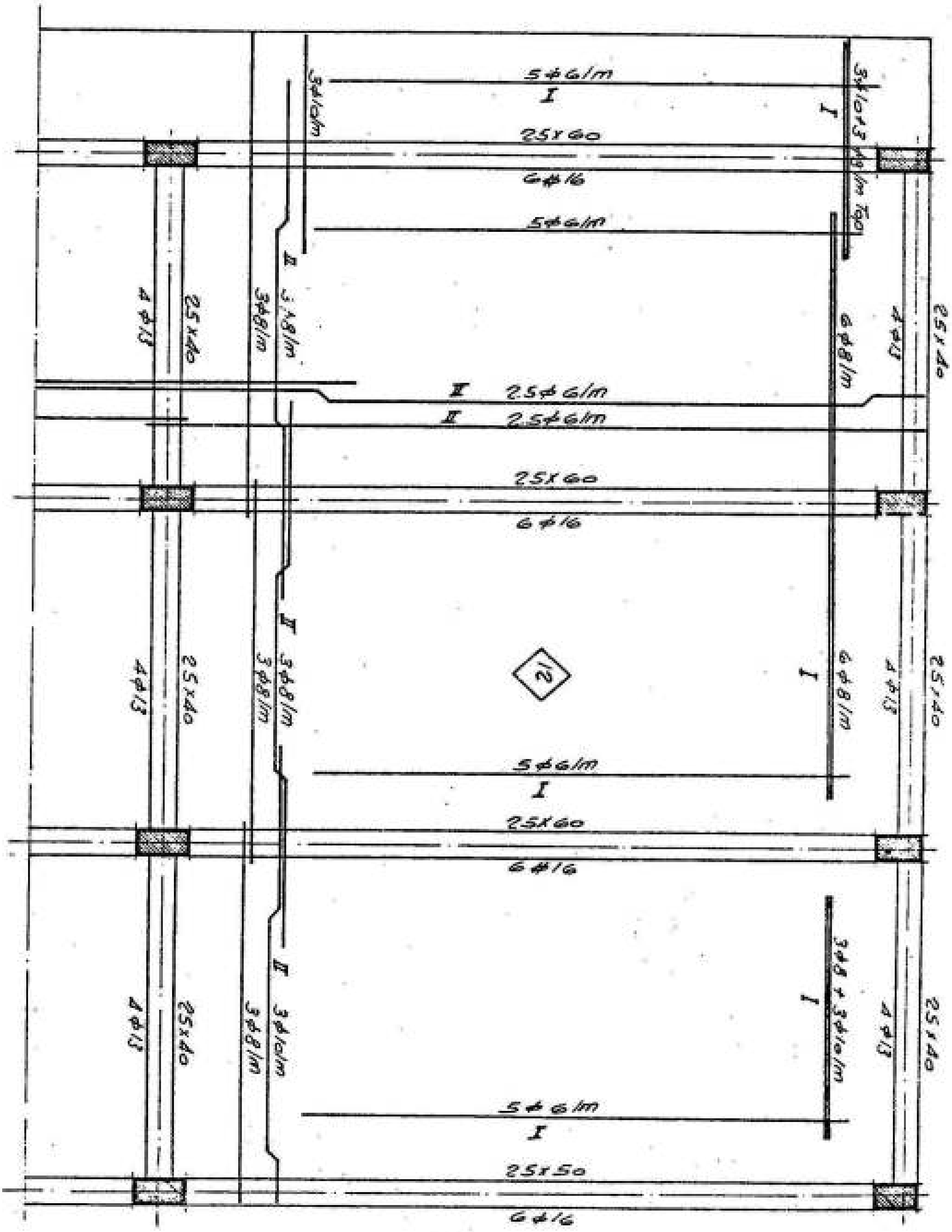


Fig 5-8 Example of a one-way slab

of two sets of parallel strips intersecting each other as shown in Fig. 5-9.

Considering the middle 1 m-wide-strips, of each direction, their deflection at the point of intersection will be the same. if the load transmitted to the short direction x is w_x and the load transmitted to the long direction y is w_y, then we should have:

$$w_x + w_y = w \quad \ldots\ldots\ 5\text{-}1$$

and

$$\frac{5}{384}\,\frac{w_x\, l_x^4}{E\,I} = \frac{5}{384}\,\frac{w_y\, l_y^4}{EI} \qquad \text{or}$$

$$\frac{w_x}{w_y} = \frac{l_y^4}{l_x^4} \quad \ldots\ldots\ldots\ldots\ 5\text{-}2$$

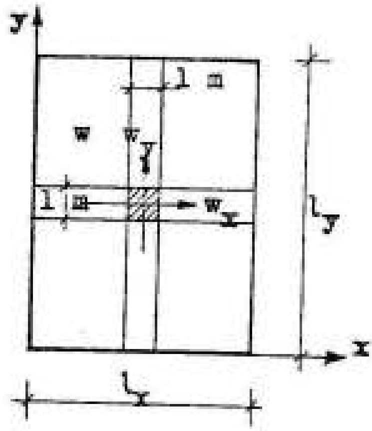


Fig. 5-9

Equations 5-1 and 5-2 give:

$$w_x = \frac{l_y^4}{l_x^4 + l_y^4}\, w \qquad \text{and} \qquad w_y = \frac{l_x^4}{l_x^4 + l_y^4}\, w \quad \ldots\ldots\ 5\text{-}3$$

It is clear that the larger part of the load is carried in the short direction and the smaller part is carried in the long direction.

This result is evidently approximate because it considers the two middle strips only assuming that they are completely seperated from the neighbouring strips. It however gives a good idea about the performance of a slab if the conditions are the same on all four sides (e.g. simply supported, partially or totally fixed).

Assuming that $w_x = \alpha\, w$, $\quad w_y = \beta\, w \quad$ and $\quad l_y / l_x = \chi$

then $\quad \alpha = \dfrac{l_y^4}{l_x^4 + l_y^4} = \dfrac{1}{(\frac{1}{\chi})^4 + 1} \quad$ similarly $\quad \beta = \dfrac{1}{\chi^4 + 1}$

The values of α and β according to Grashoff, for slabs of symmetrical edge conditions, are given in table 5-1.

<u>Table 5-1 Load distribution in two-way slabs according to Grashoff</u>

χ	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
α	.50	.595	.672	.742	.797	.834	.867	.893	.914	.928	.941
β	.50	.405	.328	.258	.203	.166	.133	.107	.086	.072	.059

Having determined w_x and w_y, the bending moments in each direction is to be calculated for its load and span (i.e. w_x and l_x or w_y and l_y).

Elastic and inelastic behavior of two-way slabs

The acual behavior of a slab is more complex than that of the two intersecting middle strips assumed by Grashoff.

However, a better analysis would result if the slab were considered as composed of several strips in each direction, held to common deflections at their intersections.

A still better approximation would consider torsional stiffness of the strips as well as their bending stiffness. With a sufficient number of strips, one may get better results that are closely equivalent to those obtained from the partial differential equation of flat plates, but the amount of labour required is excessive.

Even elastic analysis of homogeneous isotropic plates, although it provides helpful information, yet it may be considered as an exact solutión for the assumed conditions. The change of the stiffness of the slab due to different reinforcements in both directions and cracking that may develop are not recognized in this solution.

The theoretical study of slabs shows that the total load on a slab is carried not only by the bending moments in the two directions but also by the twisting moments. For this reason, bending moments in elastic slabs are smaller than would be computed in sets of unconnected strips loaded by w_x and w_y. For instance, for a simply supported square slab ($l_x = l_y = l$) subject to a uniform load w, the load in each direction, according to Grashoff is $w_x = w_y = w/2$ and the maximum bending moment is given by: $M_{max} = w\,l^2/16$ while the exact theory of bending of elastic isotropic flat plates shows that actually the maximum moment in such a square slab is only:

$$M_{max} = w\,l^2/21$$

i.e. in this case, the twisting moments relieve the bending moments by about 25%.

Reinforced concrete slabs are monolithic and if the load in sections of maximum moment is increased until this location is overstressed so that the steel yields, no failure will occur because the neighbouring strips will take over the additional load which cannot be carried by the yielding strip. Such an inelastic redistribution reduces further the maximum bending moment by a further 25%, i.e.

$$M_{max} = w\, l^2/27$$

Such values used in reinforced concrete design for different edge conditions due to uniform and triangular loading are given in the text book of tanks*. They may be used in important structures of relatively thick slabs as in tanks and bridges.

Determination of internal forces according to Egyptian code

The code provides that slabs with side ratios smaller than 2:1 shall be designed as two-way.

The precise determination of moments in two-way slabs with various conditions of continuity at the edges is mathematically formidable and not suited to design practice. For this reason the code provides simplified methods for load distribution and moments.

In order to get the redistributed inelastic bending moment in a two way slab taking its torsional resistance in consideration, the Egyptian code allows to assume $w_x = \alpha\, w$ and $w_y = \beta\, w$, where $w_x + w_y < w$. The values of α and β allowed by the Egyptian code for slabs of symmetrical edge conditions are given in table 5-2.

Table 5-2. Load distribution on two-way slabs according to Egypt. Code

χ	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
α	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.85
β *	0.35	0.29	0.25	0.21	0.18	0.16	0.14	0.12	0.11	0.09	0.08

* It has to be noted that $\beta = 0.35/\chi^2$.

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* "Theory and Design of Reinforced Concrete Tanks" by M. Hilal. Published by J. Marcou and Co. Cairo

Accordingly, the maximum bending moment in a square slab simply supported on all four sides subject to uniform load w is given by:

$$M_{max} = \frac{0.35\ w\ l^2}{8} = \frac{w\ l^2}{23}$$

which is only 10% smaller than the value of the elastic isotropic flat plates.

These coefficients are to be used for determining the field moments of slabs using the approximate values of bending moments ($M = w\ l^2/\ 10$ or $w\ l^2/\ 12$) only because the connecting moments over the edges of continuous slabs are not reduced by the torsional resistance of the slabs. The connecting moments M_c over the supports of a continuous two-way square slab would be:

$$M_c = \frac{0.5\ w\ l^2}{10} = \frac{w\ l^2}{20}$$ over support of end span and

$$M_c = \frac{0.5\ w\ l^2}{12} = \frac{w\ l^2}{24}$$ over supports of intermediate spans.

The exact values of connecting moments according to the theory of elastic flat plates are in some cases slightly bigger; e.g. in a fixed square slab.

According to Grashoff $$M- = \frac{0.5\ w\ l^2}{12} = \frac{w\ l^2}{24}$$

According to redistributed inelastic equations $$M- = \frac{w\ l^2}{19.2}$$

It can be easily seen that the exact connecting moment is bigger than the value according to Grashoff by ~ 20%, but the field moment governing the design is generally chosen bigger than the exact values, i.e:

$$M+ = \frac{w\ .\ l^2}{10+12} = \frac{0.35\ w\ l^2}{10\ +\ 12} = \frac{w\ .l^2}{30+35}$$

For heavy live loads in bridges or factories and water structures, the field moments of continuous slabs can be calculated according to the theory of elasticity and with the load distribution according to Grashoff reduced by the factors of Marcus given in table 5-3 which take the torsional resistance of the slab in consideration.

Table 5-3. Tortional reduction factors for field moments in continuous two-way slabs

$\frac{l_y}{l_x} = \lambda$	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
r	.86	.86	.87	.88	.89	.90	.91	.91	.92	.93	.94

Accordingly, the field moment in a two-way square fixed slab subject to uniform load is given by:

$$M+ = 0.86 \frac{0.5 \, w \, l^2}{24} = \frac{w \, l^2}{56}$$

which is approximately the value given by the redistributed inelastic equations, namely:

$$M+ = \frac{w \, l^2}{56.8}$$

The reduction of the field moment due to torsional resistance is usually of sequence only in exterior corners, and especially in slabs directly supported on walls, as they tend to crack the slab along 45° lines of the corner panels. For this reason, it is advisable that, in addition to the calculated reinforcement, special reinforcement shall be provided at exterior corners in both bottom and top of the slab, for a distance in each direction from the corner equal to one-fifth the long span. The reinforcement in the top of the slab shall be parallel to the diagonal from the corner. The reinforcement in the bottom of the slab shall be at right angles to the diagonal, or it may consist of bars in two directions parallel to the sides of the slab. The reinforcement in each band shall be of size and spacing equivalent to that required for the maximum field moment in the slab. Fig. 5-10.

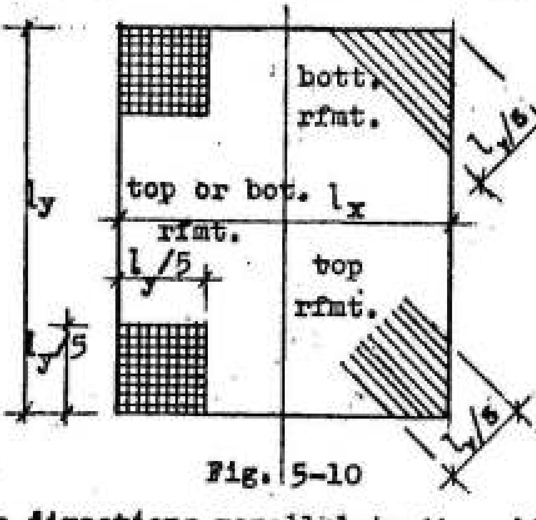

Fig. 5-10

For two-way slabs, the Egyptian code gives further the following simple methods of calculation for normal buildings.

a) This code of practice is valid only for ordinary buildings with small live load (up to 400 kg/m^2). Slabs of other structures such as bridges, liquid containers, storehouses, ... etc., are to be designed according to their corresponding special codes.

b) Minimum thickness* 8 cms

for freely supported slabs .... $t_{min} = l_x/50$

for continuous or fixed slabs ..... $t_{min} = l_x/60$

where l_x is the smaller effective span of the slab.

c) Rectangular slabs monolithically cast with beams and supported on all four sides where the length is not more than two times the breadth, and subject to uniform loads can be calculated in normal cases according to the following simplified method:

Assume l_x and l_y = theoretical spans,

m = ratio of length between points of inflection for a loaded strip in direction x to span l_x,

m_1 = same for span l_y,

= degree of rectangularity between lines of inflection of a panel

$\gamma = m_1 l_y / m\, l_x$:

The portions of the load in directions x and y are:

$$w_x = \alpha w \quad \text{and} \quad w_y = \beta w$$

The values of α and β are given in table 5-3.

d) Values of m and m_1 are to be determined from theory of elasticity. In continuous slabs, if the ratios between the spans in one direction lie between 2/3 and 3/2 the following approximation in determining the values of m and m_1 may be adopted:

If the span under consideration is continuous at one end only, m = .87

,, ,, ,, ,, ,, ,, ,, ,, both ends, m = .76.

e) In abnormal cases of loading such as heavy concentrated loads or heavy uniform partial loading as well as cases of slabs freely supported at four sides and of relatively wide spans, the above load distribu-

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* Another good rule for choosing the minimum thickness is that it is equal to the panel perimeter divided by 180.

tion is not allowed and the design should be according to acknowledged theories.

f) Reinforcement in secondary direction should not be less than 0.25 of the main reinforcement and not less than 5 Ø 6 mm/m.

g) Field moment reinforcement adjacent to a continuous edge only and for a width not exceeding ¼ of the shorter dimension of the panel may be reduced by 25%.

h) For other constructional details, refer to one-way slabs.

i) In case of slabs resting on masonry walls, the load distribution is affected according to table 5-4.

Table 5-4. Load distribution on slabs supported on masonry walls according to Marcus

γ	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
α	.396	.473	.543	.606	.660	.706	.746	.778	.806	.830	.849
β	.395	.323	.252	.212	.172	.140	.113	.093	.077	.063	.053

Illustrative example

It is required to design the floor slab shown in Fig. 5-11.

Assume: Timber floor cover + plaster .... 100 kg/m²
Live load on slab .... 400 kg/m²
Thickness of slab .... 12 cms
Concrete C160 and normal mild steel

Load: $w = 400 + 100 + 0.12 \times 2500 = 800 \text{ kg/m}^2$

Allowable stresses: (according to Egyptian code)

$\sigma_c = 50 \text{ kg/cm}^2$ and $\sigma_s = 1400 \text{ kg/cm}^2$.

The solution will be made twice; the first follows the Egyptian code assuming that the ratio of rectangularity γ equals $m_1 l_y / m l_x$ and the load-distribution-factors α and β are according to table 5-2. The bending moments and the corresponding reinforcements for each direction are then calculated with the approximate values: $M_{x,y} = \frac{w_{x,y} l^2_{x,y}}{10 \rightarrow 12}$ and the second solution will be made assuming $\gamma = l_y/l_x$ directly.

First solution. According to Egyptian code

a) Load distribution

panel	l_y	m_1	$m_1 l_y$	l_x	m_2	$m_2 l_x$	χ	α	$w_x=\alpha w$	β	$w_y=\beta w$
	ms		ms	ms		ms			kg/m²		kg/m²
S_1	5.00	0.87	4.35	4.00	0.87	3.48	1.25	.475	380	.230	184
S_2	5.00	0.87	4.35	4.00	0.76	3.04	1.43	.570	455	.177	142
S_3	5.00	0.76	3.80	4.00	0.87	3.48	1.09	.395	316	.254	203
S_4	5.00	0.76	3.80	4.00	0.76	3.04	1.25	.475	380	.230	184

b) Bending moments and reinforcements in main direction

panel	k	$M_x=\frac{w_x l_x^2}{k}$	d=t-1.5	$k_1=\frac{d}{\sqrt{M_x}}$	σ_c	k_2	$A_{sx}=\frac{M_x}{k_2 d}$	rfmt.	A_s chosen
		kgm	cm		kg/cm²		cm²		cm²
S_1	10	610	10.50	0.425	41.5	1254	4.60	6∅10	4.60
S_2	12	610	10.50	0.425	41.5	1254	4.60	6∅10	4.60
S_3	10	510	10.50	0.465	37	1268	3.80	3∅10+3∅8	3.85
S_4	12	510	10.50	0.465	37	1268	3.80	3∅10+3∅8	3.85

c) Bending moments and reinforcements in secondary direction

panel	k	$M_y=\frac{w_y l_y^2}{k}$	d=t-2.5	$k_1=\frac{d}{\sqrt{M_y}}$	σ_c	k_2	$A_{sy}=\frac{M_y}{k_2 d}$	rfmt.	A_s chosen
		kg m	cm		kg/cm²		cm²		cm²
S_1	10	460	9.50	0.443	39	1262	3.84	3∅10+3∅8	3.85
S_2	10	355	9.50	0.505	32.5	1280	2.92	6∅8	3.0
S_3	12	423	9.50	0.462	37	1268	3.52	3∅10+3∅8	3.85
S_4	12	384	9.50	0.485	34.5	1274	3.18	6∅8	3.0

Second solution

In reinforced concrete skeleton buildings with marginal beams of relatively big dimensions, one may assume that all panels of the slab are partially fixed at their edges so that the degree of rectangularity χ is equal to l_y/l_x. In this case, the tables of calculation can be reduced to one table as follows:

panel	w	l_y/l_x	α	k	M_x	t	A_{sx}	rfmt.	β	k	M_y	A_{sy}	rfmt.
	kg/m^2				kgm	cms	cm^2				kgm	cm^2	
S_1	800	1.25	.475	10	610	12	4.5	6 ø 10	.230	10	460	3.85	3ø10+3ø8
S_2	800	1.25	"	12	510	"	3.85	3ø10+3ø8	"	10	460	3.85	3ø10+3ø8
S_3	800	1.25	"	10	610	"	4.5	6 ø 10	"	12	373	3.10	6 ø 8
S_4	300	1.25	"	12	510	"	3.85	3ø10+3ø8	"	12	373	3.10	5 ø 8

In the above table, we have: $M_x = \frac{\alpha w l_x^2}{k}$ and $M_y = \frac{\beta w l_y^2}{k}$

and

$$A_{sx} = \frac{M_x}{\sigma_s \, y_{ct}} = \frac{M_x}{1400 \times 0.9\ d} = \frac{M_x}{1260 \times 0.105}$$

$$A_{sy} = \frac{M_y}{\sigma_s \, y_{ct}} = \frac{M_y}{1400 \times 0.9\ d} = \frac{M_y}{1260 \times 0.095}$$

This last solution is generally convenient for office work and gives sufficiently safe results. The details are shown in Fig. 5-11.

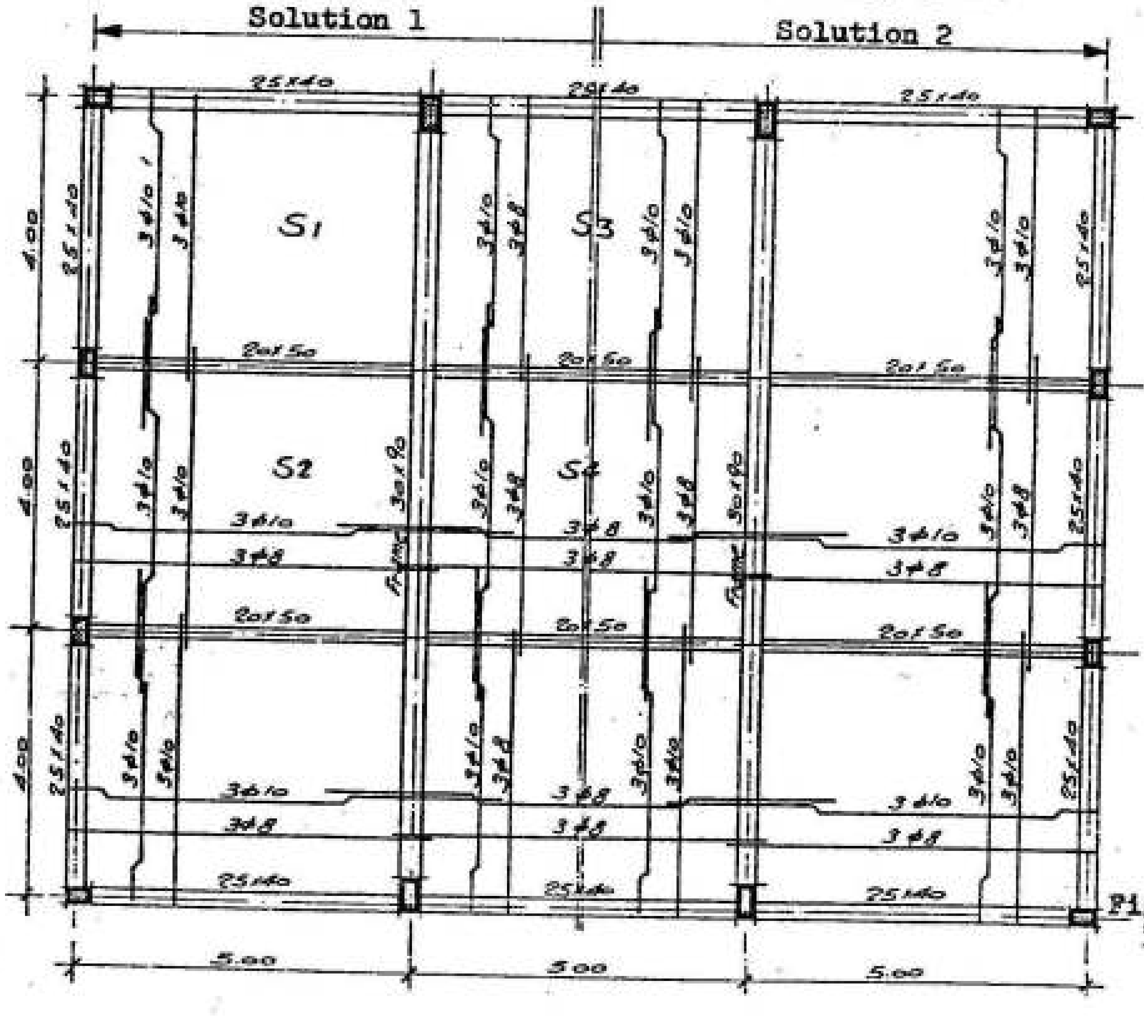


Fig. 5-11

5.4 Ribbed and Hollow-block Slabs

Design according to the Egyptian Code

The Egyptian code gives the necessary provisions for the design of one and two-way ribbed and hollow-block slabs as follows:

a) General remarks .

1- In the calculation of hollow-block slabs, the hollow-blocks are considered not to be statically acting.

2- The following conditions regarding dimensions should be complied with: Refer to Fig. 5-2.

Maximum clear spacing of ribs: e_{max} = 70 cms.

Minimum width of ribs: 5 cms or 1/3 of depth.

Minimum thickness of acting compression slab: 5 cms or e/10.

3- In case concentrated loads are acting direct on the slab between the ribs, the bearing capacity of the compression slab alone shall be checked.

b) One-way ribbed slabs (ribs in one direction):

1- Distribution bars normal to ribs are to be 20% of main reinforcement with a minimum of 3 Ø 6 mm/m and one bar 6 mm is to be placed between each two ribs.

For live loads smaller than 300 kg/m^2, and spans over 5.0 ms, at least one cross rib is to be provided in the middle of the span , of a cross section and reinforcement not less than those of the main ribs and with a top reinforcement at least ½ the bottom reinforcement.

3- For live loads bigger than 300 kg/m^2, and spans from 4.0 to 7.0 ms, one cross rib is to be provided as above; For spans above 7.0 ms, three cross ribs are to be provided. Dimensions and reinforcement as above.

c) Two-way ribbed slabs (ribs in two directions):

In this system, the compression slab may be partly omitted, i.e. , the ribs may have the form of a T-section, with a limited width of the compression flange. The load distribution is to be taken as follows

For slabs with complete cover , use the coefficients of Marcus given in table 5-4.

For ribs with cover slab partly omitted, use the coefficients of Grashoff given in table 5-1.

The following provisions apply to both one-way and two-way ribbed slabs:

1- Minimum distributing bars in the compression slab are to be 3 Ø 6 mm/m.

2- Shear forces in ribs are to be treated according to article 4.5.

3- Solid concrete part is to be provided at internal supports of continuous slabs, to resist negative bending moments.

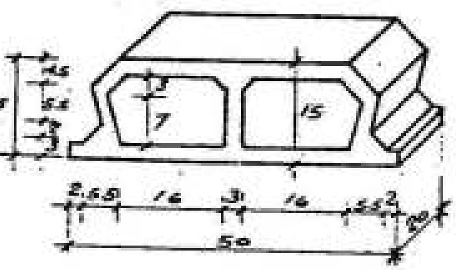
4- For effective spans and bending moments refer to article 5.2.

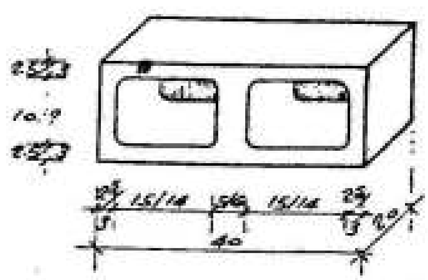
5- The breadth of the support over brickwork must not be less than 15 cms and no hollow-blocks to extend over the support.

Some types of hollow-block and ribbed slabs

The 'Misr Concrete' hollow-blocks. General data: Table 5-5

Dimensions of blocks in cms	Materials required / m^2				Dead loads kg / m^2			
	Nº. of blocks		concrete		poncit		hagarit	
	1 way	2 way	1 way	2 way	1 way	2 way	1 way	2 way
15 x 20 x 50	10	8.4	0.073	0.089	238	270	300	320
15 x 20 x 40	10.4	8.7	0.075	0.096	240	284	303	336
20 x 20 x 40	10.4	8.7	0.083	0.111	265	330	330	380
25 x 20 x 40	10	8	0.100	0.140	320	406	410	478
35 x 20 x 65	7.7	6.6	0.126	0.173	445	542	585	660





Types of ribbed slabs

The forms of one and two-way ribbed slabs are available either in steel or plastic sheets, the depths vary between 15 and 40 cms and the distance between the centers of the ribs vary between 50 and 90 cms as shown in Fig. 5-12

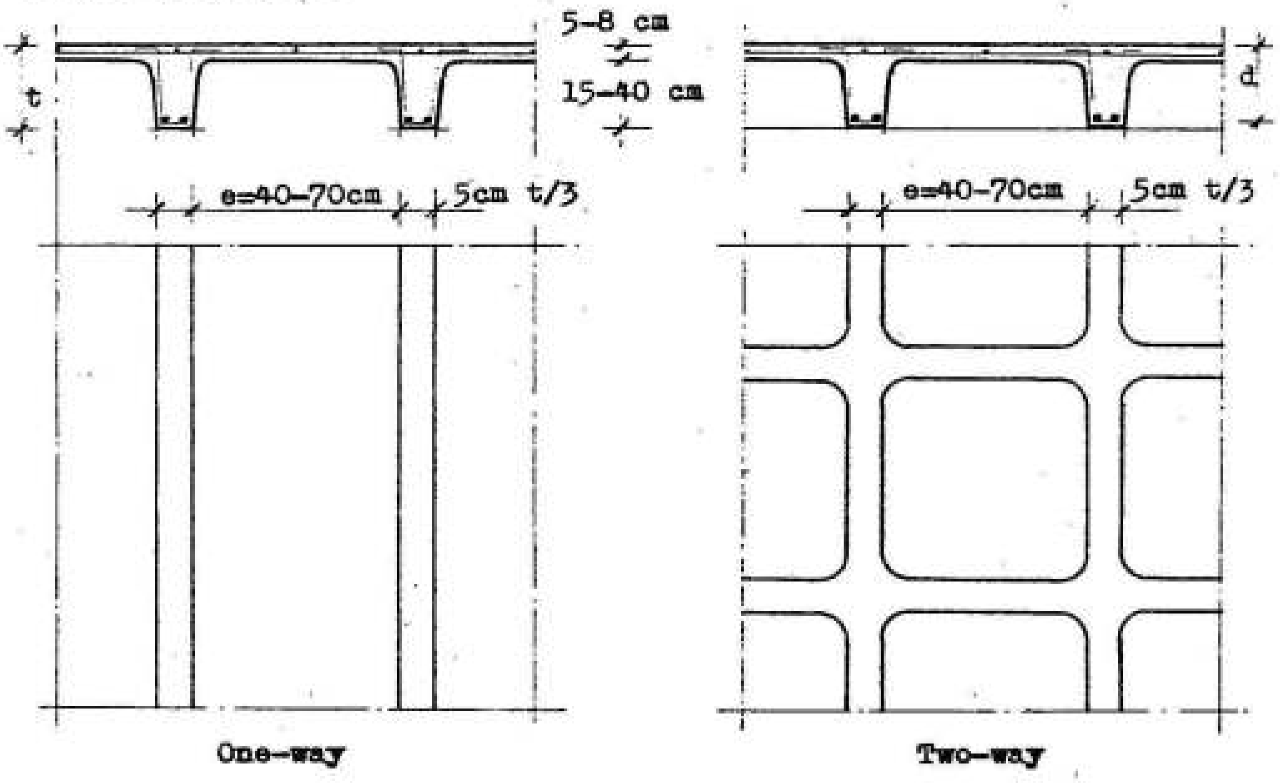


Fig. 5-12 Ribbed slabs

Illustrative example

It is required to design the one-way hollow floor slab shown in Fig. 5-13 to carry a live load of 300 kg/m^2. Assume a floor cover and plaster of 124 kg/m^2. The materials used are C180 and normal mild steel with f_y = 2300 kg/cm^2.

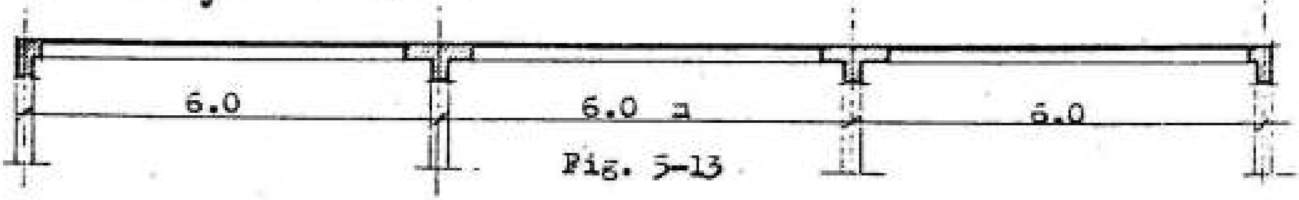


Fig. 5-13

Assume a one-way hollow slab 25 cms thick having a compression slab of 5 cms, hagarit hollow blocks 20 x 20 x 40 cms and ribs 10 cms wide; i.e. distance between center lines of ribs = 50 cms.

Loads

According to table 5-5, the own weight of the slab is 330 kg/m^2; hence:

Dead load: 124 + 330 per m^2 or $g = \frac{454}{2} = 227$ kg/m' per rib

Live load: .... 300 per m^2 or $p = \frac{300}{2} = 150$ " " "

total w = 377 " " "

Bending moments and shearing forces

The absolute maximum bending moments and shearing forces per rib (50 cms) will be calculated according to table 4-2 (case 2). Hence:

Bending moments

At 0.4 l of first span	max $M+ = (.080 \times 227 + .100 \times 150)6^2 = 1194$ kgm
" intermediate support	max $M- = (.100 \times 227 + .117 \times 150)6^2 = 1449$ "
" middle of interm. span	max $M+ = (.025 \times 227 + .075 \times 150)6^2 = 610$ "
" " " " "	max $M- = (.025 \times 227 - .050 \times 150)6^2 = 66$ "

Shearing forces

At outside support	max $Q+ = (.400 \times 227 + .450 \times 150)6 = 950$ kgs
	min $Q+ = (.400 \times 227 - .050 \times 150)6 = 500$ "
To left of interm. support	max $Q- = (.600 \times 227 + .617 \times 150)6 = 1373$ "
	min $Q- = (.600 \times 227 - .017 \times 150)6 = 802$ "
" right " " "	max $Q+ = (.500 \times 227 + .518 \times 150)6 = 1147$ "
	min $Q+ = (.500 \times 227 - .085 \times 150)6 = 605$ "

If the maximum negative bending moment at the intermediate support is reduced according to a parabolic curve, then its value will be: max. M- = 1300 kg m.

Allowable stresses

For C180, normal mild steel with $f_y = 2300$ kg/cm^2 and t = 20 cms, we have: $\sigma_c = 65$ kg/cm^2 $\sigma_s = 1400$ kg/cm^2 and $\tau = 6$ kg/cm^2

Dimensioning and reinforcements

In a hollow block slab (or a ribbed slab) of the form shown in Fig. 5-13, the field moments are resisted by the compression slab and the tension steel in the ribs, whereas the connecting moments at the supports are resisted by a solid part, because it is generally not convenient to arrange the blocks at the upper surface of the slab where the tension zone lies. In order to get a reasonable economic solution, the breadth B of the solid part should be $\leqslant$ l/6 (in our case, B $\leqslant$ 1 m). In order to get this result, it is advisable to make a redistribution of the bending moments by reducing the connecting moments over the intermediate supports by about 15%, i.e. $\Delta M^- = 0.15 \times 1449 = 220$ kg m. Accordingly, the bending moments used for the design are those shown hatched in Fig. 5-14.

Determination of breadth B of the solid part

The breadth of the solid part is to be chosen such that the moment of resistance M_r of section a-a with breadth b = 10 cms is sufficient. M_r can be determined according to equation 4-16, namely:

$$d = c\sqrt{\frac{M_r}{\sigma_c\, b}}$$

in which

d = 22.5 cms σ_c = allow. stress = 65 kg/cm^2 b = 10 cms

c is to be chosen for $\sigma_s/\sigma_c = 1400/65 = 21.5$ "   and $\alpha = 0.40$

being the maximum allowed ratio of compression steel. Hence sheet 12 gives c = 2.1 . Therefore

$$22.5 = 2.1\sqrt{\frac{M_r}{65 \times 10}} \qquad \text{giving} \qquad M_r = 750 \text{ kg m}$$

Plotting this value on the bending moment diagram, one can determine the minimum value of the breadth B of the solid part. In our case, a breadth B = 1.05 ms is ample.

Middle section of outside spans $M+$ = 1300 kg m/rib (50 cms)

$$d = k_1\sqrt{\frac{M}{b}} \qquad \text{or} \qquad 22.5 = k_1\sqrt{\frac{1300}{0.50}} \qquad \text{giving} \qquad k_1 = 0.441$$

according to table 4-5, we get: for σ_s = 1400 kg/cm^2 and $\alpha = 0$

σ_c = 39 kg/cm^2 $k_2 = 1262$ so that:

$$A_s = \frac{M}{k_2\, d} = \frac{1300}{1262 \times 0.225} = 4.56 \text{ cm}^2$$ chosen 1Ø19 + 1Ø16 (4.80 cm^2)

The shown design is based on the assumption that the neutral axis lies inside the compression slab i.e. $z \leqslant 5$ cms, if not, the compressive stresses in the compression slab must be checked; they must be smaller than the allowed value of 65 kg/cm^2. Hence

$$z = \frac{n A_s}{b}\left(\sqrt{1 + \frac{2 d b}{n A_s}} - 1\right) = \frac{15 x 4.8}{50}\left(\sqrt{1 + \frac{2 x 22.5 x 50}{15 x 4.8}} - 1\right) = 6.9 \text{ cms}$$

This value is bigger than 5 cms and can be accepted if the maximum stress in the compression slab is smaller than 65 kg/cm^2. The actual maximum compressive stress can be calculated according to sheet 4. Thus $\delta = t_s/d = 5/22.5 = 0.222$ and $\mu = A_s/bd = 100 \times 4.8/50x22.5$ or $\mu = 0.426\%$ giving $c_1 = 9.5$ and $\sigma_c = c_1 M / b d^2$ or

$$\sigma_c = \frac{9.5 \times 1300\ 00}{50 \times 22.50^2} = 49.0 \quad kg/cm^2$$

i.e. a compression slab of 5 cms is sufficient.

Middle section of intermediate span $M = 830$ kg m / rib (50 cms)

The moment in this section is much smaller than that of the previous section and the depth is the same; so that σ_c is low and one can assume $k_2 = 1300$, hence

$$A_s = \frac{830}{1300 \times 0.225} = 2.83 \quad cm^2$$ chosen 1Ø16 + 1Ø13 (3.34 cm^2)

Section over intermediate support $M = 1080$ kg m / 50 cms

$$22.5 = k_1 \sqrt{\frac{1080}{0.50}} \quad \text{or} \quad k_1\ 0.485 \quad \text{i.e.} \quad \sigma_c = 35\ kg/cm^2 \text{ and } k_2 = 1273$$

$$A_s = \frac{1080}{1273 \times 0.225} = 3.80 \quad cm^2$$ chosen 2 Ø 16 (4.02 cm^2)

Diagonal tension

The maximum diagonal tensile stress takes place at section a-a where $Q = 1217$ kgs. Hence

$$\tau_{max} = \frac{1217}{0.87 \times 10 \times 22.5} = 6.2 \quad kg/cm^2$$ acceptable.

The displaced bending moment diagram, the moment of resistance diagram and the details of the slab are shown in Fig. 5-14.

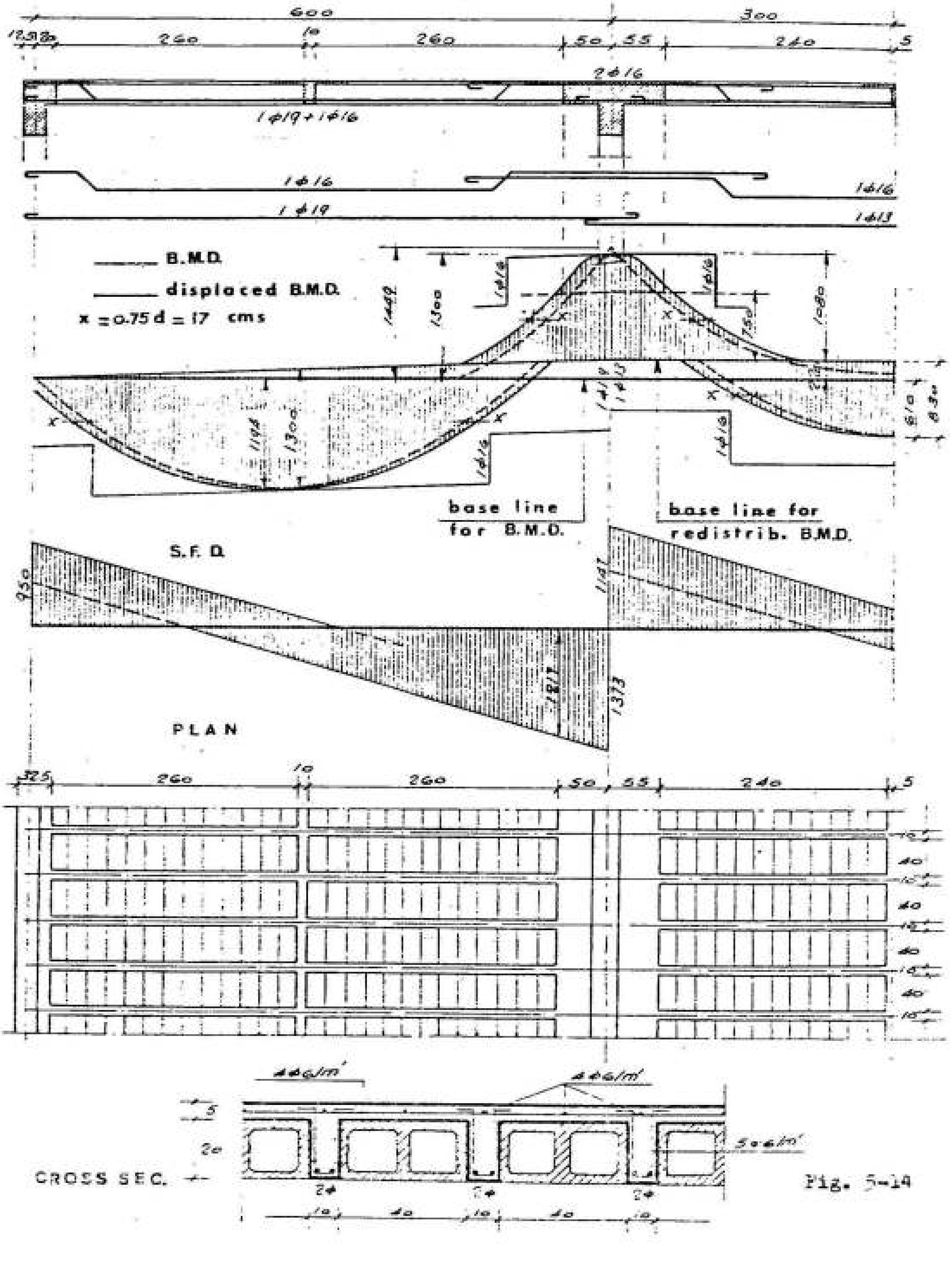

600
300
260
10
260
50
55
240
5
2φ16
1φ19+1φ16
1φ16
1φ19
1φ13
B.M.D.
displaced B.M.D.
x = 0.75 d = 17 cms
1449
1300
1080
750
1194
1300
base line for B.M.D.
base line for redistrib. B.M.D.
S.F.D.
950
1147
1373
PLAN
325
40
10
CROSS SEC.
4φ6/m'
5φ6/m'
2φ
20

Fig. 5-14

We give in the following some applications of hollow and ribbed slabs.

Fig. 5-15 shows a one-way hollow-block saw-tooth roof with a span of 7.5 ms. In the given roof, the hollow slab is supported at its lower end on a y-beam continuous over the main columns having 6.0 ms between centers, and at its upper end on a ridge beam continuous over inclined posts every 2.0 ms. The posts are again supported on the y-beam. The roof can be assumed as a one-way simply supported slab. The span being relatively big, a hollow-block slab, 25 cms thick, seems to be a convenient solution. The compression slab is 5 cms thick, the hollow blocks are 40 x 20 x 20 cms, the ribs are 10 cms wide with a distance between centers of 50 cms. The tension reinforcement of each rib is 2 Φ 16. The main ribs are connected together at their center by a distributing rib 10 cms wide and reinforced at its bottom by reinforcements of the same order as that of the main ribs and at its top by 2 ϕ 13 whose area is bigger than half the main steel. The roof is provided with a tie 20 x 20 cms and reinforced by 4 ϕ 13 to resist the horizontal thrust of the roof due to the fact that the loads are vertical and the reaction at the ridge beam is inclined and along the axis of the posts which means that the reaction of each panel at the y-beam is inclined. The horizontal components at the intermidiate y-beams cancel each other, i.e. the final load on them is vertical.

This system is generally used in factories where indirect light without sun rays is required. One can satisfy this requirement by simply arranging the windows between the posts to face the north. It is also advisable to make the inclination of the roof smaller than 30° with the horizontal to be able to concrete the slab on one shuttering. The detailing of the corners of such a roof is to be carefully done.

Fig. 5-16 shows the roof slab and the main girders of an air conditioned wollen textile factory. The air conditioning ducts, 10 ms

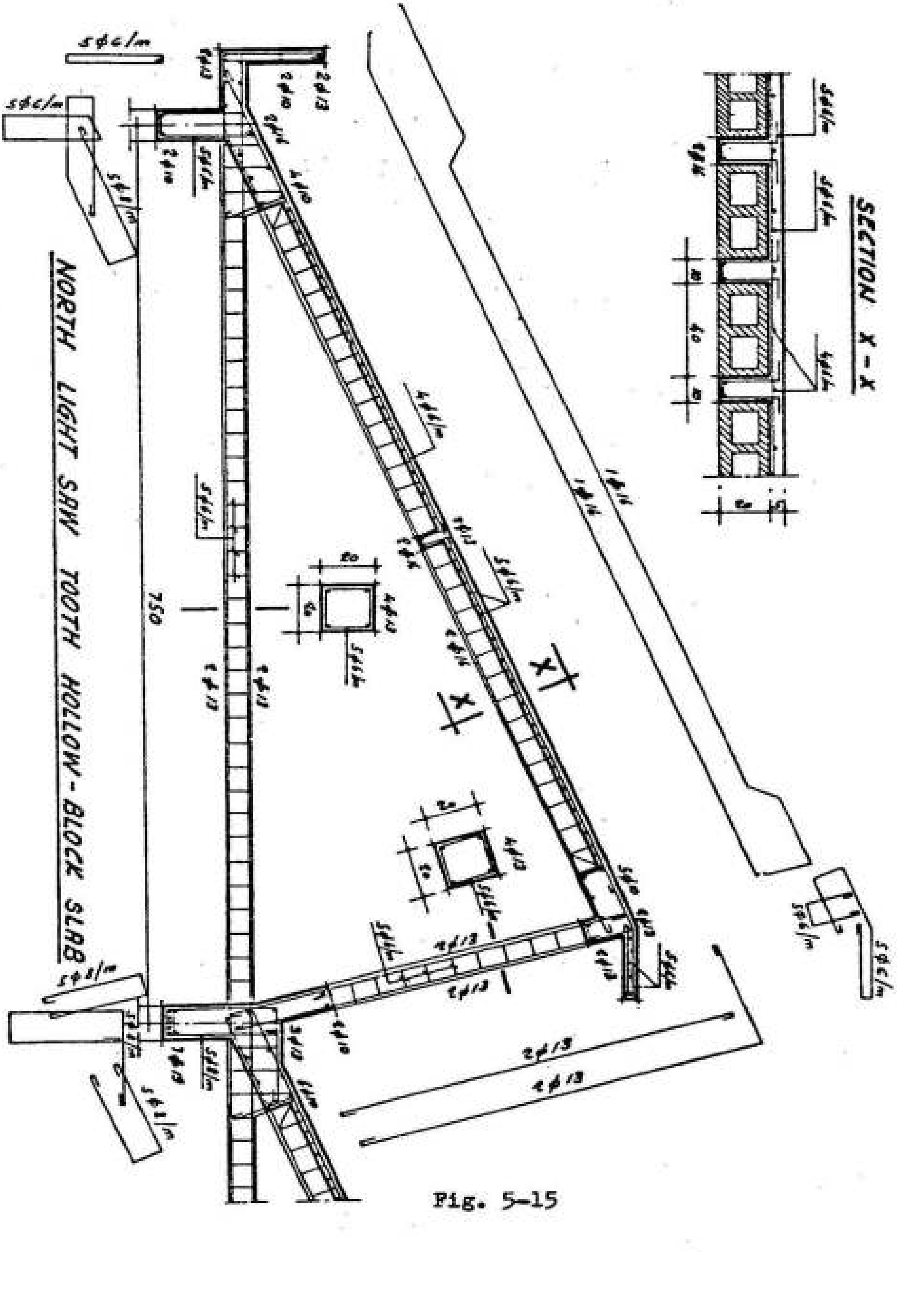


Fig. 5-15

apart, have a trapezoidal section 1.5 ms clear depth, their clear width is 1.00 m at top and 0.80 m at bottom. The supporting columns are arranged below the ducts at distances of 24 ms.

The walls, top and bottom slabs of the duct have been chosen 20 cms thick, so that it was possible to use (the 1.9 ms high) trapezoidal section of the ducts as continuous main girders 24 ms span to support a ribbed roof slab 8.60 ms clear span.

The ribbed slab is one-way and 30 cms thick. It is composed of a solid compression slab 6 cms thick and ribs arranged every 50 cms. The ribs have a trapezoidal section 24 cms deep and 8/10 cms wide. In order to distribute the load over the ribs and to assure their combined action, two stiffening ribs having the same section as the main ribs are arranged in the longitudinal direction parallel to the ducts. In order to have adequate space for resisting the connecting moments two, 40 cms long, solid parts are arranged adjacent to the ducts.

The reinforcement of the compression slab is 5Ø8/m normal to the ribs and 4Ø6/m parallel to them.

The main ribs are reinforced by 2#16 (high grade steel) at the bottom in the span and at the top over the supports. The stiffening ribs have 2#16 at bottom and 2Ø13 at top.

Another good example showing the use of one and two-way hollow-block slabs to cover wide spans is given in Fig. 5-17. It shows the details of the roof of a mosque 15 x 15 ms. The slab is assumed fixed at the perimeter of the outside octagon. Its middle octagonal part, 10 ms wide, is a two-way hollow-block slab 25 cms thick. The outside parts are 75 cms one-way hollow-block slabs to resist the negative moments of the fixation. These fixing moments are transmitted to the triangular parts at the corners and to the heavy wall beams at the central parts. Such beams can resist the fixing moment of the slab by their torsional resistance. To resist the negative moments of the roof, the slab, outside the internal octagon, is composed of a 25 cms lower comp. slab overlaid by 40 cms hollow-blocks covered by 10 cms slab.

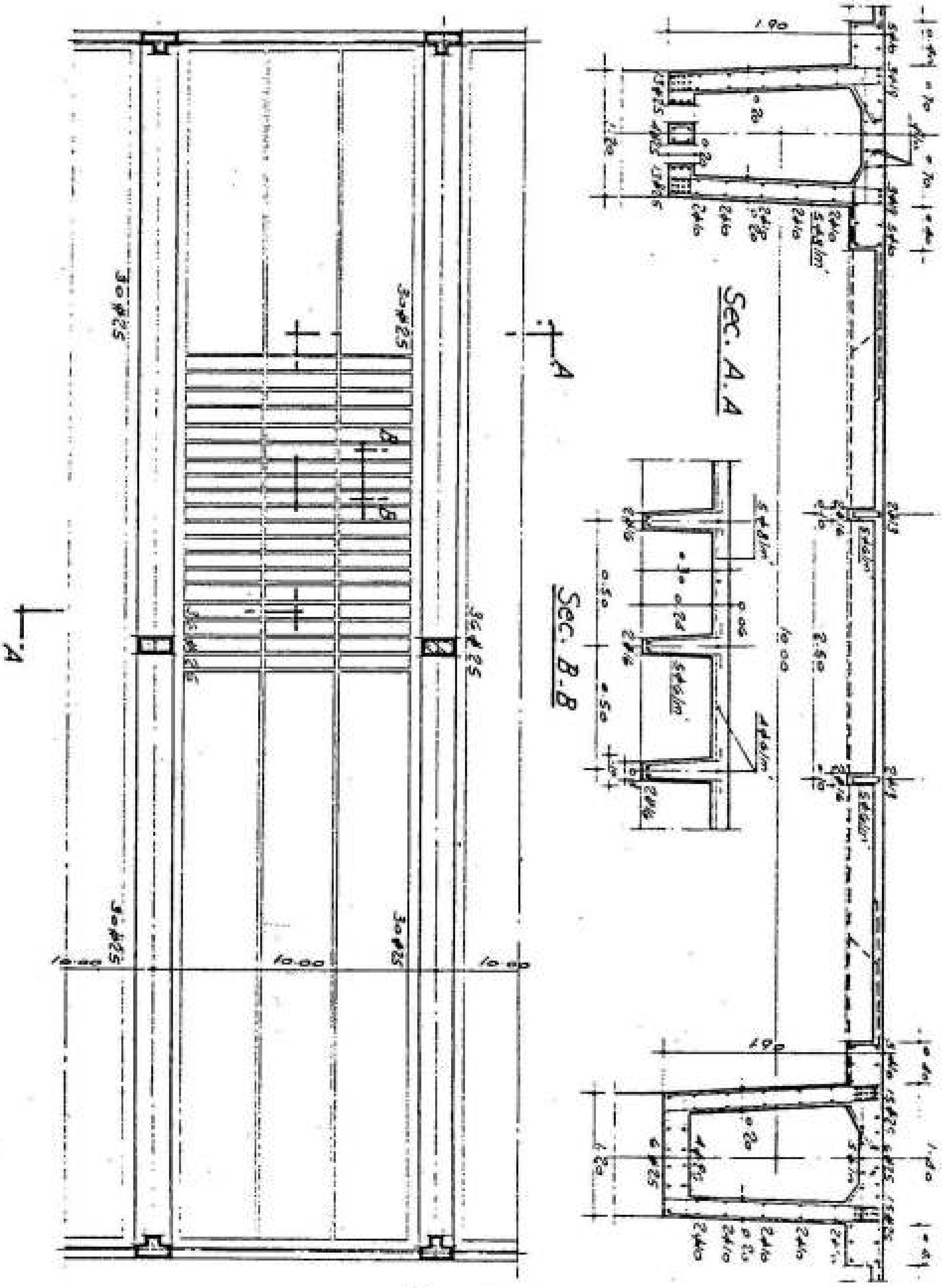


Fig. 5-16

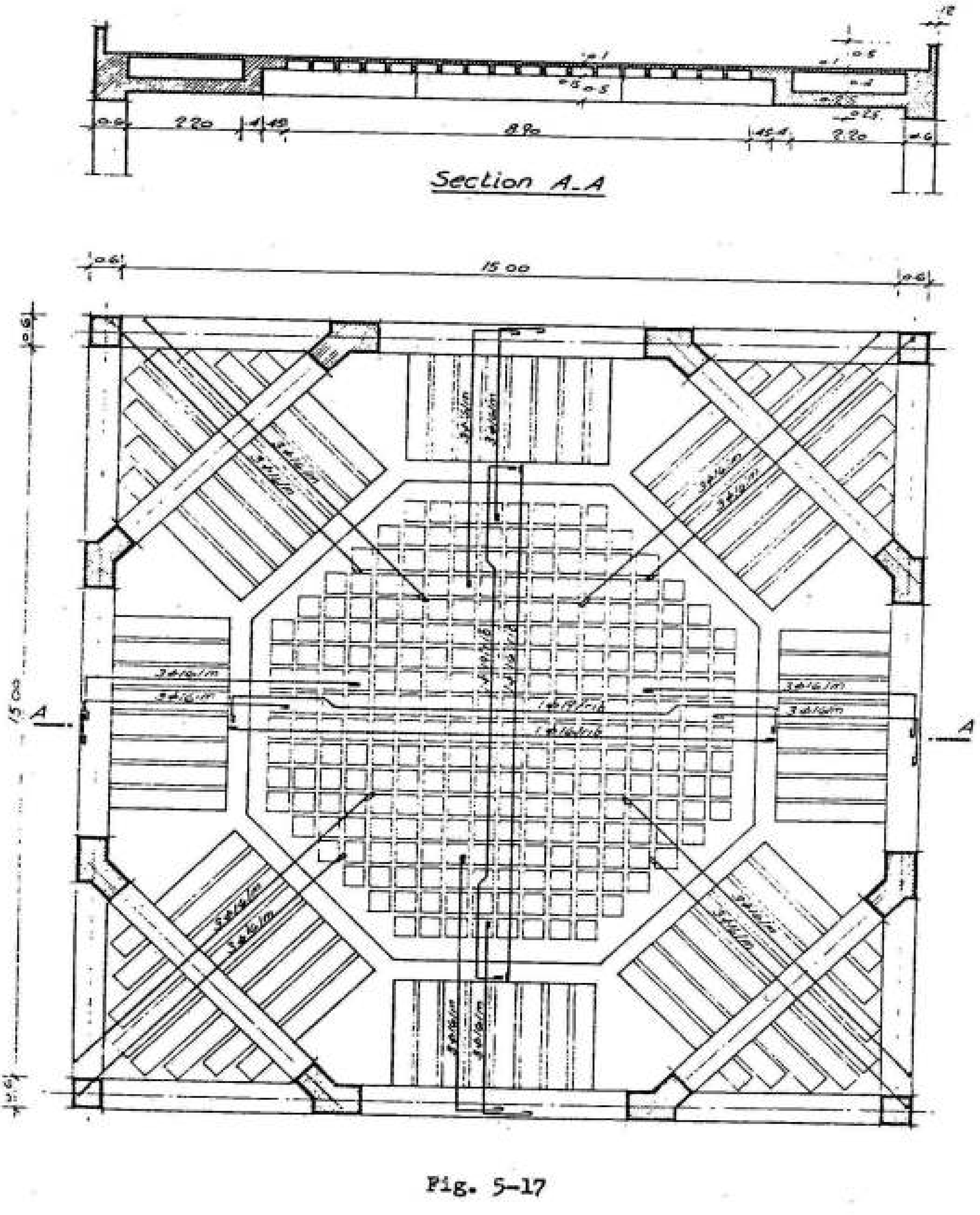


Fig. 5-17

5-5 Flat Slabs

Performance of flat slabs

A flat slab is a concrete slab generally reinforced in two directions parallel to the sides of the slab as to bring the load directly to the supporting columns, without the help of any beams or girders.

The reinforcements are arranged at the lower fiber in the middle of the spans and in the upper fiber at the supports.

In order to overcome the possible punch of the columns through the slab and to resist the high bending moments and shearing forces at the supporting columns, their heads should be according to one of the following arrangements. Fig. 5-18.

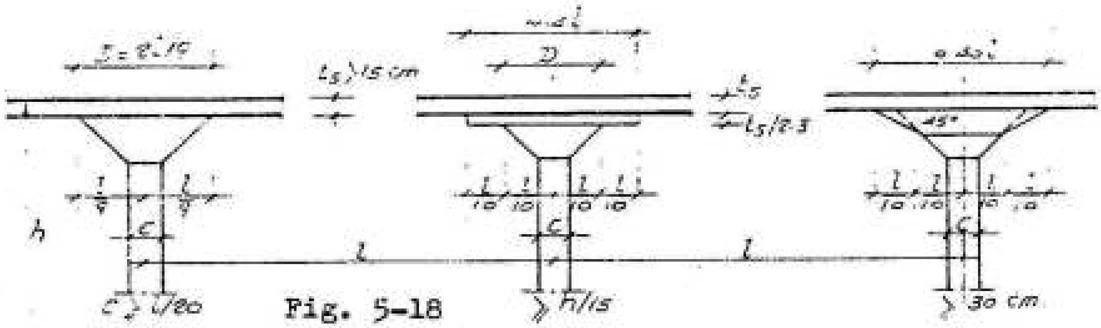


Fig. 5-18

Only the parts of a column head inside the lines of 45° can be assumed as acting.

It has been found that a flat slab with spans l_1 and l_2 behaves as if it were composed of two systems of frames or continuous beams according to the way in which the connection between the supporting columns and the slab is introduced in the calculations. If the joint between the columns and the slab is assumed rigid -as it actually is- and the relative rigidity is considered in the calculations, then the systems are continuous frames. If this rigid connection is neglected, then the systems are continuous beams. In this last case, the columns must be designed such that they can resist a specified bending moment.

One of the two systems has spans l_1 and breadth l_2, the other has spans l_2 and breadth l_1. Each system is to be calculated for the full dead and live loads. Accordingly, the bigger moments occur in the direction of the bigger spans. One should compare this with the two-way slab supported on all four sides where the short slab strips carry the larger load and larger moment. In such a slab, the long slab strips carry less load and less moment, but it should be noted that the long beams have to carry heavy reactions and large moments. Two-way slab moments in both directions are reduced because the beams help to carry the moment. In the flat slab, in contrast, the full load must be carried in both directions by the slab alone.

Flat slabs,being thin members, are not economical of steel, but they are economical in their form-work. Since formwork represents a considerable part of the cost of reinforced concrete, economy of formwork often means over-all economy.

For heavy live loads, over @ 500 kg/m^2, flat slabs have long been recognized as the most economical construction. In more recent years, flat plate floors have proved economical in multi-storey apartment buidings and warehouses. Reduced height of storeys resulting from the thin floor, absence of beams and the smooth ceiling seem to be factors in the over-all economy.

<u>Design according to Egyptian code</u>

1) <u>Notation</u>

It is generally meant by the term flat slab, a reinforced concrete beamless solid slab with or without drops, directly supported by columns with or without flared column heads. Fig. 5-19.

l_1 = Length of the panel in the direction of the span.

l_2 = Width of the panel at right angles.

l_1 and l_2 are measured from the centers of the columns.

l = The average of l_1 and l_2.

D = The diameter of the column head, or the largest circle drawn inside its section.

w = Total load per unit area of the panel.

t = Total thickness of slab.

2) Minimum dimensions_

a- The total thickness of the slab should in no case be less than the greatest of the following values:

15 cms,

l/32 for end panels without drops,

l/36 for interior panels, fully continuous and for end panels with drops,

l/40 for interior panels, fully continuous, with drops.

b- The diameter of a column (if circular), or the side length of a rectangular column should in no case be less than the greatest of the following values:

1/20 of the panel length in the same direction,

1/15 of the height of the floor,

30 cms.

c- Where column heads are provided, the heads of interior columns, and such portions of the heads of exterior columns as will lie within the building, should satisfy the following requirements:

The angle of the greatest slope of the head should not exceed 45° from the vertical.

The effective diameter D to be included in the design should not be more than 0.25 l.

Where the column and column head are not of circular cross-section, the term diameter used in this article should be deemed to mean the diameter of the largest circle which can be drawn within the section.

d- Flat slab panels should be assumed divided into field and column strips as follows:

The width of each of the field and column strips should be taken as half the width of the panel; in case of slabs with drops, the width of the column strip is equal to the width of the drop and the width of the field strip equals the rest of the panel.

3) Design of flat slabs as continuous frames

Flat slabs if not designed exactly according to theory of elasticity may be designed as follows:

a- The bending moments and shearing forces may be determined by an analysis of the structure as a continuous frame and the following assumptions may be made:

The structure may be considered as divided longitudinally and transversely into frames consisting of a row of columns and strips of slab with a width equal to the distance between the center lines of panels on each side of the row of columns.

Each frame may be analysed as a separate frame with columns above and below assumed fixed at their extremities under full dead and live load in each direction, and in positions giving maximum internal forces. The spans used in the analysis should be the distances between centers of supports and difference of moment of inertia is to be taken in consideration.

b- The slab should be designed for the bending moments so calculated at any section, except that provision need not be made for greater negative moments than those at the critical sections for shear immediatly adjacent to column, as shown in Fig. 5-19.

The bending moments for which provision is made should be divided between the column and the field strips in the proportions given in table 5-6:

Moments	Distribution of bending moments between column and field strips, expressed as percentage of total negative or positive moments	
	Column strip	Field strip
Negative moments	75 %	25 %
Positive moments	55 %	45 %

c- When the column strip is taken equal to the drop panel, then the field strip will be greater, and the distribution must be so corrected that the field strip takes greater part in proportion to the increase in its width, and the column strip smaller part, so that the the total moments are the same as above.

4) Empirical design of flat slabs subject to uniformly distributed loads.

a) This method applies only where the following conditions are satisfied:

- The slabs should comprise a series of rectangular panels of approximately constant thickness arranged in at least three rows in two directions at right angles, and the ratio of the length of a panel to its width should not exceed 4:3.
- The lengths or widths of any two adjacent panels in a series should not differ by more than 20%. End spans may be shorter, but not longer, than interior spans.
- Where adjacent spans differ, the length should always be taken as that of the longer span in calculating the bending moments.
- The drops, if any, should have a length in each direction not less than one-third of the panel length in that direction. For exterior panels, the width of drop at right-angles to non-continuous edge, measured from the center line of the column, should be equal to one half the width of drop in interior panels.
- Depth of drop should not be more than ½ and not less than ¼ of the slab thickness.

b) Critical sections for bending moments of interior panels, fully continuous are as follows:

- Positive moment along center-lines of the panels.
- Negative moments along the edges of the panel on the line joining the center of columns and around the perimeter of the column head.

c) The bending moment for which provision is made should be divided between the column and field strips as shown in table 5-7, where:

$$M_o = \frac{w\, l_2}{10}\left(l_1 - \frac{2\,D}{3}\right)^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ 5\text{-}4$$

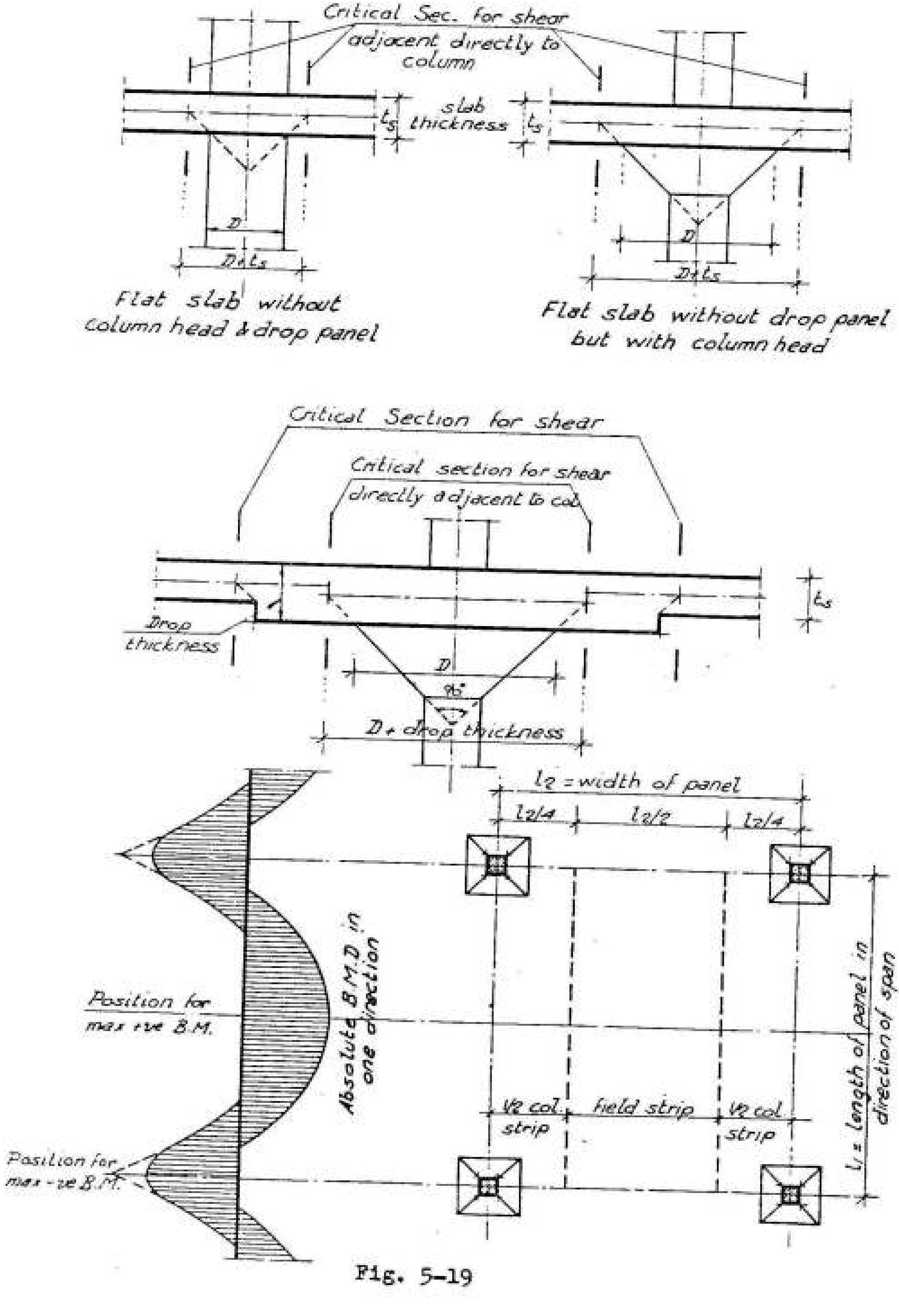

Critical Sec. for shear
adjacent directly to column
slab thickness
t_s
D
D+t_s
Flat slab without column head & drop panel
Flat slab without drop panel but with column head
Critical Section for shear
Critical section for shear directly adjacent to col.
Drop thickness
D
90°
D+ drop thickness
Absolute B.M.D in one direction
Position for max +ve B.M.
Position for max -ve B.M.
l_2 = width of panel
$l_2/4$
$l_2/2$
$l_2/4$
1/2 col. strip
field strip
1/2 col. strip
l_1 = length of panel in direction of span

Fig. 5-19

in which l_1 = length of panel in direction considered,

l_2 = " " " " perpendicular direction.

Table 5-7 Empirical distribution of bending moments in flat slabs

Distribution of B.M. in panels of flat slabs in percentage of M_o						
Strip	Col. head	End support type	Exterior panel		Interior panel	
			- ve M	+ ve M	- ve M	+ ve M
Column strip	with drop	A	45	25	50	20
		B	35			
	without drop	A	40	30	45	25
		B	30			
Field strip	with drop	A	10	20	15	15
		B	20			
	without drop	A	10	20	15	15
		B	20			

Types of end supports:

A = no beams,

B = beams with total depth equal to or more than three times the slab thickness.

Note 1: When the column strip is equal to the width of drop, the width of field strip is increased to a value greater than ½ of the breadth of the panel, and the bending moment in the field strip should be increased correspondingly. The bending moment in the column strip is to be accordingly decreased. However, the sum of both bending moments in column and field strips should not be less than the sum of bending moments (positive and negative) which is resisted by both the column and field strips together.

Note 2: When exterior panels are smaller in length than interior panels, then the bending moment may be modified accordingly.

d) In case of heavy live loads, the negative bending moment at the mid-spans of interior panels should not be less than the values given by the following relations:

Column strip: $M_{-ve} = (g - \frac{2}{3} p) \frac{l_2}{40} (l_1 - \frac{2}{3} D)^2$ 5-5

Field strip $M_{-ve} = (g - \frac{2}{3} p) \frac{l_2}{100} (l_1 - \frac{2}{3} D)^2$ 5-6

5) Bending moments in panels with and without marginal beams

a- Flat slab with marginal beam of total depth equal to or more than three times slab thickness:

* The bending moment in half column strip adjacent to the beam will be 0.25 the values in previous tables.
* The total load to be carried by the beam should comprise the loads directly on the beam plus a uniformly distributed load equal to .25 the total load on the panel.

b- Flat slab without marginal beam:

* The bending moments in half column strip will be 0.5 the values in the previous tables.

6) Bending moments in columns

a- Internal and external columns should be designed to resist bending moments equal to 50 and 90 per cent respectively of the negative moment in the column strip specified. These moments should be apportioned between the upper and lower columns in proportion to their stiffnesses. In internal columns, the direct load acting with the moment may be reduced to allow for the panel on one side to be free of imposed load.

b- In the case of external columns carrying portions of the floors and walls as a cantilevered load, the specified column moments may be reduced by the moment due to the dead load on the cantilevered portion.

7) Shear stresses

The shear stress at the critical sections shown in Fig. 5-19 should not exceed the allowable values given in table 3.2 .

8) Arrangement of reinforcements in flat slabs

a- The slabs designed according to the empirical method are to be reinforced in two directions, the reinforcement should be so disposed that each strip is reinforced over its full width.

b- In each strip or band at least one third of the positive reinforcement should extend in the lower part of the slab to a distance greater than the line joining the center line of columns.

c- The negative reinforcement in the top of slab should extend into adjacent panels for an average distance, measured from the line joining the centers of the columns, of not less than 0.25 l, and no bar should extend less than 0.20 l from this line.

d- The full area of negative reinforcement should be provided for a distance of not less than 0.20 l, measured from the line joining the centers of columns. The full area of positive reinforcement should be provided for a distance of not less than 0.25 l from the center line of the panel.

e- In flat slabs supported on columns without heads, or when the diameter of the head is less than twice the average width of the top of the column, two-thirds of the amount of reinforcement required to resist the negative moment in the column strip should be placed in a width equal to half that of column strip and central with the column and the rest to be divided on the remaining width of the column strip.

f- At all discontinuous edges, the positive and negative reinforcements should extend to maintain a proper anchorage.

9) Reinforcement of column head

Column heads are to be reinforced by reinforcements 1 and 2 shown in Fig. 5-20; they must be sufficient to resist the bending moment from the worst position and the bending moment mentioned in articles 4 and 5. The minimum rein-

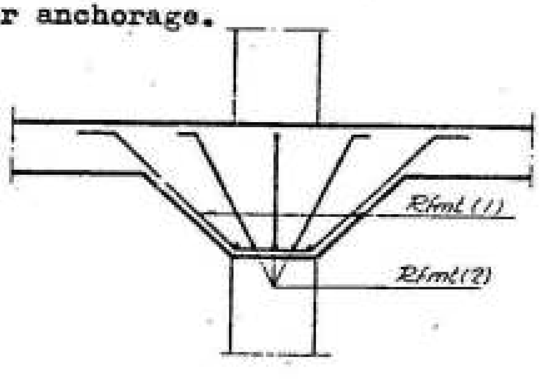


Fig. 5-20

forcement is given by:

a- For rectangular column head: 1/25 of area of negative reinforcement / m' of column strip in direction considered x length of perpendicular panel to this reinforcement.

b- For circular column head: the sum of reinforcement 1 and 2 obtained as above are to be distributed along the perimeter of the column head.

Illustrative example

The method of design of normal flat slabs is illustrated in the following example which contains the design of the roof and one of the floors of a building 20 x 28.5 ms. The height of each floor is 4 ms. The live load to be carried by each of the floors is 1100 kg/m^2. An additional load for roof cover of 200 kg/m^2 and for each of the floors of 100 kg/m^2 is to be considered in the design. The materials used are concrete C200 and normal mild steel with f_y = 2300 kg/cm^2. Fig. 5-21.

The columns are arranged every 7.00 ms in one direction and 6.50 ms in the other direction. The spans in both directions being approximately equal, the empirical method given in the Egyptian code shall be used.

a- Design of the second floor

Chosen dimensions

The spans being relatively big ($l_{max.}$ = 7.00 ms) and the live loads relatively heavy (p = 1100 kg/m^2), the slab is provided with a drop - panel.

The chosen dimensions are as follows:

Slab thickness	t_s = 20 cms	$> l_{max}/36 = 7.00/36 = 19.40$ cms
Drop panel	250x250 cms	$0.33 - 0.40\ l_{max.} = 0.357\ l_{max}$
Thickness of drop..........	8 cms	$0.25 - 0.50\ t_s = 0.40\ t_s$
Column	50 x 50 cms	$> l_{max}/20 > h/15 > 30$ cms
Column head D	150x150 cms	$\sim 2/9\ l_{max} = 156$ cms

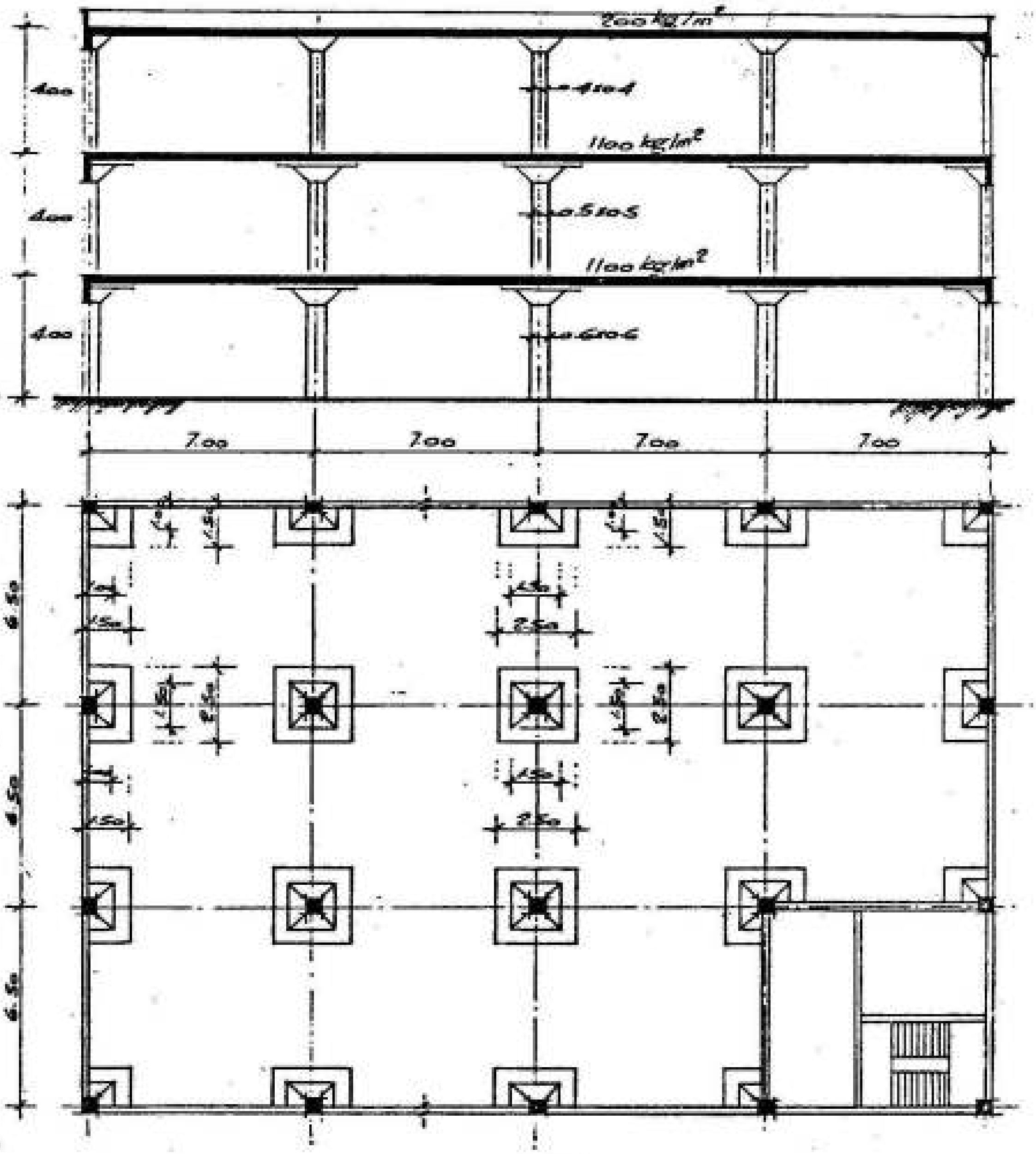


Fig. 5-21

Marginal beam .... 25 x 80 cms   t > 3 t_s = 3 x 20 = 60 cms

Loads

Floor cover ....100 kg/m²

Slab 20 cms thick 0.20 x 2500 .... 500 "

total dead load g = 600 "

live load p =1100 "

total w =1700 "

Bending moments

As the slab is provided with a marginal beam whose total depth is (30 cms) bigger than three times the slab thickness (3 x 20 = 60 cms), then case B table 5-7 governs the design.

Bending moments in longer direction: Span l_1 = 7.0 ms, breadth l_2 = 6.5 ms

According to equation 5-7, we have:

$$M_o = \frac{w\, l_2}{10}\left(l_1 - \frac{2\,D}{3}\right)^2 = \frac{1.7 \times 6.5}{10}\left(7.0 - \frac{2 \times 1.5}{3}\right)^2 \simeq 40.00 \text{ mt}$$

This bending moment shall be distributed between the column and field strips according to table 5-7 in the following manner:

Strip	Exterior panel		Interior panel	
	- ve M m t	+ ve M m t	- ve M m t	+ ve M m t
Column	.35x40 = 14.0	.25x40 = 10.0	.50x40 = 20.0	.20x40 = 8.0
Field	.20x40 = 8.0	.20x40 = 8.0	.15x40 = 6.0	.15x40 = 6.0
Total	22.0	18.0	26.0	14.0

Assuming that the breadth of the column strip is equal to the breadth of the drop panel = 2.5 ms only and not $l_2/2$ = 3.25 ms, then the breadth of the field strip shall be increased to 6.5 - 2.5 = 4 ms and the bending moment resisted by it shall be increased correspondingly i.e. by the ratio 4/3.25 = 1.23. The remaining bending moment shall be resisted by the column strip and distributed over its breadth of 2.5 ms in the following manner:

Strip	Exterior panl		Interior panel	
	- ve M m t	+ ve M m t	- ve M m t	+ ve M m t
Total M	22.0	18.0	26.0	14.0
Field 4m	1.23x8.0=9.84	1.23x8.0=9.84	1.23x6.0=7.38	1.23x6.0=7.38
" / m	9.84/4.0=2.46	9.84/4.0=2.46	7.38/4.0=1.85	7.38/4.0=1.85
Col.2.5m	22-9.84=12.16	18-9.84 =8.16	26-7.38=18.62	14-7.38 =6.62
" / m	12.16/2.50 = 4.864	8.16/2.50 = 3.264	18.62/2.50 = 7.448	6.64/2.50 = 2.658

Dimensioning

The depth of the flat slab outside the drop panels is to be determined for the maximum field moment of the exterior panel of the column strip i.e. max. M+ = 3.264 mt/m. The depth being chosen = 20 cms, hence $d = t - \text{cover} = 20 - 2.5 = 17.5$ cms and

$d = k_1\sqrt{\frac{M}{b}}$ or $17.5 = k_1\sqrt{\frac{3264}{1}}$ giving $k_1 = 0.306$

For normal mild steel $\sigma_s = 1400$ kg/cm² table 4-5 gives $\sigma_c = 62$ kg/cm² $<$ 70 kg/cm² allowed and $k_2 = 1214$

Therefore $A_s = \frac{M}{k_2\, d} = \frac{3264}{1214 \times 0.175} = 15.4$ cm² chosen 8 Ø 16/m

For the maximum negative moment at interior columns M = 7.448 mt, $t = 28$ cms , $d = 25.5$ cms and

$25.5 = k_1\sqrt{\frac{7448}{1}}$ giving $k_1 = 0.295$ i.e. $\sigma_c = 66$ kg/cm² and $k_2 = 1206$, so that $A_s = \frac{7448}{1206 \times 0.225} = 24.2$ cm² chosen 12 Ø 16/m

The reinforcements required for the other sections are to be determined in the same way; the results are given in the following table:

Strip	Position	t	d	M/m	k_1	σ_c	k_2	A_s	Rfmt
		cm	cm	kg m		kg/cm²		cm²	
Field	Ext. -ve	20	17.50	2460	.353	52	1233	11.4	3½Ø16+3½Ø13
	" +ve	20	17.50	2460	.353	52	1233	11.4	"
	Int. -ve	20	17.50	1850	.407	43	1253	8.4	7 Ø 13
	" +ve	20	17.50	1850	.407	43	1253	8.4	"
Column	Ext. -ve	28	25.50	4864	.366	49	1238	15.4	8 Ø 16
	" +ve	20	17.50	3264	.306	62	1214	15.4	"
	Int. -ve	28	25.50	7448	.295	66	1206	24.2	12 Ø 16
	" +ve	20	17.50	2656	.340	54	1229	12.3	4Ø13+4Ø16

Punching and shear stresses

The punching shear stress ($\tau_p = Q / b\, d$) on a vertical section a distance d/2 from the edge of the column head, and concentric with it, should not exceed the allowed value of 8 kg/cm^2 (table 3.2). Hence

b = 4 (column head + slab thickness) = 4 (1.50 + 0.255) = 4 x 1.755 or

$$b = 7.02 \text{ ms} \quad \text{and} \quad d = 0.255 \text{ ms}$$

The total load on the panel is given by:

$1.7 \times 7.0 \times 6.5 + 0.08 \times 2.5 \times 2.5^2 = 77.35 + 1.25 = 78.6$ tons

The punching shear at the section described above is:

$Q = 78.6 - (1.7 + 0.08 \times 2.5)\ 1.755^2 = 78.60 - 1.9 \times 1.755^2 = 72.7$ tons

hence $$\tau_p = \frac{72700}{702 \times 25.5} = 4.07 \ kg/cm^2$$ low

The punching shear stress on a vertical section a distance d/2 from the edge of the drop panel and parallel with it should not exceed 8 kg/cm^2.

$b = 4\ (2.50 + 0.175) = 4 \times 2.675 = 10.70$ ms and $d = 0.175$ ms.

$Q = 78.6 - 1.9 \times 2.675^2 = 65.2$ tons

hence

$$\tau_p = \frac{65200}{1070 \times 17.55} = 3.48 \ kg/cm^2$$ low

In checking the unit shearing stress in the slab acting as a wide beam, the presence of the drop panel over a portion of the width may be neglected.

The load per meter of slab in the 7 m span direction is:

$$1.7 \times 6.5 = 10.05 \ t/m'$$ and

The column reaction is

$$10.05 \times \frac{7}{2} = 35.18 \ \text{tons}$$

At a distance d/2 = 0.225/2 from the column head (= 0.75 + 0.1125 = 0.8625 ms from column center line) the shear Q is:

$Q = 35.18 - 10.05 \times 0.8625 = 26.51$ tons

hence

$$\tau = \frac{Q}{0.87\ b\ d} = \frac{26510}{0.87 \times 650 \times 17.5} = 2.68 \ kg/cm^2$$ low.

<u>Design of the marginal beam 7 ms span</u>

This beam has to carry its own weight + the wall load + 1/4 load of panel. i.e.

Loads

Own weight (25 x 80 cms)	0.25 x 0.6 x 2500	= 375	kg/m'
Wall load(3.2 ms high)	0.80 x 3.2 x 500	= 1280	"
Slab dead load (1/4 total)	$\frac{1}{4}$(7 x 6.5 x 600)/7	= 975	"
	total	g = 2630	"
Live load (1/4 total)	$\frac{1}{4}$ (7 x 6.5 x 1100)/7		
	or	p = 1800	"
	total	w = 4430	"

The displaced redistributed bending moment diagram, the moment of resistance diagram and the shearing force diagram as well as the details of reinforcements are shown in Fig. 5-22.

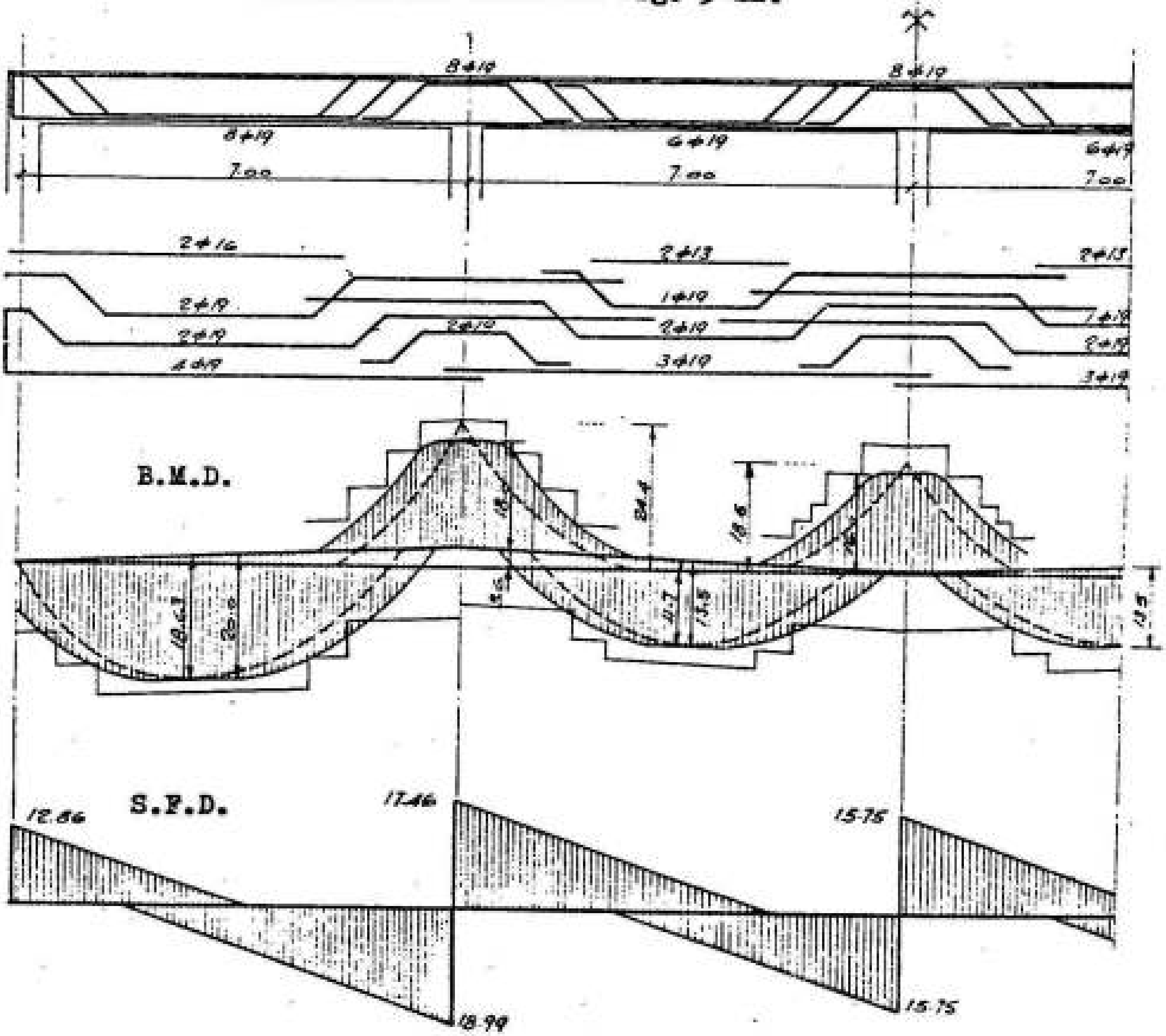


Fig. 5-22

Notes:

a- In this case, the reinforcements per meter of the column strip parallel to the given marginal beam may be reduced to half the amount of the intermediate column strips.

b- In case no marginal beam is arranged (e.g. at expansion joints of flat slabs) or when the marginal beam is made to resist its own weight and the wall load only, then the reinforcements of the column strip per meter parallel to it is to be chosen the same as those of the interior column strips.

Bending moments in shorter direction: Span l_2 = 6.5 ms, breadth l_1 = 7 ms.

$$M_o = \frac{w\ l_1}{10}\left(l_2 - \frac{2}{3}D\right)^2 = \frac{1.7 \times 7.0}{10}\left(6.5 - \frac{2 \times 1.5}{3}\right)^2 \cong 36.00 \text{ mt}$$

Distribution of M_o between column and field strips

Strip	Exterior panel		Interior panel	
	- ve M mt	+ ve M mt	- ve M mt	+ ve M mt
Column	..35x36 = 12.6	.25x36 = 9.0	.50x36 = 18.0	.20x36 = 7.2
Field	.20x36 = 7.2	.20x36 = 7.2	.15x36 = 5.4	.15x36 = 5.4
Total	19.8	16.2	23.4	12.6

The breadth of the column strip is 2.5 ms only (smaller than l/2= 3.5 ms). The breadth of the field strip = 7.0 - 2.5 = 4.5 ms and the bending moment resisted by it is increased by the ratio 4.5/3.5 = 1.27. The remaining bending moment shall be resisted by the column strip in the following manner:

Strip	Exterior panel		Interior panel	
	- ve M mt	+ ve M mt	- ve M mt	+ ve M mt
Total M	19.8	16.2	23.4	12.6
Fld/4.5m	1.27x7.2=9.14	1.27x7.2=9.14	1.27x5.4=6.86	1.27x5.4=6.86
"/ m	9.14/4.5=2.04	9.14/4.5=2.04	6.86/4.5=1.57	6.86/4.5=1.57
Col/2.5m	19.8-9.14=10.66	16.2-9.14=7.06	23.4-6.86=16.54	12.6-6.86=5.74
" / m	10.66/2.5= 4.26	7.06/2.5=2.82	16.54/2.5= 6.62	5.74/2.5=2.30

The reinforcements required in the different positions are shown in the following table

Strip	Position	t	d	M/m	k_1	σ_c	k_2	A_s	Rfmt
		cm	cm	kg m		kg/cm^2		cm^2	
Field	Ext. -ve	20	15.9	2040	.352	52	1233	10.4	3Ø16+3Ø13
	" +ve	20	15.9	2040	.352	52	1233	10.4	"
	Int. -ve	20	15.9	1570	.401	44	1250	7.9	6 Ø 13
	" +ve	20	15.9	1570	.401	44	1250	7.9	"
Column	Ext. -ve	28	23.9	4260	.366	49	1238	14.4	3Ø19+3Ø16
	" +ve	20	15.9	2820	.299	64	1210	14.7	"
	Int. -ve	28	23.9	6620	.294	66	1206	23.0	6Ø19+3Ø16
	" +ve	20	15.9	2300	.332	56	1225	11.8	6 Ø 16

The shear stresses in the slab in this direction are also low and need not be checked.

The marginal beam (6.5 ms span) can be calculated in the same way explained before.

The details of reinforcements in plan and sections are shown in Fig. 5-23.

b- <u>Design of roof slab</u>

<u>Chosen dimensions</u>

Slab thickness ....t_s = 20 cms $>$ $l_{max}/36$ = 19.4 cms

Drop panel .... no drop panel live loads are small

Column .... 40 x 40 cms $>$ $l_{max}/20 > h/15 > 30$ cms

Column head .... 150x150 cms ~ 2/9 l_{max} = 156 cms

Marginal beam 25x70 + 12x50 cms $t > 3\ t_s = 3 \times 20 = 60$ cms

<u>Loads</u>

Roof cover .... 200 kg/m^2

Slab 20 cms thick .... 500 "

D.L. g = 700 "

L.L. p = 200 " Total w = 900 kg/m^2

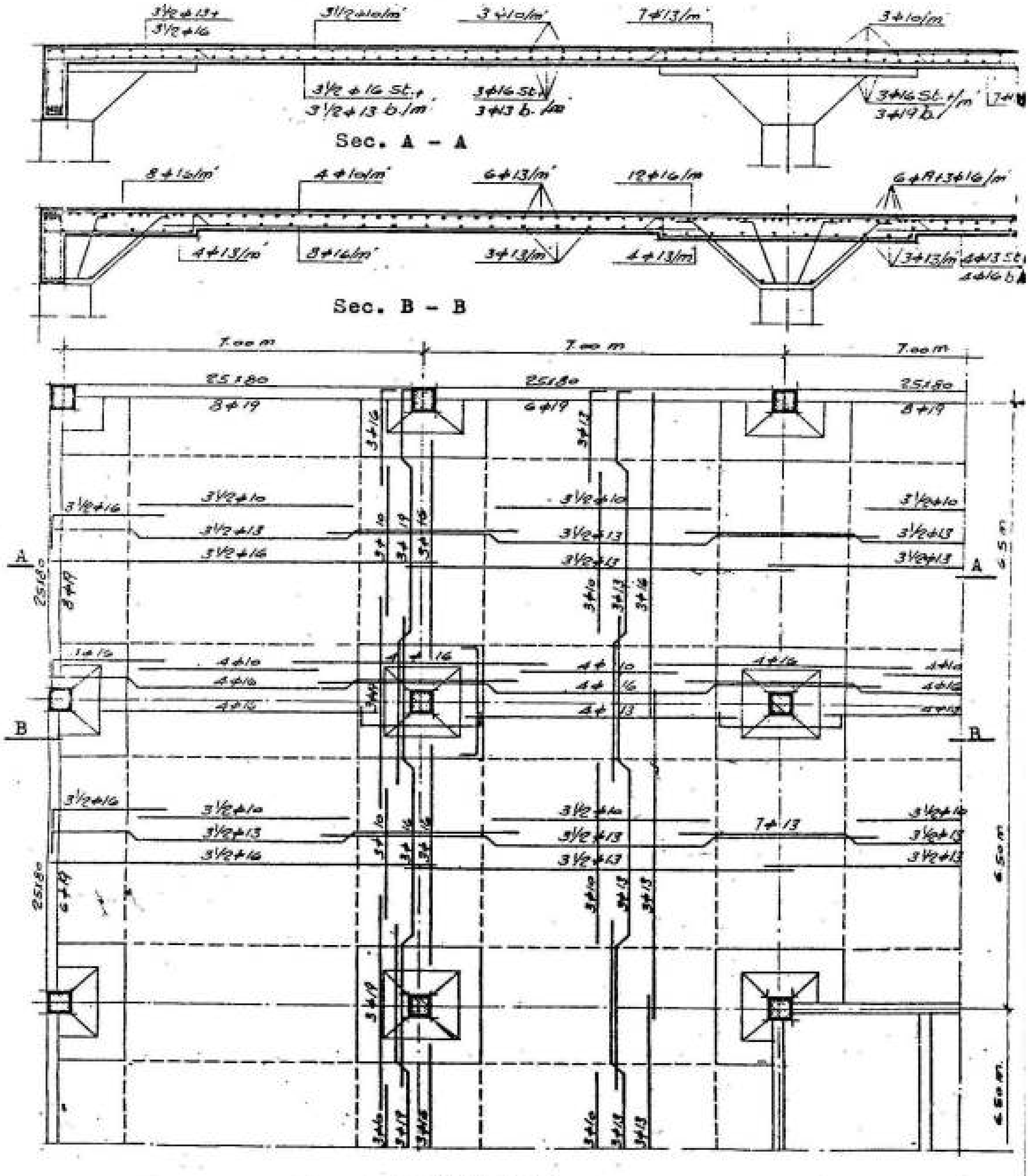


Fig. 5-23

Bending moments: According to table 5-7 case B.

Longer direction: Span l_1 = 7.0 ms and breadth l_2 = 6.5 ms.

According to equation 5-4, we have:

$$M_o = \frac{w\ l_2}{10}\ (l_1 - \frac{2\ D}{3})^2 = \frac{0.9 \times 6.5}{10}\ (7.0 - \frac{2 \times 1.5}{3})^2 = 21.06 \text{ mt}$$

This bending moment shall be distributed between the column and field strips (3.25 ms each) according to table 5-7 in the following manner:

Strip	Exterior panel		Interior panel	
	- ve M m t	+ ve M m t	- ve M m t	+ ve M m t
Col/3.25	.30x21.06=6.32	.30x21.06=6.32	.45x21.06=9.48	.25x21.06=5.27
" / m	6.32/3.25=1.95	6.32/3.25=1.95	9.48/3.25=2.92	5.27/3.25=1.62
Fld/3.25	.20x21.06=4.21	.20x21.06=4.21	.15x21.06=2.26	.15x21.06=2.26
" / m	4.21/3.25=1.29	4.21/3.25=1.29	2.26/3.25=0.69	2.26/3.25=0.69

Dimensioning and reinforcements

t = 20 cms d = 17.5 cms $k_1 = d / \sqrt{M}$ σ_c & k_2 from table 4-5.

Strip	Position		t	d	M/m	k_1	σ_c	k_2	A_s	Rfmt
			cm	cm	kg m		kg/cm²		cm²	
Column	Ext.	∓	20	17.5	1950	.396	45	1248	8.9	7 Ø 13
	Int.	-	"	"	2920	.324	58	1221	13.7	10½ Ø 13
	"	+	"	"	1620	.435	40	1260	7.4	3½Ø10+3½Ø13
Field	Ext.	∓	"	"	1290	.487	34	1275	5.8	7 Ø 10
	Int.	∓	"	"	690	.666	25	1302	3.03	7 Ø 10

Punching and shear stresses

The punching shear stress τ_p on a vertical section a distance d/2 from the edge of the column head and concentric with it can be calculated as follows:

b = 4 (1.5 + 0.175) = 4 x 1.675 = 6.7 ms and d = 17.5 ms

The total load on the panel is: 0.9 x 7.0 x 6.5 = 41.0 tons

Therefore $$\tau_p = \frac{41000}{670 \times 17.5} = 3.5 \text{ kg/cm}^2$$

The load being low, it is evident that the unit shear stress in the slab acting as a wide beam, at a distance d/2 from the column head shall also be low and needs no check.

<u>Marginal beam</u>: Span 7.0 ms , section 25 x 70 + 12 x 50 cms. Fig. 5-24

Load = Own weight + 1/4 load of panel i.e.

$$w = 2500\ (0.25 \times 0.6 + 0.12 \times 0.5) +$$
$$1/4\ (7.0 \times 6.5 \times 0.9)\ /\ 7.0 \qquad \text{or}$$
$$w = 525 + 1465 = 1990 \quad kg/m$$

The depth being relatively big and the load being relatively low, straight main reinforcement of 3 Ø 16 top and bottom shall be ample. Fig. 5-24.

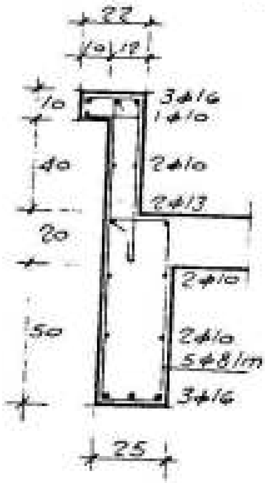


Fig. 5-24

<u>Design of shorter direction</u>

The design of the shorter direction can be done in the same way but with a span of 6.5 ms and a breadth of 7.0 ms. In the details shown in Fig. 5-25, the reinforcements of the different strips are shown as follows;

Strip	Position		Rfmt.
Column	Ext.	∓	6 Ø 13
	Int.	-	9 Ø 13
	"	+	3Ø10+3Ø13
Field	Ext.	∓	6 Ø 10
	Int.	∓	"

<u>Design of the columns</u>

When a flat slab is designed by the empirical method, the columns are designed for the following internal forces:

<u>The interior columns</u> are to be calculated either for the maximum dead and live loads N_1 plus a minimum bending moment about any of the two principal axes of the column equal to $N_1.t/10$, where t = the depth of the column: or for the maximum dead load and live loads on one side of the column under considerati. N_2 + a bending moment equal to

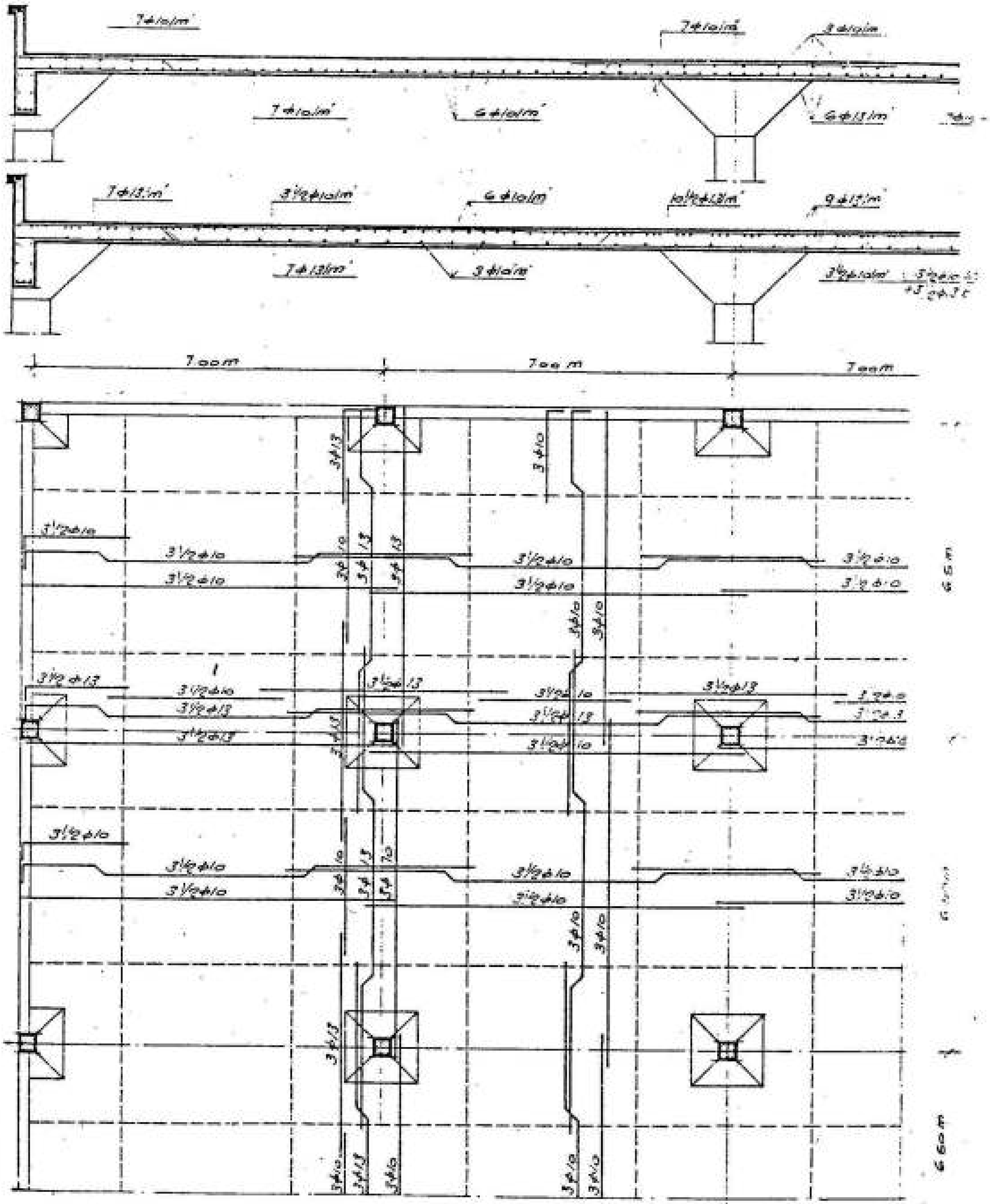

7φ10/m'
7φ10/m'
6φ10/m'
6φ13/m'
7φ13/m'
6φ10/m'
9φ13/m'
7φ13/m'
3φ10/m'
7.00m
7.00 m
7.00m
3φ13
3φ10
3½φ10
3½φ13
3φ10
3φ13

Fig. 5-25

50% of the negative moment of the column strip + a <u>minimum</u> extra bending moment equal to $N_2.t/10$.

<u>The exterior columns</u> are to be calculated for the maximum dead and live loads N + a bending moment equal to 90% of the negative moment of the column strip at the exterior column + an extra <u>minimum</u> bending moment equal to N.t/10.

In computing moments in columns due to gravity loading, the far ends of columns which are monolithic with the structure may be considered fixed. Further, the moment of inertia of the columns may be assumed infinity from the bottom of the column head to the top of the slab and the bending moment at the head is to be distributed on the columns above and below the joint under consideration relative to their stiffness.

Accordingly, the internal forces acting on the interior columns of the given example can be calculated as follows:

<u>Top column 40 x 40 cms</u>

a- Case of maximum normal force,

Max. D.L + L.L of roof................ $N_1 = 7.0 \times 6.5 \times 0.9 = 41.0$ t

Min. extra B.M. .... $M_1 = N_1 \frac{t}{10} = 41 \times \frac{.4}{10} = 1.6$ mt

b- Case of maximum bending moment,

D.L + 1/2 L.L .... $N_2 = (7.0 \times 6.5)(0.6 + \frac{0.2}{2}) = 36.3$ t

50% of -ve M of col. strip... $M_1' = 0.50 \times 9.48 = 4.74$ mt

Min extra B.M. .... $M_2' = 36.3 \times \frac{0.4}{10} = 1.45$ mt

Total M_2.............. 6.19 mt

<u>First floor column 50 x 50 cms</u>

a- Case of maximum normal force,

Max. D.L + L.L of second floor ...... $N_1 = 7.0 \times 6.5 \times 1.7 = 77.0$ t

Max. load from roof + own weight ...... $= 41.0 + 2.0 = 43.0$ t

Total N_1 .... 120.0 t

Min. extra B.M. .... $M_1 = N_1 \frac{t}{10} = 120 \times \frac{0.5}{10} = 6.0$ mt b- Case of maximum bending moment,

D.L. + 1/2 L.L. of 2nd floor $N_2 = (7.0 \times 6.5)(0.6 + \frac{1.1}{2})$ = 52.4 t

Load from roof + own weight = 36.3 + 2.0 = 38.3 t

Total N_2 .... 90.7 t

50% of -ve M of col. strip ..... = 0.5 x 18.62 = 9.31 mt

Min extra B.M. .... = $90.7 \times \frac{0.5}{10}$ = 4.54

Total M_2 .... 13.85 mt

<u>Ground floor column 60 x 60 cms</u>

a- Case of maximum normal force,

Max. D.L + L.L of first floor ...... N_1 = 7.0 x 6.5 x 1.7 = 77.0 t

Max. load from 2nd floor + own weight = 120.0 + 3.0 =123.0 t

Total N_1 .... 200.0 t

Min extra B.M. .... $M_1 = 200.0 \times \frac{0.6}{10}$ = 12.0 mt

b- Case of maximum bending moment,

D.L + 1/2 L.L of 1st floor $N_2 = (7.0 \times 6.5)(0.6 + \frac{1.1}{2})$ = 52.4 t

Load from 2nd floor + own weight = 90.7 + 3.0 = 93.7 t

Total N_2 .... 146.1 t

50% of -ve M of col. strip ..... = 0.50 x 18.62= 9.31 mt

Min extra B.M. .... = $146.1 \times \frac{0.6}{10}$ = 8.77 mt

Total M_2............ 18.08 mt

The bending moments of case a shall be assumed positive or negative and constant all over the column of any of the floors.

The bending moments on the columns of the different floors of case b are shown in Fig. 5-26 a, b, and c. The total bending moment diagram acting on any of the interior columns is given in d.

The internal forces in an exterior column can be treated in a similar way, noticing that the maximum bending moments and normal forces, normal to the outside face of the column under consideration, take place at the same time. For an outside intermediate column, the two cases (a and b) explained for interior columns have to be investigated; but this last case is generally not governing.

It is however advisable to reinforce each of the columns, interior or exterior, symmetrically, so that unsymmetrical loading can be resisted.

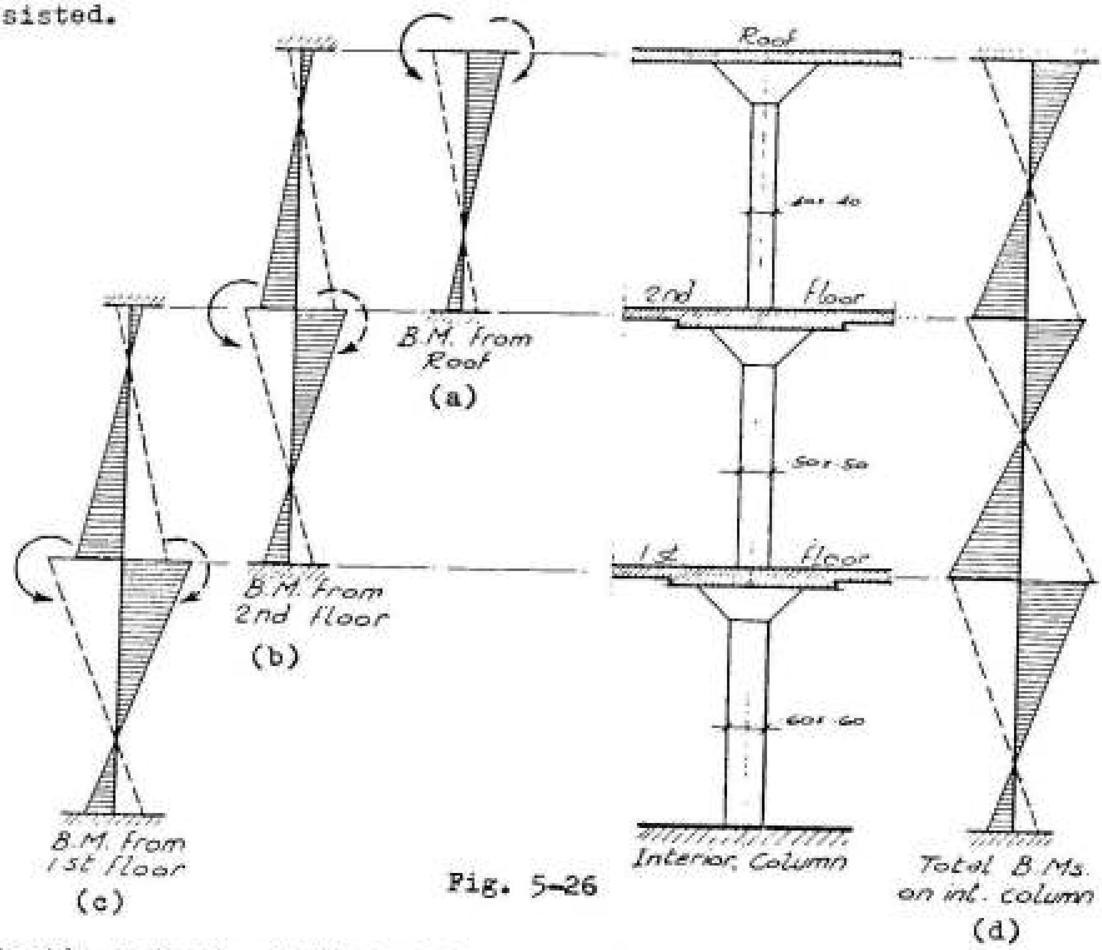


Fig. 5-26

Elastic analysis of flat slabs as continuous frames

The design of flat slabs according to the empirical method is allowed only if the basic conditions specified in the code are satisfied; in other cases, an elastic frame analysis is essential. The chief complication of such an analysis is the determination of the relative stiffness of the slab and the columns.

Slabs and columns are usually variable section members and the solution is simplified if we assume that the value of 1/EI for slabs is taken as zero for the width of the column head. Likewise, for columns 1/EI is considered zero from the bottom of the column head to the top of the floor above.

Ribbed and hollow-block flat slabs

Recently, it has been possible to construct ribbed and hollow-block flat slabs as that shown in Fig. 5-27. In such slabs, the existanceof a solid part with or without drop panel at the column heads is essential. The 8.0 x 8.0 ms panels of the floor shown in figure are ribbed, 28 cms thick, distance between center lines of ribs is 50 cms and covered by an 8 cms compression slab.

It can be noted that the middle part of the column strip is also ribbed due to the fact that the negative moments there are low.

Basically, the design of such slabs is the same as that of solid flat slabs with proper consideration of the principles of ribbed and hollow-block slabs explained in article 5-4.

5-6 Flat Plates

Flat plates are reinforced concrete solid, ribbed or hollow slabs supported directly on columns without the help of beams or girders. In this system, it is not necessary to arrange the columns in rows as in flat slabs, but they may be irregular as shown in part b of the building given in Fig. 5-28 which shows the general layout of a twenty storey building in Cairo. The building is 45 x 45 ms and is composed of two main parts. One part @ 23 x 21 ms is used for offices and the second part @ 45 x 24 ms is residential.

In the section of offices, the columns are arranged in rows at l_1 = 7.2 ms in one direction and l_2 = 4.8 ms in the other direction. The live load of the offices (not including the weight of the partitions) is 300 kg/m^2. The position of the interior partitions of this section is not fixed. For convenience of design, it was decided to make this part as a flat slab supported directly on columns without capitals. As the conditions for the application of the empirical method of flat slabs are not satisfied ($l_1/l_2 > 1.33$), it was calculated as a continuous frame of variable moment of inertia.

In order to have a consistent design, the residential section of the

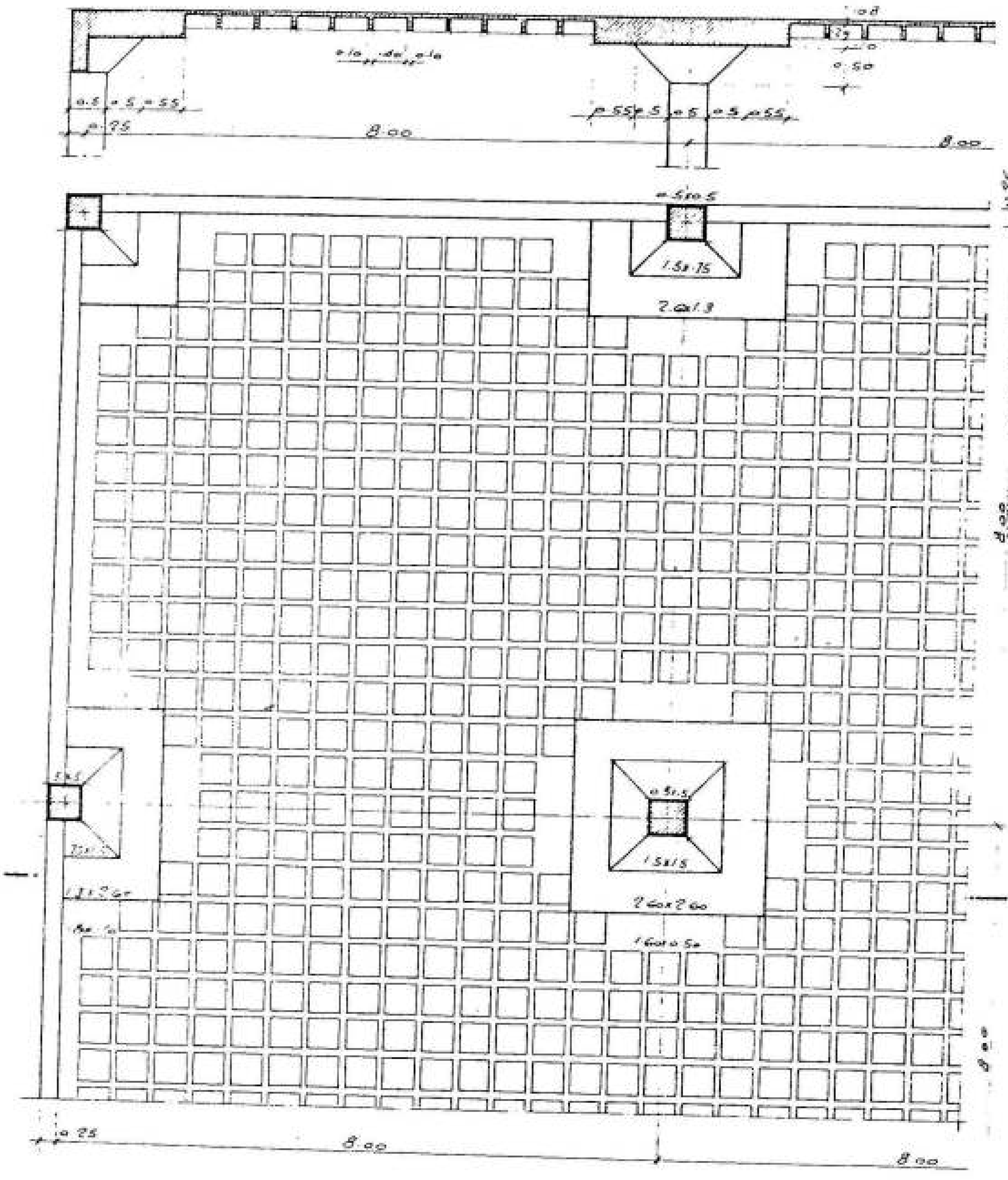


Fig. 5-27

building, Fig. 5-28 b, was also made as a flat plate with stiffening beams around open spaces and supporting beams,which have to give the required architectural effect, at the facade. Both the supporting columns and the walls in this section of the building are irregular. Such slabs are analized by "the yield line theory of flat plates"*. Their thickness is generally 16 to 30 cms and reinforced by a mesh 6 to 7 Ø 13 to 16 mm/m in the lower surface, the splices lie on the lines connecting the columns. The upper surface, except in the central part of the panels, is reinforced by a similar mesh spliced mid-way between columns; moreover, the top reinforcement must be increased over the columns, for a distance about 1/4 panels from each side, to resist the high bending moments and shearing forces there.

In the shown building, the wind pressure is resisted by the floor slabs as rigid diaphragms transmitting the wind load to the reinforced concrete stair-case and outside walls.

Yield-line analysis has been verified by tests; the calculated load normally underestimates the actual load results and the engineers may use the yield-line methods with confidence; it gives a conservative estimate of strength in moment and the slabs designed by this method will be entirely satisfactory at working loads.

Flat-plate floors have been found economical and otherwise advantageous for such uses as apartment and office buildings where the spans are moderate, and the loads relatively light. In this system , the construction depth for each floor is held to the absolute minimum, with resultant savings in the overall height of the building.

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* Refer for example to:

1- "Reinforced Concrete Fundamentals" by Ferguson. Published by: John Wiley and Son.

2- "Design of Concrete Structures" by Winter. Urquhart. O'Rourke. Nilson. Published by: Mc Graw-Hill Company.

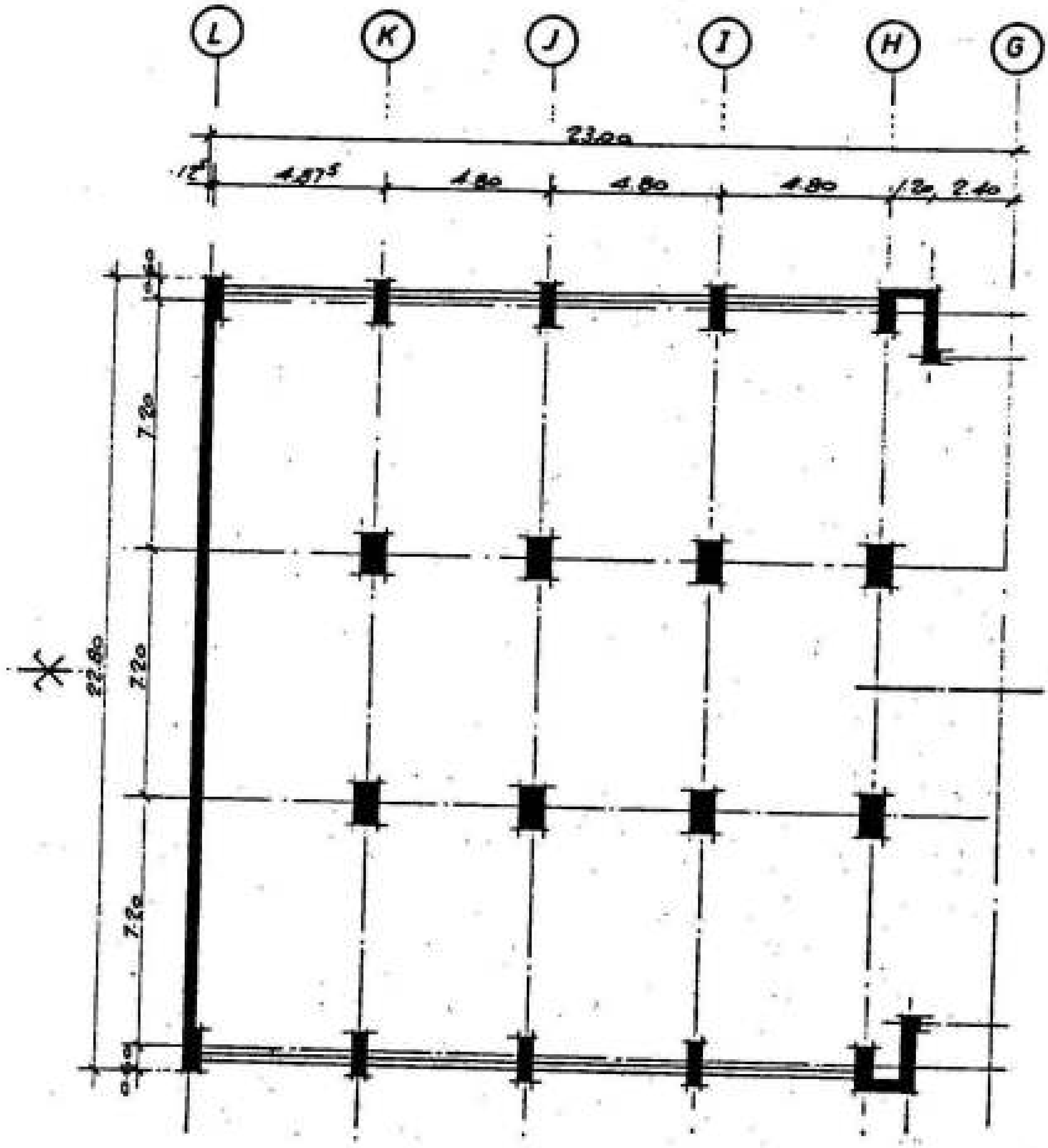


Fig. 5-28 (a)

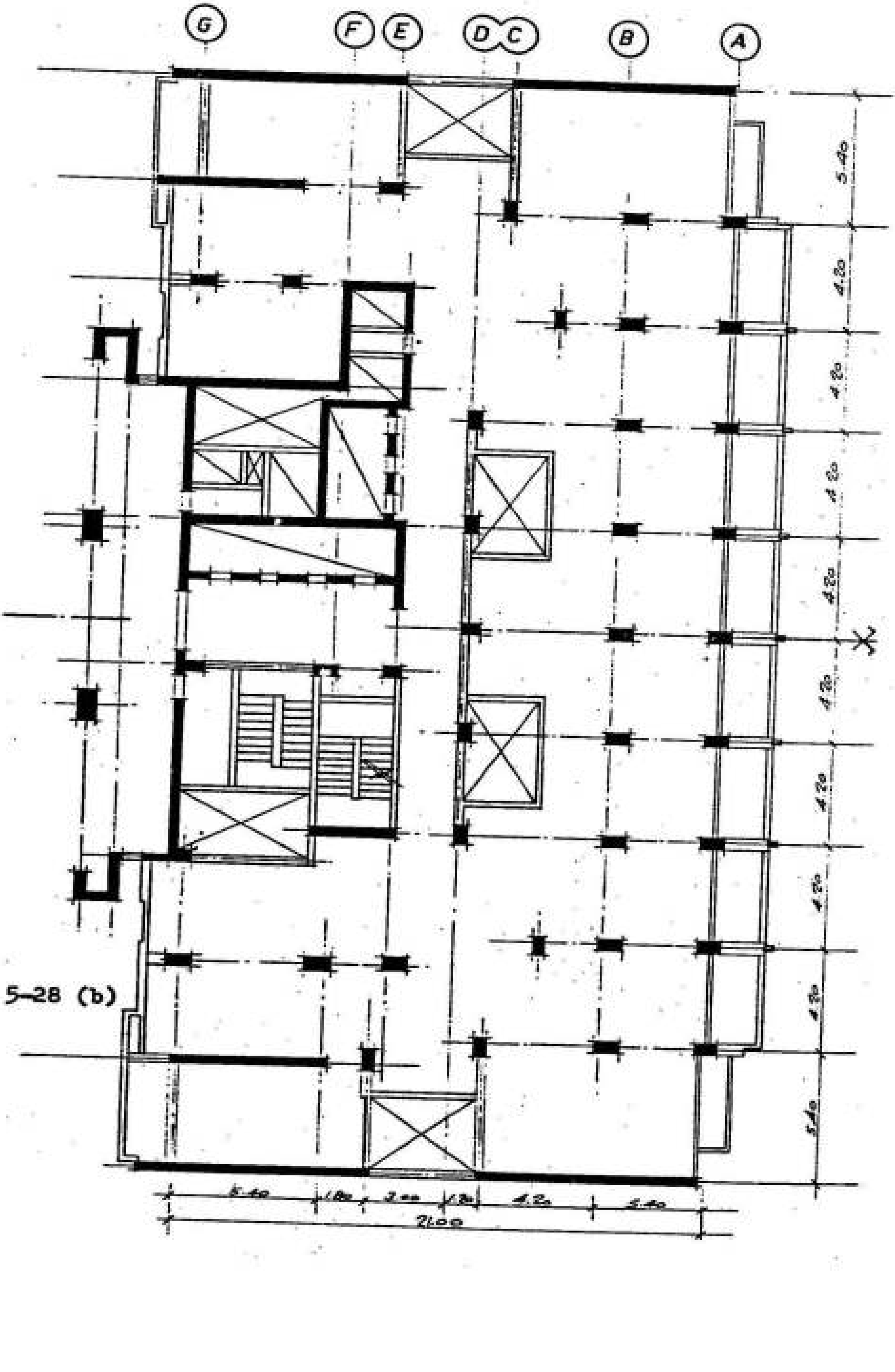


Fig. 5-28 (b)

CHAPTER 6

REINFORCED CONCRETE COLUMNS

AND

ELEMENTS SUBJECT TO ECCENTRIC FORCES

6.1 General Considerations

Columns are the main supporting elements of structures, and hence, they must be carefully designed. They are mainly subject to normal compressive forces with or without eccentricity. However, as has been previously explained in 2.3, no columns are likely to be purely axially loaded even if design calculations show a compression member to be free from any bending moments. Imperfections of construction, such as slight beam eccentricities and deviation from straightness or verticality, will cause unintentional bending moments. For this reason, it is recommended to design compression members for an adequate minimum eccentricity e equal to 1/10 of depth (or breadth) of cross section of member. Therefore, a column is an element that is generally subject to eccentric compression.

If the normal stresses in the section of a column are all compressive or if the tensile stresses are smaller than the tensile strength of the concrete, "cases of small eccentricity", one can assume that the full concrete section is acting (refer to 2.7 a).

In cases of "big eccentricity", giving high tensile stresses, such that the concrete in tension is cracked, then the concrete in compression only shall be assumed as acting and the concrete in tension is neglected (refer to 2.7 b).

In case of elements subject to eccentric tension, if the normal force lies inside the section, then the compressive stresses are either zero or small and may be neglected, this case is called "eccentric tension with small eccentricity". If the normal tensile force is outside the section, then we have a case of "eccentric tension with big eccentricity".

In the following, the dimensioning of sections subject to eccentric forces with small and big eccentricity, both by the W.S.D. and the U.S.D.-methods will be given.

6.2 Dimensioning by the W.S.D.-method

In cases of eccentric compression, the allowable compressive stresses in concrete have to be increased gradually from σ_{co} of axial compression to σ_c of simple bending according to e/t or e_s/d, where e, e_s t and d have the same definitions as given before, in the following manner. Table 6-1.

Table 6-1. Allowable comp. stresses in sections subject to eccentric forces

Case of ecc. loading		All. stresses σ_c in kg/cm^2 for concrete						
		C160	C180	C200	C225	C250	C275	C300
Axial comp.	$\frac{e}{t} = 0$	45	50	55	60	65	70	75
Small ecc.	$0 < \frac{e}{t} < .33$	50	55	60	65	70	75	80
Med. ecc.	$.8 < \frac{e_s}{d} < 1.5$	55	60	65	70	75	80	85
Big ecc.	$\frac{e_s}{d} > 1.5$	60	65	70	75	80	85	90

1) Eccentric compression with small eccentricity

In this case, the whole concrete section is assumed as statically acting and the normal stresses can be computed by the equations of Navier, 2-99. Fig. 2-39.

Application to rectangular sections

A rectangular section can be computed according to equations 2-99 if the normal force N lies inside the middle two-thirds of the section in which case: e=t/3. Such sections are generally designed with symmetrical reinforcement and with the maximum stress σ_c in the section equal to the allowable value. Table 6-1.

The following methods will be confined to the case of symmetrical reinforcements only ($A_s = A_s'$). Fig. 6-1. Sheet 15.

Virtual area .... $A_v = b\ t + 2\ n\ A_s$

Virtual moment of inertia .... $I_v = \frac{b\ t^3}{12} + 2\ n\ A_s\ a^2$

Assuming $A_s = \mu\ b\ t$ $a = 0.447\ t$ i.e. $a^2 = 0.2\ t^2$ & $n = 15$

Then $A_v = b\ t\ (1 + 30\mu)$ & $I_v = b\ t^3\ (\frac{1}{12} + 6\mu)$

therefore

$$\sigma_c = \frac{N}{b\ t\ (1 + 30\mu)} + \frac{N\ e \times 0.5\ t}{b\ t^3\ (\frac{1}{12} + 6\mu)}$$

or

$$\sigma_c = \frac{N}{b\ t\ (1 + 30\mu)} + \frac{N}{b\ t\ (1 + \frac{1}{6} + 12\mu)} \cdot \frac{e}{t} \quad \ldots\ldots 6\text{-}1$$

<u>Case 1</u> Determination of steel reinforcement for given dimensions and concrete quality. i.e.

Given b, t, σ_c, N and e. Required μ and the corresponding A_s such that $A_s = A_s'$.

Assume: $\frac{N}{\sigma_c\ b\ t} = k$ and $\frac{e}{t} = \zeta$ then

$$1 = \frac{k}{1 + 30\mu} + \frac{k\zeta}{\frac{1}{6} + 12\mu} \qquad 6\text{-}2$$

In sheet 15, the values of μ are given as factors of $k = \frac{N}{\sigma_c\ b\ t}$ and $\zeta = \frac{e}{t}$.

<u>Example</u>

Given: b = 30 cms, t = 60 cms, N = 100 tons, e = 15 cms, concrete C250 and normal mild steel with $f_y = 2300\ kg/cm^2$.

Required: A_s.

$e/t = 15/60 = 0.25 < 0.33$ a case of small eccentricity.

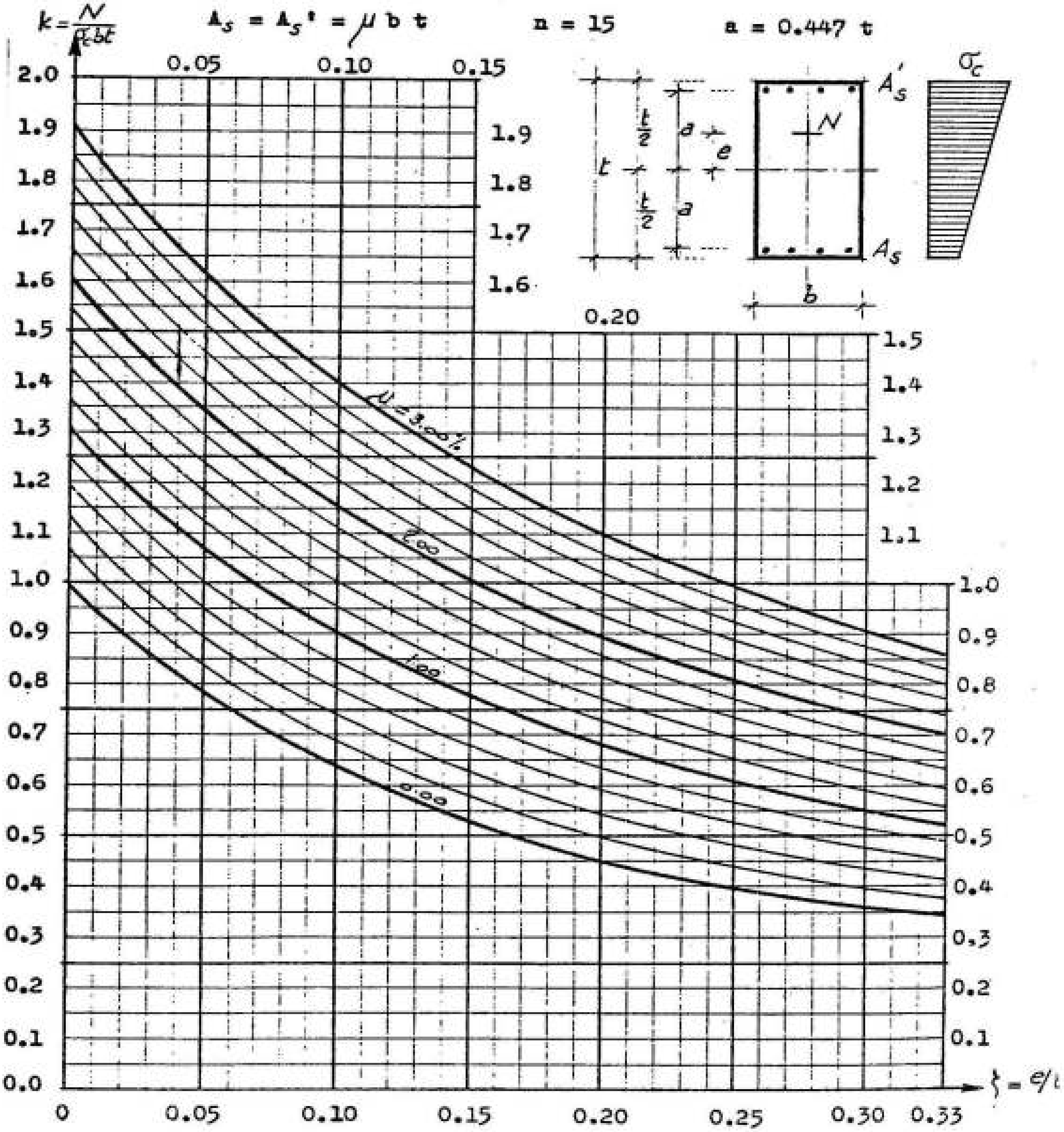
According to table 6-1, the allow. concrete comp. stress = 70 kg/cm².

Therefore $k = \frac{N}{\sigma_c\ b\ t} = \frac{100\ 000}{70 \times 30 \times 60} = 0.795$

SHEET 15

Case 1: ECCENTRIC FORCES WITH SMALL ECCENTRICITY (W.S.D-METHOD)

Dimensioning of Symmetrically Reinforced Rectangular Sections



For $\zeta = e/t = 0.25$ and $k = 0.795$ sheet 15 gives $\mu = 1.75\%$ so that $A_s = A_s' = 30 \times 60 \times 1.75 / 100 = 31.50\ cm^2$ on each side.

Case 2 Determination of dimensions for a given ratio of steel and concrete quality. i.e.

Given: b, ratio of steel reinforcement μ such that $A_s = A_s'$, σ_c, N and e. Required t.

The breadth b is to be assumed. The depth t is to be determined in the following manner:

Equation 1-6 gives:

$$\sigma_c = \frac{N}{b\ t} \cdot \frac{1}{1 + 30\mu} + \frac{N\ e}{b\ t^2} \cdot \frac{1}{\frac{1}{6} + 12\mu}$$

Assuming $\frac{N}{\sigma_c\ b} = \chi$, $\frac{1}{1 + 30\mu} = \psi$ and $\frac{1}{\frac{1}{6} + 12\mu} = \delta$, then

$$t = \frac{\chi\psi}{2} + \sqrt{\frac{\chi^2\psi^2}{4} + \chi e \delta} \quad \text{............ 6-3}$$

The values of ψ and δ are given in table 6-2 and the curves shown in sheet 16.

Example

If, in the previous example, the maximum ratio of steel in the cross section must not exceed 2.0% (i.e. $\mu = 1.0\%$), determine the required depth t.

$$\chi = \frac{N}{\sigma_c\ b} = \frac{100\ 000}{70 \times 30} = 47.6$$

Hence, for $\mu = 1.0\%$, $\psi = 0.769$ and $\delta = 3.485$ so that

$$t = \frac{47.6 \times 0.769}{2} + \sqrt{\frac{47.6^2 \times 0.769^2}{4} + 47.6 \times 15 \times 3.485} = 72.4 \text{ cms}$$

depth chosen = 75 cms

and $A_s = A_s' = 30 \times 72.4 \times 1.0 / 100 = 21.7\ cm^2$ 6 Ø 22

2) Eccentric tension with small eccentricity

Due to a normal tensile force N acting between the reinforcements, the magnitude of A_s and A_s' is inversely proportional to their distance from the line of action of N. Fig. 6-2.

SHEET 16

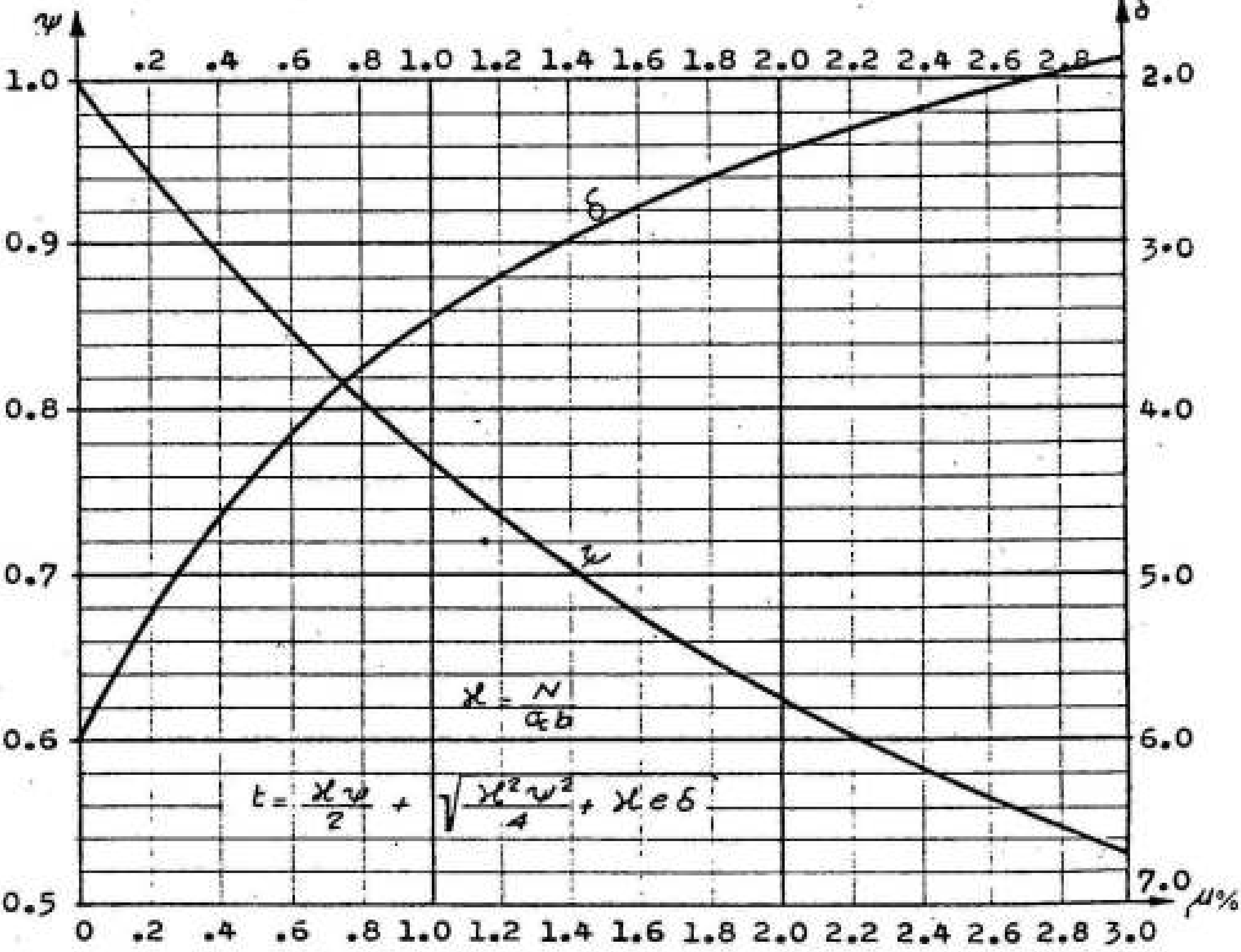
Case 2: ECCENTRIC FORCES WITH SMALL ECCENTRICITY (W.S.D. - METHOD)

Dimensioning of Symmetrically Reinforced Rectangular Sections

Table 6-2: Values of ψ and δ for different μ percentages

μ%	0.4	0.6	0.8	1.0	1.2	1.4	1.6
ψ	0.893	0.847	0.806	0.769	0.735	0.704	0.676
δ	4.658	4.189	3.807	3.485	3.215	2.988	2.788
μ%	1.8	2.0	2.2	2.4	2.6	2.8	3.0
ψ	0.649	0.625	0.602	0.581	0.562	0.544	0.526
δ	2.613	2.459	2.322	2.199	2.089	1.989	1.937

$A_s = A_s' = \mu\, b\, t$ $\qquad$ $n = 15$ $\qquad$ $a = 0.447\, t$



Taking moments about the upper reinforcements:

$T\ d = N\ (e + d/2)$ $\qquad$ $T = A_s\ \sigma_s$ $\qquad$ then

$$A_s = \frac{N}{\sigma_s}\ (1/2 + e/d) \qquad 6\text{-}4$$

Taking moments about the lower reinforcements:

$$A_s' = \frac{N}{\sigma_s}\ (1/2 - e/d) \qquad 6\text{-}5$$

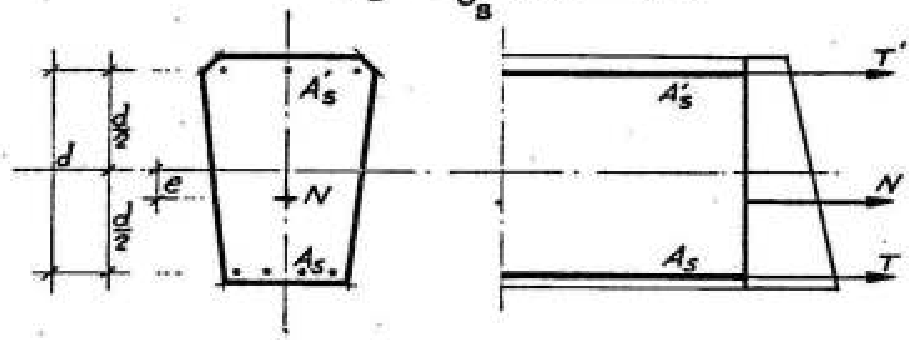


Fig. 6-2

3) Eccentric compression with big eccentricity

a) Rectangular sections with tension reinforcements only ($A_s' = 0$).

Fig. 6-3

Given: M, N, b, σ_c and σ_s. Required d and A_s.

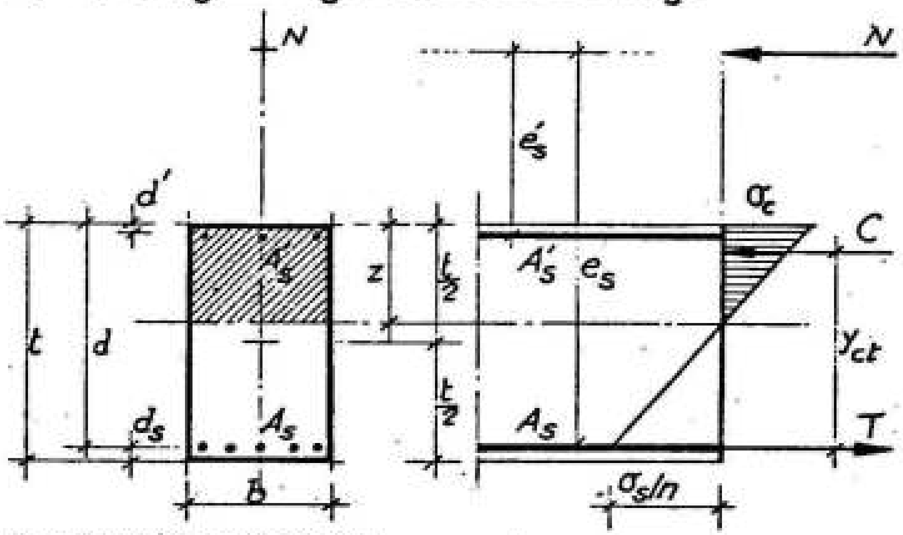


Fig. 6-3

It has been found that:

$$\frac{n\ \sigma_c}{\sigma_s} = \frac{z}{d - z} \qquad \text{or} \qquad z = \frac{n\ \sigma_c}{\sigma_s + n\ \sigma_c}\ d$$

and $\qquad y_{ct} = d - z/3$

Assuming $z = \xi\ d$ $\qquad$ then $\qquad \xi = \frac{z}{d} = \frac{n\ \sigma_c}{\sigma_s + n\ \sigma_c}$

Taking moments about the tension steel, we get:

$$M_s = N\,e_s = C\,y_{ct} = \sigma_c \frac{b\,z}{2}(d - z/3) \qquad \text{in which}$$

$$e_s = e + t/2 - d_s \quad \text{and} \quad e = M/N \qquad \text{so that}$$

$$\underline{d = k_1 \sqrt{\frac{M_s}{b}}} \quad \text{.......... (6-6 a)} \quad \text{in which} \quad k_1 = \sqrt{\frac{2}{\xi(1-\frac{\xi}{3})\,\sigma_c}}$$

or

$$\underline{d = c \sqrt{\frac{M_s}{\sigma_c\,b}}} \quad \text{......... (6-6 b)} \quad \text{in which} \quad c = \sqrt{\frac{2}{\xi\,(1-\xi/3)}}$$

Taking moments about the center of compression, we get:

$$T\,y_{ct} = N\,(e_s - y_{ct}) = M_s - N\,y_{ct}, \quad \text{but } T = A_s\,\sigma_s \quad \text{and } y_{ct} = d - \frac{z}{3}$$

so that

$$A_s\,\sigma_s = \frac{M_s}{d - z/3} - N = \frac{M_s}{d\,(1 - \xi/3)} - N \qquad \text{or}$$

$$\underline{A_s = \frac{M_s}{k_2\,d} - \frac{N}{\sigma_s}} \quad \text{........ (6-7 a)} \quad \text{in which} \quad k_2 = \sigma_s\,(1 - \xi/3)$$

The area of steel can also be determined as follows:
If the section were subject to a bending moment equal to M_s only, then the area of tension steel A_{so} that would have been required is:

$$A_{so} = \mu_o\,b\,d = \frac{M_s}{k_2\,d}$$

Therefore, the area of tension steel required for an eccentric compression N can be given in the form:

$$\underline{A_s = \mu_o\,b\,d - \frac{N}{\sigma_s}} \quad \text{.................. (6-7 b)}$$

k_1 & k_2 and c & μ_o have the same values given in case of simple bending. For k_1 and k_2 refer to table 4-5 and sheet 11; for c and μ_o refer to sheet 12.

It has to be noted that $M_s = N\,e_s > M = N\,e$, hence the depth required for a section subject to a bending moment M and a normal compressive force N is bigger than that required for a section subject to a bending moment only, the increase depends on the ratio of N with respect to M. The assumed depth can however, after one trial, be corrected.

The area of tension steel is smaller than that which would have been required if the section were subject to M only.

Example

Given M = 15 mt, N = 10 t (compression), b = 30 cms. Concrete C200 and normal mild steel with min. f_y = 2300 kg /cm^2. Required d and A_s. According to table 3.2, σ_c = 70 kg/cm^2 and σ_s = 1400 kg/cm^2.
e = M in cm t / N in t = 1500 / 10 = 150 cms

Assume that we have a case of big eccentricity; if the section were subject to a bending moment M = 15 000 kg m only, then

$$d_o = c\sqrt{\frac{M}{\sigma_c\ b}}$$ in which

Table 4-4 gives, for σ_s/σ_c = 1400/70 = 20, c = 2.33. So that

$$d_o = 2.33\sqrt{\frac{15\ 000}{70 \times 0.30}} = 62.40 \text{ cms}$$

The depth d is somewhat bigger than d_o. Assume d = 70 cms and t = 73 cms. Hence, $e_s = e + t/2 - d_s = 150 + 73/2 - 3 = 183.5$ cms, and

$M_s = N\ e_s = 10\ 000 \times 1.835 = 18350$ kg m so that

$$70 = c\sqrt{\frac{18350}{70 \times 0.3}}$$ giving c = 2.37

according to table 4-4, we get for c = 2.37, the values:

$r = \sigma_s/\sigma_c = 21.25$ and $\mu_e = 0.992\%$ hence

σ_c= 1400/21.25 = 66 kg/cm^2 and the area of tension steel is

A_s= 30 x 70 x 0.992 / 100 - 10 / 1.4 = 13.70 cm^2 chosen 5 Ø 19

As e_s/d = 183.5 / 70 = 2.62 $\rangle$ 1.5, then the solution as a case of a section with big eccentricity is justified.

It is clear that the solution is similar to that of simple bending using M_s instead of M and the area of tension steel A_s is reduced by the amount N/σ_s.

b) <u>Rectangular sections with double reinforcements</u>. Fig. 6-3

1- <u>Use of curves</u>

It is easy to prove that in case of rectangular sections subject to eccentric forces with big eccentricity, the equations 6-6 and 6-7 are valid* so that the depth d and the reinforcements A_s & A_s' can be determined from the relations:

$$d = c\sqrt{\frac{M_s}{\sigma_c b}}, \quad A_s = \mu_o\, b\, d - N/\sigma_s \text{ and } A_s' = \alpha\, \mu_o\, b\, d \; .... (6-8)$$

where c and μ_o have the same values given in case of simple bending. (Sheet 12).

<u>Example</u>

If in the previous example, the maximum depth t is limited to 67 cms only, determine A_s and A_s'.

$d = t - \text{cover} = 67 - 3 = 64$ cms , $e = M/N = 1500/10 = 150$ cms

i.e. $e_s = 150 + \frac{t}{2} - 3 = 179$ cms , $M_s = 10\,000 \times 1.79 = 17900$ kgm

As the section is of limited depth, it must be designed fo the maximum allowed $\sigma_c = 70$ kg/cm^2 and $\sigma_s = 1400$ kg/cm^2. Hence,

$$d = 64 = c\sqrt{\frac{17900}{70 \times 0.30}} \qquad \text{giving} \qquad c = 2.11$$

For $r = \sigma_s/\sigma_c = 1400/70 = 20$ and $c = 2.11$, sheet 12 gives:

$\alpha = 0.22$ and $\mu_o = 1.25\%$ so that

$A_s = 30 \times 64 \times 1.25 / 100 - 10/1.4 = 24.0 - 7.1 = 16.9$ cm^2

chosen 6 Ø 19

$A_s' = 0.22 \times 24.0 = 5.28$ cm^2 chosen 3 Ø 16

2- <u>Direct method</u>

Having determined $z = \xi\, d = n\, d / r + n$, the reinforcements can be directly determined by taking moments of external and internal forces about the compression and tension reinforcements, thus

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* Refer to "Design of Reinforced Concrete Rectangular Sections" by M. Hilal. Published in 1944 by the Renaissance Bookshop. Cairo.

$$N\ e_s' \doteq M_s' = A_s\ \sigma_s\ (d - d') - \sigma_c\ \frac{z\ b}{2}\ (z/3 - d') \qquad \text{or}$$

$$A_s = \frac{M_s' + \sigma_c\ \frac{b\ z}{2}\ (\ z/3 - d'\ )}{\sigma_s\ (d - d')} \qquad \text{..............} \quad (6\text{-}9)$$

and

$$N\ e_s\ = M_s\ = A_s'\ \sigma_s'\ (d - d') + \sigma_c\ \frac{zb}{2}\ (d - z/3) \qquad \text{or}$$

$$A_s' = \frac{M_s - \sigma_c\ \tfrac{1}{2}\ b\ z\ (d - z/3)}{\sigma_s'\ (d - d')} \qquad \text{..............} \quad (6\text{-}10)$$

where

$$\sigma_s' = n\ \sigma_c\ \frac{z - d'}{z} \qquad \text{.................} \quad (6\text{-}11)$$

Minimum steel

In sections of limited depth subject to eccentric compression with medium eccentricity ($0.8 < e_s/d < 1.5$) the minimum area of steel ($A_s + A_s'$) does not always correspond to maximum σ_s but to a smaller stress. This can be explained as follows: Fig. 6-4.

In cases of medium eccentricity, the normal compression force N is relatively high and σ_s is generally low, Fig. 6-4 a. If the section is designed for the max. allowed values of σ_c and σ_s, Fig. 6-4 b, the height of the compression zone is assumed smaller than what it should be ($z_2 < z_1$).

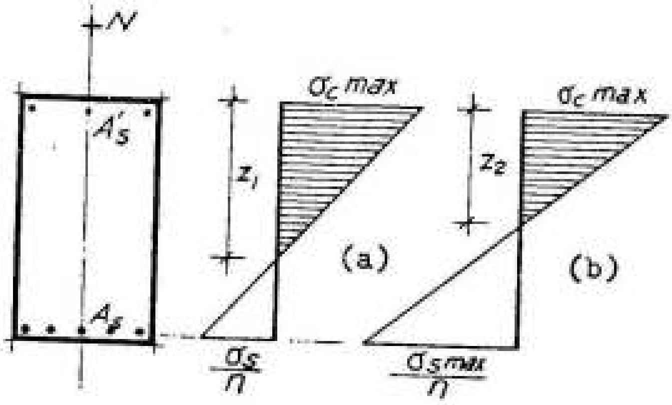

Fig. 6-4

The result is that one may need for the compression reinforcement A_s' an amount bigger than that required for the tension steel A_s; in some other cases, one may get high values for A_s' and negative values for A_s. Both cases give a weak design because A_s must be bigger than A' or equal to it.

The design for minimum ($A_s + A_s'$) gives generally a convenient design.

The stress σ_s giving minimum ($A_s + A_s'$) can be determined from the following relation:

$$\frac{d\,A_s}{d\,\sigma_s} + \frac{d\,A_s'}{d\,\sigma_s} = 0$$

The compressive stress σ_c should be assumed equal to the maximum llowable value, otherwise, no compression reinforcements would have een required.

As σ_s is a function of z, the condition for minimum reinforcements an also be given in the form:

$$\frac{d\,A_s}{dz} + \frac{d\,A_s'}{dz} = 0$$

Substituting σ_s in equation 6-9 by the value $n\,\sigma_c\,(d - z)/z$, we get:

$$A_s = \frac{M_s'\,z + \sigma_c\,½\,b\,z^2\,(z/3 - d')}{n\,\sigma_c\,(d - d')(d - z)} \qquad 6\text{-}9\text{ a}$$

Substituting σ_s' in equation 6-10 by the value $n\,\sigma_c\,(z - d')/z$, we get:

$$A_s' = \frac{M_s\,z - \sigma_c\,½\,b\,z^2\,(d - z/3)}{n\,\sigma_c\,(d - d')\,(z - d')} \qquad 6\text{-}10\text{ a}$$

Differentiating these two relations with respect to z and equating to zero, we get:

$$\frac{M_s'\,d}{(d - z)^2} - \frac{M_s\,d'}{(z - d')^2} + \sigma_c\,b\left[-\frac{z^3}{3} + \frac{z^2}{2}(d + d') - d\,d'\,z\right] x \left[\frac{1}{(d - z)^2} - \frac{1}{(z - d')^2}\right] = 0 \qquad 6\text{-}12$$

Assuming $z = \xi\,d$, $\sigma_s/\sigma_c = r$, $\sigma_s/n\sigma_c = \nu$, then $\xi = 1/(1 + \nu)$

and $d'/d = \beta = 0.08$, $e_s/d = \zeta$, then

$$M_s' = M_s\,e_s'/e_s = M_s'\left[\frac{e_s - (d - d')}{e_s}\right] = M_s\left(1 - \frac{1 - \beta}{\zeta}\right)$$

$$= M_s\,\frac{\zeta - 0.92}{\zeta}$$

Substituting these values in equation 6-12, we get:

$$\frac{M_s}{\sigma_c\,b\,d^2}\left[\frac{\zeta - 0.92}{\zeta} \cdot \frac{1}{\nu^2} - \frac{0.08}{(0.92 - 0.08\,\nu)^2}\right] = \left[\frac{0.24\,\nu^2 - 1.14\,\nu - 0.38}{3\,(1 + \nu)^3}\right]\left[\frac{1}{\nu^2} - \frac{1}{(0.92 - 0.08\,\nu)^2}\right]$$

but $d = c\sqrt{\dfrac{M_s}{\sigma_c\ b}}$ then $\dfrac{M_s}{\sigma_c\ b\ d^2} = \dfrac{1}{c^2}$ and

$$\frac{\zeta - 0.92}{\zeta} \cdot \frac{1}{\nu^2} - \frac{0.08}{(0.92 - 0.08\,\nu)^2} = c^2 \left[\frac{0.24\,\nu^2 - 1.14\,\nu - 0.38}{3\,(1+\nu)^3} \right] x \left[\frac{1}{\nu^2} - \frac{1}{(0.92 - 0.08\,\nu)^2} \right] \qquad 6\text{-}13$$

This equation shows that ν and respectively $\sigma_s/n\ \sigma_c = \nu$ is a function of $1/c^2 = M_s/\sigma_c\ b\ d^2$ and $\zeta = e_s/d$.

Therefore, for each value of c and $\zeta = e_s/d$, the value of $r = \sigma_s/\sigma_c$ corresponding to the minimum reinforcements can be determined. Joining the points having the same value of $\zeta = e_s/d$ together, we get the dotted series of curves shown in sheet 12.

The relation between σ_s and $A_s + A_s'$ is shown in Fig. 6-5. It is clear from the figure that although the minimum value of $A_s + A_s'$ corresponds to a certain value of σ_s, yet it does not change much for any other close value of σ_{s_0}.

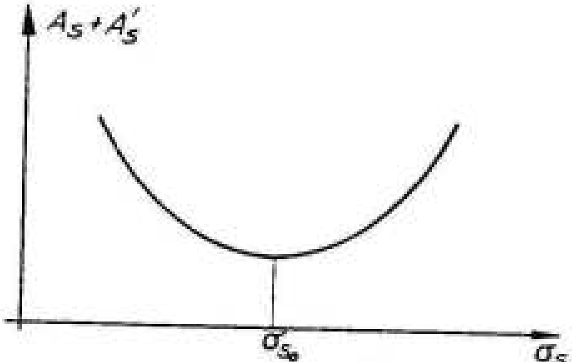

Fig. 6-5

The main change is in the distribution of $A_s + A_s'$ between the tension and compression sides of the section, that one may calculate $A_s + A_s'$ for any reasonable value of σ_s and distribute this total amount in a convenient manner, e.g. $A_s' = A_s$ or $A_s' = \frac{1}{2} A_s$ as can be seen from the following example.

<u>Example</u>

Given t=65 cms, d=61 cms, b= 30 cms, N=24 t, and M=12 mt.

The materials used are: C200 & normal mild steel with f_y=2300 kg/cm^2.

Required: A_s and A_s' so that : $A_s + A_s'$ is minimum. Fig. 6-6.

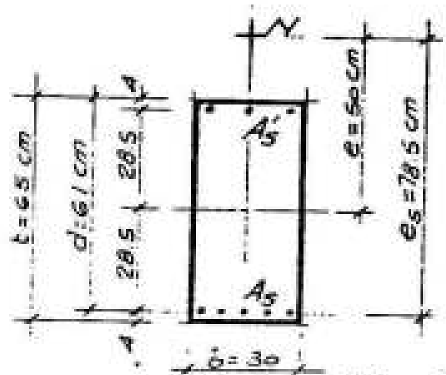

Fig. 6-6

$e = M/N = 1200/24 = 50$ cms , $e_s = 50 + 65/2 - 4 = 78.50$ cms

i.e. $e_s/d = 78.5/61 = 1.285$ and $M_s = 24\ 000 \times .785 = 18840$ kg m

so that $61 = c\sqrt{\frac{18840}{70 \times 0.3}}$ giving $c = 2.04$ and with $e_s/d = 1.285$

sheet 12 gives: $r = \sigma_s/\sigma_c = 16.5$ i.e. $\sigma_s = 70 \times 16.5 = 1155$ kg/cm^2

further $\alpha = 0.18$ and $\mu_o = 1.70$ % so that

$$A_s = \frac{1.7}{100} \times 30 \times 65 - \frac{24}{1.155} = 33.1 - 20.9 = 12.2 \text{ cm}^2$$

$$A_s' = 0.18 \times 33.1 = \underline{6.0} \text{ " } @ \tfrac{1}{2} A_s$$

$$A_s + A_s' = 18.2 \text{ "}$$

Assuming $\sigma_s = 1400$ kg/cm^2 and $\sigma_c = 70$ kg/cm^2 i.e. $r = 20$

For $c = 2.04$ and $r = 20$, sheet 12 gives $\alpha = 0.37$ and $\mu_o = 1.385$%

so that

$$A_s = \frac{1.385}{100} \times 30 \times 65 - \frac{24}{1.4} = 27.0 - 17.1 = 9.9 \text{ cm}^2$$

$$A_s' = 0.37 \times 27.0 = \underline{10.0} \text{ " } @ = A_s$$

$$A_s + A_s' = 19.9 \text{ "}$$

This example shows that the minimum amount of $A_s + A_s'$ equals 18.2 cms^2 and corresponds to $\sigma_s = 1155$ kg/cm^2. For a stress $\sigma_s = 1400$ kg/cm^2 the total amount of $A_s + A_s'$ equals 19.9 cm^2 which is only @ 5% more than the minimum value; but in the first solution A_s' was @ ½ A_s and in the second solution A_s' was approximately equal to A_s. The choice depends on the convenience of the design.

4) <u>Eccentric tension with big eccentricity</u>

Proceeding in the same way as in case of eccentric compression, we get for a rectangular section, the following relations: Fig. 6-7.

$e = M/N$, $e_s = e - \frac{t}{2} + \text{cover} < e$, $M_s = N\, e_s < M$

$$\underline{d = c\sqrt{\frac{M_s}{\sigma_c\, b}}}\quad d_o \;,\quad \underline{A_s = \mu_o\, b\, d + \frac{N}{\sigma_s}} \quad \text{and} \quad \underline{A_s' = \alpha\, \mu_o\, b\, d} \qquad 6\text{-}14$$

It has to be noted that in sections subject to eccentric tension

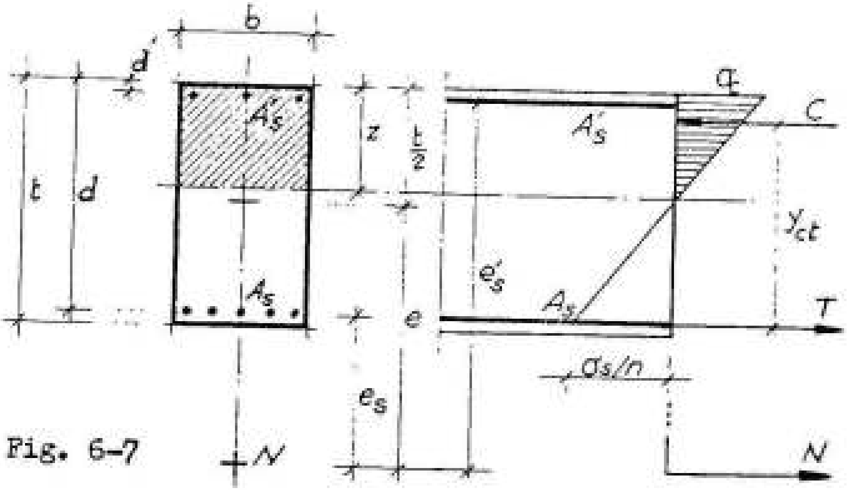

Fig. 6-7

$e_s < e$, $M_s < M$, $d < d_o$ and $A_s > A_{so}$ in which d_o and A_{so} are the depth and area of steel which would have been required if the section were subject to M only.

Example

Given M = 15 mt, N = 10 t (tension), b = 30 cms, $\sigma_c = 70$ kg/cm^2 and $\sigma_s = 1400$ kg/cm^2. Required t and A_s. (Assume $A_s' = 0$).

e = M in t cm/N in t = 1500/10 = 150 cms assumed big eccentricity.

$d_o = 2.33\sqrt{\frac{15000}{70 \times 0.30}} = 62.4$ cms. Assume d = 57 cms $< d_o$ and t = 60 cm

$e_s = e - t/2 +$ cover $= 150 - 60/2 + 3 = 125$ cms $< e$, so that

$M_s = N\ e_s = 10\ 000 \times 1.25 = 12500$ kg m $< M$ and

$57 = c\sqrt{\frac{12500}{70 \times 0.30}}$ giving c = 2.38 so that sheet 12 gives:

for $\alpha = 0$, $r = \frac{\sigma_s}{\sigma_c} = 20$ or $\sigma_c = \frac{1400}{20} = 70$ kg/cm^2 & $\mu_o = 1.07\%$

i.e. $A_s = \mu_o\ b\ d + \frac{N}{\sigma_s} = \frac{1.07}{100} \times 30 \times 57 + \frac{10}{1.4} = 18.3 + 7.1 = 25.4$ cm^2.

Assume that max. t = 55 cms, (d = 51.5 cms), then

$e_s = 150 - \frac{55}{2} + 3.5 = 126$ cms and $M_s = 10\ 000 \times 1.26 = 12600$ kg m

$51.5 = c\sqrt{\frac{12600}{70 \times 0.30}}$ giving c = 2.10 so that sheet 12 gives:

for $\frac{\sigma_s}{\sigma_c} = \frac{1400}{20} = 20$ and c = 2.1, $\alpha = 0.25$ and $\mu_o = 1.25\%$

i.e. $A_s = \frac{1.25}{100} \times 30 \times 51.5 + \frac{10}{1.4} = 19.3 + 7.1 = 26.40 \quad cm^2$

$$A_s' = \alpha \mu_o\, b\, d = 0.25 \times 19.3 = \underline{4.80} \quad "$$

$$\text{total} = 31.20 \quad "$$

The minimum area of steel $A_s + A_s'$ for sections of limited depth subject to eccentric tesion corresponds generally to the maximum allowable values of σ_c and σ_s.

6-3 Dimensioning by the U.S.D.-method*

Safety provisions

It was pointed out in Art. 3.2b that in ultimate strength design adequate safety margins are established by applying overload factors to the design loads and strength-reduction factors to the theoretical ultimate strengths computed for members assumed to be perfect in workmanship and made of materials having exactly the specified strength. It has also been shown that the strength-reduction factors are different for different stressed members.

For eccentrically compressed members, the reduction factors Ω were given by the following:

For tied members: $\Omega = 0.70$ and for spirally reinforced members: $\Omega = .75$. The difference between the two values reflects the added safety furnished by the greater toughness of spirally reinforced members.

There are various reasons why these coefficients are considerably lower than those of bending where $\Omega = 0.90$. For one, the strength of under-reinforced members subject to bending is not much affected by variations in concrete strength, since it depends primarily on the yield strength of the steel, while the strength of axially loaded members depends strongly on the concrete compression strength. Because the compression strength of concrete under site conditions is less closely controlled than the yield strength of mill-produced steel, a

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* 'Design of Concrete Structures' by Winter. Urquhart. O'Rourke. Nilson Published by McGraw-Hill Book Company.

larger occasional strength deficiency must be allowed for. This is particularly true for columns, in which the concrete being placed from the top down the long, narrow form, is more subject to segregation than in horizontally cast beams. Finally, the consequences of a column failure, say in a lower storey, would be more catastrophic than that of a single beam in a floor system of the same building.

The capacity-reduction factors for compressed members may be increased linearly from 0.7 in axially loaded members and members with small eccentricity to 0.8 in members with medium eccentricity and 0.9 in members with big eccentricity and simple bending.

Interaction diagram

The study of the failure of members subject to eccentric forces in general has shown the considerable complexity of behavior and of corresponding strength calculations. The situation is more easily visualized if the results of the corresponding calculations are graphically illustrated in the so-called 'interaction diagram' proposed by Whitney.

For a given cross section and reinforcement, an interaction diagram has the general shape shown in Fig. 6-8 plotted in terms of ultimate axial loads as ordinates and ultimate moments as abcissas. Any point on the curve, such as point a, represents a pair of values N_{au} and M_{au} which, according to the ultimate-strength theory, will just fail the member.

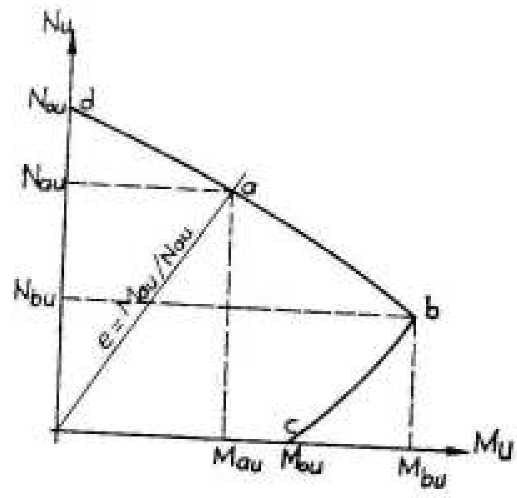


Fig. 6-8

For concentric compression ($M_u = 0$), the curve starts at d with the strength N_{ou} of a centrically loaded member. the portion d b belongs to that range of small eccentricities in which failure is initiated by crushing of the concrete. Point b represents the balanced conditions i.e., under the simultaneous action of the load N_{bu} and the corresponding moment M_{bu}, the concrete will reach its limiting strain (say 0.003) simultaneously with the tension steel reaching its yield stress f_y. The portion b c represents the range in which failure is initiated by yielding of the tension steel. Finally the end point c refers to the moment capacity M_{ou} in simple bending, i.e. when $N_u = 0$. Any inclined line through the origin 0 has a slope whose reciprocal represents the centroidal eccentricity corresponding to the particular combination of ultimate values N_u and M_u as shown in Fig. 6-8; that is $e = M_u/N_u$.

In order to simplify the design of eccentrically loaded members, the curves shown in sheets 17,18&19 are drawn. They are somewhat approximate in that they have been calculated on the assumption that the compression reinforcement has yielded when ultimate strength is reached; an assumption which is satisfied in most cases. In the given graphs however, instead of plots of N_u versus M_u, convenient nondimensional parameters have been selected ($N_u/\Omega b t f_{cp}$ versus $N_u e/\Omega b t^2 f_{cp}$) which, taken in conjunction with the radial lines for various e/t ratios and the interaction curves drawn for various values of μ_t m, for dimensioning or analysis of sections, where

$$\mu_t = \frac{A_s + A_s'}{b\ t} \quad \text{........ 6-15} \qquad \& \qquad m = \frac{f_y}{0.85\ f_{cp}} \quad \text{........ 6-16}$$

Examples

A column 30 x 65 cms is to be designed for N_g = 8 t, N_p = 16 t, M_g = 4 t and M_p = 8 mt. Assume a minimum guaranteed cube strength of concrete = 160 kg/cm², giving f_{cp} = 135 kg/cm², and normal mild steel with f_y = 2300 kg/cm².

$N_u = 1.5\ N_g + 1.8\ N_p = 1.5 \times 8 + 1.8 \times 16 = 12 + 28.8 = 40.8$ t

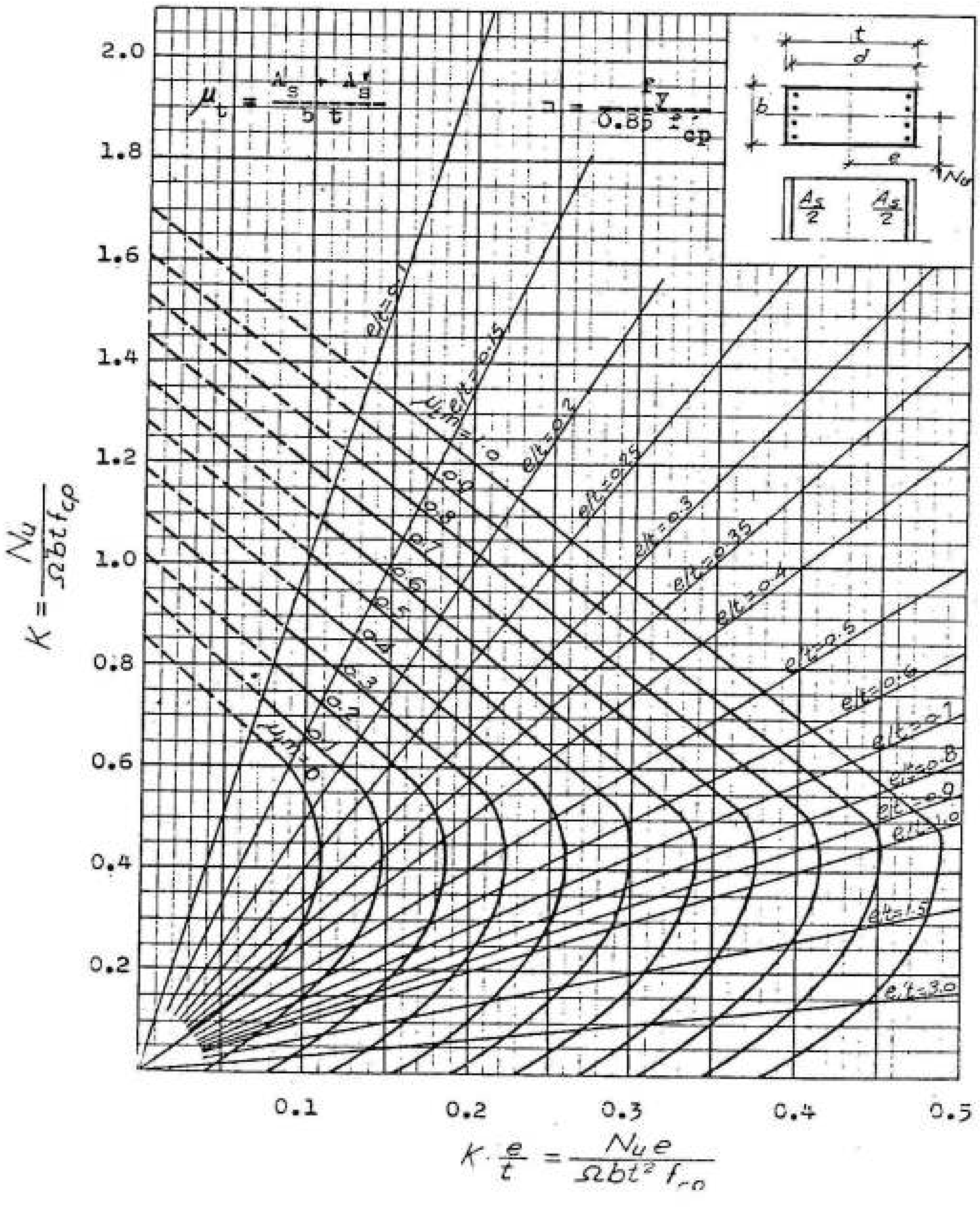
$M_u = 1.5\ M_g + 1.8\ M_p = 1.5 \times 4 + 1.8 \times 8 = 6 + 14.4 = 20.4$ m t

$e = M_u/N_u = 20.4/40.8 = 0.5$ ms , $e/t = 0.50/0.65 = 0.77$

$e_s = e + t/2 - \text{cover} = 50 + 65/2 - 4 = 78.5$ cms, $e_s/d = 78.5/61 =$ 1.29 which means that we have a case of medium eccentricity & $\Omega = 0.90$

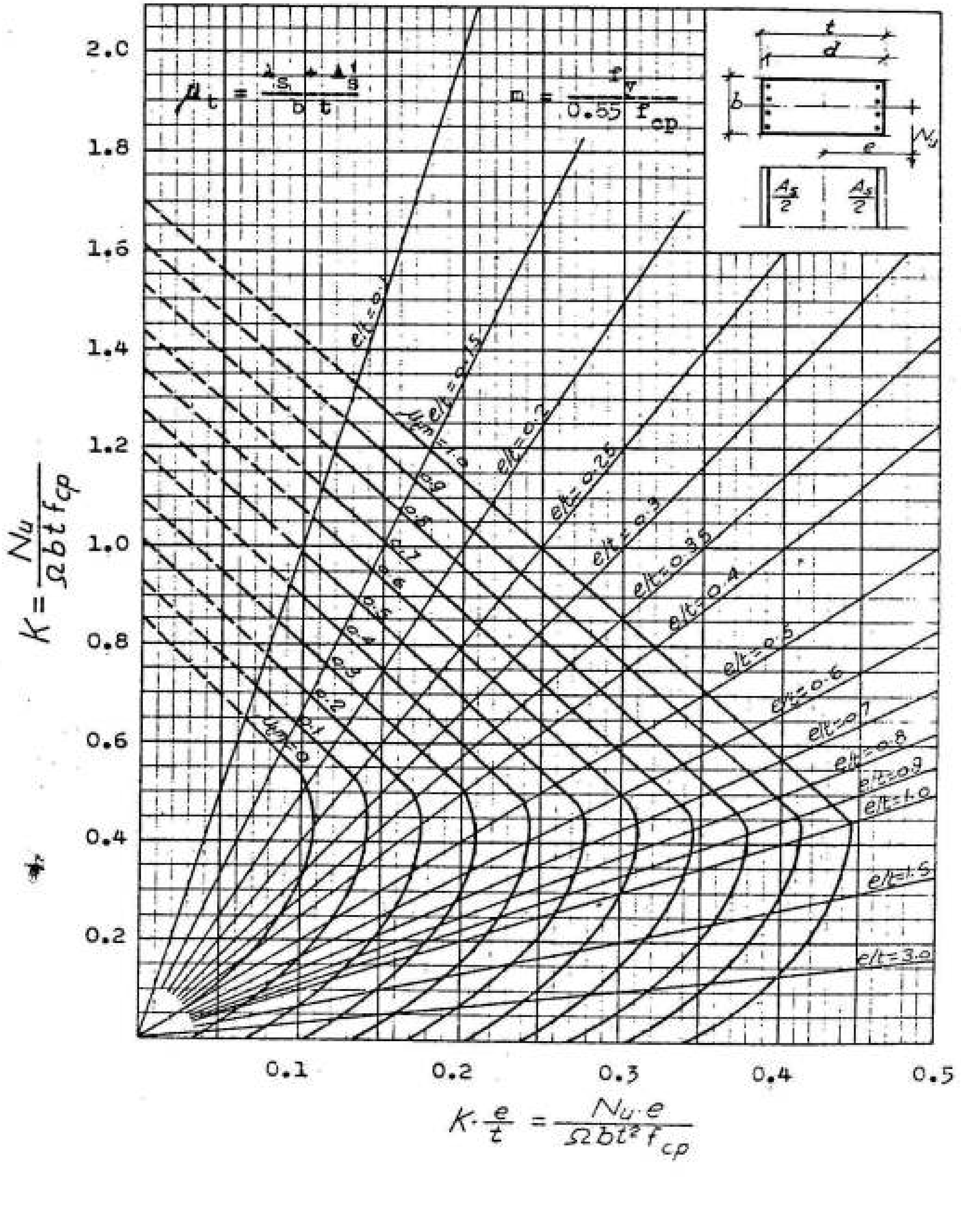
SHEET 17

INTERACTION DIAGRAM FOR DESIGN OF SYMMETRICALLY REINFORCED RECTANGULAR SECTIONS SUBJECT TO ECCENTRIC FORCES. (U.S.D-METHOD). For d/t = 0.95



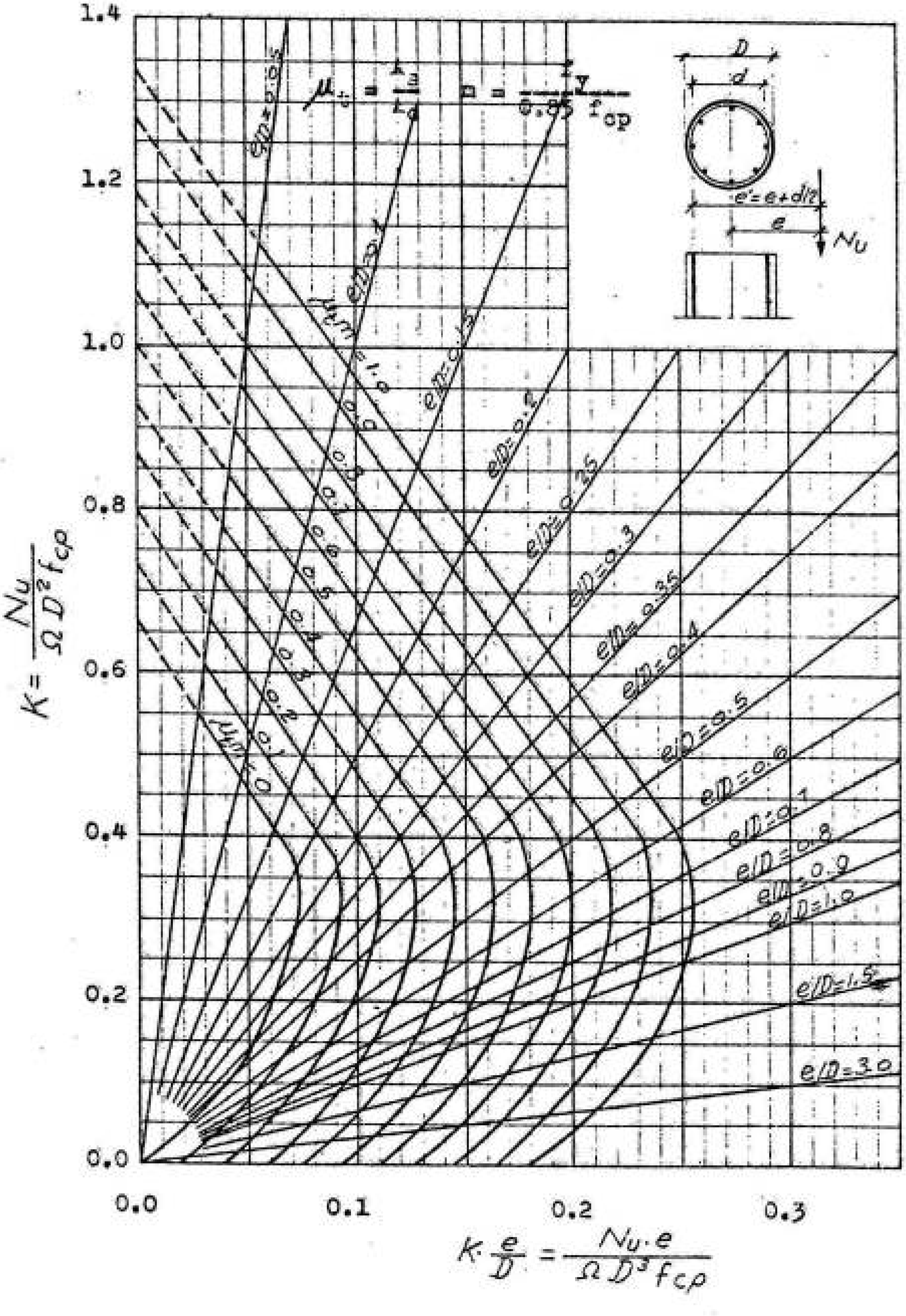
SHEET 18

INTERACTION DIAGRAM FOR DESIGN OF SYMMETRICALLY REINFORCED RECTANGULAR SECTIONS SUBJECT TO ECCENTRIC FORCES.(U.S.D-METHOD). For d/t = 0.90



SHEET 19

INTERACTION DIAGRAM FOR DESIGN OF SYMMETRICALLY REINFORCED CIRCUL SECTIONS SUBJECT TO ECCENTRIC FORCES. (U.S.D-METHOD). For d/D = 0.



$d/t = 61/65 = 0.94$ so that sheet 17 can be used.

$$K = \frac{N_u}{\Omega\ b\ t\ f_{cp}} = \frac{40800}{0.8 \times 30 \times 65 \times 135} = 0.194\ ,\quad K\frac{e}{t} = .194 \times .77 = 0.149$$

Sheet 17 gives $\mu_t\ m = 0.2$ where $m = \frac{f_y}{0.85\ f_{cp}} = \frac{2300}{0.85 \times 135} = 20$

Therefore $\mu_t \times 20 = 0.20$ i.e. $\mu_t = 0.01$ and

as equation 6-15gives: $A_s + A_s' =$ total $A_s = \mu_t\ b\ t$ then

$A_s + A_s' = 0.01 \times 30 \times 65 = 19.5\ cm^2$ and $A_s = A_s' = 19.5/2 = 9.75\ cm^2$.

It is however possible to reinforce the section such that $A_s' = ½\ A_s$.

Hence $A_s = \frac{2}{3} \times 19.5 = 13.0\ cm^2$

and $A_s' = \frac{1}{3} \times 19.5 = \underline{6.5}\ cm^2$

total $= 19.5\ cm^2$

For this new distribution of reinforcements, the ultimate load that can be carried by the section, may be checked by the method given at Fig. 2-46. Hence

Assuming that failure will be initiated by yielding of both tension and compression steel, then $\sigma_s = \sigma_s' = f_y = 2300\ kg/cm^2$, and

$$T = A_s\ f_y = 13.0 \times 2.3 = 29.90\ t$$

$$C_s = A_s'\ f_y = 6.5 \times 2.3 = 14.95\ t$$

$$C_c = 0.85\ f_{cp}\ y\ b = 0.85 \times 0.135 \times 30\ y = 3.44\ y$$

The resultant of the internal forces $(C_s + C_c - T)$ should be equal to N_u/Ω. Hence

$N_u/\Omega = C_s + C_c - T$ where $\Omega = 0.8$ and

$$\frac{N_u}{0.8} = 14.95 + 3.44\ y - 29.9 = 3.44\ y - 14.95$$ or

$$y = \frac{N_u + 11.96}{2.752}$$

The magnitude of y (and respectively x) can be determined by taking moments about the point of application of N_u. Hence

$$T \times 78.5 = C_s \times 21.5 + C_c\ (y/2 + 17.5) \quad \text{or}$$

$$29.9 \times 78.5 = 14.95 \times 21.5 + 3.44\ y\ (y/2 + 17.5) \quad \text{or}$$

$y = 21.0$ cms i.e. $z = y/0.85 = 21.0/0.85 = 24.75$ cms

giving $N_u = 21.0 \times 2.752 - 11.96 = 57.79 - 11.96 = 45.83 > 40.8$ t

The given calculations can only be correct if the strains in the compression and tension steel ε_s' and ε_s are bigger than its yield strain ε_y. Fig. 6-9.

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{2300}{2100\ 000} = \frac{1.10}{1000}$$

$$\varepsilon_s' = \varepsilon_c \frac{(z - d')}{z} \quad \text{or}$$

$$= \frac{.003\ (24.75 - 4)}{24.75} = \frac{2.52}{1000}$$

$$\varepsilon_s = \varepsilon_c \frac{(d - z)}{z} \quad \text{or}$$

$$= \frac{.003\ (61 - 24.75)}{24.75} = \frac{4.40}{1000}$$

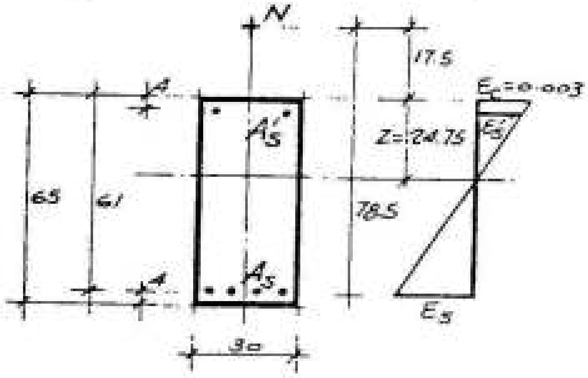


Fig. 6-9

ε_s' and ε_s are both bigger than ε_y. Therefore , both compression and tension reinforcements are in a yield state at failure.

Dimensioning of rect. sec^s. with tension reinforcements only subject to eccentric forces

It is however possible to design rectangular sections with tension reinforcements only subject to eccentric forces with big eccentricities by using the following equations:

$$d = c\sqrt{\frac{M_{su}}{f_{cp}\ b}} \quad \text{and} \quad A_s = \frac{M_{su}^{*}}{f_y\ \eta\ d} \pm \frac{N_u}{\Omega f_y} \quad \text{............ 6-17}$$

-ve sign for eccentric compression and +ve sign for eccentric tension. Values of c and η are given in table 4-8 and sheet 14. The application of these equations are shown in the following illustrative examples.

Illustrative examples

1) Given b = 30 cms, M_u = 50 mt, N_u = 25 t (compression), f_{cp} = 165 kg/cm^2 and f_y = 2300 kg/cm^2. Required t and A_s.

$e = M_u/N_u = 50/25 = 2.00$ ms   assume a case of big eccentricity.

If the section were subjected to a bending moment M_u only, then a convenient depth wuold be:

$$d_o = 2.65\sqrt{\frac{50 \times 10^5}{165 \times 30}} = 84.4 \text{ cm} \quad \text{assume } d = 90 \text{ cm} \ \& \ t = 95 \text{ cm}$$

then $e_s = 200 + 95/2 - 5 = 242.5$ cms & $M_{su} = 25 \times 2.425 = 60.6$ mt

$$\text{i.e.} \quad 90 = c\sqrt{\frac{60.6 \times 10^5}{165 \times 30}} \quad \text{or} \quad c = 2.57 \simeq 2.65$$

Therefore, the chosen depth is acceptable. Table 4-8 gives: $\eta = 0.8$ and

$$A_s = \frac{60.6 \times 10^5}{0.8 \times 2300 \times 90}^{*} - \frac{25}{0.9 \times 2.3} = 36.8 - 12.1 = 24.7 \text{ cm}^2.$$

2) If the maximum depth t equals 70 cms (d = 65 cms) and assuming d'= 4 cms. Determine A_s and A_s'.

$e_s = 200 + 70/2 - 5 = 230$ cms   and $M_{su} = 25 \times 2.30 = 57.50$ mt

$$65 = c\sqrt{\frac{57.50 \times 10^5}{165 \times 30}} \quad \text{giving} \quad c = 1.91 < 2.$$

Hence, compression reinforcement is required.

$$M_{su1} = 0.25 \, f_{cp} \, b \, d^2 = 0.25 \times 165 \times 30 \times 65^2 = 52.5 \times 10^5 = 52.5 \text{ mt}$$

$$N_{u1} = M_{su1}/e_s = 52.5/2.3 = 22.8 \text{ t}, \quad N_{u2} = N_u - N_{u1} = 25 - 22.8 = 2.2 \text{ t}$$

$$A_{s1} = \frac{M^{*}_{su1}}{0.71 \, f_y \, d} - \frac{N_{u1}}{\Omega f_y} = \frac{52.5 \times 10^5}{0.71 \times 2300 \times 65} - \frac{22.80}{0.9 \times 2.3} = 38.50 \text{ cm}^2$$

N_{u2} is resisted by additional tension steel A_{s2} and comp. steel A_s'. Taking moment of N_{u2} about line of action of tension steel, we get:

$$N_{u2} \, e_s = \Omega A_s' \, f_y \, (d - d') \quad \text{hence} \quad A_s' = \frac{N_{u2} \, e_s}{\Omega f_y \, (d - d')} \quad \text{i.e.}$$

compression steel A_s' is given by: $$A_s' = \frac{2.2 \times 230}{0.9 \times 2.3 \, (65 - 4)} = 4 \text{ cm}^2$$

Taking moment of N_{u2} about line of action of comp. steel, we get:

$N_{u2} \, e_s' = \Omega \, A_{s2} \, f_y \, (d - d')$ where $e_s' = e_s - (d - d') = 230 - 61 = 169$ cm.

---

* The capacity-reduction factor Ω is already included in this part of the equation.

Therefore: $A_{s2} = \dfrac{N_{u2}\ e_s'}{\Omega f_y\ (d - d')} = \dfrac{2.2 \times 169}{0.9 \times 2.3 \times 61} = 3.07 \quad cm^2$

Total tension steel $A_s = 38.5 + 3.07 = 41.57 \ cm^2$

The depth being small - c = 1.92 - deflection must be checked.

3) Assume that $N_u = 25$ t, given in example.1, is tension. Determine t and A_s . Use high grade steel with $f_y = 3600\ kg/cm^2$.

$d_o = 84.4$ cms. Assume d = 75 cms & t = 80 cms, then

$e_s = 200 - \frac{80}{2} + 5 = 165$ cms and $M_{su} = 25 \times 1.65 = 41{,}25$ mt i.e.

$75 = c \sqrt{\dfrac{41.25 \times 10^5}{165 \times 30}}$ or $c = 2.61 \simeq 2.65$ therefore

the chosen depth is convenient and table 4-8 gives: $\gamma = 0.81$,

then $A_s = \dfrac{41.25 \times 10^5}{0.81 \times 3600 \times 75} + \dfrac{25}{0.9 \times 3.6} = 18.9 + 7.7 = 26.6\ cm^2$.

4) If in the last example of eccentric tension, the maximum depth t is equal to 60 cms (d = 55 cms). Assume d' = 4 cms. Determine A_s & A_s'

$e_s = 200 - \frac{60}{2} + 5 = 175$ cms and $M_{su} = 25 \times 1.75 = 43.75$ mt i.e.

$55 = c \sqrt{\dfrac{43.75 \times 10^5}{165 \times 30}}$ or $c = 1.86 < 2$ comp. rfts are required.

$M_{su1} = 0.25\ f_{cp}\ b\ d^2 = 0.25 \times 165 \times 30 \times 55^2 = 36.4 \times 10^5$ kg cm

$N_{u1} = 36.4/1.75 = 20.8$ t therefore $N_{u2} = 25 - 20.8 = 4.2$ t

$A_{s1} = \dfrac{36.4 \times 10^5}{0.71 \times 3600 \times 55} + \dfrac{20.8}{0.9 \times 3.6} = 25.9 + 6.4 = 32.3 \quad cm^2$

Compression steel* A_s' is given by: $A_s' = \dfrac{4.2 \times 175}{0.9 \times 2.3\ (55 - 4)} = 7 \quad cm^2$

Additional tension steel A_{s2} is given by:

$$A_{s2} = \frac{4.2\ (175 + 51)}{0.9 \times 3.6\ (55 - 4)} = 5.7\ cm^2$$

Total tension steel A_s is given by: $A_s = 32.3 + 5.7 = 38.0 \quad cm^2$.

The ratio of total steel in the section is @ 3% , the design is uneconomic.

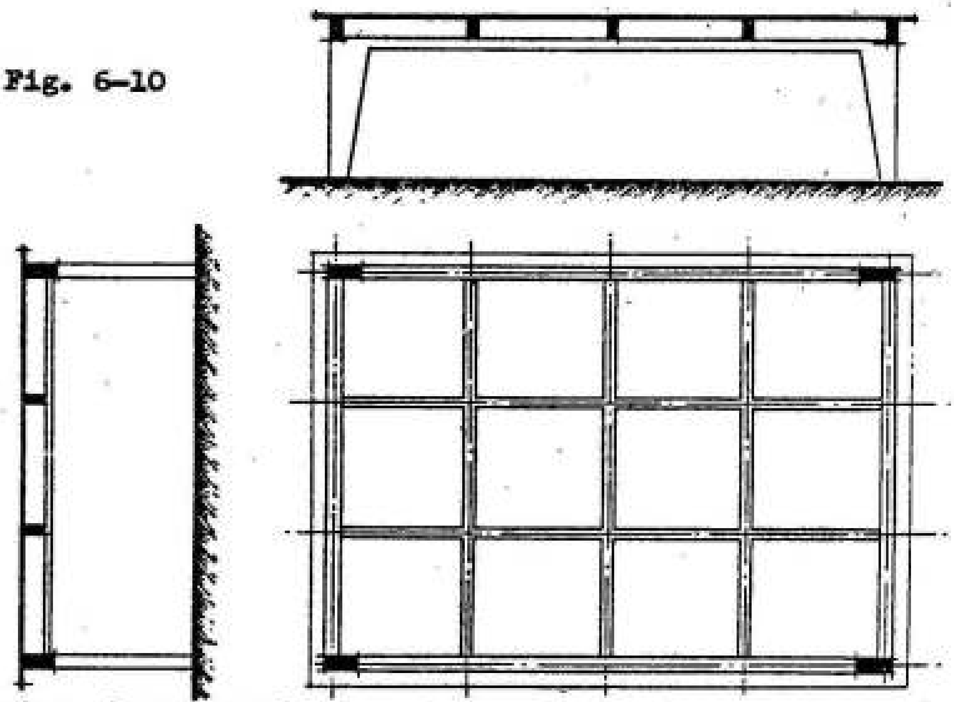
The depth being small -c = 1.86- deflection must be checked.

* Normal mild.

6-4 Columns in biaxial bending

The methods discussed permit rectangular columns to be designed or analyzed if bending is present about only one of the two principal axes of the section. This gives the general case in the majority of structures. There are other situations, in which axial compression is accompanied by simultaneous bending about both principal axes of the section. Such is the case, for instance, in corner columns of building frames. Fig. 6-10.

Fig. 6-10



Dimensioning of rectangular sections by the U.S.D.-method

Fig. 6-11 shows the cross-section of one of the corner columns of the frames shown in Fig. 6-10 under the simultaneous action of a concentric load N and biaxial bending by the moments M_x and M_y. The resulting action is the same as when the load N is applied with eccentricities:

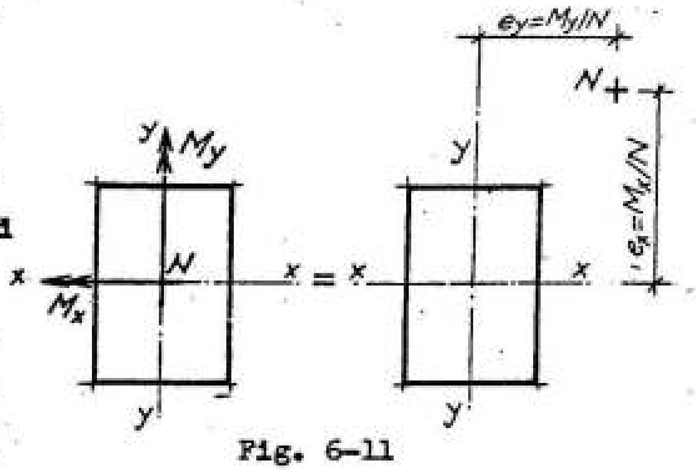


Fig. 6-11

$$e_x = M_x/N \qquad \text{and} \qquad e_y = M_y/N$$

about the centroidal x and y axes, respectively as shown.

It is possible to design such a biaxially eccentric column on the basis of the general assumptions of ultimate-strength theory that have been utilized in all other situations (rectangular equivalent stress block of depth $y = 0.85\ z$, ultimate concrete strain $\epsilon_{cu} = 0.003$, ... etc.).

a) <u>Rectangular sections with tension reinforcements only</u>: Fig. 6-12

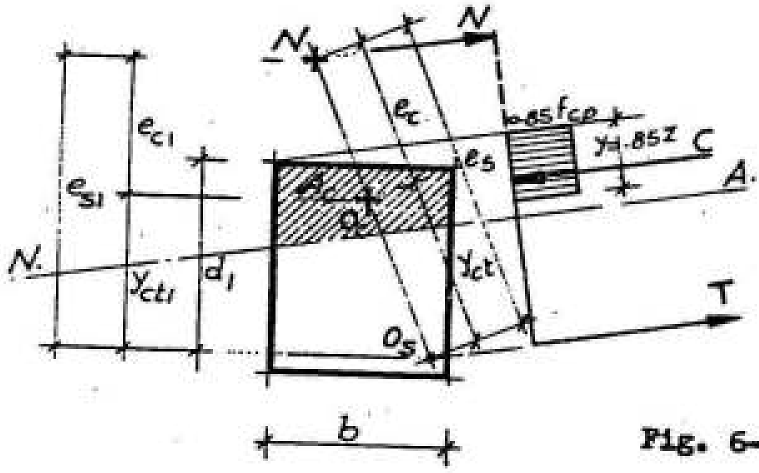


Fig. 6-12

The dimensions and area of tension steel can be determined as follows:

Conditions of equilibrium at failure:

$N_u - C + T = 0$ or $N_u - 0.85\ f_{cp}\ A_c + A_s\ f_y = 0$ 6-18

$N_u\ e_s = C\ y_{ct}$ or $N_u\ e_s = 0.85\ f_{cp}\ A_c\ y_{ct}$ 6-19

These two equations give:

$$N_u - \frac{N_u\ e_s}{y_{ct}} + A_s\ f_y = 0 \quad \text{or} \quad A_s = \frac{N_u\ e_s}{f_y\ y_{ct}} - \frac{N_u}{f_y} \qquad 6\text{-}20$$

Assuming: $A_s = \mu\ b\ d_1$ and $y_{ct1} = \eta\ d_1$ where

y_{ct1} = the vertical distance (parallel to d_1) between O_c & O_s where

O_c = the center of gravity of the concrete compression zone,

O_s = " " " " " " tension reinforcement, and

$A_c = \mu_c\, b\, d_1$. Equation 6-19 can be given in the form:

$N_u\, e_s = 0.85\, f_{cp} \cdot \mu_c\, b\, d_1 \cdot y_{ct}$ but Fig 6-11 shows that $\frac{e_s}{y_{ct}} = \frac{e_{s1}}{y_{ct1}}$

so that

$N_u\, e_{s1} = 0.85\, f_{cp} \cdot \mu_c\, b\, d_1 \cdot y_{ct1}$ or $M_{su1} = 0.85\, f_{cp}\, \mu_c\, \eta\, b\, d_1^2$

Introducing the capacity-reduction factor $\Omega = 0.9$, then

$M_{su1} = \Omega .\ 0.85\, f_{cp}\, \mu_c\, \eta\, b\, d_1^2 = 0.765\, f_{cp}\, \mu_c\, \eta\, b\, d_1^2$ therefore

$d_1 = c_1 \sqrt{\frac{M_{su1}}{f_{cp}\, b}}$ .... 6-12 in which $c_1 = \sqrt{\frac{1}{0.765\, \mu_c\, \eta}}$ 6-22

Replacing e_s/y_{ct} in equation 6-20 by e_{s1}/y_{ct1} and introducing the capacity-reduction factor Ω, then

$A_s = \frac{N_u\, e_{s1}}{\Omega\, f_y\, y_{ct1}} - \frac{N_u}{\Omega\, f_y}$ assuming $\Omega\, y_{ct1} = 0.9\, \eta\, d_1 = \eta_1\, d_1$

then

$A_s = \frac{M_{su1}}{f_y\, \eta_1\, d_1} - \frac{N_u}{\Omega\, f_y}$ 6-22 where $\eta_1 = 0.9\eta = 0.9\, y_{ct1}/d_1$

If the values of μ_c and η are assumed in advance then d_1 and A_s can be determined. The assumption is to be checked such that condition 6-18 is satisfied. Introducing the capacity-reduction factor $\Omega = 0.9$, we get:

$\Omega N_u - 0.85\, f_{cp}\, \mu_c\, b\, d_1 + A_s\, f_y = 0$

Assuming $f_y/0.85\, f_{cp} = m$ and knowing that $A_s = \mu\, b\, d_1$, then

$$\Omega\, N_u - \frac{f_y}{m}\mu_c\, b\, d_1 + \mu b\, d_1\, f_y = 0$$ or

$$\mu_c \equiv \frac{\Omega\, N_u}{0.85\, f_{cp} b\, d_1} + \mu m \qquad 6\text{-}23$$

For μ and c_1, values given in table 4-8 can aproximately be used; (assume $c_1 = c$).

The above design shows that rectangular sections subject to concentric compression plus double bending can be designed as if they were subject to N_u and M_{su1} only.

Examples

1) Determine the tension steel required for a rectangular section 50 x 80 cms subject to $N_u = 30$ t, $M_{u1} = 60$ mt and $M_{u2} = 28$ mt taken about the horizontal and vertical axes of the section.

The materials used are: concrete with a minimum guaranteed prism strength $f_{cp} = 165$ kg/cm^2 and high grade steel with $f_y = 3600$ kg/cm^2.

$e_1 = \frac{M_{u1}}{N_{u1}} = \frac{60}{30} = 2.0$ ms and $e_{s1} = 2.0 + \frac{0.80}{2} - 0.10 = 2.3$ ms

$M_{su1} = 30 \times 2.30 = 69.0$ mt and $d_1 = 80 - 10 = 70$ cms, then

$70 = c_1 \sqrt{\frac{69 \times 10^5}{165 \times 50}}$ giving $c_1 = 2.43 > 2$ depth acceptable.

Table 4-8 gives $\eta = 0.785$ therefore $\eta_1 = 0.9 \times 0.785 = 0.706$, then

$A_s = \frac{69 \times 10^5}{3600 \times 0.706 \times 70} - \frac{30}{0.9 \times 3.6} = 39.0 - 9.3 = 29.7$ cm^2 chosen 8 ∅ 22 ($A_s = 30.4$ cm^2).

Equation 6-22 gives:

$$c = \sqrt{\frac{1}{0.765\, \mu_c\, \eta}}$$ where

$c = 2.43$ and $\eta = 0.785$ so that $\mu_c = 0.28$

Equation 6-23 gives $\mu_c \geq \frac{\Omega N_u}{0.85\, f_{cp}\, b\, d_1} + \mu m$ but

$\mu = \frac{A_s}{b\, d_1} = \frac{30.4}{50 \times 70} = 0.87\%$ and $m = \frac{f_y}{0.85\, f_{cp}} = \frac{3600}{0.85 \times 165} = 25.8$

Hence, the right hand side of equation 6-23 gives:

$$\frac{0.9 \times 30\,000}{0.85 \times 165 \times 50 \times 70} + \frac{0.87}{100} \times 25.8 = 0.055 + 0.224 = 0.279$$

i.e., condition 6-23 is satisfied:

2) It is required to determine the area of steel and the minimum depth of cross section of a column 50 cms wide, if tension reinforcements only are used for $N_u = 100$ t, $M_{u1} = 60$ mt and $M_{u2} = 30$ mt.

Assume $f_{cp} = 165$ kg/cm^2 and $f_y = 3600$ kg/cm^2.

The solution can be done as follows:

$e_1 = \frac{M_{u1}}{N_u} = \frac{60}{100} = 0.6$ ms; assume $t_1 = 75$ cms, i.e. $d_1 = 75 - 10 = 65$ cms

$e_{s1} = 60 + 75/2 - 10 = 87.5$ cms and $M_{su1} = 100 \times 0.875 = 87.5$ mt

For minimum depth $c_1 = 2$, $\eta = 0.71$ i.e. $\eta_1 = 0.9 \times 0.71 = 0.639$

so that min. $d_1 = 2\sqrt{\frac{87.5 \times 10^5}{165 \times 50}} = 65$ cms i.e. $t_1 = 75$ cms

and $A_s = \frac{87.5 \times 10^5}{3600 \times 0.639 \times 65} - \frac{100}{0.9 \times 3.6} = 58.6 - 30.9 = 27.7$ cm²

chosen 8 # 22 (30.4 cm²).

Check according to equation 6-23:

Equation 6-22 gives $2 = \sqrt{\frac{1}{0.765\mu_c \times 0.71}}$ i.e. $\mu_c = 0.46$

but

$\mu = \frac{30.4}{50 \times 65} = 0.933\%$ and $m = \frac{3600}{0.85 \times 165} = 25.8$ so that

$$\frac{\Omega N_u}{0.85 f_{cp} b d_1} + \mu m = \frac{0.9 \times 100\,000}{0.85 \times 165 \times 50 \times 65} + \frac{0.933}{100} \times 25.8 = 0.21 + 0.25$$

giving 0.46 which is the same as μ_c.

b) Rectangular sections with symmetrical reinforcement

An approximate U.S.D.-method has been developed by Boris Bresler[x] and has been verified by tests. Bresler's equation can be given in the form:

$$\frac{1}{N_u} = \frac{1}{N_{xu}} + \frac{1}{N_{yu}} - \frac{1}{N_{ou}} \quad \text{.......... 6-24}$$

in which

N_u = ultimate load for biaxial bending with eccentricities e_x and e_y,

N_{xu} = " " when only eccentricity e_x is present ($e_y = 0$),

N_{yu} = " " " " " e_y " " ($e_x = 0$) and

N_{ou} = " " for concentrically loaded column ($e_x = e_y = 0$).

Equation 6-24 can be used in the following manner:

Given the service normal load N and the biaxial bending moments M_x and

---

x "Design criteria for reinforced concrete columns under axial load and biaxial bending". By Boris Bresler. Journal of ACI. Vol.57. 1960.

M_y about the two principal axes x and y, one proceeds in the design as follows:

1- Determine $\bar{N}_u$, M_{xu} and M_{yu}. $U = 1.5\ G + 1.8\ P$.

2- Compute $e_x = M_{xu}/\bar{N}_u$ and $e_y = M_{yu}/\bar{N}_u$.

3- Assume dimensions and reinforcements of cross section. Compute e/t.

4- Calculate $m = f_y/0.85\ f_{cp}$, μ_{tx} and μ_{ty} for each two opposite sides in the directions x and y.

5- Compute $N_{ou} = 0.85\ f_{cp}\ A_c + f_y\ A_s$, N_{xu} and N_{yu} for uniaxial bending about the x and y axes respectively from the interaction diagrams given in sheets 17 and 18.

6- Determine N_u from equation 6-24. The design ultimate load $\bar{N}_u$ must be smaller than or equal to $\Omega N_u = 0.7\ N_u$.

Equation 6-24 is valid provided $N_u \geq 0.1\ N_{ou}$. They are not reliable where biaxial bending is prevalent and is accompanied by an axial force smaller than $N_{ou}/10$. In the case of strongly prevalent bending, failure is initiated by yielding of the steel, and the situation corresponds to the lower tenth of the interaction diagram. In this range, it is safe and satisfactorily accurate to neglect the axial force entirely and to calculate the section for biaxial bending only.

Examples

1) Design a short column subject to the following service loads:

$N_g = 40$ t, $N_p = 22.2$ t, $M_{xg} = 10$ mt, $M_{xp} = 5$ mt, $M_{yg} = 5$ mt and $M_{yp} = 2.5$ mt. The materials used are concrete with $f_{cp} = 165$ kg/cm^2 and high grade steel with $f_y = 3600$ kg/cm^2.

$\bar{N}_u = 1.5 \times 40 + 1.8 \times 22.2 = 60 + 40 = 100$ t

$M_{xu} = 1.5 \times 10 + 1.8 \times 5.0 = 15 + 9 = 24$ mt $e_x = \frac{24}{100} = 0.24$

$M_{yu} = 1.5 \times 5 + 1.8 \times 2.5 = 7.5 + 4.5 = 12$ mt $e_y = \frac{12}{100} = 0.12$

Assume cross section 40 x 65 cms, reinforced by 14 ∅ 19. Fig. 6-13.

Therefore: $e_x/t = 24/65 = 0.37$ and $e_y/b = 12/40 = 0.30$

$$m = \frac{f_y}{0.85\ f_{cp}} = \frac{3600}{0.85 \times 165} = 25.8$$

$$\mu_{tx} = \frac{A_s\ (10 \# 19)}{b\ t} = \frac{28.4}{40 \times 65} = 1.09\%$$

$$(m\ \mu_t)_x = \frac{25.8 \times 1.09}{100} = 0.281$$

$$\mu_{ty} = \frac{A_s\ (8 \# 19)}{b\ t} = \frac{22.7}{40 \times 65} = 0.885\%$$

$$(m\ \mu_t)_y = \frac{25.8 \times 0.885}{100} = 0.228$$

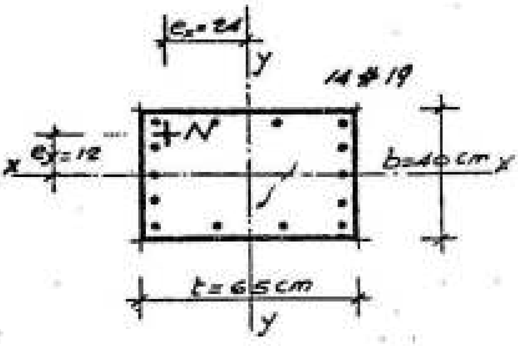


Fig. 6-13

$N_{ou} = 0.85\ f_{cp}\ A_c + f_y\ A_s = 0.85 \times 165 \times 40 \times 65 + 3600 \times 39.7 = 507\ 000$ kg

Using the interaction diagram, sheet 17, we get:

For $e_x/t = 0.37$ and $(m\ \mu_t)_x = 0.281$ $K_x = 0.475$ so that

$$K_x = 0.475 = \frac{N_{xu}}{b\ t\ f_{cp}} = \frac{N_{xu}}{40 \times 65 \times 165} \quad \text{or} \quad N_{xu} = 204\ 000 \text{ kg} \quad \text{and}$$

For $e_y/b = 0.30$ and $(m\ \mu_t)_y = 0.228$ $K_y = 0.520$ so that

$$K_y = 0.520 = \frac{N_{yu}}{b\ t\ f_{cp}} = \frac{N_{yu}}{40 \times 65 \times 165} \quad \text{or} \quad N_{yu} = 278\ 000 \text{ kg}$$

Equation 6-24 gives:

$$\frac{1}{N_u} = \frac{1}{N_{xu}} + \frac{1}{N_{yu}} - \frac{1}{N_{ou}} \quad \text{or} \quad \frac{1}{N_u} = \frac{1}{204} + \frac{1}{278} - \frac{1}{507} \quad \text{giving}$$

$N_u = 153$ tons so that $\bar{N}_u = 0.7 \times 153 = 107.1$ tons $>$ 100 tons.

This means that the section and reinforcement chosen are convenient.

2) Assuming an average load factor of 1.6 and a capacity reduction factor of 0.7, determine the safe axial service load of a column having the same section and reinforcement as that of Fig. 6-13. Materials used are C250 and high grade steel with $f_y = 3600$ kg/cm^2.

Minimum guaranteed cube strength of concrete C250 $\simeq 0.8 \times 250 = 200$ kg/cm^2. Corresponding $f_{cp} = 165$ kg/cm^2.

$N_{ou} = 0.85\ f_{cp}\ A_c + f_y\ A_s = .85 \times 165 \times 40 \times 65 + 3600 \times 39.7 = 507\ 000$ kg

Min. e_x/t or $e_y/b = 0.1$ and $(m\ \mu_t)_x = 0.281$ & $(m\ \mu_t)_y = 0.228$.

Sheet 17 gives:

$$K_x = 0.86 = \frac{N_{xu}}{b\ t\ f_{cp}} = \frac{N_{xu}}{40 \times 65 \times 165} \qquad \text{or} \qquad N_{xu} = 370\ 000 \ \text{kgs}$$

$$K_y = 0.77 = \frac{N_{yu}}{b\ t\ f_{cp}} = \frac{N_{yu}}{40 \times 65 \times 165} \qquad \text{or} \qquad N_{yu} = 330\ 000 \ \text{kgs}$$

Equation 6-24 gives:

$$\frac{1}{N_u} = \frac{1}{N_{xu}} + \frac{1}{N_{yu}} - \frac{1}{N_{ou}} \qquad \text{or} \qquad \frac{1}{N_u} = \frac{1}{370} + \frac{1}{330} - \frac{1}{507} \qquad \text{giving}$$

$$N_u = 269\ t \simeq 0.53\ N_{ou}$$

$$\text{The safe load} = \frac{0.7\ N_u}{1.6} = 117 \ \ t$$

If the bending moments due to the minimum eccentricities t/10 and b/10 were not considered, then the safe load would have been

$$\frac{0.7\ .\ N_{ou}}{1.6} = \frac{0.7 \times 507}{1.6} = 221 \ \ t$$

which is about double as much as the case of biaxial loading.

6-5 Buckling of slender compression members

The UNESCO* proposes the following simple method for checking the critical strength of slender members subject to compression.

The analysis of the critical strength of uniaxial or biaxial bending in the limit state of instability, i.e., the calculation of the limiting values of the buckling moments and the corresponding direct (longitudinal) forces is reducible to the ultimate strength analysis by conventionally introducing a complementary eccentricity (in the direction of one or the two pricipal axes of the section) to the direct force.

For practical purposes, the above assumption consists in transforming the analysis for the limit state of instability into the usual of the ultimate limit state for eccentric compression.

Consider a member subjected to that longitudinal compression. At

* UNESCO "Reinforced Concrete". An International Manual. Published 1971

any particular cross section of that member the direct force has a total eccentricity comprising:

a- the known or intentional eccentricity due to deliberate structural arrangements;

b- the accidental or inevitable eccentricity due to inaccuracies of construction;

c- the deflection due to the flexural deformations arising from the two foregoing eccentricity components (a) and (b).

It thus appears that, in terms of actual structural behaviour, concentric compression is merely a hypothetical ideal condition; in reality, bending always complicates the issue. For this reason the validity for applying the standard theory of elastic buckling of reinforced concrete members may well be questioned. Accordingly, for checking a member with regard to the limit state of instability, it appears advisable to turn to an analogy calculation based on the knowledge of the maximum deformations of the member in the ultimate limit state and enabling the critical buckling strength to be estimated in a simple manner by applying the usual ultimate strength analysis for eccentric compression.

In short, the analysis for buckling involves adding bending moments (referred to as the 'additional bending moments') to the initial loading system on which the design of the section for the ultimate state is based.

The analysis of the critical strength in the limit state of instability is nessesary for all compression members whose Euler slenderness ratio λ exceeds 40. Otherwise the ultimate strength analysis (i.e. the ordinary analysis of the member in the limit state of failure) will suffice.

In accordance with Euler's theory, the slenderness ratio λ of a member is defined as the ratio of the effective length l_b to the radius of gyration i of the concrete cross section in the corresponding plane:

$$\lambda = l_b/i$$

The effective length l_b depends upon the structural features in any particular case. If l denotes the geometrical length of the member under consideration, the effective length l_b is equal to:

$l_b = 2\ l$ if the member is free at one end and fixed at the other;

$l_b = l$ if the member is hinged at both ends;

$l_b = l/\sqrt{2}$ if the member is hinged at one end and fixed at the other;

$l_b = l/2$ if the member can be considered fixed at both ends.

In the frequently encountered case of a multi-storey building in which the continuity of the columns and their sections is assured, the effective length l_b may be equal to:

$l_b = l/\sqrt{2}$ if the ends of the column are either rigidly fixed to a foundation block or are joined to floor beams which have the same moment of inertia as the column in the direction considered and which extend on both sides of it;

$l_b = l/1.15$ in all other cases.

The geometrical length of the member under consideration can be chosen according to Fig. 6 14 for the given special constructional systems:

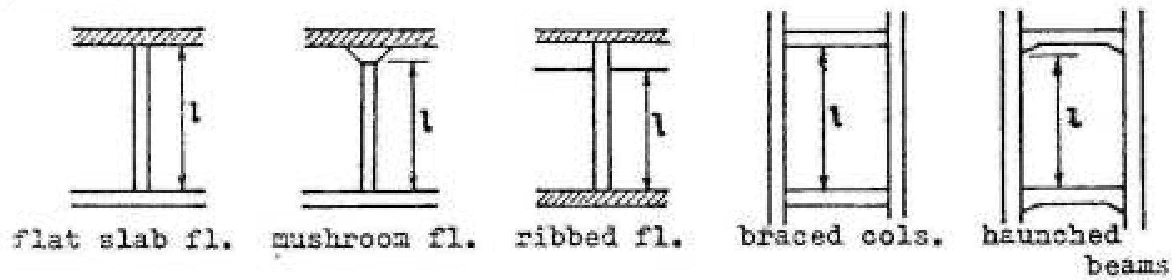


Fig. 6-14

Limit state of instability of axially loaded columns

The analysis of the critical strength of columns for concentric compression in the limit state of instability, i.e., the calculation of the limiting value of the force associated with instability, is reduced to the ultimate strength analysis of such columns under the effect of the additional moment:

$$M_c = N\ t\ E_c\ /\ 3000\ \sigma_e$$

where N is the axial load on the column, t the total geometrical depth of the section (measured parallel to the buckling plane), E_c the instantaneous modulus of elasticity of the concrete, and σ_e the Euler critical stress.

The Euler critical stress for columns of constant cross section is given by the formula:

$$\sigma_e = \pi^2 E_c I / l_b^2 A_c$$

where I is the moment of inertia of the concrete cross section in the buckling plane, and A_c is the cross sectional area of the concrete. On introducing the radius of gyration i and the slenderness ratio , the expression for the Euler critical stress may be written in the following simple form:

$$\sigma_e = \pi^2 E_c / \lambda^2$$

The additional moment to be introduced in the ultimate strength analysis is therefore:

$$M_c = N \lambda^2 t / 30\,000$$

The analysis of the critical strength for concentric compression in the limit state of instability is thus reduced to the analysis of the ultimate strength for eccentric compression by application of a conventionally introduced eccentricity e_c :

$$e_c = \lambda^2 t / 30\,000 = k\, t \qquad 6\text{-}25$$

to the compressive direct force N, calculated according to the theory of first order.

The Egyptian code gives the values of k for rectangular and cir - cular sections as shown in table 6-3.

Table 6-3. Values of k for rectangular and circular sections (diam. D)

$\lambda = l_b/i$	42	50	55.4	62.2	69	76	83	90	97
l_b/t	12	14	16	18	20	22	24	26	28
l_b/D	10.4	12.1	13.9	15.6	17.3	19.1	20.8	22.5	24.3
k	.059	.083	.102	.129	.165	.193	.230	.270	.314

Limit state of stability of eccentrically loaded columns

The analysis of the critical strength of columns for eccentric compression in the limit state of instability, i.e., the calculation of the limiting value of the buckling moment and the corresponding direct (longitudinal) force, is reduced to the ultimate strength analysis of such columns by the introduction of an additional moment:

$$M_c = N\,E_c\,(t + e_c)\,/\,3000\,\sigma_e$$

which should be added to the initial system of loading (M and N) as obtained by application of the theory of first order to the structure considered. This calculation procedure is applicable only on condition that the initial eccentricity e_o of the direct force N does not exceed the total geometrical depth t of the section (measured parallel to the buckling plane), i.e., $e_o < t$.

For columns of constant cross section the additional moment to be introduced into the ultimate strength analysis is (with due reference to the expression for the Euler critical stress given above):

$$M_c = N\,\lambda^2\,(t + e_o)\,/\,30\,000$$

The analysis of the critical strength for eccentric compression in the limit state of instability is thus reduced to the analysis of the ultimate strength for eccentric compression by application of a conventionally introduced complementary eccentricity e_c:

$$e_c = \lambda^2\,(t + e_o)\,/\,30\,000 = k\,(t + e_o) \qquad 6\text{-}26$$

to the compessive direct force N, calculated according to the theory of first order.

The determination of the load capacity of a slender column subject to normal forces and biaxial bending about the principal axes of the column due to small eccentricities can now be easily done as shown in the following illustrative example.

Example

Assuming that the free height l of the axially loaded column given in example 2 is 7.35 ms, determine the safe load.

Assume $l_b = l/1.15$, then the buckling length of the column is given by:

$$l_b = 7.35/1.15 = 6.4 \text{ ms}$$

As $\frac{l_b}{t} = \frac{6.40}{0.65} = 9.82$, then there is no danger of buckling in x-direction, but $\frac{l_b}{b} = \frac{6.40}{0.40} = 16 > 12$, then buckling must be considered in y- "

According to table 6-3, $e_{cy} = k\,b = 0.102 \times 40 = 4.08$ cms so that total $e_y = 0.1 \times 40 + 4.08 = 8.08$ cms and $\frac{e_y}{b} = \frac{8.08}{40} = 0.202$

We have further: $\frac{e_x}{b} = 0.1$ and

$$(m\,\mu_t)_y = 0.228 \qquad (m\,\mu_t)_x = 0.281$$

Sheet 17 gives:

$$K_x = 0.86 = \frac{N_{xu}}{b\,t\,f_{cp}} = \frac{N_{xu}}{40 \times 65 \times 165} \quad \text{or} \quad N_{xu} = 370\,000 \text{ kgs} \quad \text{and}$$

$$K_y = 0.67 = \frac{N_{yu}}{b\,t\,f_{cp}} = \frac{N_{yu}}{40 \times 65 \times 165} \quad \text{or} \quad N_{yu} = 288\,000 \text{ kgs}$$

Equation 6-24 gives:

$$\frac{1}{N_u} = \frac{1}{N_{xu}} + \frac{1}{N_{yu}} - \frac{1}{N_{ou}} \quad \text{or} \quad \frac{1}{N_u} = \frac{1}{370} + \frac{1}{288} - \frac{1}{507} \quad \text{so that}$$

$N_u = 239$ t i.e. the safe load is $N = \frac{0.7 \times 239}{1.6} = 104.5$ tons

6-6 Bending moments in columns of buildings

Girders of reinforced concrete buildings are generally cast monolithically with the supporting columns. If no exact calculation as a building frame is done, it is allowed to consider them as freely supported on the interior columns and rigidly connected to the exterior ones. This means that, in case of approximately equal spans, interior columns may be assumed as axially loaded; whereas exterior columns are subject to normal forces and bending moments. The bending moments at the joint of girders with exterior columns may approximately be estimated in the following manner: Fig. 6-15.

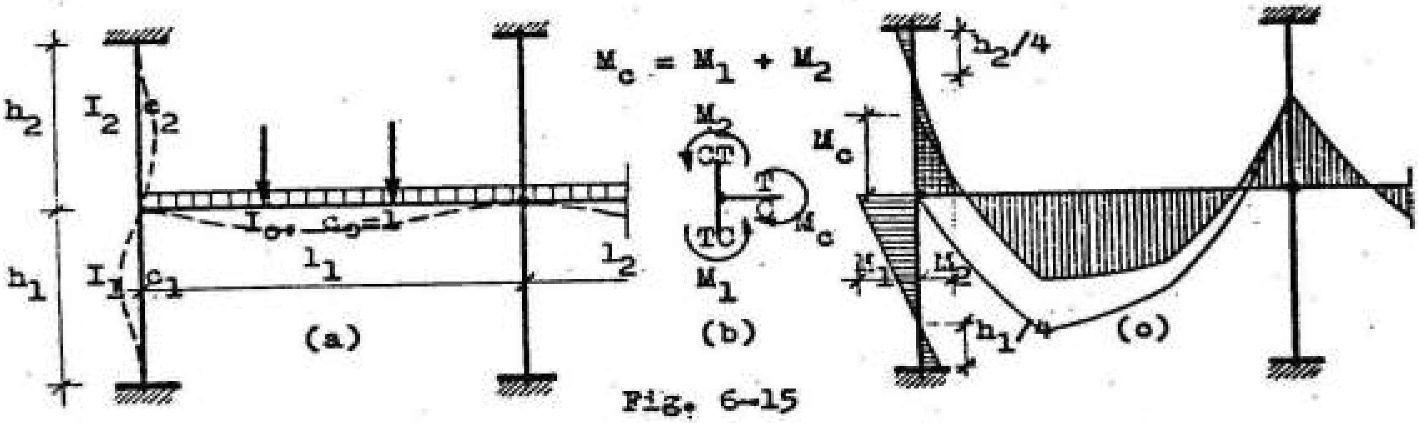


Fig. 6-15

Assume I_o, I_1 and I_2 to be the moments of inertia of the girder of span l, the lower column of height h_1, the upper column of height h_2. Their relative rigidity can be given by the factors:

$$c_o = \frac{I_o}{l} \cdot \frac{l}{I_o} = 1 \qquad \text{reference value;}$$

$$c_1 = \frac{I_1}{h_1} \cdot \frac{l}{I_o} \qquad \text{relative rigidity of lower column;}$$

$$c_2 = \frac{I_2}{h_2} \cdot \frac{l}{I_o} \qquad \text{relative rigidity of upper column.}$$

Then, the fixing moment at the exterior column is given by:

$$M_c = \bar{M} \cdot \frac{c_1 + c_2}{1 + c_1 + c_2} \qquad 6\text{-}27$$

in which $\bar{M}$ is the fixed-end moment of the loaded span l_1. This moment will be distributed on the columns in proportion to their relative rigidity c. Thus

$$M_1 = M_c \cdot \frac{c_1}{c_1 + c_2} \quad \text{and} \quad M_2 = M_c \cdot \frac{c_2}{c_1 + c_2} \qquad 6\text{-}28$$

For girders of top floor: $c_2 = 0$ and $M_1 = M_c$

The sense of the moments can be adjusted by the system of arrows shown in Fig. 6-15. M_c, calculated according to equation 6-27, causes tensile stresses at the upper fiber of the girder and compressive stresses at its lower fibers. It will be represented by an arrow starting from the tension side and ending on the compression side. Further, it will be resisted by M_1 and M_2 such that they are in equilibrium with it in magnitude and sense. Hence

$$M_1 + M_2 = M_c \qquad 6\text{-}29$$

Their sense is opposite to M_c and thus defining the tension and compression sides of the columns at their connection with the girder. This can however be verified by imagining the elastic line due to the loads on the girder and shown dotted in Fig. 6-15.

The interior columns are to be designed for the maximum normal force N plus minimum bending moments due to accidental eccentricities (e_x = t/10 and e_y = b/10). Exterior columns are to be designed for the maximum normal force N plus the maximum bending moments due to loading on gider according to Fig. 6-15 plus bending moments due to accidental eccentricities.

6-7 Constructional details and remarks

1) Types of columns

The main types of columns are:

a) longitudinally reinforced columns (sometimes called tied columns) with main longitudinal steel and thin lateral ties (sometimes called hoops or stirrups);

b) spirally reinforced columns with main longitudinal steel and closely spaced spirals;

c) cross reinforced columns with main longitudinal steel and closely spaced thin lateral ties in the form of successive meshes;

d) composite columns, in which a structural steel or cast iron member is thoroughly encased in a concrete column of type a or b.

In the following, we shall deal with columns of types a,b and c.

The longitudinal reinforcements should not be less than 0.8% of the cross section of a column and are generally arranged symmetrical on every two opposite sides of the cross section.

Lateral reinforcement, in the form of ties or spirals, serves several functions. For one, such reinforcement is needed to hold the longitudinal bars in position in the forms while the concrete is being placed. For another, transverse reinforcement is needed to prevent the highly stressed, slender longitudinal bars from buckling outward by bursting the thin concrete cover. Closely spaced spirals and mesh ties evidently serve these two functions. However, in any tie plane, a sufficient number of ties must be provided to position and hold all bars. On the other hand, in columns with many longitudinal bars, if the column section is crossed by too many ties, they interfere with the placement of concrete in the forms. To achieve adequate tying, yet hold the number of ties to a minimum, the Egyptian code gives many rules for tie arrangement.

Fig.6-16 gives the arrangement of longitudinal and lateral reinforcement in different types of columns.

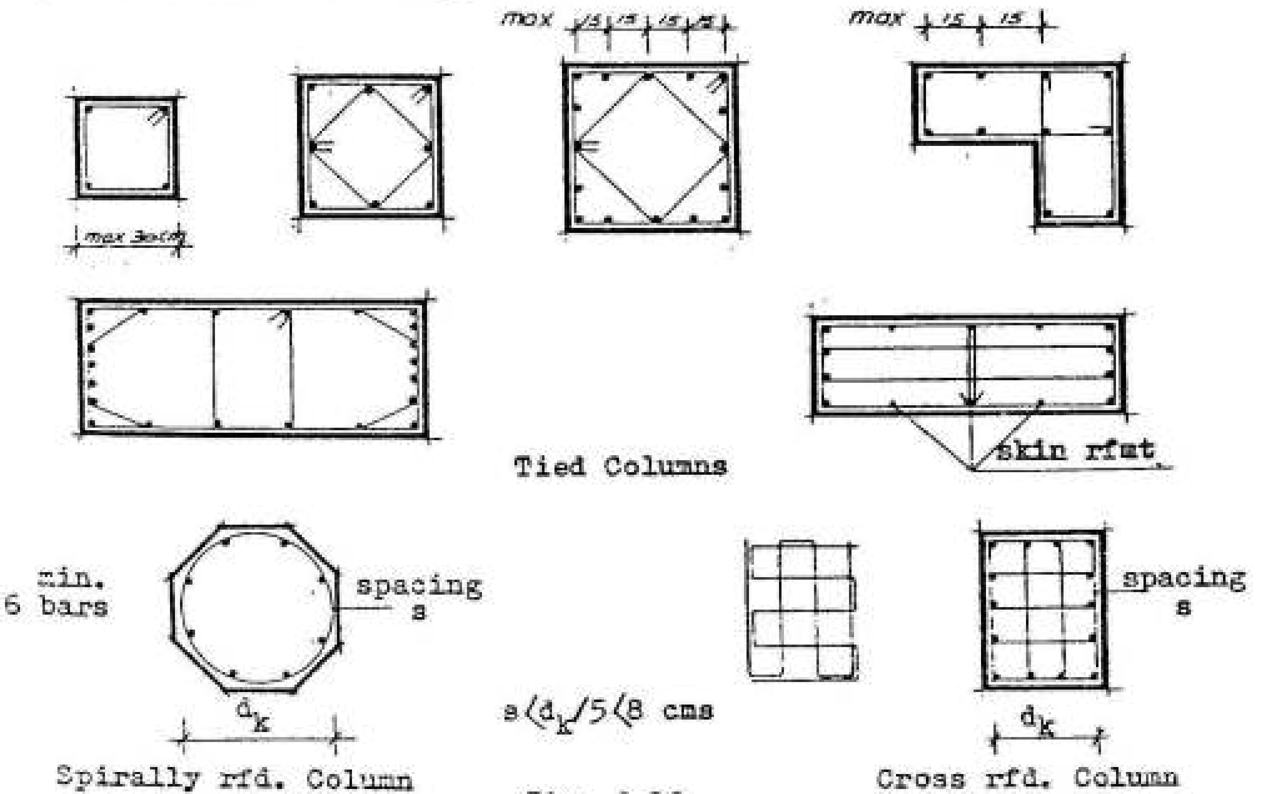


Fig. 6-16

It has to be noted that in tied columns, the ties are so arranged that every corner bar and alternate longitudinal bars have lateral support provided no bar is farther than 15 cms from a laterally sup - ported bar; that the corner of a tie has an included angle of not more than 135°.

Skin reinforcements (min. 0.05% of cross section) can be arranged along the unreinforced sides of the column without the need of cross ties through the breadth of the section.

2) Laps of longitudinal reinforcements

For convenience of construction and execution, laps of column re-inforcements are generally made just above floors by one of the fol-lowing methods: Fig. 6-17.

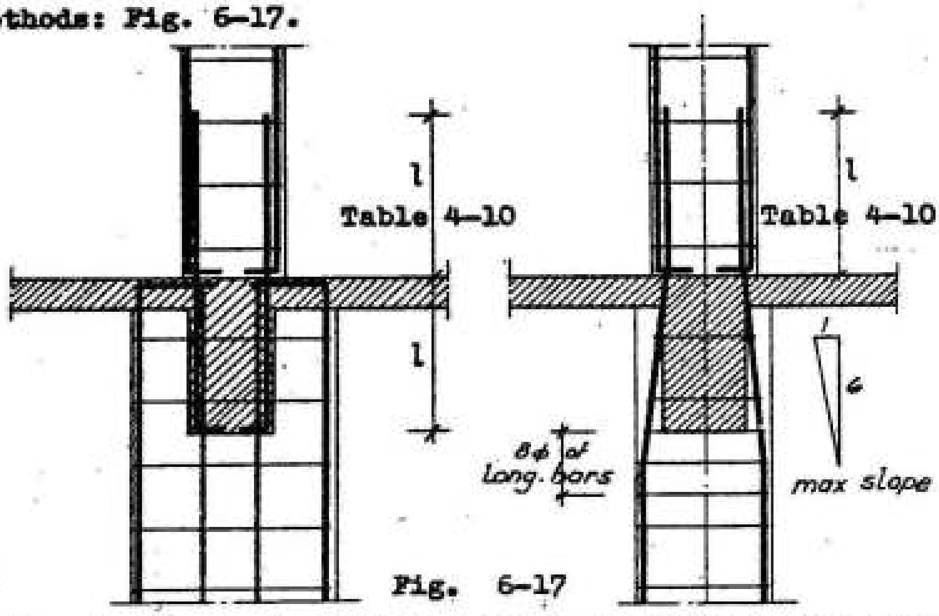


Fig. 6-17

If a lapped compression splice is used, and if the bars are offset at the splice, the slope of the inclined axis of the bar is not to exceed 1:6 , and the portions of the bar above and below the offset are to be parallel to the axis of the column. Horizontal support at the offset bends, provided by ties, spirals, or parts of the floor construction, should be adequate to carry a horizontal thrust assumed equal to 1½ times the horizontal component of the nominal stress in the inclined portion of the bar.

3) Requirements of the Egyptian code

The Egyptian code gives, under the same heading, the following restrictions:

a) The minimum amount of longitudinal reinforcement in columns with ordinary hoops is 0.8% of the required concrete area but not less than 0.4% of the actual area if $l_b/b < 15$, or $l_b/i < 50$. For higher slender-ratio it is recommended to choose the minimum reinforcement according to the following relations:

Min percentage of steel

$$\text{min. } A_s = 0.25 + 0.015\ l_b/i \qquad 6\text{-}30\ a$$

of the required area of concrete.

For rectangular sections

$$\text{min. } A_s = 0.25 + 0.050\ l_b/b \qquad 6\text{-}30\ b$$

of the required area of concrete for special cases of light live load.

b) Minimum dimensions of a supporting column are 20 x 20 cms. smaller dimensions are permitted in special cases of light loads and the side ratio of the rectangular cross section should not exceed 4:1 or else the eccentricity of loads (if any) is taken into consideration.

c) All columns shall have one longitudinal bar at each corner.

d) Maximum side length of column in which only corner bars are used is 35 cms., otherwise, intermediate bars are placed with maximum spacing of 30 cms. These bars must be held by special hoops.

e) Maximum spacing of hoops to be the least of:

15 times the diameter of the smallest longitudinal bar;

the least side of column;

25 cms.

f) Minimum diameter of longitudinal bar is 13 mm.

g) Minimum diameter of hoops to be ¼ diameter of biggest longitudinal bar and not less than 6 mm. and the least volume of hoops is 0.25% of volume of concrete.

h) Hoops as well as spirals are to be placed also within depths of beams.

i) Spirals shall have a circular or nearly circular form.

j) Maximum pitch of spirals is 8 cms. or 1/5 core of section whichever is the smaller. The minimum pitch is 3 cms. The pitch is to be maintained constant.

k) The use of additional reinforcement with a view of reducing the column dimensions is expensive. An economic method for obtaining a smaller section is to use a richer concrete mix.

l) Maximum percentage of steel for concrete up to C180 or less is 3% and for higher grade concretes is 6%.

m) Minimum splices are to be 25 diameter in length with a minimum of 40 cms. For columns under full compression, where welding is more economical, splices may be dispensed with. The sectional area of dowel bars, shall not be less than these of columns themselves.

n) Loads on columns should be properly calculated, taking advantage of the permissible reduction in live loads in multi-storey buildings.

o) In both tension and compression sides of rectangular columns, subjected to small eccentricity (small eccentricity is that which gives tensile stresses not more than ¼ the compressive stresses), the minimum area of steel is to be 0.2% of the actual concrete section on condition that it is not less than 0.8% of the required section.

PART III

STAIRS, PANELLED BEAMS, FOOTINGS, RETAINING WALLS AND TORSION

CHAPTER 7

REINFORCED CONCRETE STAIRS

7.1 General Considerations

A reinforced concrete stair is generally composed of inclined slabs for the flights and horizontal slabs for the landings supported on broken, inclined or horizontal beams and columns.

The slope of the inclined slabs, depends on the number of the steps in each flight and the dimensions of each step, i.e. its rise and going.

To have a comfortable stair, it is advisable not to arrange more than 12 steps in one flight with the following dimensions for each step: (Fig. 7-1)

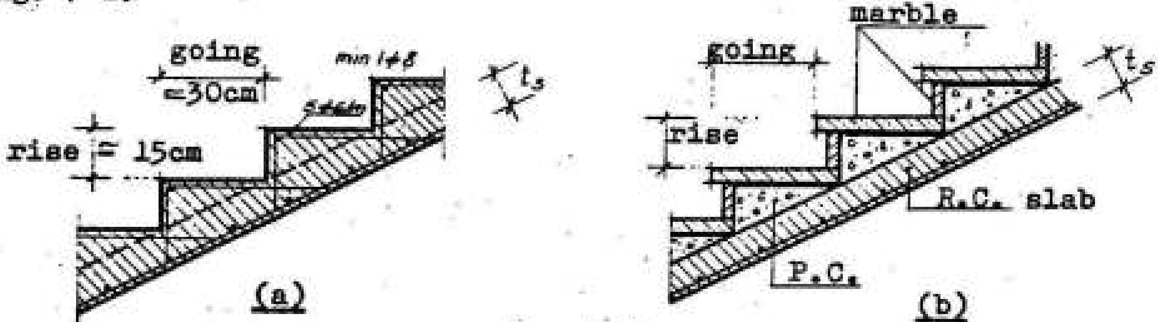


Fig. 7-1

Rise: 14 - 20 cms (normal 15 - 16 cms), going: 25 - 35 cms (normal 28-30 cms).

The top surface of the inclined slab of a stair may be stepped as shown in Fig. 7-1 a, in which case, the stair can be used during the construction of the building. In order to protect the outside corners of the steps and to prevent their chipping off, at least 1 ∅ 8 and stirrups 5 ∅ 6 mm/m must be arranged as shown in Fig. 7-1 a.

The correct execution of such stepped slabs is time consuming and some prefer to make them with a constant thickness t_s and formulate the steps either by plain concrete poured when covering the steps with the stair cover, e.g. marble tiles or by using reinforced mosaic triangular steps. (Fig. 7-1 b).

One has to note that the tiles on the landings of stepped stairs need 6 cms fill while they are constructed directly on the reinforced concrete steps using 1 cm mortar in between. (Fig. 7-2, details a & b)

7.2 Loads

In addition to the dead weight of the different elements of the stair, a live load 150 kg/m^2 more than that of the serviced floors, and with a maximum of 500 kg/m^2, is assumed so that the total load/m^2 lies generally between 900 and 1100 kg/m^2 horizontal; a concentrated load of 100 kgs on the edge of any step and eventually a line load of 100 kg/m on the edge of the landing has to be considered in the calculations of cantilever stairs.

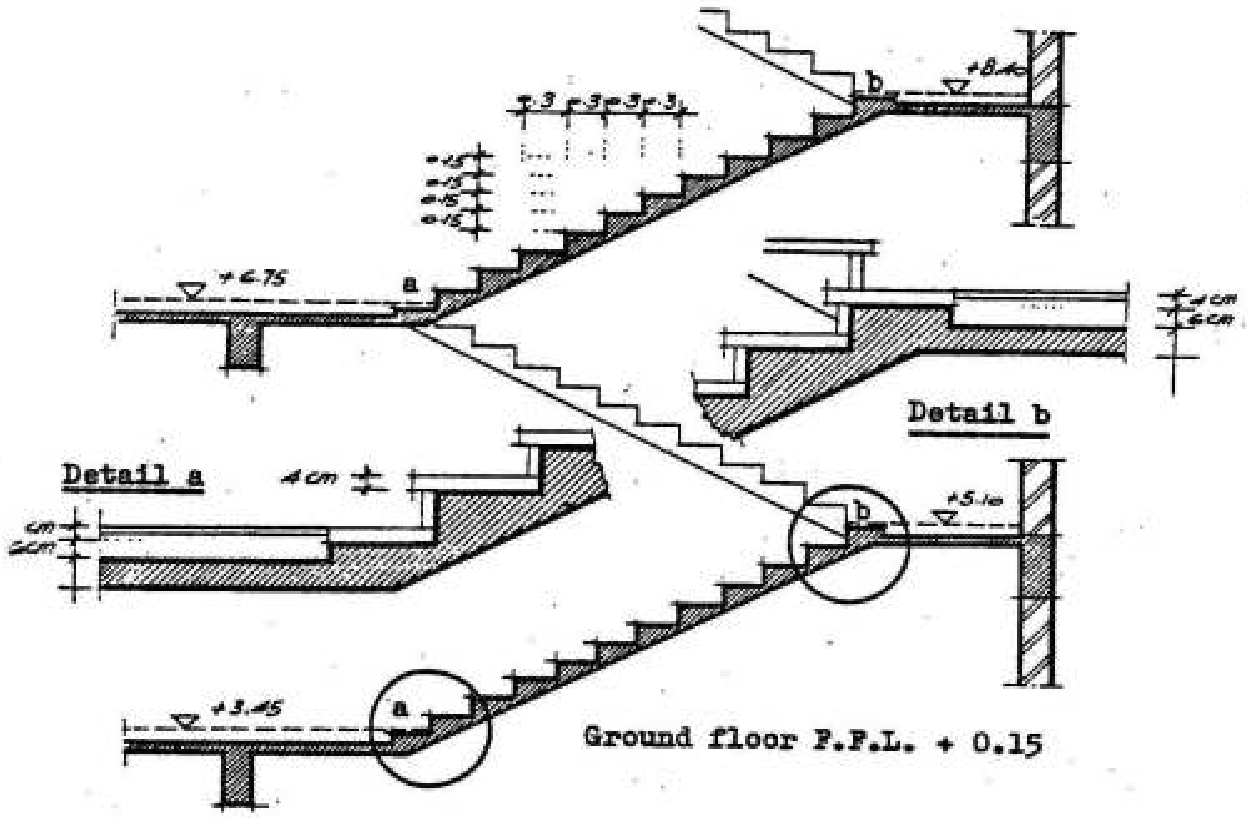
The main structural types of reinforced concrete stairs are:

7.3 Cantilever Stairs

In cases where there is no possibility to make any beams, except the wall-beams, to support the stair (Fig. 7-2), one may use this type of stairs in which the stair slab is assumed as a simple cantilever fixed in the wall-beam. The fixing moment of the slab, being resisted by torsion in the wall-beam and knowing that the torsional resistance of reinforced concrete is low, it is recommended, in this case, not to have big spans for the wall-beams. In the case of the stair shown in Fig. 7-2, if this system is used, it might be advisable to arrange a column at the middle of each longitudinal wall-beam (shown dotted in Fig. 7-2).

Example

It is required to design the stair shown in Fig. 7-2 for an apart-


Detail a
Detail b
+6.75
+8.40
+5.10
+3.45
4cm
Ground floor F.F.L. + 0.15

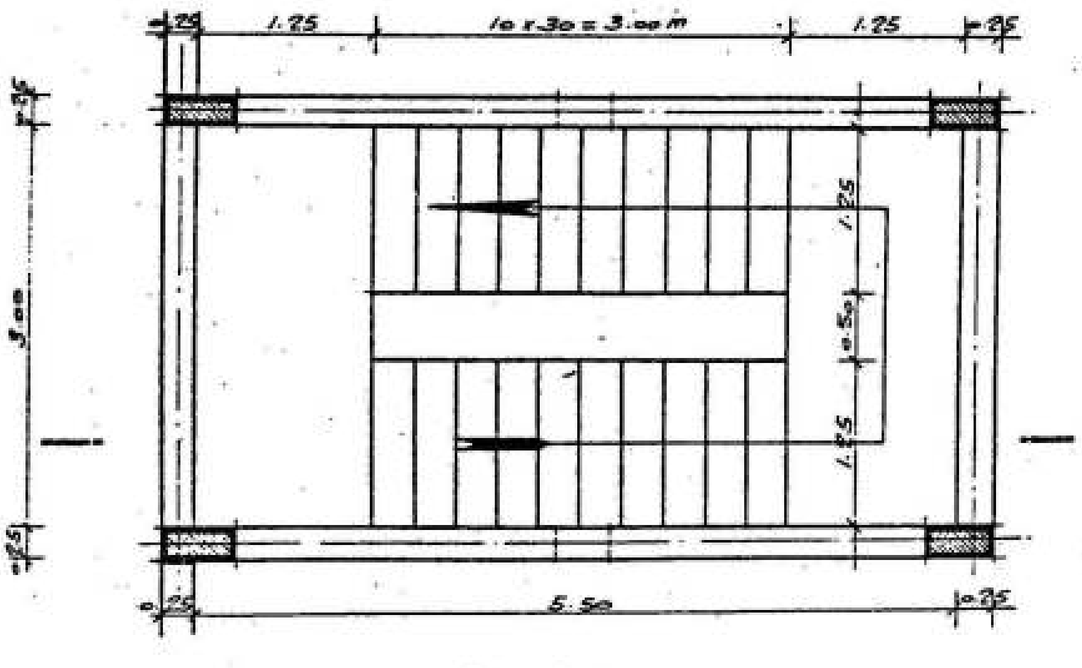

1.25
10 x .30 = 3.00 m
1.25
5.50

Fig. 7-2

ment building; the materials used are C200 and normal mild steel . The stair will be assumed of the cantilever type (Fig. 7-3).

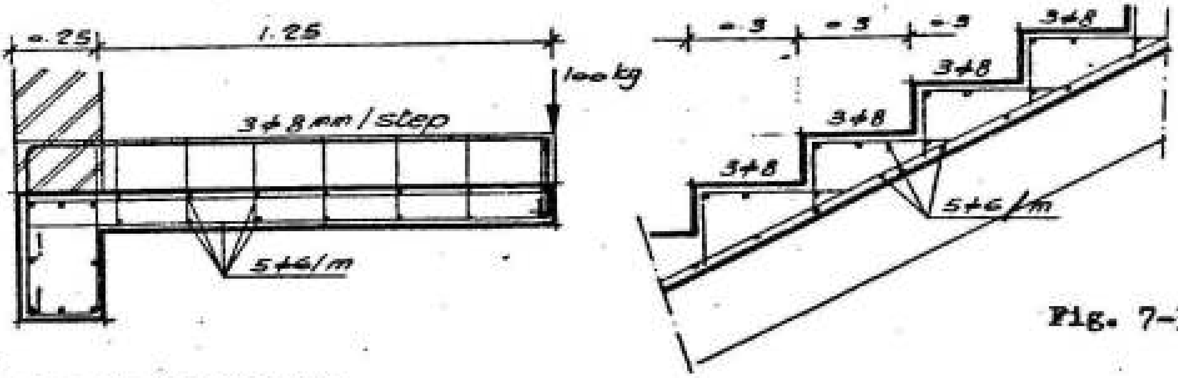


Fig. 7-3

Design of the flight

Each step in the flight, shown heavy, will be replaced by a rectangular one 30 x 20 cms as shown dotted in Fig. 7-4.

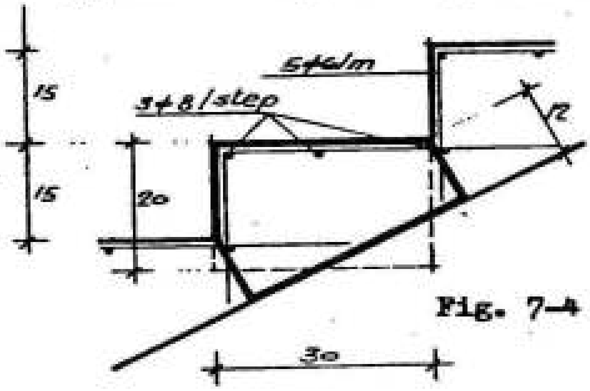


Fig. 7-4

Each of the assumed steps (30 x 20 cms) will be designed for its own weight, cover and plaster, a uniform load of 350 kg/m^2 and a concentrated load of 100 kg at its free edge. Therefore, the uniform load per step is given by:

Own weight	0.30 x 0.20 x 2500	= 150 kg/m
Cover 4 cms	0.30 x 0.04 x 2000	= 24 "
Plaster 2 cms	0.30 x 0.02 x 1500	= 9 "
Uniform live load	0.30 x 350	= 105 "
		288 "

Bending moment: $M_{max} = 288 \times \frac{1.25^2}{2} + 100 \times 1.25 = 350$ kgm/step

Dimensioning and reinforcement: t is assumed 20 cms, therefore d = 17.5 cms

$d = k_1\sqrt{M/b}$ or $17.5 = k_1\sqrt{\frac{350}{0.30}}$ giving $k_1 = 0.513$

According to table 4-5: For $\sigma_s = 1400$ kg/cm^2, $\sigma_c = 32.5$ kg/cm^2 and $k_2 = 1280$. So that $A_s = \frac{M}{k_2\ d} = \frac{350}{1280 \times 0.175} = 1.56$ cm^2 3 Ø 8 / step.

Design of landing

The thickness of the landing is chosen 12 cms. Assuming a concentrated load of 100 kg/m at the free edge, then

w = own weight + 6 cms sand + 4 cms cover + 2 cms plaster + 350 kg/m^2

= 0.12 x 2500 + 0.06 x 1500 + 0.04 x 2000 + 0.02 x 1500 + 350 or

$w = 850$ kg/m^2 and $M_{max} = 850 \times \frac{1.25^2}{2} + 100 \times 1.25 = 790$ kgm/m .

d = 10 cms then $10 = k_1 \sqrt{790}$ or $k_1 = 0.355$

Table 4-5 gives $\sigma_c = 52 < 60$ kg/cm^2 and $k_2 = 1245$ so that

$A_s = \frac{790}{1245 \times 0.10} = 6.35$ cm^2/m chosen 8 Ø 10 mm/m

Distributers 0.20 A_s = 0.20 x 6.35 = 1.27 cm^2 chosen 5 Ø 6 mm/m.

In addition, a mesh 5 Ø 6 mm/m is arranged in the lower fiber of the flights and landings. The details are shown in Fig. 7-3.

7.4 Cross-supported Stairs

If in a stair, it is possible to arrange the string beams B_1 and the cross beams B_2 (Fig. 7-5), then, both the flights and the landings are cross-supported; they act as simple beams with the span equal to the breadth of the flight or landing. This system is supposed to be very economic because the span of the stair is generally small.

For the case shown in Fig. 7-5, the flight acts as a simple one-way slab of span 1.30 ms supported on the wall-beam on one side and on the string beam on the other side.

In normal cases, a thickness t_s of 8 cms and main cross reinforcement of 6 Ø 8 mm/m are generally sufficient for the slab. Distributers may be chosen 5 Ø 6 mm/m. The details of reinforcements of a cross - section in the flight is shown in Fig. 7-6.

The string beams B_1 are inclined straight simple beams supported on the landing horizontal beams B_2. Their horizontal span is 3.15 ms.

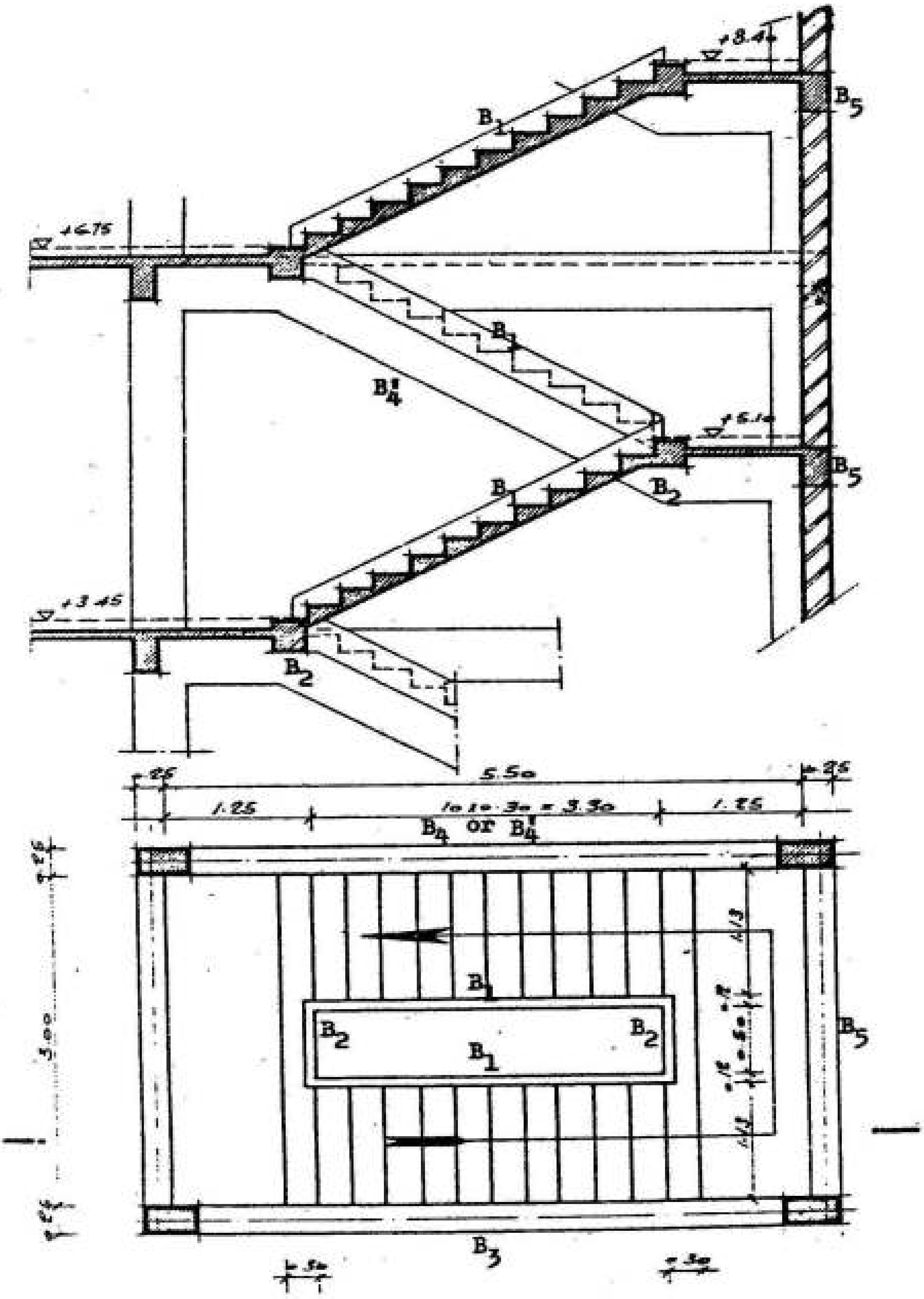

+8.4
B5
B1
+6.75
B1
B4'
+5.10
B1
B2
B5
+3.45
B2
5.50
.25
1.25
10 x .30 = 3.30
1.25
.25
B4 or B4'
B1
B2
B2
B1
B5
3.00
B3

Fig. 7-5

They carry their own weight plus the loads from the stairs. They may be

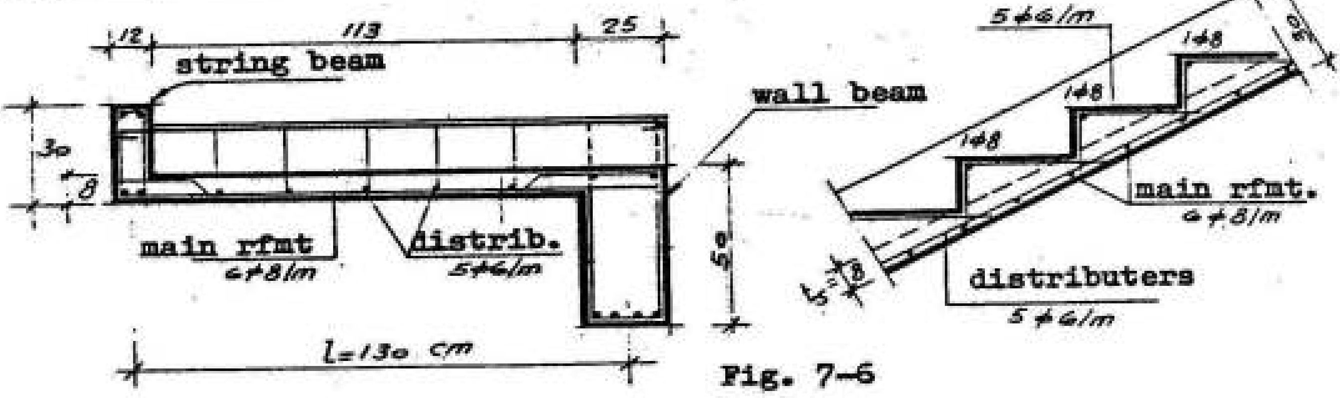


Fig. 7-6

below the stair slab or inverted as shown in Fig. 7-6.

The horizontal landing beams B_2 are simply supported on the wall-beams B_3 and B_4; span = 1.05 x 3.0 = 3.15 ms . They carry, in addition to their own weight plus the load from the landing, the two concentrated reactions of beam B_1 (Fig. 7-7).

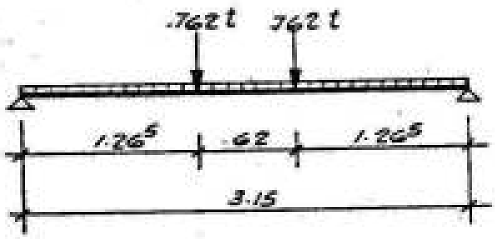
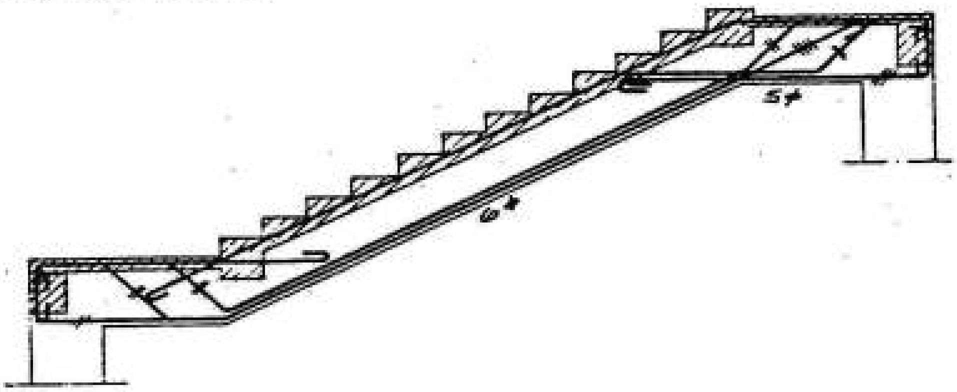


Fig. 7-7

The wall-beams are either straight (e.g. beam B_5) or broken (e.g. beams B_3 and B_4). They may be simple (e.g. B_3 and B_4) or continuous with the floor beams (e.g. beam B_4'). They carry their own weight, the wall load, the load from the stair slab plus the concentrated reactions of the landing beams B_2. Sketches of the reinforcements are shown in Figs. 7-8 a, and 7-8 b.



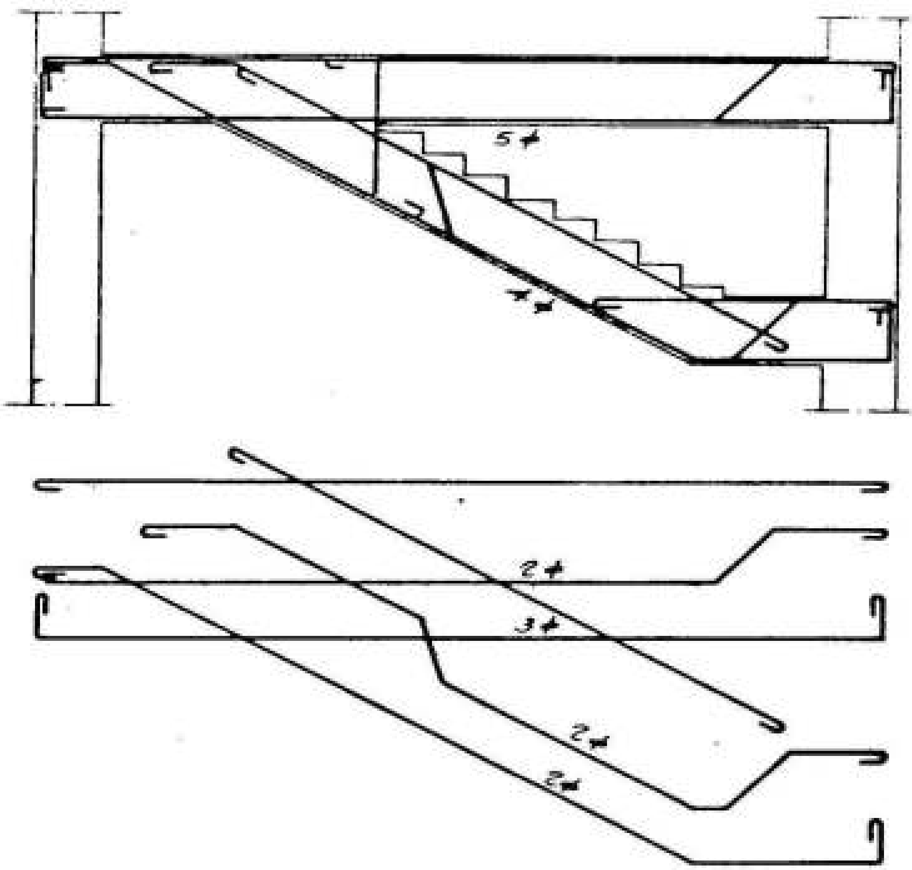


Fig. 7-8 b

7.5 Longitudinally Supported Stairs

In some cases, it is only possible to support the stair in the longitudinal direction (Fig. 7-9). In such cases, the span of the stair slab is l_s. This being relatively big, the bending moments ,the thickness t_s and the reinforcements will be big and the design may be uneconomic.

If l_s is say 4.50 ms and the load is @ 1100 kg/m^2, the maximum moment $M_{max} = 1100 \times 4.5^2/8 = 2750$ kgm . This needs a thickness t_s of @ 20 cms and main steel of ⌀ 16 mm every 15 cms.

Hence, this system is to be used only if there is no other more convenient system.

The details of reinforcements are shown in Fig. 7-10.

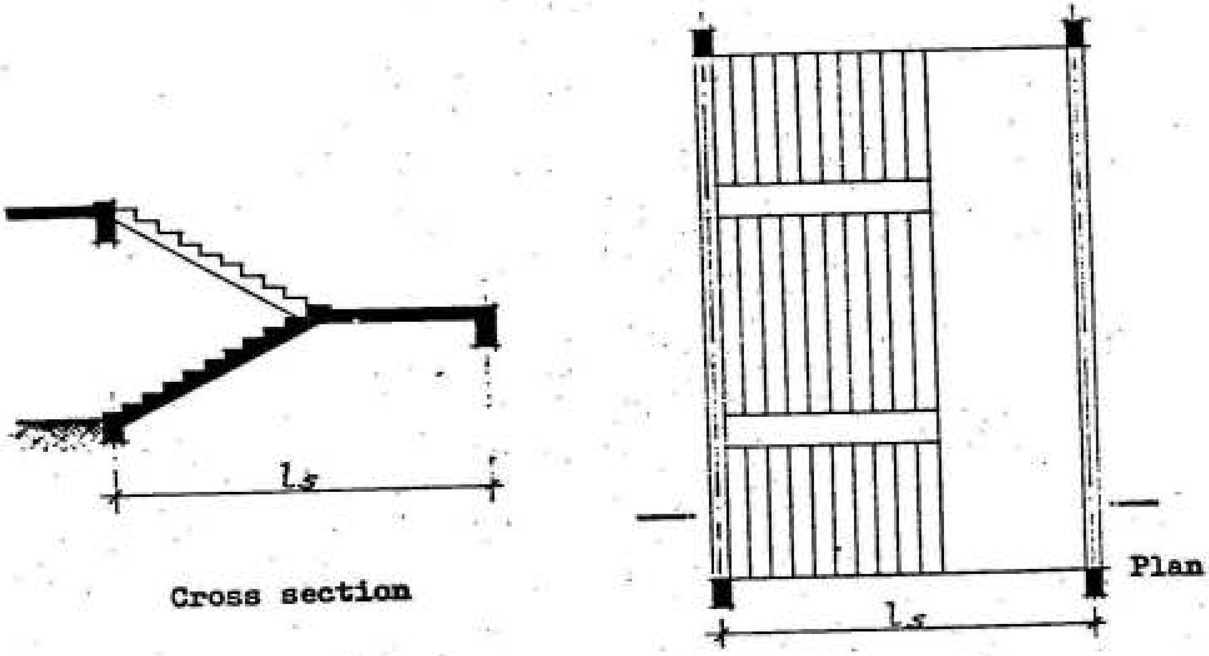


Fig. 7-9

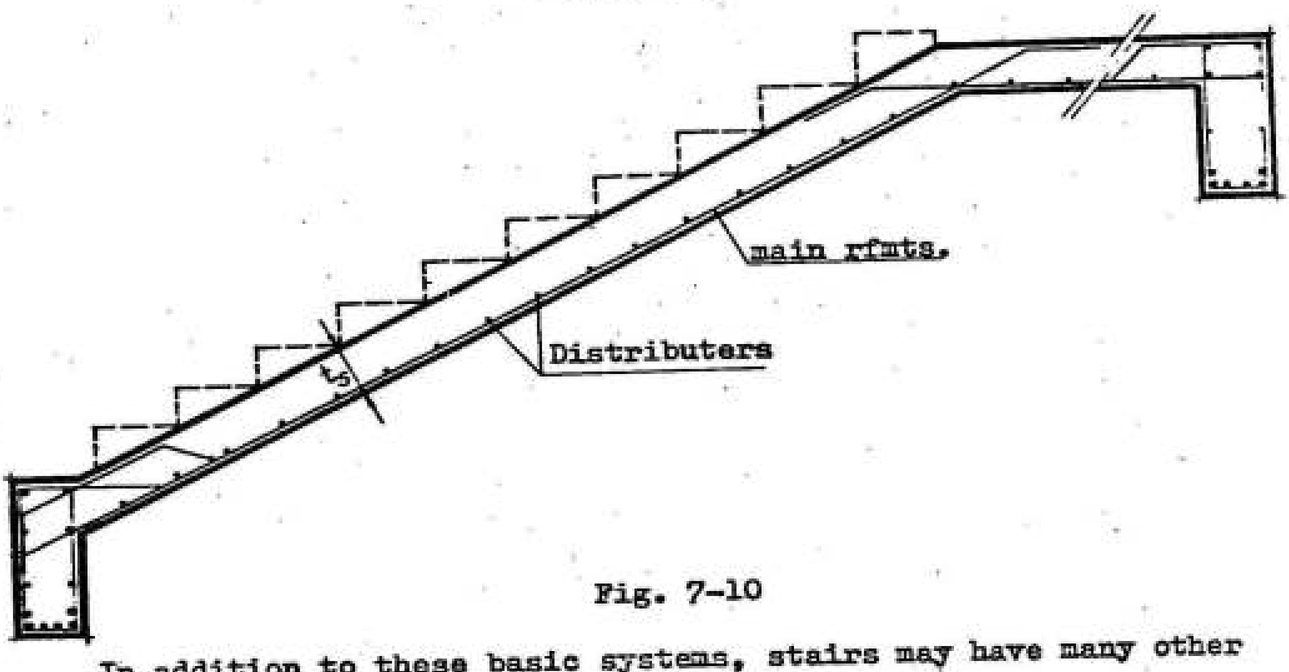
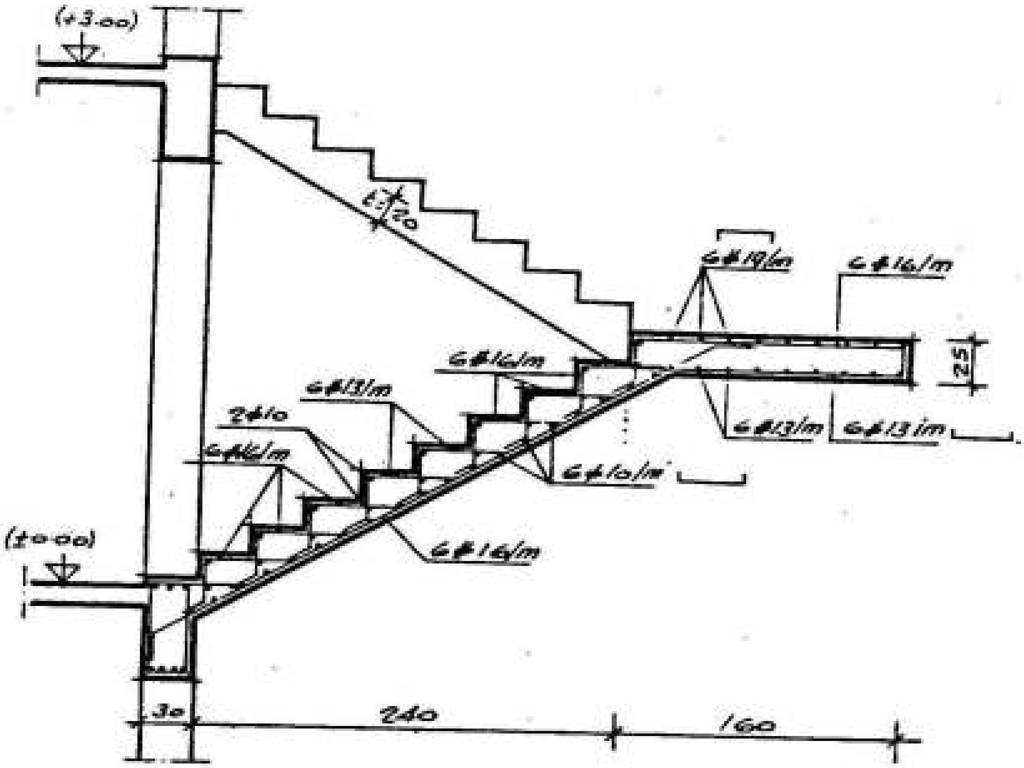


Fig. 7-10

In addition to these basic systems, stairs may have many other structural forms as, for example, the free standing stair shown in Fig. 7-11. This stair is supported on the lower and upper floors without any supporting beams or columns. Its slab is subject to axial forces, bending moments, shearing forces and torsional moments. The displacements of the intermediate landing have a big effect on the internal forces which must be carefully determined.



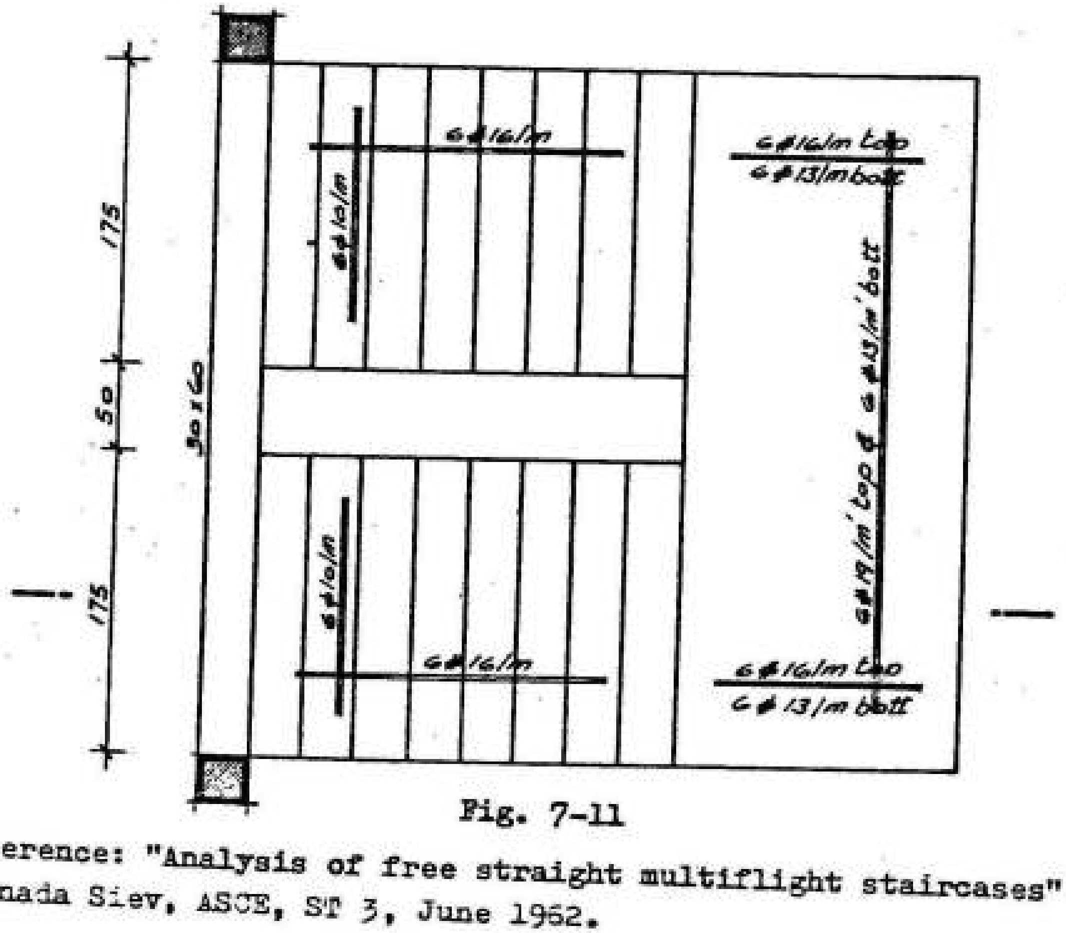


Fig. 7-11

* Reference: "Analysis of free straight multiflight staircases" by Auinada Siev, ASCE, ST 3, June 1962.

CHAPTER 8

PANELLED BEAMS

8.1 General Idea

In relatively big areas with $l_y/l_x \leqslant 1.5$, it might be convenient to support the covering slab by a system of beams of the same dimensions, called 'panelled beams' similar to those shown in Fig. 8-1.

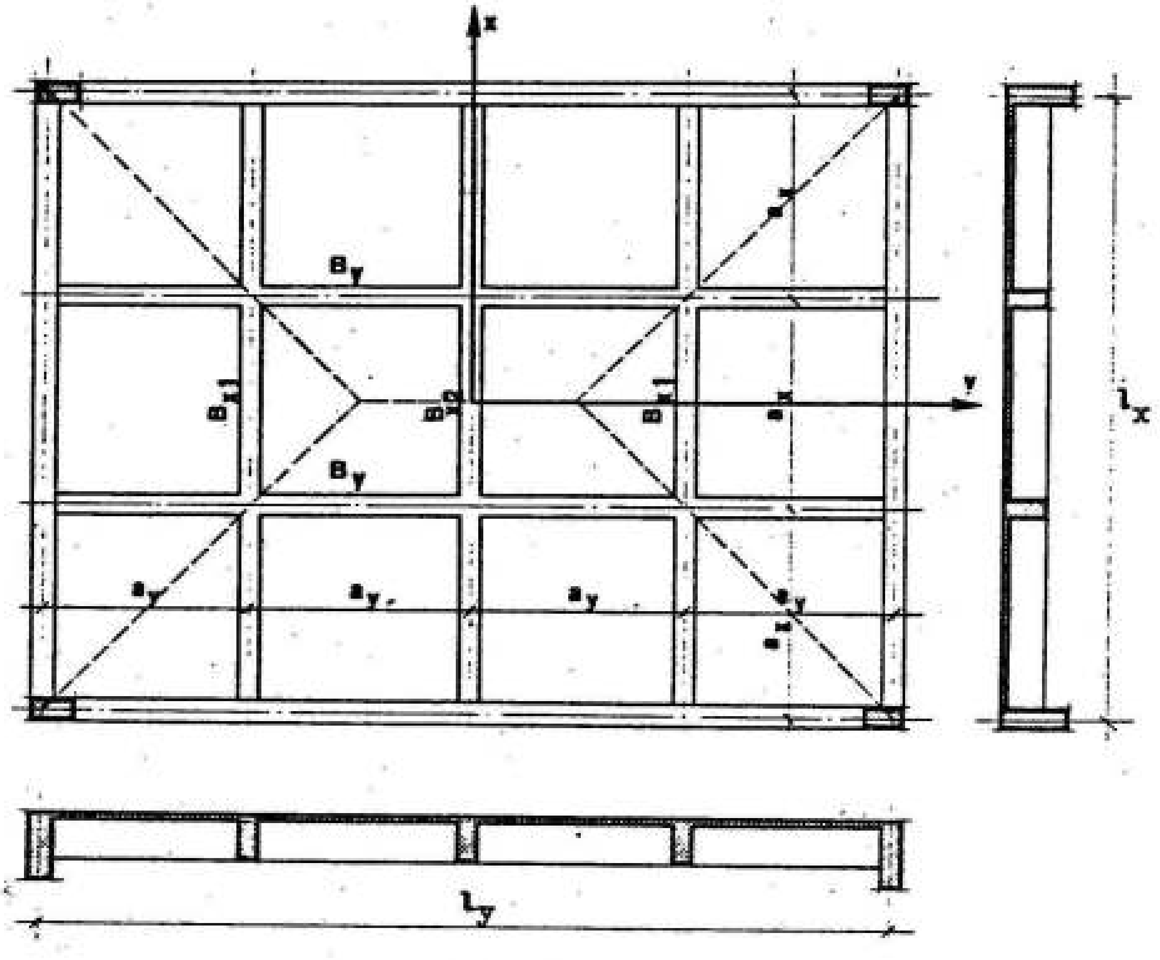


Fig. 8-1

8.2 Approximate Method of Calculation of Simple Panelled Beams

One of the simple approximate solutions of such a floor is to assume that it behaves like a two-way slab with spans l_x and l_y in which the load is transmitted to the outside main girders through the beams B_{x1}, B_{x2}, B_y ..... etc. according to the dotted lines shown in Fig. 8-1; this means that, if the torsional resistance is neglected, the load on the floor w will be distributed on both directions x and y in the ratio w_x and w_y where $w_x + w_y = w$ and $w_x = \alpha w$, $w_y = \beta w$.

The values of α and β depend on the ratio $\chi = l_y/l_x$ and are to be extracted, according to Grashoff, from table 5-1; i.e., the bigger load and bending moment shall be in the shorter direction x and the smaller load and bending moment shall be in the longer direction y. Each of the beams B_x is to be dimensioned for the corresponding total bending moments of the strip a_y and each of the beams B_y is to be dimensioned for the total bending moments of the strip a_x.

In order to simplify the calculations, the parabolic bending moments of a two-way simple slab will be replaced by trapeziums in the form shown in Fig. 8-2. That is:

The maximum bendingmoments and reactions of beam B_{x1} are equal to the areas M_{x1} and W_{x1} respectively,

The maximum bending moments and reactions of beam B_{x2} are equal to the areas M_{x2} and W_{x2} respectively, and

The maximum bending moments and reactions of beam B_y are equal to the areas M_y and W_y respectively. ... etc.

8.3 Example of a Simple Panelled Floor

A reinforced concrete floor with l_y = 12.00 ms and l_x = 9.00 ms is simply supported on all four sides, carries a live load of 300 kg/m^2 and is supported by a system of panelled beams, spaced at 3.00 ms in both directions as shown in Fig. 8-2. Design the beams and determine

$w_x = 0.76 \times 900 = 684\ kg/m^2$ and $w_y = 900 - 684 = 216\ kg/m^2$

Bending moments

Maximum bending moment in beam B_{x2} is given by:

$$M_{x2} = a_y \frac{w_x\, l_x^2}{8} = 3.0 \times \frac{684 \times 9^2}{8} = 3.0 \times 6940 = 20920 \quad kg\ m$$

Maximum bending moment in beam B_{x1} is given by:

$$M_{x1} = \text{area } M_{x1} = 0.96\ M_{x2} = 0.96 \times 20920 = 20000 \quad kg\ m$$

Neglecting the small reduction due to the trapezoidal distribution of bending moments, then

Maximum bending moment in beam B_y is:

$$M_y = a_x \frac{w_y\, l_y^2}{8} = 3.0 \times \frac{216 \times 12^2}{8} = 3.0 \times 3900 = 11700 \quad kg\ m$$

Shearing forces and reactions

$$W_{x2} = a_y \frac{w\, l_x}{2} = 3.0 \times \frac{900 \times 9}{2} = 3.0 \times 4050 = 12150 \quad kgs$$

$$W_{x1} = 1/2\ (4050 + 4050 \times 1.5/4.5) \times 3.0 = 8100 \quad kgs$$

$$W_y = 1/2\ (4050 + 4050 \times 1.5/4.5) \times 3.0 = 8100 \quad kgs$$

Design of beams

Beams B_x shall be designed for max. $M_x = 20900$ kg m; they behave as T-sections with breadth of flange $B = 12\ t_s + b = 12 \times 8 + 25 = 121$ cms

Assume $d = 73 - 6 = 67$ cms then $67 = k_1 \sqrt{\frac{20900}{1.21}}$ giving $k_1 = 0.51$. According to table 4-5, $\sigma_c = 38\ kg/cm^2$ and $k_2 = 1850$

so that $A_{sx} = \frac{20900}{1850 \times 0.67} = 16.9\ cm^2$ chosen $6\ \phi\ 19$

Beams B_y: $d = 65$ cms then $65 = k_1 \sqrt{\frac{11700}{1.21}}$ giving $k_1 = 0.66$. According to table 4-5, $\sigma_c = 28\ kg/cm^2$ and $k_2 = 1900$

so that $A_{sy} = \frac{11700}{1900 \times 0.65} = 9.5\ cm^2$ chosen $5\ \phi\ 16$

Normal mild steel may however be used in the longitudinal beams, in which case: σ_c is low , $k_2 = 1300$ and

$$A_{sy} = \frac{11700}{1300 \times 0.65} = 14.0 \ cm^2 \qquad \text{chosen } 5 \phi 19.$$

Shear stresses

$$\tau_x = \frac{12\ 150}{0.87 \times 25 \times 67} = 8.35 \ kg/cm^2 \ \& \ \tau_y = \frac{8100}{0.87 \times 25 \times 65} = 5.82 \ kg/cm^2$$

Both values are not high and the corresponding diagonal tension is to be determined in the normal way.

Details of beams

The details of beams B_x and B_y are shown in Fig. 8-3.

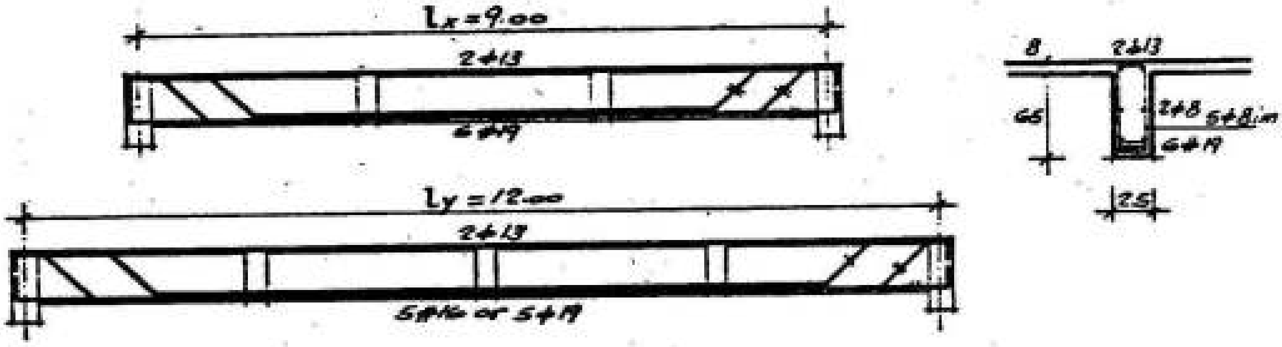


Fig. 8-3

Reactions of panelled beams

The reactions of the panelled beams B_x and B_y on the outside main girders are shown in Fig. 8-4.

Fig. 8-4

8.4. Continuous Panelled Beams

Continuous panelled beams behave as continuous or partially fixed slabs in which the moment distribution is as shown in Fig. 8-5.

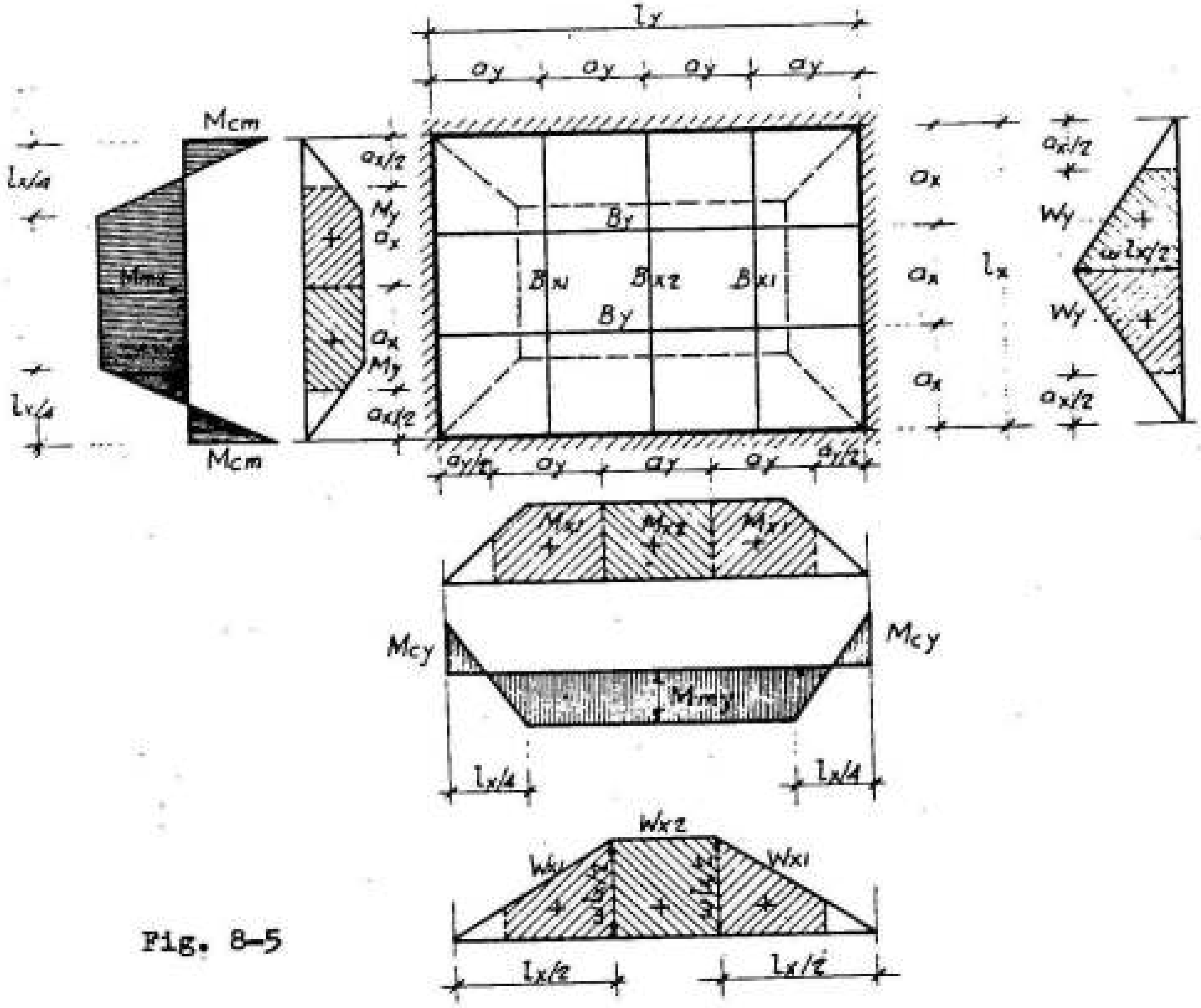


Fig. 8-5

The figure shows that the bending moments of continuous panelled beams is the same as that of continuous slabs for one meter strip multiplied by a_y in the x-direction and by a_x in the y-direction.

For symmetrical conditions at the supports, the reactions of panelled beams are the same as the previous case.

CHAPER 9

REINFORCED CONCRETE FOOTINGS

The footing is the underground part of the structure which transmits the load of the superstructure to the underlying soil. It must be so chosen that the settlement is small and as uniform as possible. Relatively big differential settléments may cause severe damage to the superstructure and hence are to be avoided.

In order to satisfy these requirements, it is essential to transmit the load to a firm stratum and to spread the load on a sufficiently large area so that the bearing pressure is within the allowable limits.

If adequate firm layers are near ground surface, spread footings give the most economic solution.

We recognise:

9.1 WALL FOOTING

It is simply a strip of reinforced concrete wider than the wall to distribute its pressure on the soil as shown in Fig. 9-1.

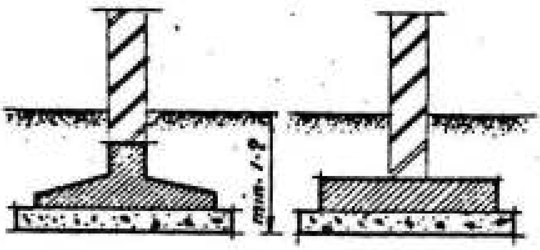


Fig. 9-1

It is generally not allowed to construct a reinforced concrete footing directly on clayey soils; a plain concrete layer 20 to 40 cms (and sometimes more) underneath the reinforced concrete footing is recommended as it leads generally to a more economic solution. The area of the plain concrete footing is simply equal to the load on the wall plus the own weight of the footing divided by the allowed soil pressure. The own weight of the footing may, in this stage be estimated by 4 to 6 % of the wall load, the former value applies to the stronger type of soil. The area of the reinforced concrete footing is chosen such that the bearing pressure underneath it is smaller than 5 to 10 kg/cm^2 and the maximum tensile stress σ_t in the lower surface of the plain concrete footing is smaller than @ 4.0 kg/cm^2.

This last condition gives the projection x of the plain concrete footing relative to its thickness t according to Fig. 9-2 as follows:

Assuming that the allowable soil pressure is σ_b, then the maximum bending moment at section a - a is given by:

$$M_a = \tfrac{1}{2}\sigma_b x^2$$

and the maximum tensile stress σ_t at the bottom of section a - a is

$$\sigma_t = 6 M_a/t^2$$

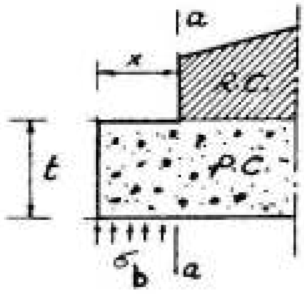


Fig. 9-2

Substituting for M_a, we get:

$$\sigma_t = 3\,\sigma_b\, x^2/t^2$$

Therefore

$$(x/t)^2 = \sigma_t/3\,\sigma_b$$

Assuming, for example σ_t = 3.0 kg/cm², then

$$x/t = \sqrt{1/\sigma_b}$$

Hence, one may safely assume that:

For σ_b=	1	1.25	1.50	2.00	2.50	3.00	kg/cm²
x =	1	0.90	0.80	0.70	0.63	0.58	t

The governing factors for the choice of the dimensions of the reinforced concrete footings are the economic considerations, the chosen thickness of the plain concrete footing and the allowable soil pressure, rather than the allowable bearing stress on the plain concrete footing.

Design of Wall Footing

If the reinforced concrete footing is supporting a concrete wall or connected to a reinforced concrete wall beam, then the max. moment and max. shear take place in section I-I at the face of the wall.

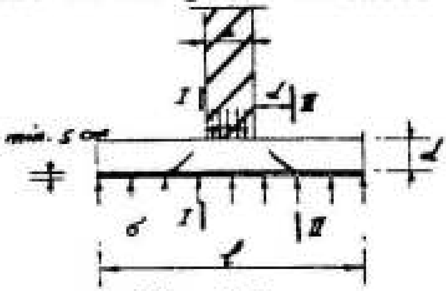


Fig. 9-3

$M_I = \frac{1}{8}\,\sigma\,(l - a)^2$ $\qquad d = k_1\sqrt{M_1/b}$ $\qquad t = d + \phi/2 + 5$ cm

$Q_I = \frac{1}{2} \sigma \ (l - a)$ $\qquad \tau_b = Q_I / \Sigma\, O\, y_{CT}$ $\qquad \tau_b$ = bond stress

The max. shear stress takes place at Sec. II - II at a distance d from the face of the wall.

$Q_{II} = \sigma (\frac{l - a}{2} - d)$ $\qquad \tau = Q_{II} / b\, y_{CT} < 6 - 8 \ kg/cm^2$

For masonry walls directly supported on the footing, the maximum moment is assumed to take place at a section midway between the centre line and the face of the column.

9.2 ISOLATED COLUMN FOOTINGS

They are simple cantilever slabs projecting out from the column in both directions and loaded upward by the soil pressure.Single isolated column footings are generally square in plan. Footings supporting rectangular columns may be rectangular. It is recommended to choose them such that the projections from the face of the column are approximately the same in both directions. They may be of constant or variable thickness as shown in Fig. 9-4.

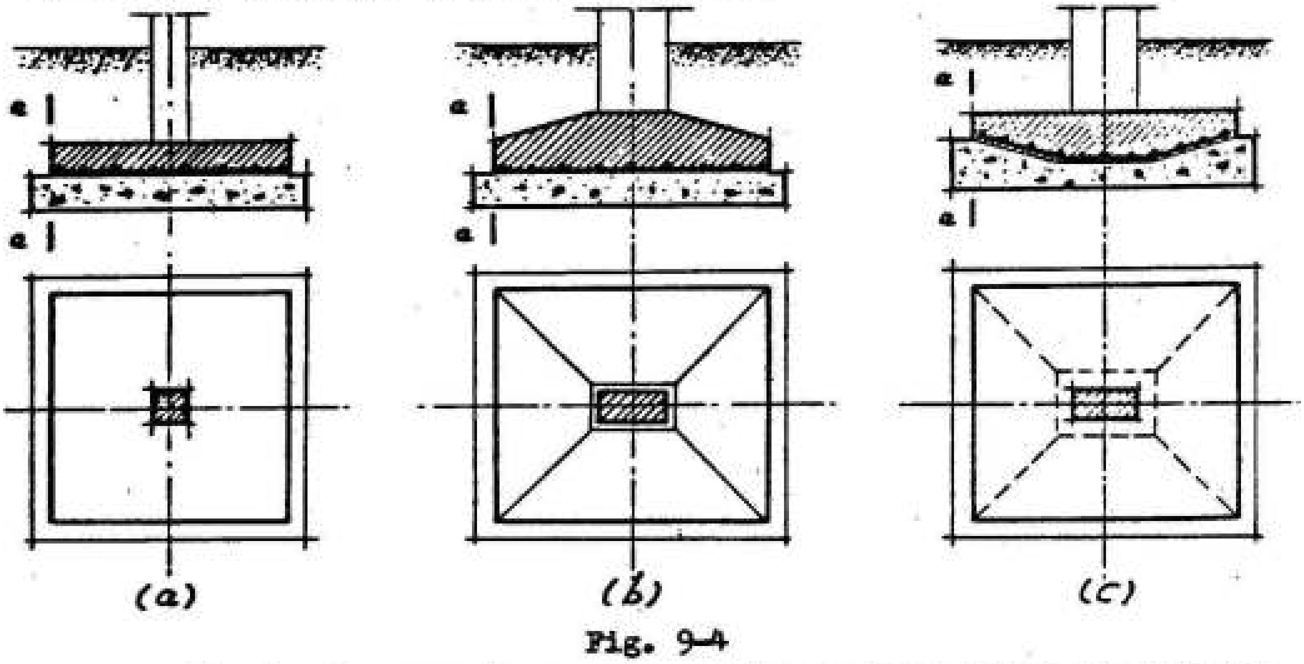


Fig. 9-4

Due to the upward pressure acting on the reinforced concrete footing, corresponding tension stresses are caused in its bottom surface. Such footings are therefore reinforced by two layers of steel perpendicular to each other and generally parallel to the edges; the fibers of the footing above the neutral axis are subject to compressive stresses with max. values at the extreme upper fiber. As these compressive stresses are resisted by the concrete it seems to the author that the form shown in figure 9-4c is structurally more convenient than the normal form, shown in figure 9-4b., generally used in footings of variable thickness. In this new form, the upper fiber that is most stressed extends over the whole width of the footing; in addition, the thickness of the plain concrete footing is maximum at section a - a (fig. 9-4c.) where the maximum bending moments and the maximum tensile stresses σ_t occur.

In order to prevent differential settlements of footings on medium soils it is recommended to join them by stiff plate or vierendeel girders as can be seen in the following two examples :

Example 1 : <u>Foundations of the Extension of El Haram at Medina.</u>

Fig. 9-5

The soil at the position of El Haram at Medina is fill for a depth of 4 ms. underlaid by medium clays that can support an allowable bearing pressure of 1.5 kg/cm^2 . In order to avoid any noticeable differential settlements, it was decided to connect the columns in both directions by stiff girders. In order to satisfy this requirements, the girders were chosen 0.25 ms. thick and 1.50 ms. deep giving a very big moment of inertia and reinforced with 4 Φ 22 at top & 4 Φ 22 at bottom, the vertical stirrups were 5 Φ 8/m and the horizontal reinforcements around the openings were 4 Φ 10 giving a total amount of $\sim$ 100 kg steel per meter cube of concrete. In order to increase the stiffness of the whole system of foundations against differential settlement, the footings and the portions of the columns under the ground were rotated 45° as shown in figure 9-5 to give gradual smooth effective connection with the columns.

Example 2 : <u>Foundations of the T - V Buildings at Mansurah and Abis</u>

The site of these buildings is composed of 1.2 ms. agricultural soil underlaid by weak clays that can support 0.5 kg/cm^2 soil pressure only. The ground water level lies at 1.0 m from the ground surface and the good incompressible sandy layers exist at 10 - 20 ms from it. The finished floor level lies at + 1.5 ms. from the ground level. It has been found unfeasible and very uneconomic to use deep pile foundations. It was decided to construct the foundations in the following manner (Fig. 9-6).

1) The foundation level was chosen at 1.2 ms. from ground level at the top surface of the medium clay.

2) The foundations of the columns were isolated footings composed of a plain concrete layer 25 cms. thick - so that its top

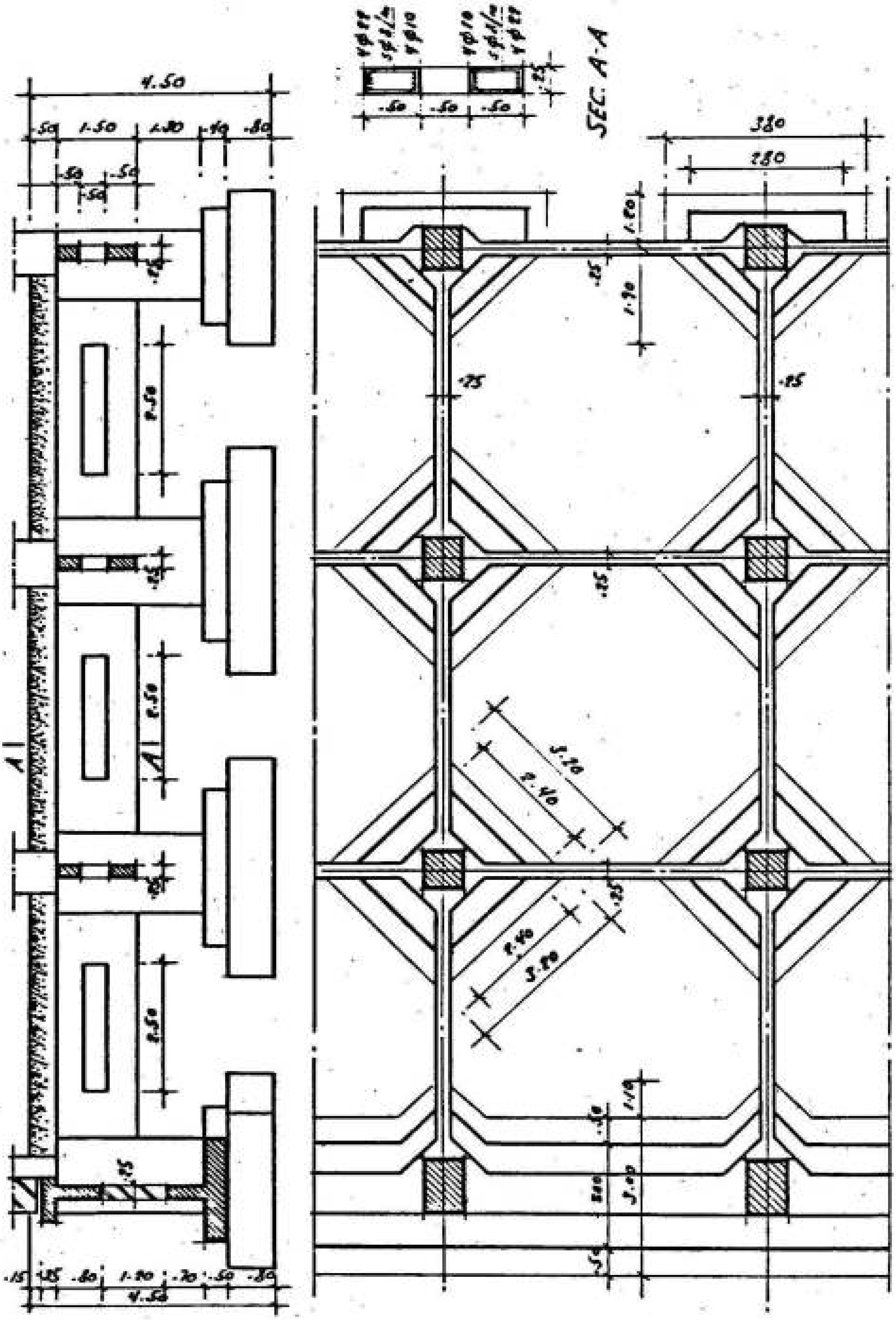


Fig. 9-5

surface is 5 cms above the ground water - and a reinforced concrete square or rectangular footing 30 - 40 cms thick.

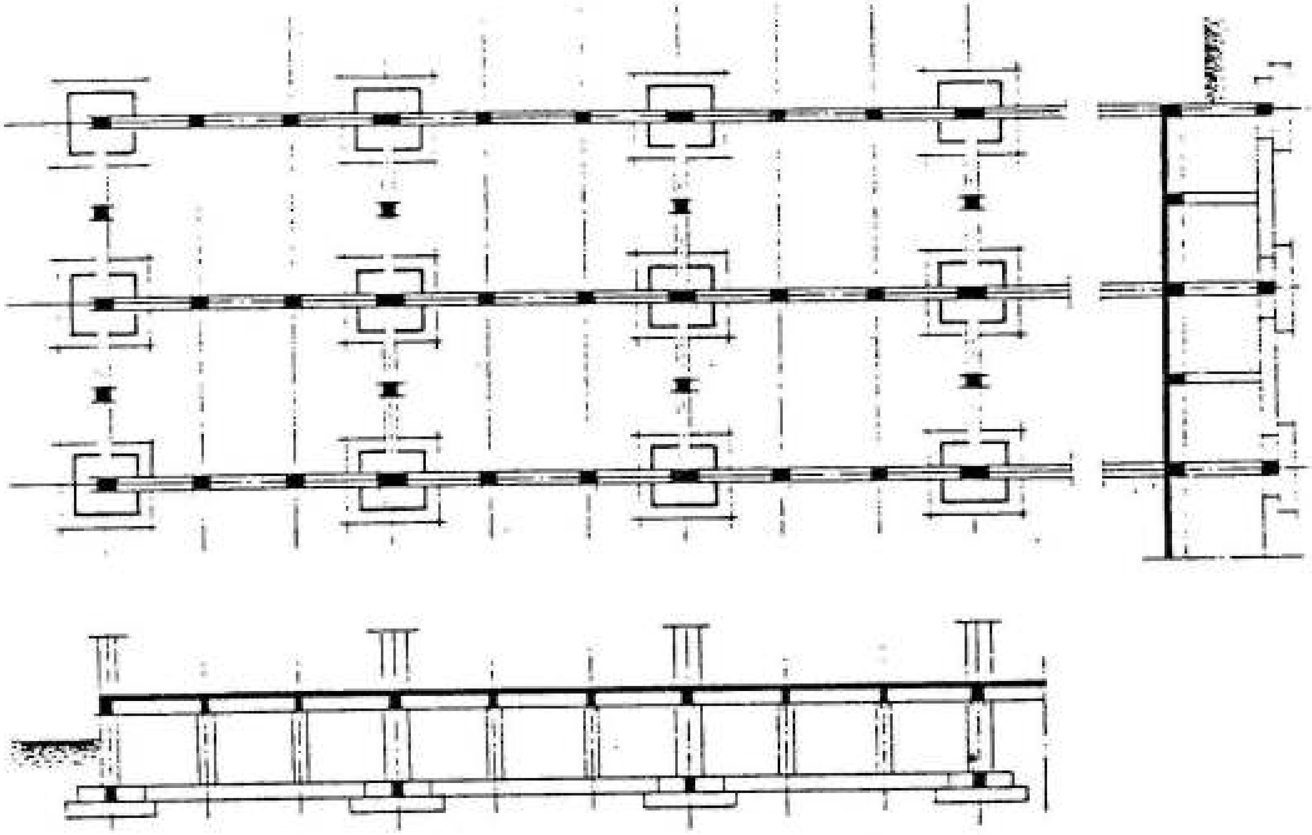


Fig. 9-6

3) The bearing pressure on the soil was chosen 0.5 kg/cm^2

4) In order to prevent the possible differential settlements, the main columns were connected in both directions by Vierendeel girders with their bottom chord at the level of the reinforced concrete footings and top chord at the ground floor level, the verticals were arranged every 2 - 3 ms.

5) The ground floor was made of reinforced concrete ; the volume underneath was left without fill to allow for any settlement without exerting any upward pressure on the ground floor.

6) The openings in the outside and some main intermediate Vierendeel girders were built in with masonry. In order to get a complete interaction between the masonry and the reinforced concrete, the walls were first built on the bottom chord, with keys at the position of the columns and posts. After placing their longitudinal reinforcements and hoops, their concrete was poured in the masonry then the top chord was constructed on the top of the wall.

Design of Isolated Column Footing

The simplest design is given by the A.C.I. code of practice and its application is explained by Winter , Urquhart, O'Rourke, Nilson in their text book on " Design of Concrete Structures "

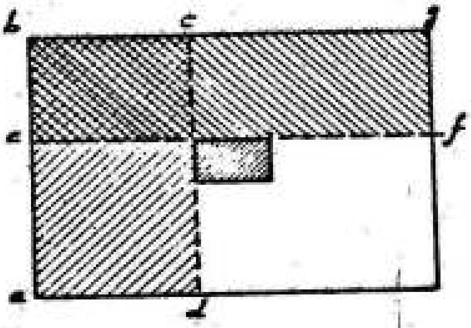


Fig. 9-7

The bending moment on any vertical section through the footing say c-d. is that caused by the upward pressure on the area to one side of the section i.e. the area a b c d. The reinforcement perpendicular to that section, i.e. the bars running in the long direction are calculated from this bending moment. Similarly, the moment about section e-f is caused by the pressure on the area b e f g , and the reinforcement in the short direction, i.e. perpendicular to e-f is calculated for this bending moment. In footings supporting reinforced concrete columns, these critical sections for bending are located at the faces of the column as shown in figure 9-7.

In square footings, the reinforcement is uniformly distributed over the width of the footing in each of the two layers, i.e. the spacing of the bars is constant.

In rectangular footings, the reinforcement in the long direction is again uniformly distributed over the shorter width. In locating the bars in the short direction, one has to consider that the support provided to the footing by the column is concentrated near the middle. Consequently, the curvature of the footing is sharpest i.e., the moment per meter largest, immediately under the column and it decreases in the long direction with increasing distance from the column. For this reason a larger steel area per longitudinal meter is needed in the central portion than near the far ends of the footing. The A C I code provides, therefore, that :

In the case of the reinforcement in the short direction that portion (of the reinforcement) determined by the following equation shall be uniformly distributed across a band-width centered with respect to the column and having a width equal to the length of the short side of the footing. The remainder of the reinforcement shall be uniformly distributed in the outer portion of the footing.

$$\frac{\text{Reinforcement in band width B}}{\text{Total reinforcement in short direct.}} = \frac{2}{S + 1}$$

in which S is the ratio of the long side to the short side of the footing.

The critical sections for bond are the same as those for bending

A column supported by the slab of a simple footing tends to punch through that slab because of the shear stresses which act in the footing around the perimeter of the column, If failure occurs, the fracture takes the shape of a truncated pyramid. (Fig. 9-8) with sides sloping outward at approximately 45° The average shear stress in the concrete which fails in this manner can be taken as that which acts on vertical planes laid through the footing at a perimeter a distance d/2 from the faces of the column (vertical section through a b c d). therefore the nominal shear stress in the concrete in this critical perimeter section is

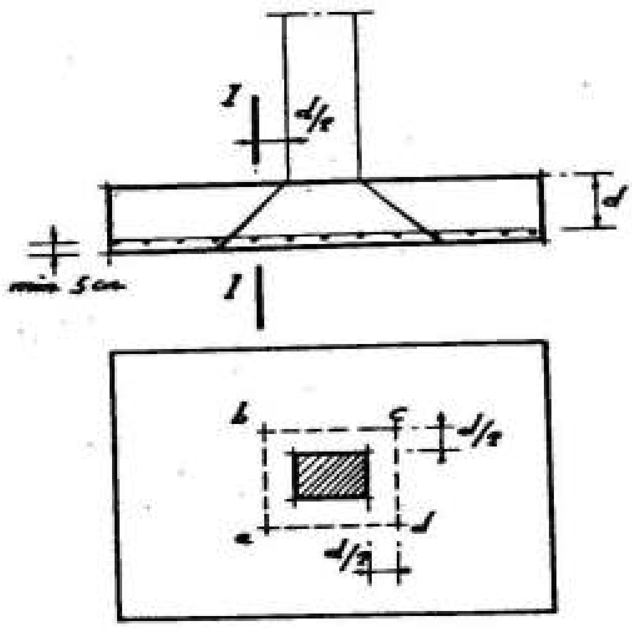


Fig. 9-8

$$\tau_p = Q_1 / b\, d \quad 6 \div 8 \ \text{kg/cm}^2$$

where b = length of perimeter of critical section (a b c d in Fig. 9-8).

Q_1 = shear force acting on the same section i.e.column load minus net upward soil pressure on area a b c d .

It has to be noted that the maximum bearing stresses are to be calculated for the column load plus the weight of the footing, while the internal forces, B. Ms. and S. Fs., are to be calculated for the net bearing pressure ; i.e. the column load only divided by the area of the footing.

Example :

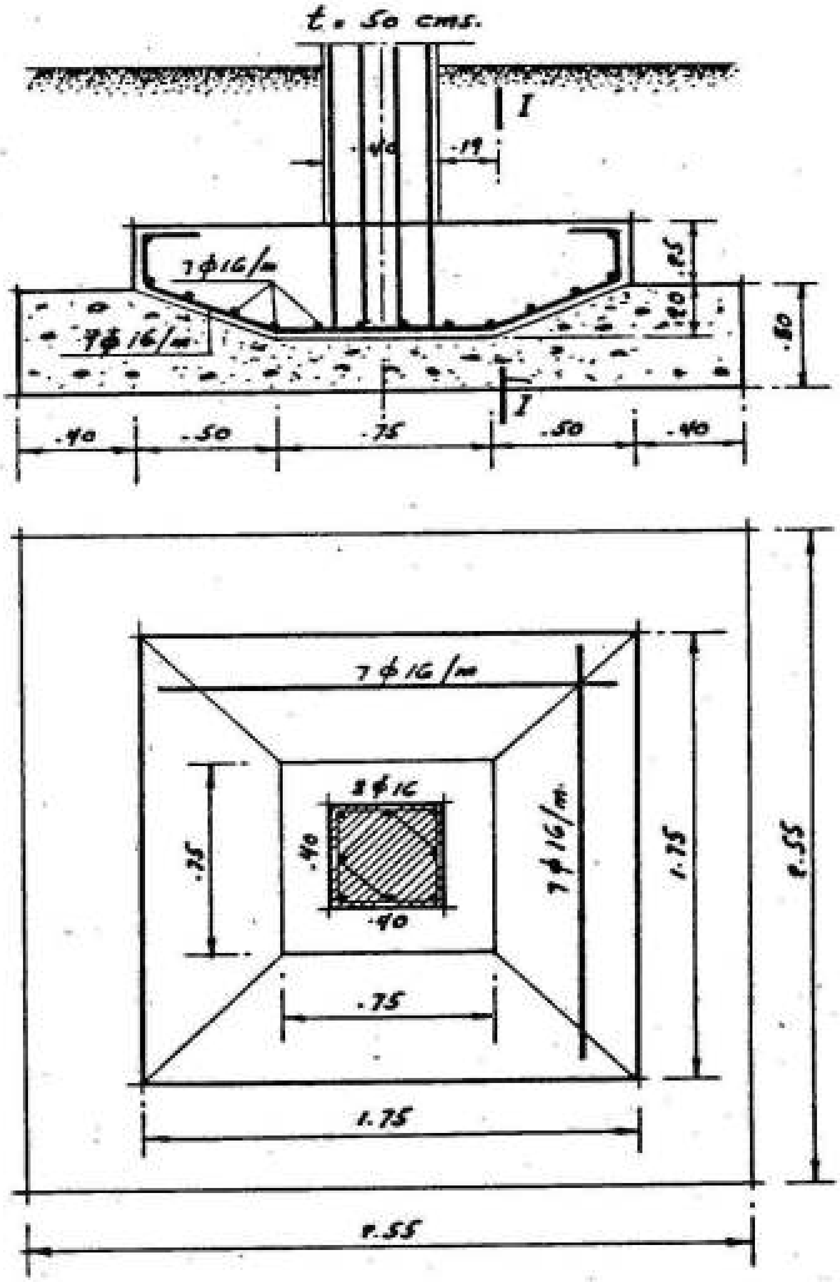
It is required to design an isolated footing for a column 40 x 40 cms. reinforced by 8 Φ 16 and supporting an axial load of

N = 90 tons. Assume the allowable stress is equal to 1.5 kg/cm^2 at a depth of 1.5 ms. from ground surface.

Assume own weight of footing ~ 8% of the load, therefore

The total load N_t = 97^t

Area of plain concrete footing = N_t / σ_b = 97/15 = 6.45 m^2



DETAILS OF AN ISOLATED FOOTING

Fig. 9-9

The plain concrete footing is chosen 2.55 x 2.55 ms, with a maximum thickness at the edge of 50 cms.

According to table (page 18), one finds that for $\sigma_b = 1.5$ kg/cm^2, x = 0.8 t and the reinforced concrete footing is therefore 1.75 x 1.75 ms. The net bearing stress on the plain concrete is accordingly given by: $90 / 1.75 \times 1.75 = 29.6$ t/m^2.

Max. B.M. at face of column: $M_{max} = \frac{29.6}{2} (\frac{1.75}{2} - 0.2)^2 = 6.75$ mt .

Assuming $\sigma_c = 50$ kg/cm^2 and $\sigma_s = 1400$ kg/cm^2, the depth required for bending moment is given by:

$d = 0.361 \sqrt{6750} = 30$ cms chosen thickness t = 40 cms.

The punching shear stress is to be checked at section I-I (Fig. 9-9) at a distanse d/2 = 16.5 cms from the face of the column. Hence

Total shear along perimeter $Q_I = 90 - 0.73^2 \times 29.6 = 74.20$ tons

Punching shear stress $\tau_p = 74200 / 4x73x33 = 7.7$ kg/cm^2 big.

Try a total max. depth t = 45 cms i.e. d = 38 cms; then :

The punching shear is $Q_I = 90 - 0.78^2 \times 29.6 = 72.0$ tons and

Pun hing shear stress $\tau_p = 72000 / 4x78x38 = 6.0$ kg/cm^2 safe.

Required tension steel is then $A_s = 6750 / 1265 \times 0.38 = 14$ cm^2 chosen 7 Ø 16/m

The details of the footing are shown in Fig. 9-9.

9.3 COMBINED FOOTINGS

Combined footings are used under two or more columns if they are closely spaced and heavily loaded so that if single footings were provided, they would completely or nearly merge. Fig. 9-10.

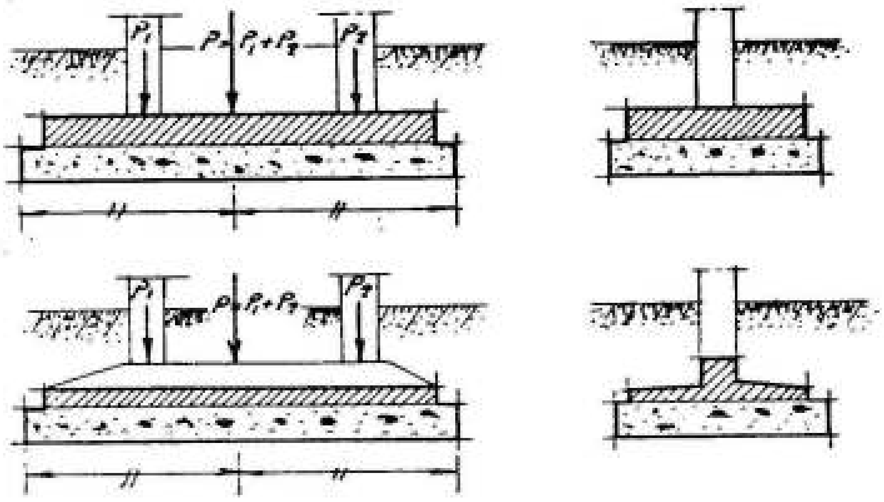


Fig. 9-10

They are also used if footings of exterior columns cannot project sufficiently beyond the face of the column. (Fig. 9-11)

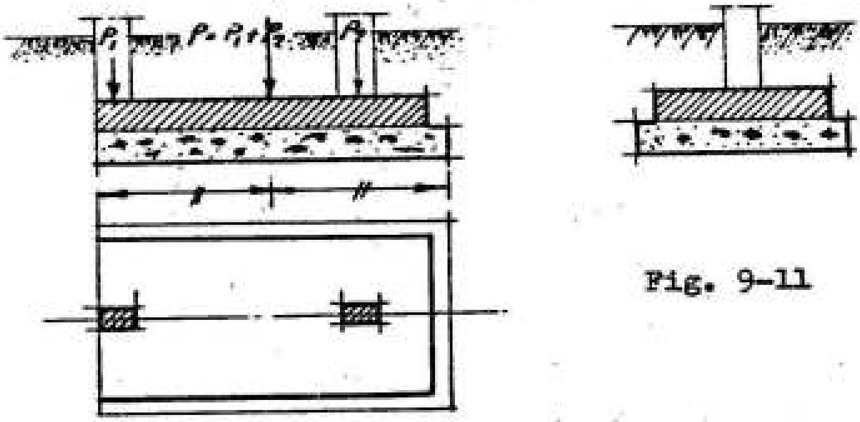


Fig. 9-11

In order to prevent tilting of the combined footings, it is recommended to have the line of action of the resultant of the column loads coinciding on the center of gravity of the footing.

Design of Combined Column Footings

An exterior column 40 x 60 cms. with a total load of 100 tons and an interior column 40 x 75 cms. with a total load of 180 tons are to be supported on a combined rectangular footing whose exterior cannot protrude beyond the outer face of the exterior column. The distance center to center of columns is 4 ms. The allowable soil pressure is 2.0 kg/cm^2. (Fig. 9-12)

The center of gravity of the total load lies at a distance c from the interior column where

c = 100 x 4 / 280 = 1.43 ms

In order to have uniform pressure on the soil, the total load of 280^t must coincide on the center of gravity of the plain concrete footing. Hence

Length of plain concrete footing = 5.74 ms.

Assuming own weight of footing ~ 10% of total load N_t, then

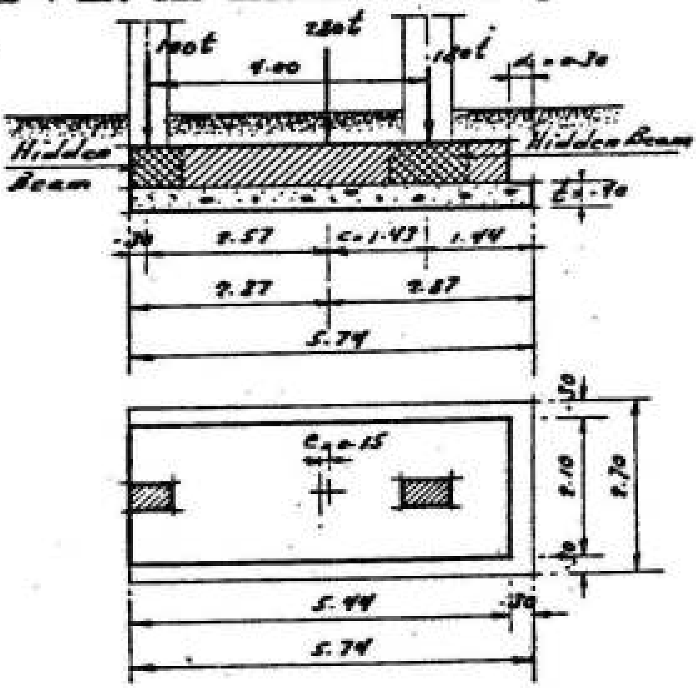


Fig. 9-12

N_t = 280 + 28 = 308 ton

Area of plain concrete footing = N_t / σ_b = 308 / 20 = 15.4 m^2

Breadth of plain conc. footing = 15.4 / 5.74 ≅ 2.7 ms

For a soil stress of 2 kg/cm^2 , the distance x according to table on page (18) is equal to 0.7 t. Assuming t = 40 cms. then x ≅ 30 cm

The dimensions of the reinforced concrete combined footing are :

5.44 x 2.10 ms.

The total load of 280 tons is in this manner eccentric with respect to the reinforced concrete footing.

The eccentricity e = 5.44 / 2 - (1.43 + 1.44 - 0.3)=2.72 - 2.57=0.15ms

The net bearing pressure on the plain concrete footing is therefore given by (Fig. 9-13)

$$\sigma_{\frac{1}{2}} = \frac{280}{5.44 \times 2.1} \pm \frac{6 \times 280 \times 0.15}{2.10 \times 5.44}$$

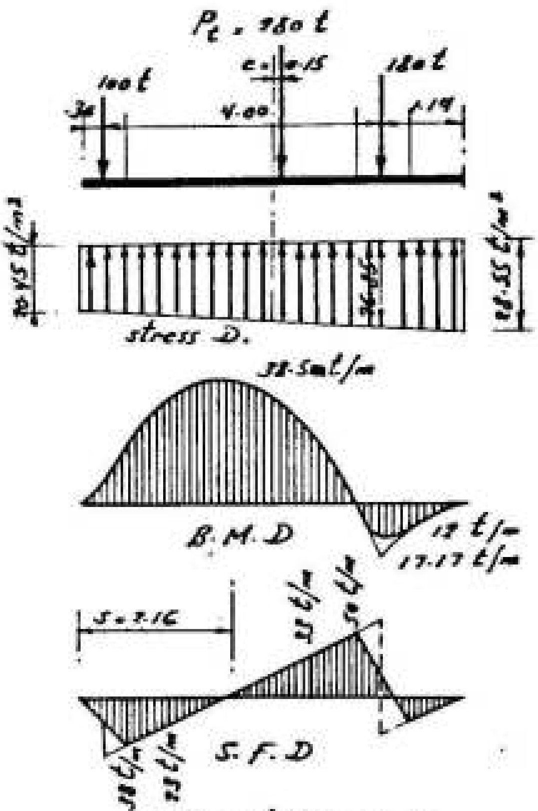
$$= 24.5 \qquad \pm \; 4.05$$

Hence

σ_1 = 24.5 + 4.05 = 28.55 t/m^2

σ_2 = 24.5 - 4.05 = 20.45 "

In the longitudinal direction,the footing represents an upward loaded beam spanning between columns and cantilevering beyond the interior column. Since this beam is considerably wider than the columns, the columns loads are distributed crosswise by transverse hidden beams, one under each column. It may however be assumed that the effective width of this beam is equal to that of the column plus, half the depth of the footing on each side.



Internal Forces in continuous Footing

Fig. 9-13

The maximum cantilever moment M_c at center line of column is given by :

$$M_c = 26.85 \times \frac{1.14^2}{2} + (28.55 - 26.85) \times \frac{1.14}{2} \times \frac{1.14}{3} = 17.17 \text{ mt/m.}$$

The maximum field moment M_m lies at point of zero shear which lies at a distance s from the left edge of the footing.

Hence

$$\frac{100}{2.1} = 20.45\ s + (\ 28.55 - 20.45\ )\ \frac{s}{5.44} : \frac{s}{2}$$

or

$$47.55 = 20.45\ s + 8.1 \times \frac{s^2}{10.88} \qquad \text{and} \qquad s = 2.16\ \text{ms}$$

$$M_m = 47.55\ (2.16 - 0.3) - 20.45 \times \frac{2.16^2}{2} - 8.1 \times \frac{2.16}{5.44} \times \frac{2.16}{3} = 38.5\text{mt/m}$$

The load of the exterior column per meter = 100/2.1 = 47.55 t
" " " " interior " " " = 180/2.1 = 85.55 t

The shearing force diagram is approx. as shown in Fig. 9-13.
The depth of the reinforced concrete footing is to be determined for the maximum field moment M_m = 38.5 mt.

Hence for σ_c = 70 kg/cm^2 σ_s = 2000 kg/cm^2

d_m = 0.32 $\sqrt{38\ 500}$ = 62.5 cms t = 70 cms.

A_s = 38500/1800 x .625 = 34.2 cm^2 9 Φ 22/m

Nominal mild steel reinforcement at center line of intermediate column.

A_s = 12000/1300 x .625 = 14.7 cm^2

The minimum allowed tension steel is 0.25% of the section,hence min. A_s = 0.25 x 100 x 62.5 = 15.6 cm^2 chosen 5 Φ 16 + 5 Φ 13

The reinforcements in the cross direction of the footing are to be minimum 20% of the longitudinal reinforcements.

Hence,
Cross reinforcement in top fiber : 0.2 x 34.2 = 6.84 cm^2 5 Φ 13/m
From the shear diagram of Fig. 9-13, the shearing force Q. at a distance d to the left of the left face of the interior column is approximately equal to 33 tons. The maximum diagonal tensile stress is therefore

τ = Q / 0.87 b d = 33000/0.87 x 100 x 62.5 = 6.0 kg/cm^2 safe.

Additionally as in isolated footings, punching shear should be checked on a perimeter section a distance d/2 around the column. Of the two columns, the exterior one with a three sided perimeter a distance d/2 from the column is more critical in regard to this " punching shear ". The perimeter is

b = 2 (60 + 62.5/2) + (40 + 62.5) = 2 x 91.25 + 102.50 = 285 cms and the punching force Q_p , being the column load minus the soil pressure within the perimeter, is

Q_p = 100 - 0.9125 x 1.025 x 20.45 = 100 - 19 = 81 t.

Consequently, the punching shear stress on the perimeter section is given by :

$\tau_p = Q_p / b\,d = 81000 / 285 \times 62.5 = 4.55$ kg/cm^2

Considerably less than the allowable value of 6 kg/cm^2

The width of the transverse beam under the interior column can now be established by the previous rules. It is given by :

b = 75 + 62.5 = 137.5 cms.

The net upward load per meter of the transverse beam is :

180 / 2.1 = 85.5 t/m

The moment at the edge of the interior column is :

$M = 85.5 \times 0.85^2 / 2 = 31$ mt.

Since the transverse bars are placed on top of the longitudinal bars, the actual value of d is $\simeq$ 60 cms,

and $60 = k_1 \sqrt{31000 / 1.375}$ or $k_1 = 0.40$

For $\sigma_s = 2000$ kg/cm^2 , $\sigma_c = 52$ kg/cm^2 and $k_2 = 1800$

So that

$A_s = 31000 / 1800 \times 0.6 = 28.8$ cm^2 chosen 8ϕ 22

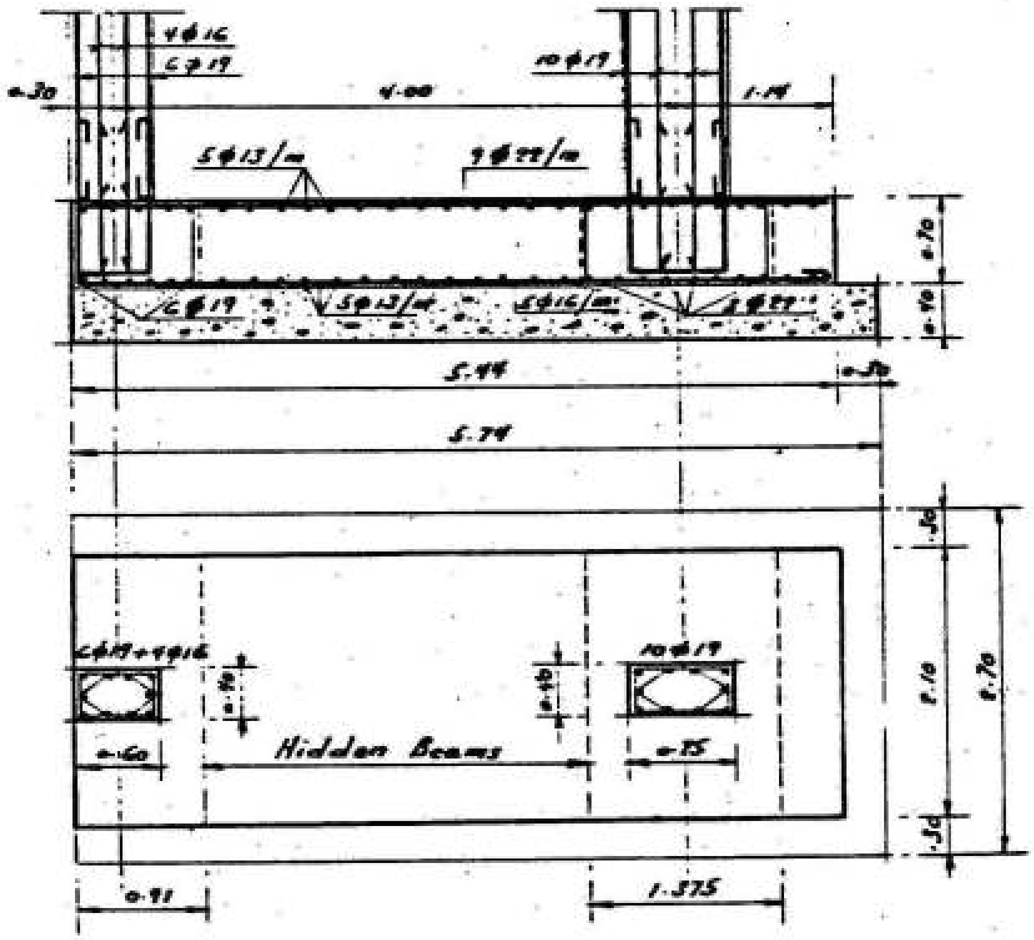
Similarly, the width of the transverse beam under the exterior column is 60 + 62.5 / 2 $\simeq$ 91 cms. and the required reinforcement is 6 ϕ 19.

The details are shown diagramatically in Fig. 9-14.

<u>Another Solution Using a Strap Footing</u>

It is however possible to support the columns on isolated footings, axial under the intermediate column and eccentric under the exterior column such that it does not project beyond the property line. In this manner, the stress distribution will be uneven under the exterior footing and tipping is liable to take place. To counteract this tendency, the exterior footings is connected by a beam or strap to the nearest interior footing. Such a system is called a " Strap footing " . Fig. 9-15.

The footing areas must be so proportioned such that they satisfy the following two conditions :



DETAILS OF A COMBINED FOOTING

Fig. 9-14

1) The pressure under each of the footings is uniform and is the same under both footings.

2) The centroid of the combined area of the two footings coincides with the resultant of the column loads. This condition can be satisfied after some trial computions. The strap is generally constructed

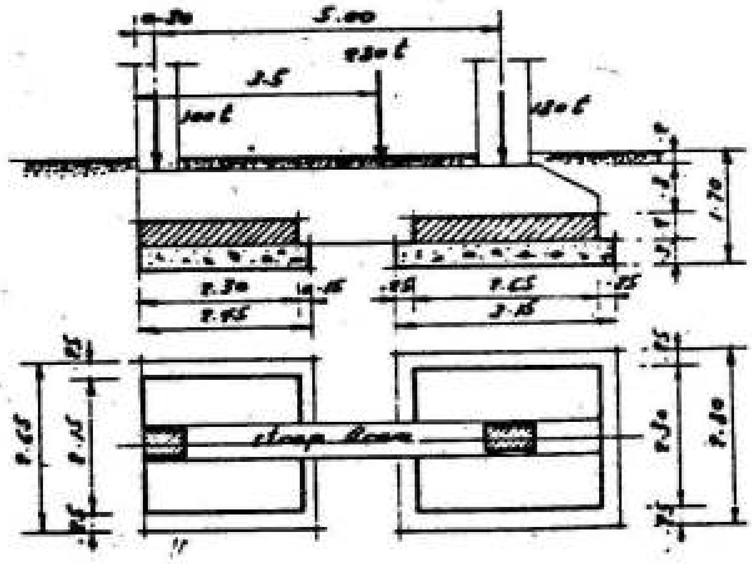


Fig. 9-15

in such a manner that it will not bear directly on the soil.

This new solution generally leads to less concrete quantities especially if the distance between the columns is relatively big.

In order to illustrate the design of this type of footing the main steps will be shown for the data of the previous example, assuming the distance between the columns is 5.0 ms.

The total load including the weight of the footings can however be assumed as in previous example i.e. N_t = 308 t

The total required bearing area is therefore 308/20 = 15.4 m^2

The resultant of the column loads - 280 tons lies at distance from the outer face of the exterior column equal to :

(180 x 5.3 + 100 x 0.3)/ 280 = 984/280 = 3.5 ms.

The shown arrangement gives for the plain concrete footings under intermediate column an area 3.15 x 2.8 = 8.82 m^2
and under exterior column an area 2.45 x 2.65 = 6.50 m^2

Total area = 15.32 m^2

Their center of gravity lies at a distance from the exterior column equal to (8.82 x 5.3 + 6.5 x 1.225) / 15.32 = 3.55 m

The same conditions must also be satisfied for the reinforced concrete footings in which case we have :

Area of R.C. footing under interm. column	2.65 x 2.3	=	6.10 m^2
" " " " exterior "	2.30 x 2.15	=	4.90 m^2
	Total area	=	11.00 m^2

The bearing stress on the plain concrete footings is therefore given by 308/11 = 28.0 t/m^2

Their center of gravity lies at a distance from the outside face of the exterior column equal to (6.1 x 5.3 + 4.9 x 1.176) /11.0 = 3.46ms

The previous calculation shows that the required conditions are satisfied for the soil pressure and the bearing pressure on the plain concrete.

To compute the moments and shears in the various parts of the strap footing, it is first necessary to determine the intensity and location of all external forces and reactions. These are shown in fig. 9-16. Since the footing areas are so arranged that the pressure on each of them is uniform, the resultants of these net pressures, R_e and R_i act at the centers of the respective footings. Assuming the their reactions on the outside main girders. Assume concrete C200 and high grade steel.

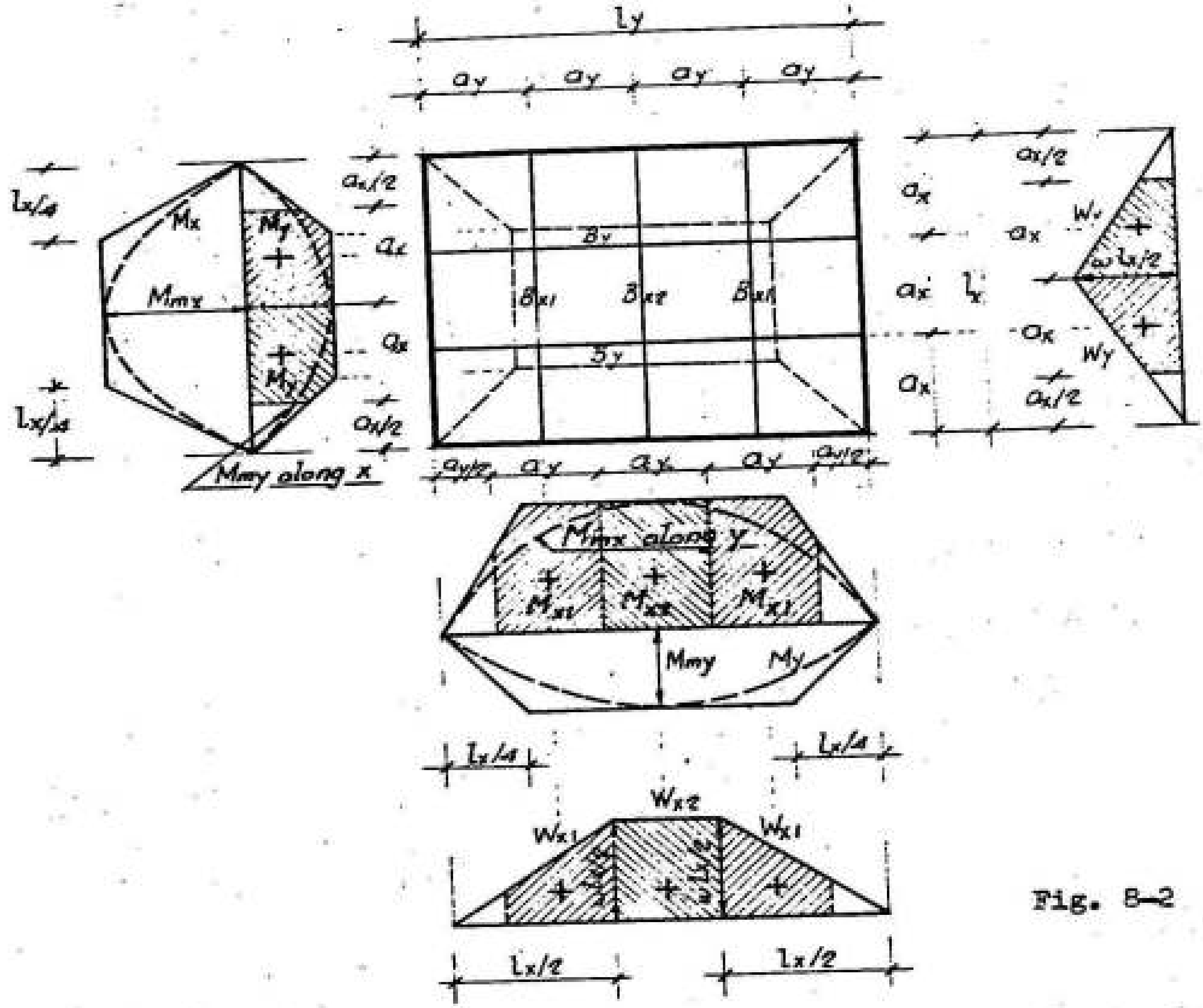


Fig. 8-2

Loads

Assume slab 8 cms thick .... load = 200 kg/m^2

Light floor cover and normal plaster .... load = 130 "

Assume panelled beams 25 x 73 cms

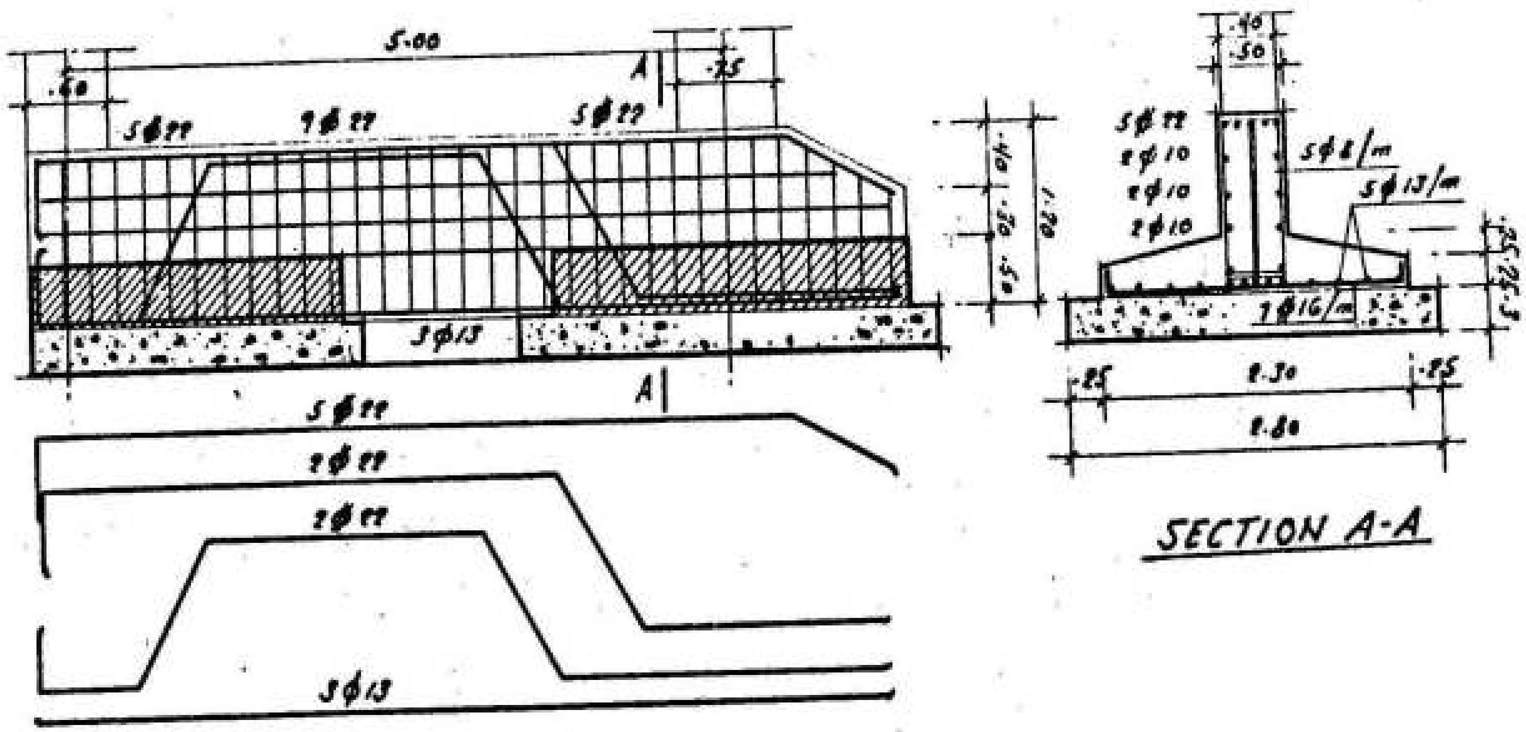
Average load = 0.25 x 0.65 x 2500 (3 + 3) /9 = 270 "

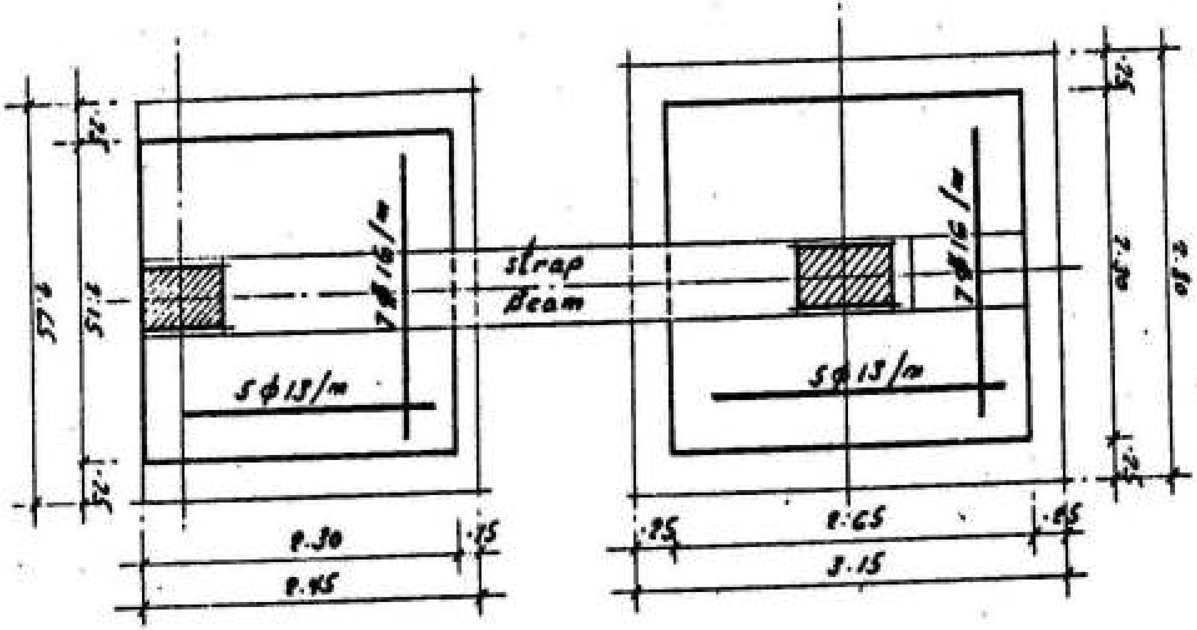
Total load = 200 + 130 + 270 + 300 i.e. w = 900 "

Load distribution

l_y = 12.00 ms , l_x = 9.00 ms $\chi = l_y/l_x = 1.33$ table 5-1 gives:

$\alpha = 0.76$ & $\beta = 0.24$ so that





DETAILS

OF

A STRAP FOOTING

Fig. 9-17

$d = 0.40\sqrt{11\,300} = 43$ cms   chosen t = 50 cms

$A_s = 11300 / 1818 \times .43 = 19.2\ cm^2$ chosen 7 Φ 16 / m.

The details of reinforcements in longitudinal and cross directions are shown in Fig. 9-17.

9-4) DESIGN OF A CONTINUOUS FOOTING

A) Rigid Continuous Footing

If the footing is rigid, the soil pressure can be assumed as uniformly distributed and the problem is statically determinate. The internal forces can be easily determined as shown in the following example :

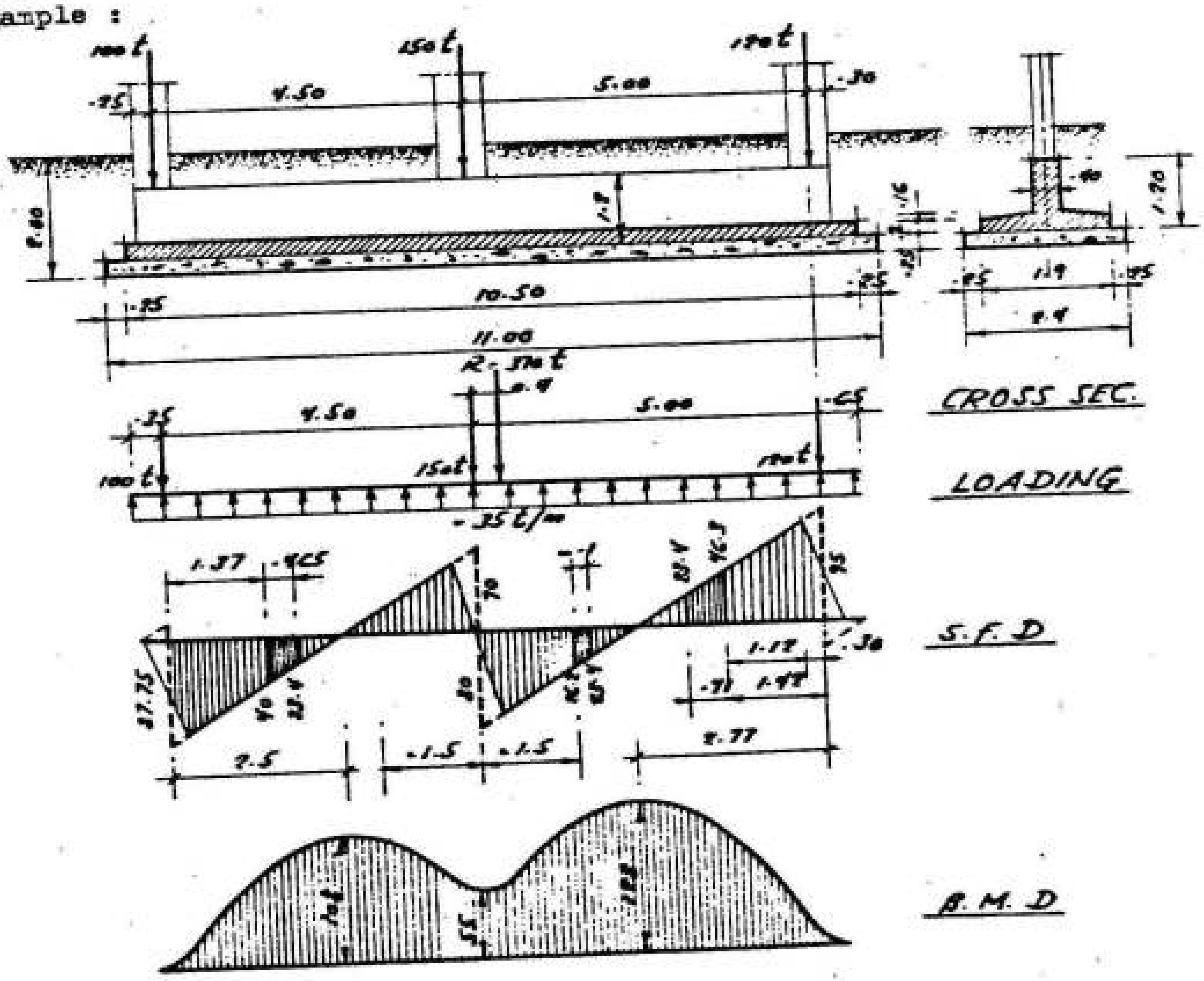


Fig. 9-18

Example :

It is required to design a continuous footing for the three columns shown in (Fig. 9-18) ; assuming that the allowable bearing stress on the soil is 1.5 kg/cm^2.

The footing is composed of a rigid continuous girder, connecting the three columns, provided with a flange to distribute the load

Total column loads = 100 + 150 + 120 = 370 t

Assuming own weigh of footing 8 % of total column loads, then

Total column loads plus own weight of footing = 1.08 x 370 ≃ 400 t

Area of plain concrete footing = N_t / σ_b = 400 / 15 = 26.6m^2

It is chosen 11.00 ms. long and 2.4 ms. wide .

In order to have uniform stress on the soil, the resultant of the column loads must coincide on the centroid of the plain concrete footing.

The thickness of the plain concrete footing is chosen 30 cms and the soil pressure being 1.5 kg/cm^2 , then the reinforced concrete footing can be chosen 10.5 ms. long and 1.9 ms. wide, giving a net bearing pressure on the plain concrete footing equal to

$$370 / 10.5 \times 1.9 = 18.5 \ t/m^2$$

Assuming the breadth of the main continuous girder = 40 cms, the bending moment of the foot slab at the face of the girder is given by

$M = 18.5 \times 0.75^2 / 2 = 5.20$ mt/m

Assuming σ_c = 50 kg/cm^2 and σ_s = 2000 kg/cm^2, we get :

$d = 0.40 \sqrt{5200} = 29.0$ cms chosen t = 36 cms and

$A_s = 5200 / 1818 \times .29 = 10.0 \ cm^2$ chosen 5 Φ 16 / m

Intensity of pressure under main girder = 370 / 10.5 = 35t/m

The column loads being known, then the girder is statically determinate so that the bending moments and shearing forces can be directly determined.

The girder behaves as a T - beam with an effective width of 1.9 ms. or 4 times breadth of web = 4 x 0.4 = 1.60 ms. or ¼ of span 5/4 = <u>1.25</u> ms. for 5 m. span and 4.5 / 4 = <u>1.12</u> for $4^{\underline{m}}$ span whichever is smaller.

Therefore for the 5 m. span M_{max} = 128 mt. and b = 1.25 ms.

Assuming σ_c = 60 kg/cm^2 and σ_s = 2000 kg/cm^2 , then

$d = 0.347 \sqrt{128\,000 / 1.25} = 111$ cms. chosen 120 cms

$A_s = 128\,000 / 1800 \times 1.25 = 63.5$ cm^2 chosen 13 ϕ 25

For the 4 m span

$A_s = 108\,000 / 1800 \times 1.12 = 53.5$ cm^2 chosen 11 ϕ 25

Max. diagonal tensile stress lies at a distance d from the inner face of the right column where Q = 46.3 tons.

Therefore

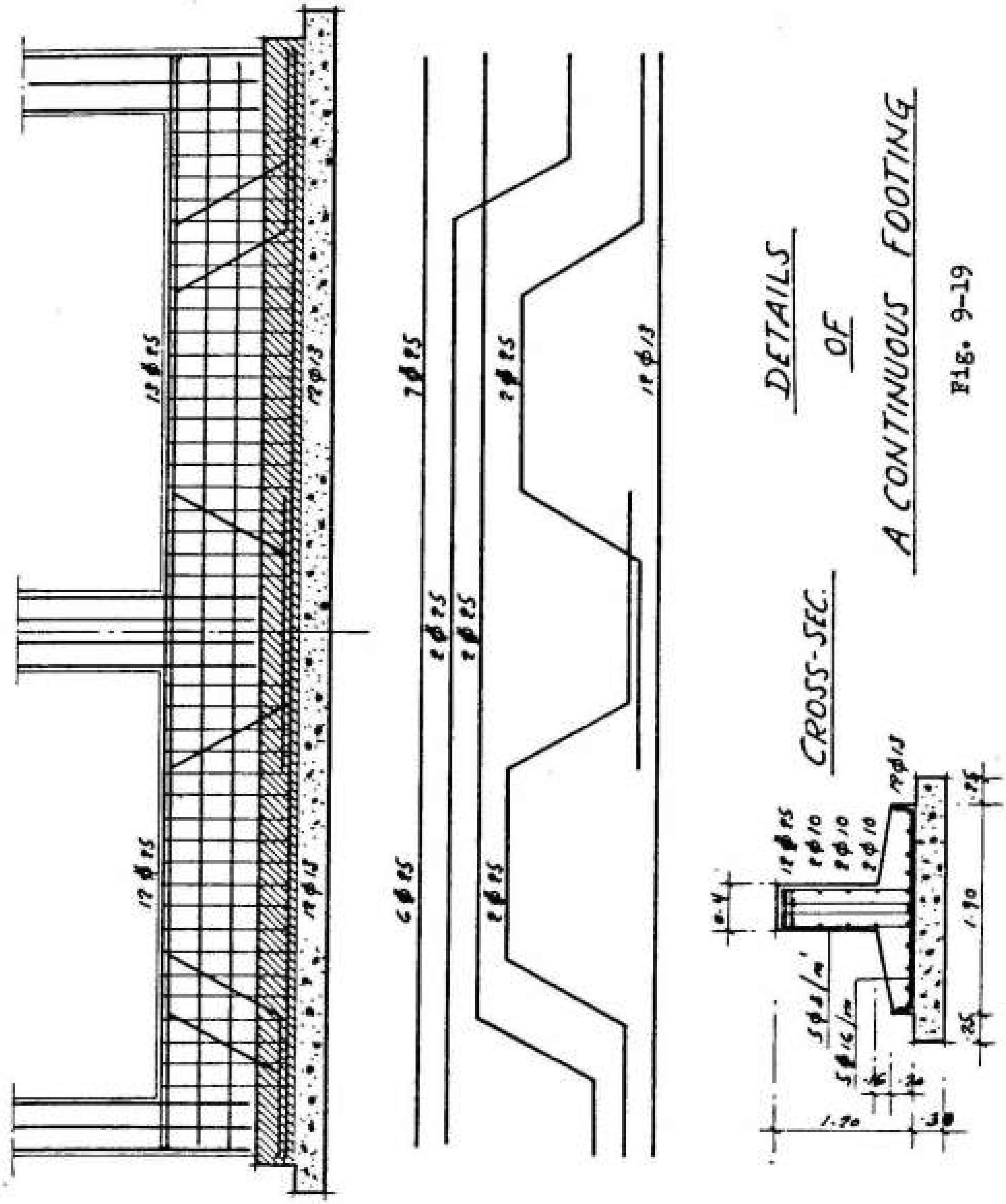
$\tau_{max} = Q / .87\ b\ d = 46300 / .87 \times 40 \times 122 = 11.8$ kg/cm^2 i.e. web reinforcements are required.

Assuming that the max. allowable shear stress which does not need bent bars is 6 kg/cm^2 , then the corresponding shearing force is given by:

$Q = 0.87 \times 40 \times 112 \times 6 = 23400$ kgs.

The zones of high shear stresses are shown in (Fig. 9-18) heavily hatched.

The details of reinforcements are shown in Fig. 9-19.



DETAILS OF A CONTINUOUS FOOTING

Fig. 9-19

3. <u>Continuous Footing on Elastic Foundation</u>

The assumption of equal distribution of soil pressure gives generally high internal forces ; the footing can however be solved as a beam on elastic foundation* if the characteristies of the soil are known as follows :

1) Choose the subgrade modulus k_o according to the kind of soil.
2) Determine the characteristic value $n = \sqrt[4]{k / 4 E I}$ in which:

   $k = b k_o$

   b = constant width of beam in contact with the soil

   EI = flexural rigidity of the beam
3) Calculate the deflections and internal forces for the beam under consideration under the given column loads.
4) Complete the design in the normal manner.

The bending moments and the bearing pressure of the continuous footing shown in (Fig. 9-20) treated as a beam on elastic foundation will be given.

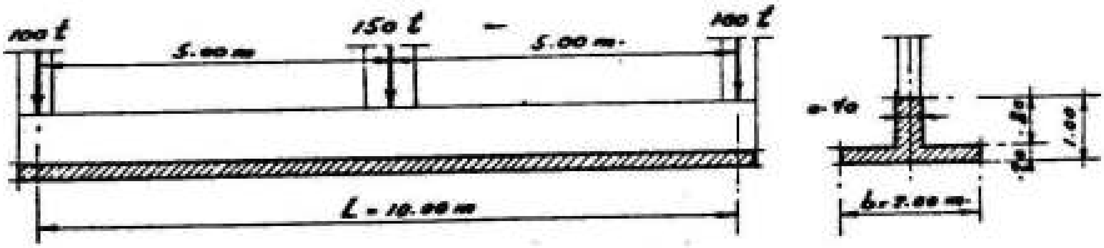


Fig. 9-20

Assume $k_o = 5$ kg/cm^3 and $E = 200\,000$ kg/cm^2

The moment of inertia of the section of the beam $I = .314x200x100 = .0628x100^4$ cm^4

and $k = 200 \times 5 = 1000$ kg/cm^2

The characteristic value $n = \sqrt[4]{1000/4x200000x.0628x100^4} = 3.76/1000$ cm

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* M. HILAL. Theory and design of reinforced concrete tanks.

  HETENYI . Beams on elastic foundation. University of Michigan Studies.

II-24

So that

$nL = \frac{3.76}{1000} \times 1000 = 3.76$ $\qquad \frac{nL}{2} = \frac{3.76}{2} = 1.88$

sin nL	= − 0.580		sin nL/2	= 0.953
cos nL	= − 0.815		cos nL/2	= 0.304
sinh nL	= − 21.463		sinh nL/2	= 3.201
cosh nL	= 21.486		cosh nL/2	= 3.353

For the two equal concentrated loads P = 100 ton at each end :

Deflection at end points : $y_A = y_B = \frac{2\,P\,n}{k} \cdot \frac{\cosh nL + \cos nL}{\sinh nL + \sin nL}$ or

$$y_A = y_B = \frac{2 \times 100\,000 \times 3.76}{1000 \times 1000} \cdot \frac{21.486 - 0.815}{21.463 - 0.580} = 0.752 \cdot \frac{20.671}{20.883} = 0.745\text{cm}$$

Deflection at the middle : $y_C = \frac{4\,P\,n}{k} \cdot \frac{\cosh \frac{nL}{2} \; \cos \frac{nL}{2}}{\sinh nL + \sin nL}$ or

$$y_C = \frac{4 \times 100\,000 \times 3.76}{1000 \times 1000} \cdot \frac{3.353\,(-0.304)}{20.883} = -0.073 \text{ cms}$$

Bending moment at the middle : $M_C = -\frac{2\,P}{n} \cdot \frac{\sinh \frac{nL}{2} \; \sin \frac{nL}{2}}{\sinh nL + \sin nL}$ or

$$M_C = -\frac{2 \times 100\,000 \times 1000}{3.76} \cdot \frac{3.201 \times 0.953}{20.883} = -77.4 \times 10^5 \text{ kg.cm} = -77.4\text{mt}$$

The deflection y_D and bending moment M_D at the quarter point D , a distance x = 0.25 L from one end and x' = 0.75L from the other end are given by :

$$y_D = \frac{2\,P\,n}{k} \cdot \frac{\cosh nx \; \cos nx' + \cosh nx' \; \cos nx}{\sinh nL + \sin nL} \quad \text{and}$$

$$M_D = -\frac{P}{n} \cdot \frac{\sinh nx \; \sin nx' + \sinh nx' \; \sin nx}{\sinh nL + \sin nL}$$

in which

For x = 0.25 L	= 250 cm		x' = 0.75 L	= 750 cm
$nx = \frac{3.76}{1000} \times 250$	= 0.94		$nx' = \frac{3.76}{1000} \times 750$	= 2.82
sin nx	= 0.808		sin nx'	= 0.316
cos nx	= 0.590		cos nx'	= −0.949
sinh nx	= 1.085		sinh nx'	= 8.359
cosh nx	= 1.475		cosh nx'	= 8.418

So that

$$y_D = \frac{2 \times 100\,000 \times 3.76}{1000 \times 1000} \cdot \frac{-1.475 \times 0.949 + 8.418 \times 0.59}{20.883} = 0.128 \text{ cms}$$

$$M_D = -\frac{100\,000 \times 1000}{3.76} \cdot \frac{1.085 \times 0.316 + 8.359 \times 0.808}{20.883} = -90.5 \times 10^5 \text{kgcm}$$

$= -90.5$ mt

For the concentrated load $P = 150$ ton at the middle :

Deflection at end points : $y_A = y_B = \frac{2\,P\,n}{k} \cdot \frac{\cosh\frac{nL}{2}\ \cos\frac{nL}{2}}{\sinh nL + \sin nL}$ or

$$y_A = y_B = \frac{2 \times 150\,000 \times 3.76}{1000 \times 1000} \cdot \frac{3.353\ (-0.304)}{20.883} = -0.055 \text{ cms}$$

Deflection at the middle $y_C = \frac{P\,n}{2\,k} \cdot \frac{\cosh nL + \cos nL + 2}{\sinh nL + \sin nL}$ or

$$y_C = \frac{150\,000 \times 3.76}{2 \times 1000 \times 1000} \cdot \frac{21.486 - 0.815 + 2}{20.883} = 0.306 \text{ cm}$$

Max. bending moment at the middle : $M_C = \frac{P}{4\,n} \cdot \frac{\cosh nL - \cos nL}{\sinh nL + \sin nL}$ or

$$M_C = \frac{150\,000 \times 1000}{4 \times 3.76} \cdot \frac{21.486 + 0.815}{20.883} = 106.5 \times 10^5 \text{ kg.cm} = 106.5 \text{ m.t.}$$

The deflection y_D and bending moment M_D at the quarter point D at a distance $x = 0.25\,L$ from the center line ($x' = L - x = 0.75\,L$) are given by

$$y_D = \frac{P\,n}{2\,k\,(\sinh nL + \sin nL)} \cdot (\cos nx\ \cos nx' + \cos nx\ \cosh nx' - \sinh nx\ \sin nx' + \sin nx\ \sinh nx' + 2\cosh nx\ \cos nx')$$

$$y_D = \frac{150\,000 \times 3.76}{2 \times 1000 \times 1000 \times 20.883} \cdot (-1.475 \times 0.949 + 0.59 \times 8.418 - 1.085 \times 0.316 + 0.808 \times 8.359 + 2 \times 1.475 \times 0.59) = 0.159 \text{ cms}$$

and

$$M_D = \frac{P}{4\,n\,(\sinh nL + \sin nL)} \cdot \Big[\sinh nx\ (\sin nx - \sin nx') - \cosh nx\ (\cos nx + \cos nx') + \sin nx\ (\sinh nx - \sinh nx') + \cos nx\ (\cosh nx + \cosh nx') \Big]$$

or

$$M_D = \frac{150\,000 \times 1000}{4 \times 3.76 \times 20.883} \cdot \Big[1.085\ (0.808 - 0.316) - 1.475\ (0.590 - 0.949) + 0.808\ (1.085 - 8.359) + 0.590\ (1.475 + 8.418) \Big]$$

or

$$M_D = 4.76 \times 10^5\ (0.534 + 0.530 - 5.88 + 6.07) = 6 \times 10^5 \text{ kg.cm} = 6 \text{ mt}$$

The results of the previous calculations are given in the following table and Fig. 9-21.

Section at		Middle C	Quarter D	Edges A & B
Settlement y in cms	P at edges	- 0.073	0.128	0.745
	P at middle	0.306	0.159	- 0.055
	total	0.233	0.287	0.690
Stress σ in Kg/cm^2	$= k_o\ y_{tot}$	1.165	1.435	3.45
Moment M in m t	P at edges	- 77.4	- 90.5	-
	P at middle	+ 106.5	+ 6.0	-
	total	+ 29.1	- 84.5	

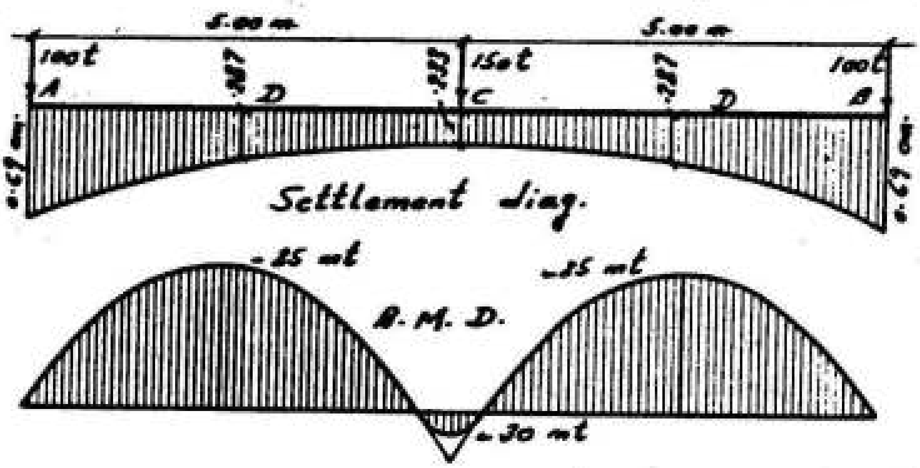


Fig. 9-21

The maximum bending moments are ca 2/3 the values of a rigid beam, which means a big economy. A depth of 1.00 m for the main girder is sufficient and the main high grade tension steel is therefore.

A_s = 85000 / 1800 x 0.92 = 51 cm^2 chosen 10 Φ 25

The details of reinforcements are similar to those shown in Fig. 9-19.

The solution of the girder as rigid leads to about 20% increase both in the steel and concrete quantities.

9.5 RAFT FOUNDATIONS

Under poor soil conditions and in buildings that must have a water tight basement, it is sometimes desirable to support the entire structure on a single reinforced concrete slab. Such a foundation is often called a floating or raft foundation. It may be constructed as an inverted flat slab with or without column heads or of the beam and girder type as shown in Fig. 9-22.

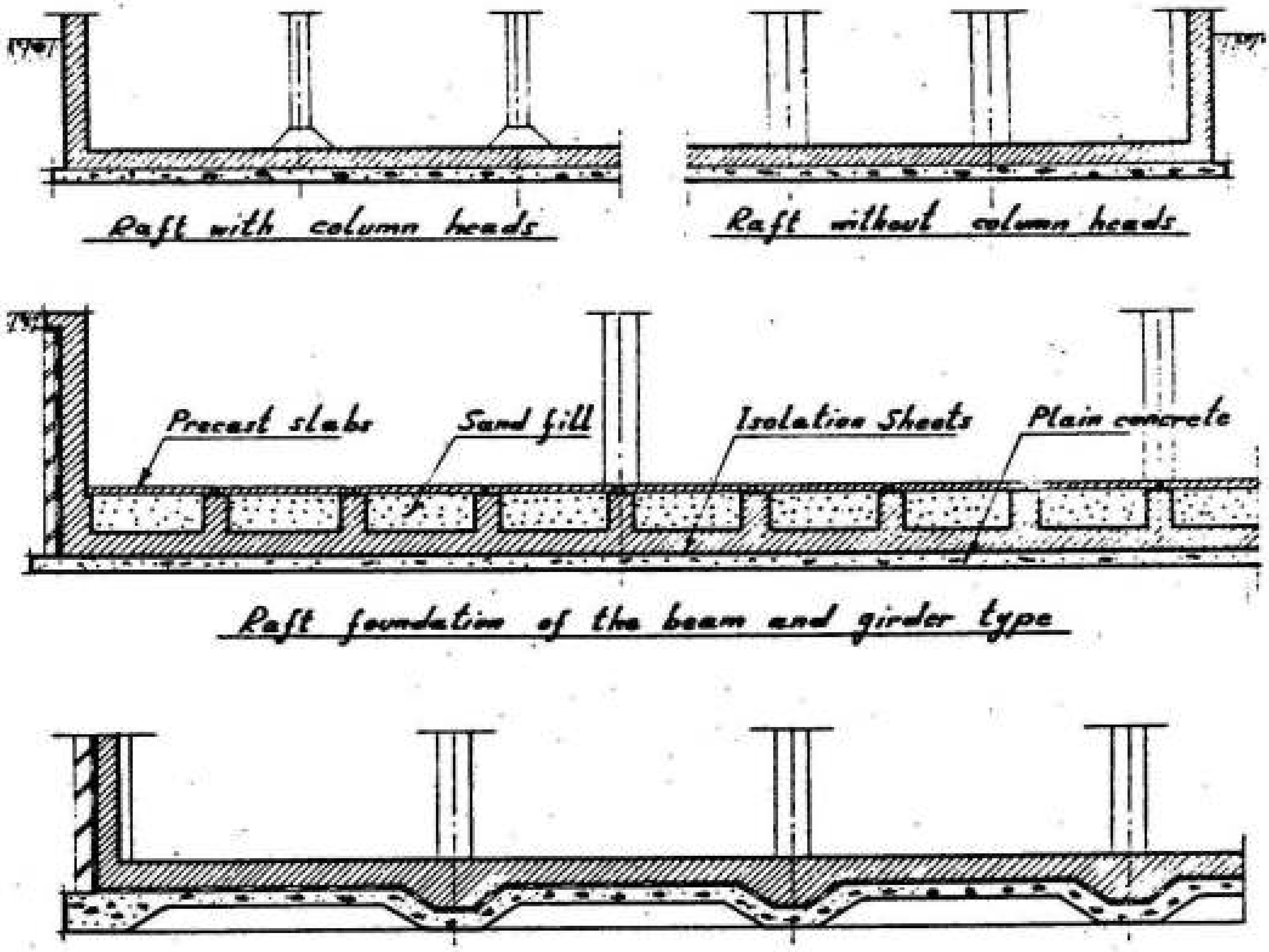


Fig. 9-22

The uniform distribution of pressure under isolated footings is a normal assumption and leads generally to acceptable results, whereas it may lead in long continuous footing and raft foundations to very uneconomic solutions.

The analysis of a raft* includes many complicated problems which cannot be easily solved.

The main problem is the relationship between the stress σ on the soil and the possible compressibility y which does not depend on the subgrade modulus k only i.e. $\sigma = k\,y$ as was generally assumed in the

* Beton Kalender 1958

classic theory, but, as has been found, it depends further on the shear modulus and the shear resistance of the soil. As a result of these last factors, unloaded zones around the loaded area will be stressed and compacted. The characteristics and thickness of the different compressible layers under the raft, to a minimum depth equal to the breadth of the building, are governing factors. If the layers are inclined the factors included in the analysis are much increased.

The stress distribution depends also on the rigidity of the superstructure.

It has also been found that the soil pressure is generally not uniform under uniformly distributed loads due to the possible distribution of the pressures inside the soil.

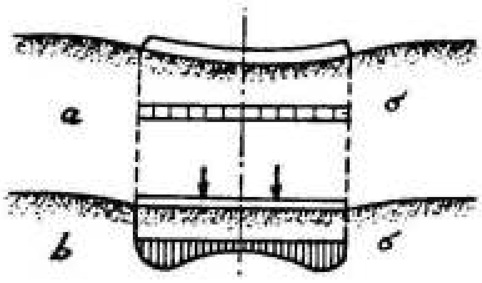


Fig. 9-23

The settlement of sandy layers under uniformly distributed pressure is not uniform as shown in (Fig. 9-23 a). If it is required to have uniform settlement then the loads must be heavier at the edges. We arrive to the same result if we have a very rigid superstructure (e.g. a reinforced concrete silo) giving a uniform settlement, the soil pressure will be bigger at the edges even under uniform load as shown in (Fig. 9-23 b) If the loads from the superstructure are distributed in the same manner as the soil pressure, then the raft is not subject to any moments. Accordingly, we arrive to the following conclusion :

If it is assumed that the soil pressure under a rigid raft is uniform then the corresponding dimensioning will be on the safer side if the loads are concentrated towards the edges and on the unsafe side if the loads are concentrated towards the center.

<u>Rigid Footings</u>

It is however possible to assume that the soil pressure is linear in cases where the exact analysis does not lead to appreciable economy. This is the case in relatively thick footings and in statically determinate cases where the column loads are independent from the elastic deformation of the footing, e.g. the foundations of a tower according to Fig. 9-24 a, b and c.

In the following we will discuss the behavior of a square raft supporting four corner columns - case a - or a square reinforced concrete wall - case b - calculated under the assumption of linear stress

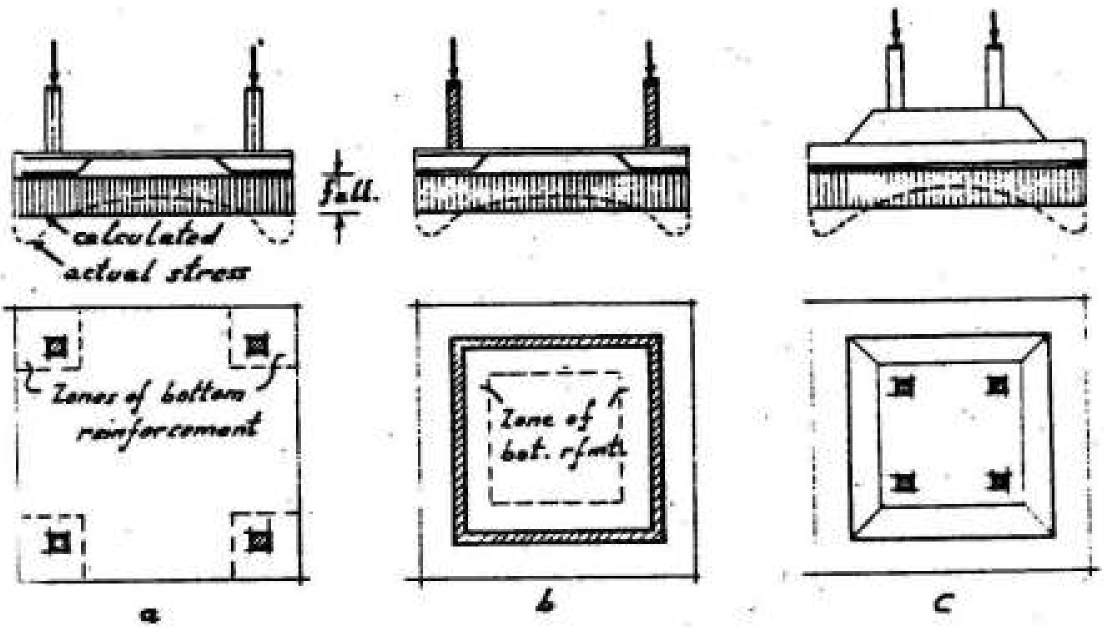


Fig. 9-24

distribution. The actual stresses - shown dotted - will be at the edges much bigger and generally exceed the allowable values.

Accordingly, the actual max. values of the bending moments at the center are smaller than the assumed. At failure the sand grains under the edges move horizontally outwards, the soil stresses will tend again to be uniformly distributed and conform with the fundamental assumption of the design.

Case c behaves in a different way because the loads concentrate towards the center of the raft. If the maximum cantilever moment of such a raft is calculated on the basis of a linear stress distribution then the real moment due to the actual stress distribution will be much bigger i.e. the actual maximum stress at the edges will be bigger than the allowed values and the section of the raft will be under-dimensioned. It is therefore recommended to treat this case by more exact methods although a redistribution of the moments may give accectable results with a bigger possibility of cracks on the lower surface of the raft. For the design of rafts as that shown in case c, the internal forces in sections parallel to the edges and diagonal sections are to be considered.

In rigid foundations, it is generally allowed to calculate the internal forces over the whole cross-section and to distribute the reinforcements uniformly.

It is however allowed to consider the foundations of relatively long structures supported on thin rafts or even on isolated footings as rigid if the structure itself is rigid e.g. a silo as that shown in Fig. 9-25.

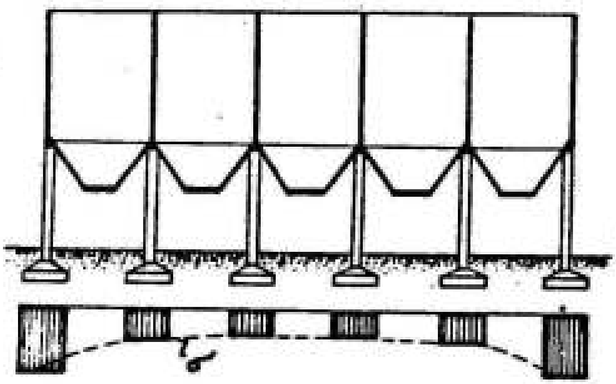


Fig. 9-25

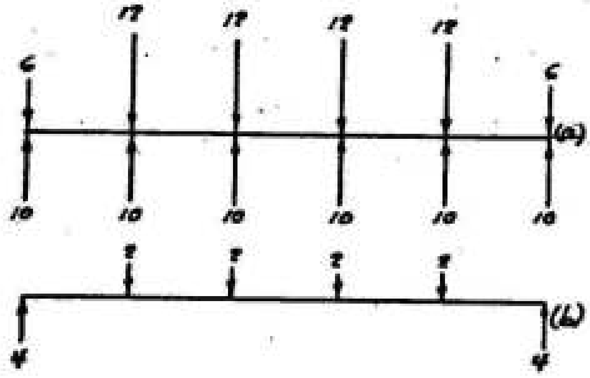


Fig. 9-26

It is a common error, that the stress σ under the footings of such structures is calculated from the relation $\sigma = P/A$ where P is the column load from the superstructure, because the soil pressure, in this case, depends mainly on the total load of the structure. Due to the big rigidity of the structure, the vertical displacement of the different footings will be the same and the soil pressures will be nearly equal if we disregard the pressure increase towards the edges. In this manner, the downward loads are not equal to the upward reactions Fig. 9-26 a. The difference shown in Fig. 9-26 b creates bending moments in the superstructure. This can be explained as follows: Assume that the shown silo weighs 60 load units then each of the four intermediate columns carries 12 units while each of the exterior columns carries 6 units. If we assume further that the soil pressure under all footing is the same and that they have equal areas then the upward reaction from each footing will be equal to 10 units. The bending moments due to the difference between the downward loads and the upward reactions must be resisted by the superstructure. If such bending moments are neglected cracks may be developed. The internal stresses will however be bigger if we assume that the soil stress under the outside footings is bigger than that under the inside ones.

Considering these additional stresses leads to an uneconomic solution and a study of the possible settlements gives a better solution.

A load P is uniformly distributed over a circular area of radius a. The soil pressure due to this load will be distributed inside the soil in the form of a frustum of a cone inclined 45° to the horizontal as shown in Fig. 9-27. The pressure σ_x at any depth x is therefore given by :

$$\sigma_x = P / \pi (a + x)^2$$

Elements above the frustum are not stressed and follow, without compaction the movement of the frustum.

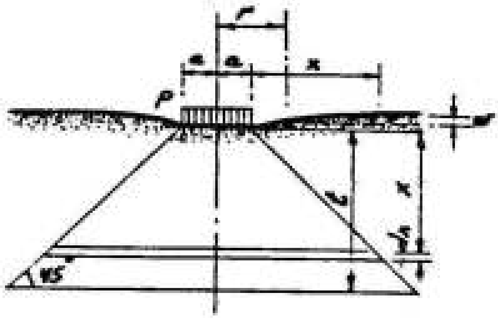


Fig. 9-27

The settlement w of a point at a distance r from the center of the load is equal to the compaction of the frustum below $x = r - a$. If the total thickness of the compressible layer is t , we get

$$w = \int_{x=r+a}^{x=t} \sigma_x \, dx / E = P / \pi E \, . \int_{r+a}^{t} dx / (a + x)^2$$

in which E is the modulus of elasticity of the soil. Therefore

$$w = \frac{P}{\pi E} \left(\frac{1}{r} - \frac{1}{a + t} \right) \text{ for } r \begin{cases} \leqslant a + t \\ \geqslant a \end{cases}$$

Outside $r = a + t$, $w = 0$ while under the load, the settlement w_o is equal to w for $r = a$, hence

$$w_o = \frac{P_o}{\pi E} \cdot \frac{t}{a (a + t)}$$

The settlement w_o under a load P_o is increased by any loads P_i that act within a distance equal to $r = a_o + t$

$$w_o = \frac{1}{\pi E} \left[\frac{P_o \, t}{a_o (a_o + t)} + \sum {}_i \left(\frac{1}{r_i} - \frac{1}{a_i + t} \right) \right]$$

In which i is a figure indicating all footings that exist within a circle of radius $a_o = t$. In order to simplify the calculations the dimensions of the neighbouring footings may be neglected. Hence, according to Fig. 9-28, we get :

$$\pi E w = \frac{P \, t}{a (a + t)} + \sum \frac{P_i}{r_i} - \frac{1}{t} \sum P_i \qquad (A)$$

In which

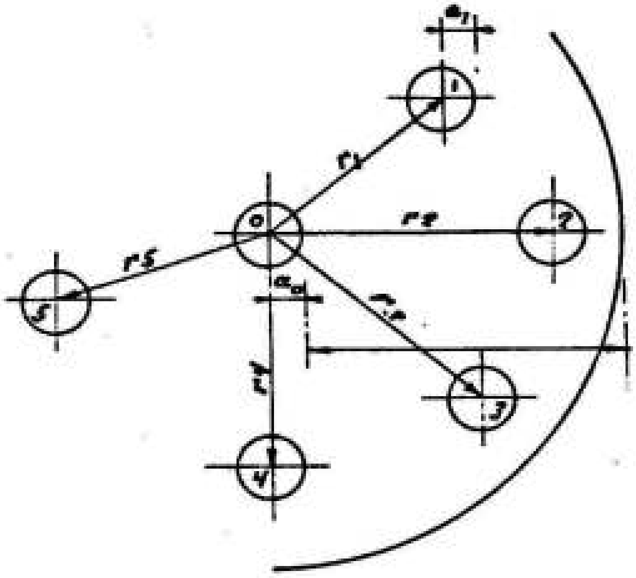


Fig. 9-28

w	=	settlement	of footing under consideration
P	=	load	
a	=	radius	
P_i	=	load	of neighbouring footings whose $r_i \leqslant a + t$
r_i	=	distance	
t	=	thickness of compressible layers	
E	=	modulus of elasticity of soil	

For determining the required areas of the footings of a rigid structure, one can proceed as follows :

1) Determine the area of the middle edge footing for the allowable soil pressure, from which one can get the required value of a.
2) For this footing calculate the value of $\pi E w$ according to equation A.
3) For the neighbouring footings choose " a " such that they give the same value for $\pi E w$.

Solving equation A, we get :

$$a = \sqrt{t^2 / 4 + \alpha} - t/2 \qquad \text{(B)}$$

in which

$$\alpha = P t / \left(\pi E w - \Sigma \frac{P_i}{r_i} + \frac{1}{t_i} \Sigma P_i \right)$$

Example :

The columns supporting a silo are arranged as shown in figure 9-29.

The loads are :
on middle footing 200^t
on outside " 115^t
on corner " 65^t
assume t = 8 m
allow. soil pressure
$\sigma = 30\ t/m^2$

We determine first the area of footing 1 for the allow. soil stress σ, thus

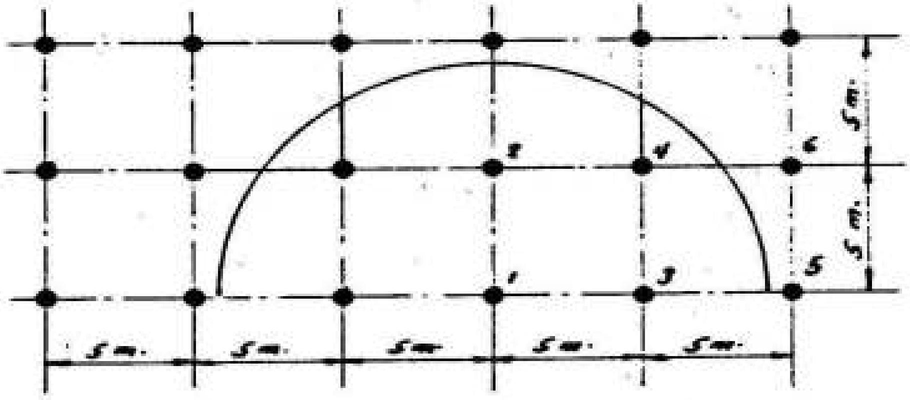


Fig. 9-29

$A_1 = P_1 / \sigma = 115/30 = 3.83\ m^2$ or $a_1 = 1.10$ m.

According to equation A, we get for this footing :

$$\pi \Sigma w = \frac{115 \times 8}{1.1 \times 9.1} + 2(\frac{115}{5} + \frac{200}{5} + 2\,\frac{200}{5\sqrt{2}}) - \frac{1}{8}\,(2x115x3x200) = 131\ t/m$$

According to equation B, we get e.g. for footing 2 :

$$\alpha = 200 \times 8 / \left[131 - (2\,\frac{115}{5} + 2\,\frac{200}{5} + 4\,\frac{115}{5\sqrt{2}}) + \frac{1}{8}\,(2x115+2x200+4 \times 115)\right] = 1600 / (131 - 191 + 136) = 21.1$$

$$a_2 = \sqrt{8^2 / 4 + 21.1} - 8/2 \qquad = 2.09\ m \quad or \quad A_2 = 13.7 m^2$$

The soil stress under footing 2 is given by :

$$\sigma_2 = \frac{200}{13.7} = 14.6\ t/m^2 \quad \text{much smaller than} \quad \sigma_{all} = 30\ t/m^2$$

Similar calculations for the other footings lead to the following results

Footing	1	2	3	4	5	6
$\sigma\ t/m^2$	30	15	34	17	68	37

The different footings will be subject to the same settlement under the stresses shown in the table.

There is however no danger from the high stresses under footings 5 and 6 so long as they correspond to the same settlement as the other footings.

When calculating the settlement due to the total load of the silos, only half the dead weight of the footings is to be considered.

A building with masonry longitudinal and cross walls may be considered as a rigid building even if it is constructed on a relatively thin raft.

Rafts Under Elastic Structures

In an ideal elastic structure the settlements of the raft follow the deflection of the structure. This ideal case never exists. The protection of the weak structural elements due to such deflections is possible by taking the convenient means. For example, in elastic skeleton buildings built on medium soils, not to build the walls before finishing the other dead loads.

The exact calculation of an elastic raft is not existing and very complicated. The solution must prove the equilibrium of the forces and the stability of the structure. It is extremely difficult to satisfy the deformation conditions.

Due to the big factors included in the problem, it is not possible to say that an exact solution gives results nearer to the actual behavior of the structure than a reasonable approximate solution governed by practical experience.

When a complete raft is assumed as elastic, this does not mean that it is not possible to divide it into rigid strips as will be assumed in the following approximate solution.

The raft shown in (Fig. 9-30). is assumed to be divided to rigid strips under which the stress is linear but may vary from panel to panel. According to this assumption, we will have equilibrium between the loads from the columns and the reactions from the raft but the deformation conditions are not satisfied.

For the design of a raft foundation in the form of an inverted flat slab one can proceed as follows ; Fig. 9-30.

1) Divide the slab into strips of breadth b.

2) Distribute the load P linearly on the area a.b ; where a is the distance between the centers of the panels.

3) Determine the bending moments in any row of columns assuming hinges at the middle of the spans. This is to be done for both directions of the slab.

If we design the slab for these moments then the bearing capacity may be satisfied but the deformation conditions are not fulfilled

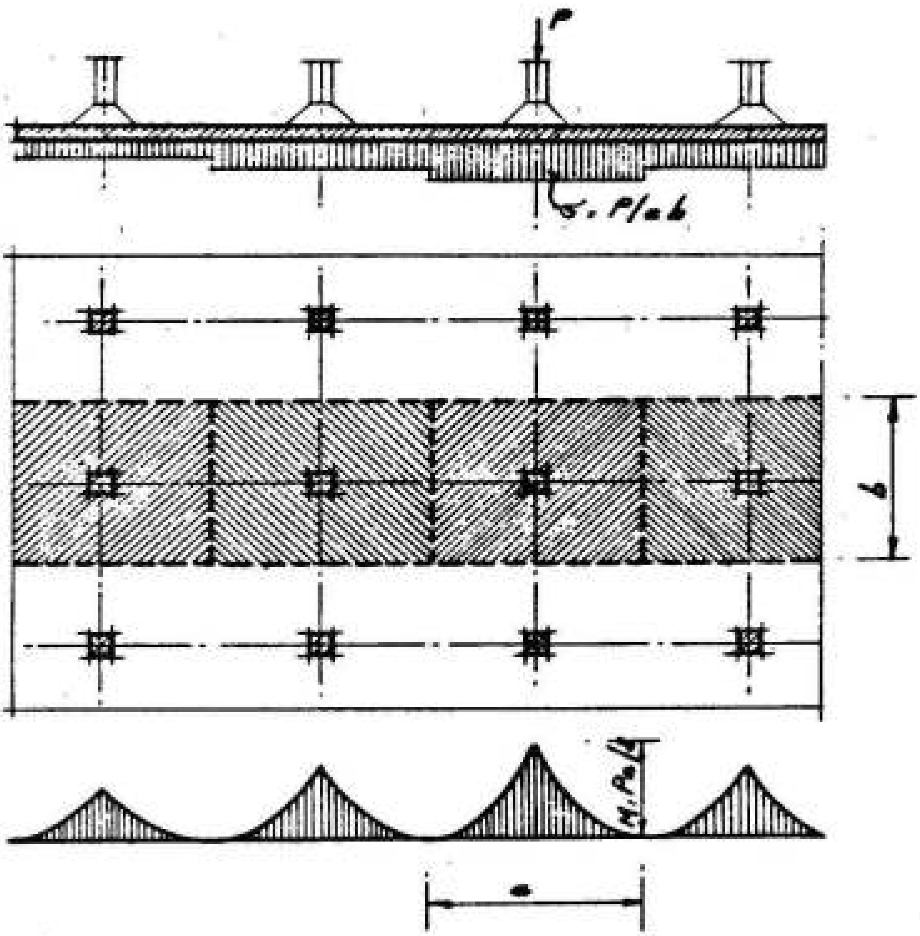


Fig. 9-30

because the whole slab will be subject to tensile stresses on the lower surface and hence must deflect in a form (concave upwards) which cannot be the case in long slabs.

One can however assume that a long slab of equal spans and subject to equal column loads deflects in the form shown in Fig. 9-31 ; flat in the middle part and concave upwards for a length equal to the thickness of the compressible layers below the raft.

In the straight part, shift the closing line of the bending moments such that it passes through the lower third point of the connecting moments. In the curved part, length t, the closing line remains unshifted or even shifted downwards.

The resulting bending moments are then distributed between the column and the field strips according to the known principles of flat slabs.

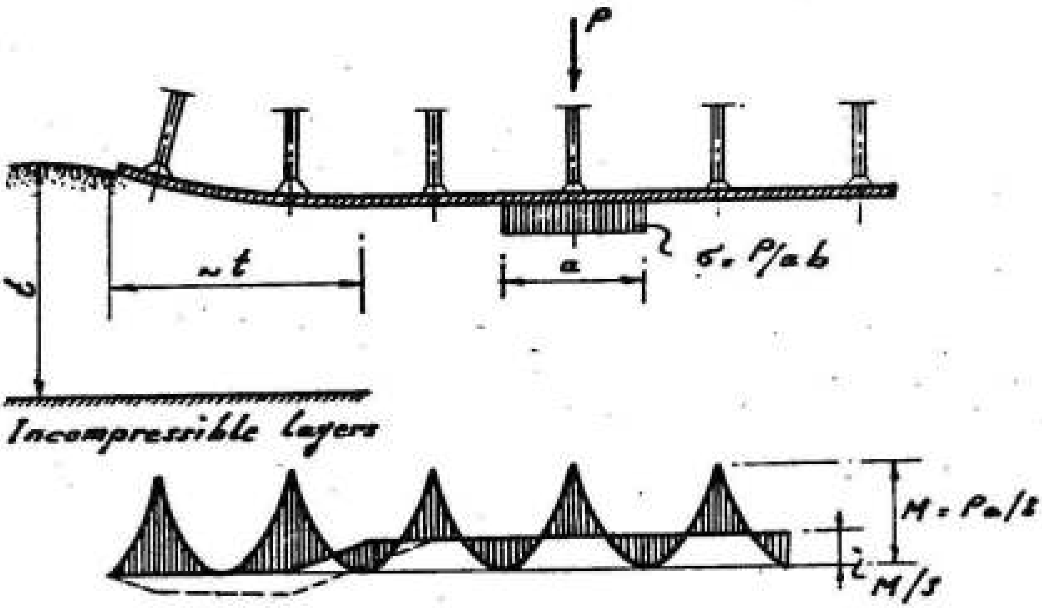


Fig. 9-31

For raft foundations of the form shown in (Fig. 9-31) one may assume that the upward soil pressure acting on the slabs is equal to the average stress of four corner columns.

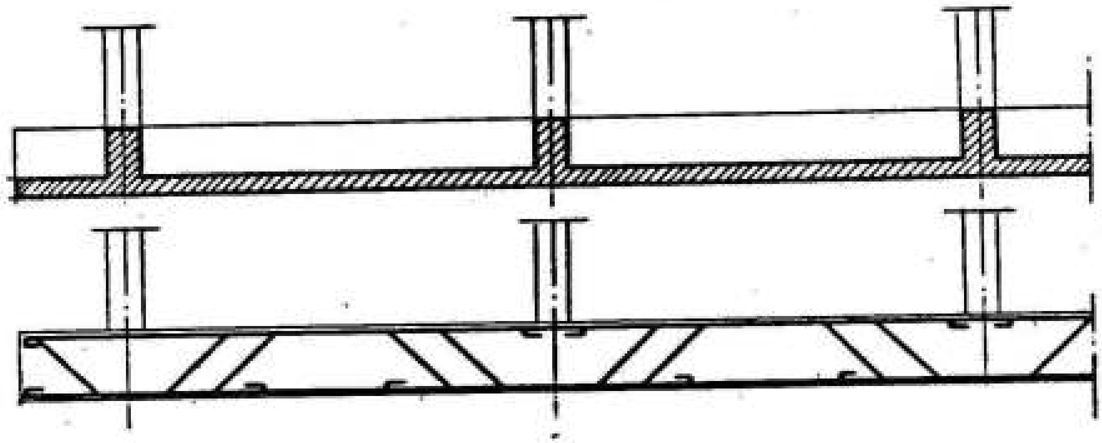


Fig. 9-32

The moments on the beams may be determined according to the principles given as an explanation for Fig. 9-31.

The typical reinforcement of a continuous beam with approximately equal spans supporting a raft is shown in Fig. 9-32.

9.6 DEEP ISOLATED FOOTINGS (Fig. 9-33)

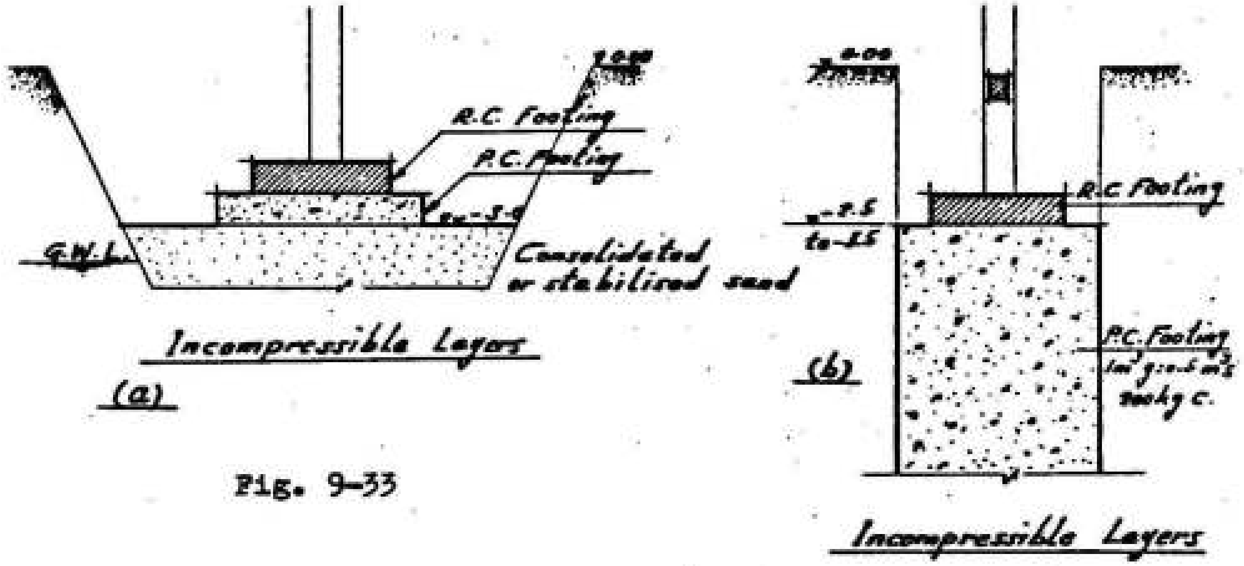


Fig. 9-33

If the incompressible layers are deep (4 to 8 ms. and sometimes more) and it is possible to excavate to these layers without difficulties one may use deep isolated footings as shown in (Fig. 9-33). In order to reduce the costs of the foundations, one may replace a part of the excavated soil by consolidated sand as shown in Fig. 9-33a. In cases of fill or very weak soils such layers of sand must be stabilised by adding 100 kgs. cement to every cubic meter of sand in order to prevent any lateral movement under the column load. In normal cases, the pit under the footing is filled in with plain concrete composed of 1 m^3 gravel or broken stones : 0.5 m^3 sand : 200 kgs. cement. The price of the column per meter being generally smaller than the corresponding price of the plain concrete, it is recommended to fill the pit with plain concrete to such a depth that the reinforced concrete footing can be easily constructed and concreted (say 2.5-3.5 ms). In such cases, the columns must be connected by semelles at ground level to reduce their buckling length to reasonable limits (Fig. 9-33b).

9.7 PILE FOUNDATIONS

If the incompressible layers are deep and it is not possible to excavate for the foundations to these layers, pile foundations may be used.

It is recommended to use bearing piles transmitting the load of column to the incompressible layers by bearing.

In buildings, generally cast-in-situ plain concrete, hand or mechanical piles are used.

The bearing capacity of the pile is equal to the area of cross-section of the pile multiplied by an allowable compressive stress σ_{co} varying between 30 kg/cm^2 for hand piles and 40 kg/cm^2 for mechanical piles. Thus

For hand	piles	25	cms	diameter,	the	bearing	capacity	=	15^t
"	"	30	"	"	"	"	"	=	20^t
mech.	"	30	"	"	"	"	"	=	25^t
"	"	40	"	"	"	"	"	=	$40 - 45^t$
"	"	45	"	"	"	"	"	=	$45 - 50^t$

The bearing capacity of mechanical piles is to be checked by the refusal (the downward movement of the steel pipe as a result of the ten final strokes of the hammer measured during the driving process) of the pipe. If the piles reach an incompressible sand layer, the refusal is generally smaller than ~ 2.5 cm.

Load tests for 1.5 times the pile load, made according to specifications give a further guarantee.

Every column or group of columns is supported by a group of piles placed symmetrical to the centroid of the load. The spacing between the piles = 2.7 to 3 times pile diameter. The load is transferred from the column to the piles by a pile cap.

The pile cap must be as near as possible to the ground surface. Its top surface is generally chosen ca 10-20 cms. below ground. The size of the pile cap is determined by the required number of piles and by the spacing between them. The tops of the piles must be securely embedded in the cap. For this purpose the bottom of the cap is located not less than 7.0 cm below the top of the piles, and the distance from the center of outside piles is to be made not less than ϕ/2+10 to 20 cms. where ϕ is the diameter of the pile. The bottom reinforcement mesh is located at a distance of 3.0 cms. above the tops of the piles.

It is recommended to reinforce the upper 2.0 ms. of cast in situ piles by 5 ϕ 13 /m longitudinal bars in order to connect the piles with their cap. Such bars must protrude a distance 40 - 50 cms. inside the cap as shown in (Figs. 9-34 to 9-40).

It is further essential to break the upper 30 - 40 cms. of the pile and to construct the pile cap on a clean rough surface of the pile that is free from any fill impurities and with the connecting 13mm. bars protruding from the pile.

Design of Pile Caps

Cap for a Single Pile

It is not recommended to use a single pile under a column except in special cases of relatively light loads because if the axis of the column is not exactly coinciding on the axis of the pile, a matter that can never be guaranteed, bending moments may be created in the pile. As cast in situ piles are made of plain concrete, they cannot resist any appreciable bending moments. For this reason it is absolutely essential to connect single piles by semelles, arranged in two perpendicular directions, which are capable to resist any eccentricity between column and pile axes by bending in the semelles and not by bending in the piles.

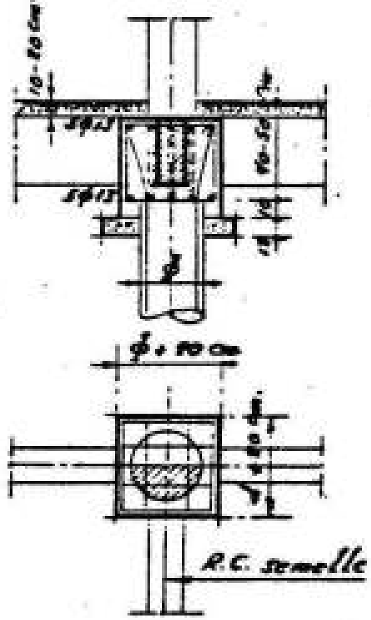


Fig. 9-34

The cap of a single pile is a reinforced concrete prism having a side dimension equal to Φ + 20 cms. and a depth equal to 50 - 60 cms, reinforced by 5 closed stirrups Φ 13 mms. in each direction as shown in (Fig. 9-34).

Cap for Two Piles

If the effective depth of the cap is smaller than half the spacing between the piles; it is designed as a simple beam subject to a single concentrated load in the middle of the span L. The load can be assumed as distributed on a length (a + d) where " a " is the side length of the column The critical section for bending is that at the face of the column. The maximum diagonaltensile stress lies at a distance d from the face of the pile.

The cap is in this case reinforced by longitudinal main bars and vertical stirrups satisfying the requiements of beams. Fig. 9-35.

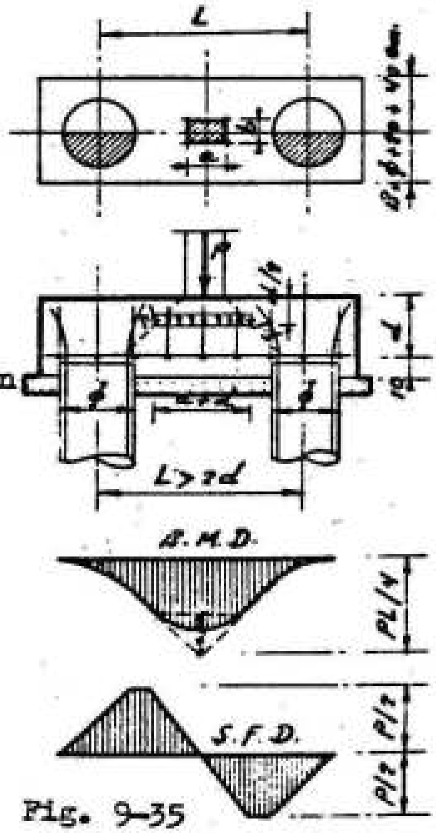
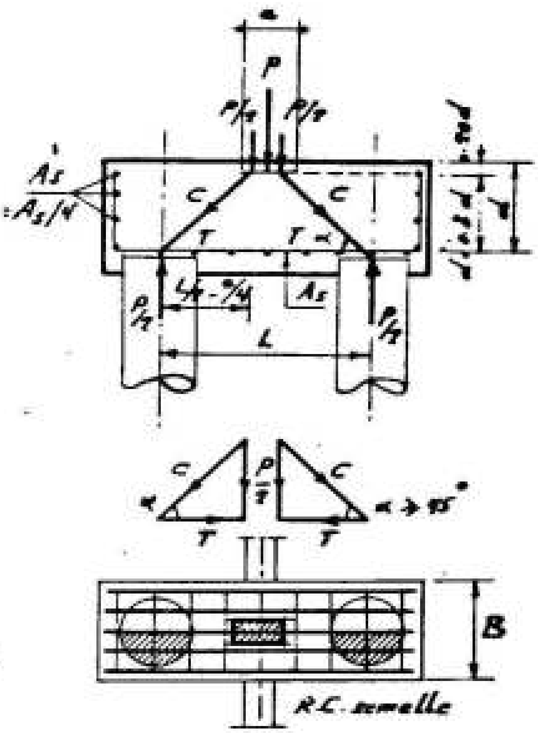


Fig. 9-35

If the depth d' (= 0.8 d) is bigger than L/2; the tension in the pile cap can be determined by direct resolution of forces as shown in (Fig. 9-36), The effective depth is chosen here equal to $d' = 0.8\ d$ for the following reason:

We have a construction joint which cannot resist any horizontal forces at the top surface of the pile cap, but due to the change of direction of the force from vertical (for P/2) to inclined - (for C) a horizontal force H, that needs some concrete to resist it, is created.

The longitudinal reinforcement $A_s = T/\sigma_s$. Due to the tension trajectories horizontal stirrups along the vertical sides of the cap of total area A'_s are to be arranged, where



$$A'_s = T/4\ \sigma_s$$

Fig. 9-36

Vertical stirrups with a minimum area of cross-section of 0.15% of the cross-section B d are to be arranged

Example

Design a pile cap for a column supporting a load of

a) 60 tons and having a cross-section of 30 x 40 cms.

b) 90 " " " " " " " 40 x 40 cms.

Assume plain concrete cast-in-situ piles 40 cms. diameter and 45 tons capacity . Use concrete C200, and normal mild steel for the reinforcements.

Number of piles = 2 for both cases, spacing : L = 1.10 m.

Hence,

Total length of pile cap = 1.70 m and total breadth: B = 0.60 m.

Case a : P = 60 tons

Assume total depth of cap = 55 cms.

Therefore :

$$d = 55 - 10 = 45 \text{ cms} < L/2$$

Hence we use the beam method for the design of the cap. The load P=60 tons is assumed to be distributed on a length of 40 + 45 = 85 cms.

and on the full breadth of the cap.

Intensity of load $w = 60 / 0.85 = 70$ t/m'

Neglecting the effect of the own weight of the cap, the bending moment at the face of the column due to its load is given by :

$M = 30 (0.55 - 0.20) - 70 \times 0.225^2 / 2 = 10.5 - 1.8 = 8.7$ m t

$45 = k_1 \sqrt{8700 / 0.6}$ hence $k_1 = 0.373$

For $\sigma_s = 1400$ kg/cm^2 and $\alpha = 0$

we get $\sigma_c = 49$ kg/cm^2 and $k_2 = 1240$

$A_s = 8700 / 1240 \times .45 = 15.6$ cm^2 chosen 4 Φ 22"

choosing for the vertical stirrups 5 Φ 8 mm/m of the ⊔ - form, then

$A_{st} = 2 \times 5 \times 0.5 = 5$ cm^2 ≈ 0.15 % B d $= 0.15 \times 60 \times 45/100 \approx 5$ cm^2

No diagonal tension is liable to take place, because the critical section at a distance d from the face of the pile lies inside the column.

Case b : P = 90 tons

Assume total depth of cap = 70 cms

Therefore : $d = 70 - 10 = 60$ cms and

$d' = 0.8\ d = 48$ cms

Further $L/2 - a/4 = 55 - 10 = 45$ cms $< d' = 48$ cms

Hence $\alpha > 45°$ and

the tension in the longitudinal reinforcement of the cap can be determined from a triangle of forces (refer to Fig. 9-35).

$T / P/2 = (L/2 - a/4) / d'$ or

$T / 45 = 45 / 48$ or $T = 42$ ton and

$A_s = T / \sigma_s = 42 / 1.4 = 30$ cm^2 chosen 6 Φ 25

area of cross-section A_s' of horizontal bars along perimeter :

$A_s' = A_s/4 = 30 / 4 = 7.5$ cm^2 chosen 3 horizontal stirrups Φ 13 mm.

giving an area $A_s' = 6 \times 1.32 = 7.90$ cm^2

choosing for the vertical stirrups 6 Φ 8 mm/m of the ⊔ -form, then

$A_{st} = 2 \times 6 \times 0.5 = 6.0$ cm^2 $> 0.15 \times 60 \times 60 / 100 = 5.4$ cm^2

In order to resist the possible bending moments due to any eccentricity between the column axis and the axis of the piles, it is essential to connect pile caps for two piles by a semelle along their shorter direction.

Caps for any number of piles can be calculated according to one of the two above-mentioned methods depending on the spacing and number of piles, the thickness of the cap and the magnitude of the angle α .

If the thickness is small and α is smaller than 45°, the beam method is used assuming that the column load is distributed on an area (a + d) (b + d). Whereas, if the thickness is big and the angle $\alpha > 45°$ the tension in the cap and the corresponding tension reinforcement is determined by direct resolution of the column load. The reinforcement in this last method is arranged in a circulage form and is accordingly called " the circulage method ". It can be conveniently used for caps on 3, 4, 5, 6 and 7 piles arranged symmetrically with respect to the column.

In caps on three or more piles, the punching shear on a perimeter section d [(a + d) (b + d)] must be checked.

THE BEAM METHOD

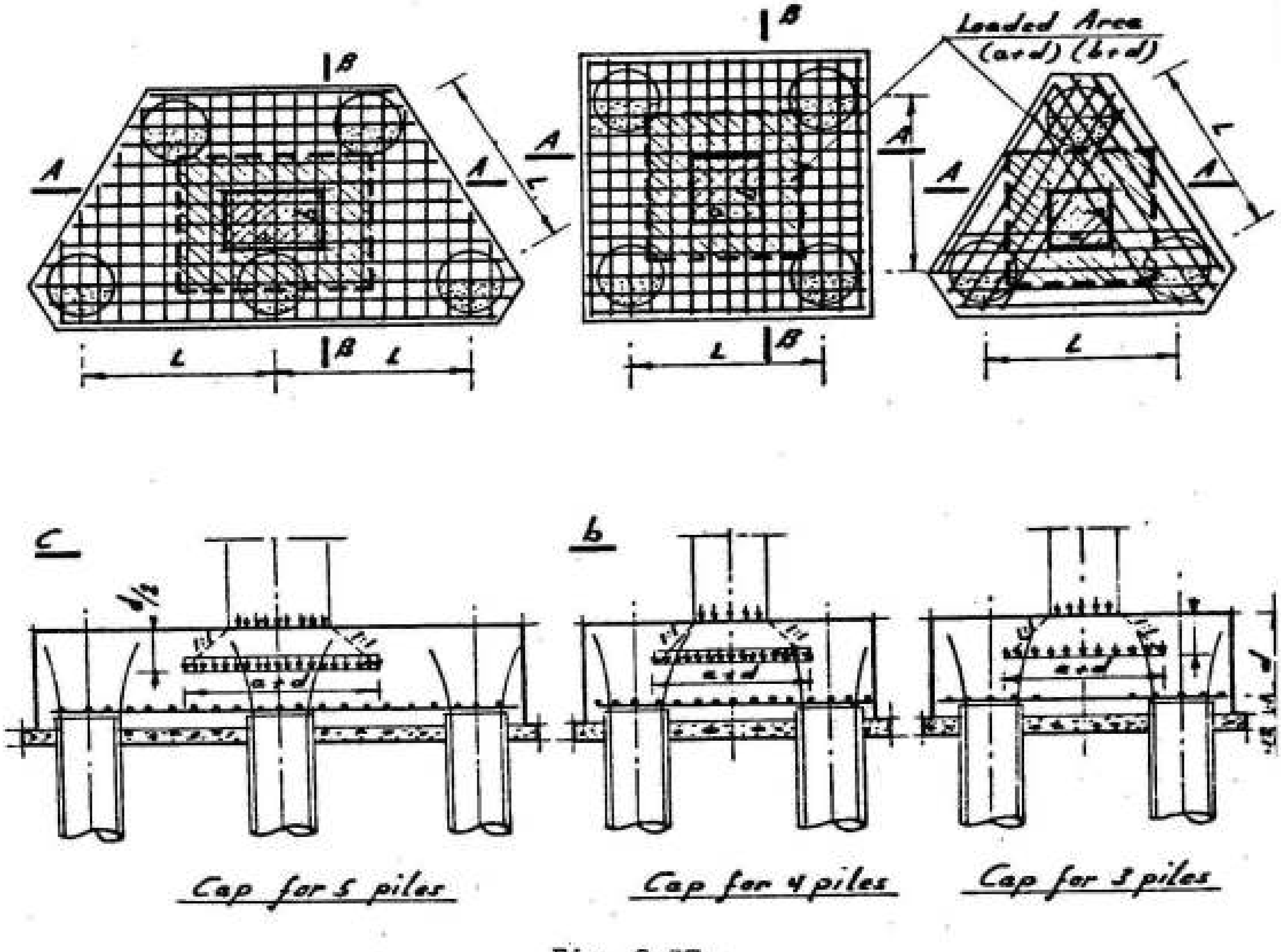


Fig. 9-37

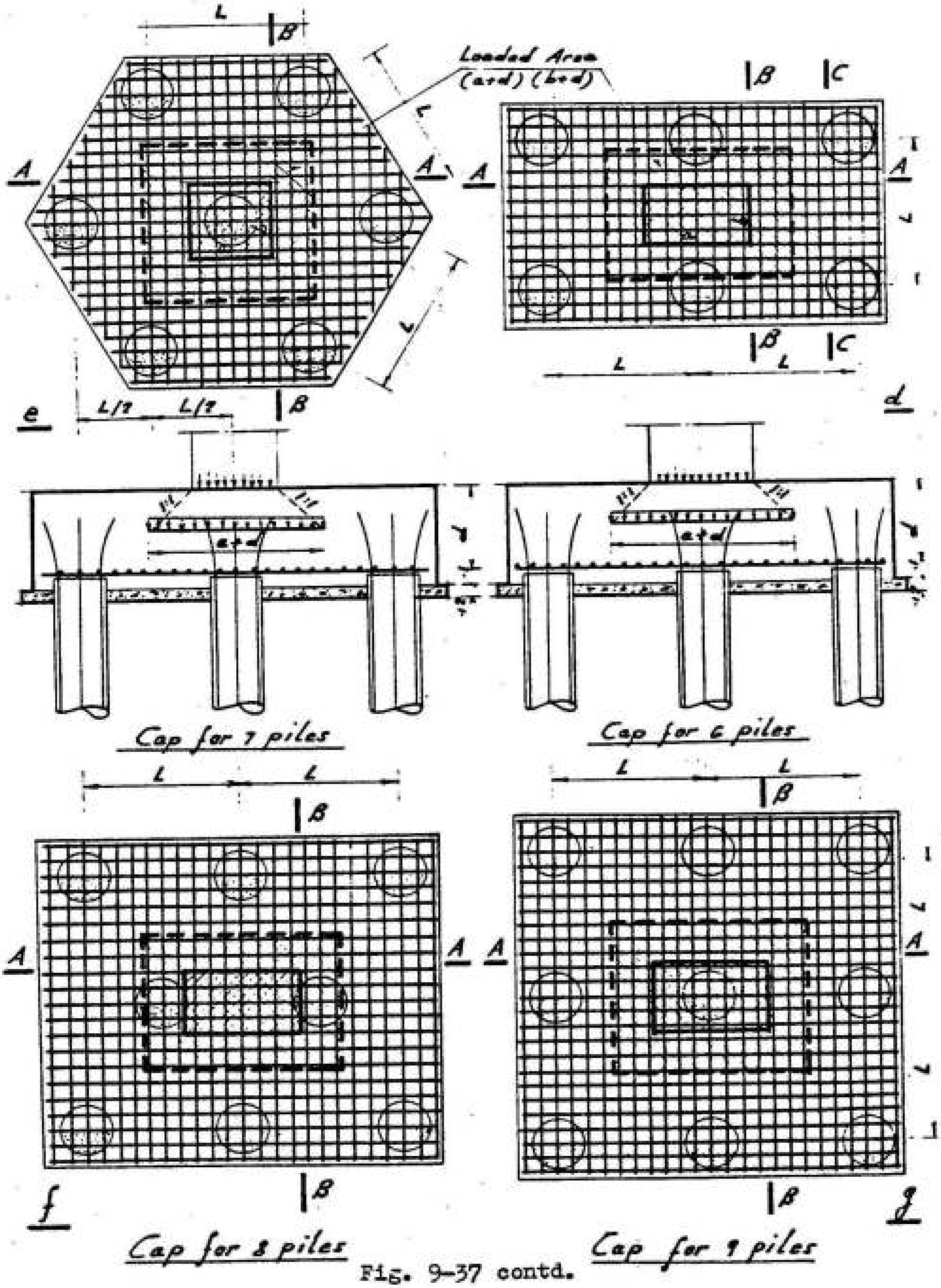


Fig. 9-37 contd.

The bending moments are to be computed at the face of the column for section A-A and B-B with the concentrated pile loads acting upwards and the column load distributed on an area (a + d)(b + d) acting downwards. In computing the shearing forces, the pile load is to be assumed as uniformly distributed on the area of cross-section of the pile. (Fig. 9-37).

If in any section under consideration, the main tension reinforcement is not arranged normal to the section - e.g. section A-A in the cap for three piles, only the component of the area of steel normal to the section can be assumed as resisting the moment, i.e. if the moment on section A-A is M, then the required area of cross-sec. of the tension steel is given by: $A_s = M / y_{CT}\ \sigma_s$. Reinforcement is placed parallel to lines connecting the center lines of the piles, i.e. inclined 30° to the direction of the moment. If the area of cross-section of the steel reinforcement parallel to any side is A_{s1}, then:

$$A_{s1} = A_s / 2 \cos 30° = 0.58\ A_s$$

It is however possible to arrange the reinforcements in the form of a rectangular mesh as shown in Fig. 9-38 provided the above condition is satisfied for any of the sections B-B.

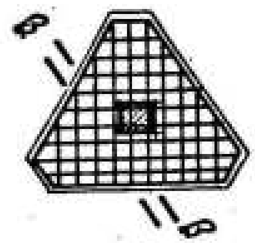


Fig. 9-38

Example

It is required to design a pile cap for a 40 x 80 cms column, carrying 230 tons if the piles are 40 cms diameter and having a capacity of 40 tons. Use concrete C200 and high grade steel for the main reinforcement.

Solution

Assume that the own weight of the pile cap = 10 tons, i.e.
Total load on piles = 230 + 10 = 240 tons so that the column can be supported by six piles. If they are arranged in the form shown in Fig. 9-37 d and the spacing between the piles is L = 110 cms., then

the total length of the pile cap is 290 cms, and

" " breadth " " " " " 180 cms.

Assuming that the total depth of the cap is 85 cms ; then the theoretical depth d may be assumed equal to 75 cms.

The column load will be distributed on an area (a + d) (b + d)
or (.80 +.75) (.40 + .75) = 1.55 x 1.15 = 1.78 m^2
load intensity w = 230 / 1.78 = 129 t/m^2
Bending moment at section B - B
due to pile loads : 0.7 x 240/3 = 56 m t
due to own weight of cap 2.5 x 0.85 x 1.80 x 1.05^2 /2 = 2.1mt
due to distributed column load 129 x 1.15 x 0.375^2 / 2 = 10.4 "
So that the bending moment at section B - B is
56 - (2.1 + 10.4) = 43.5 "
Shearing force at section CC : 240/3 - 1.8x0.55x.85x2.5 = 77.9 t
Dimensioning :

$75 = k_1 \sqrt{43500/1.8}$ i.e. $k_1 = 0.485$ so that

For $\sigma_s = 2000$ kg/cm^2 $\alpha = 0$ we get

$\sigma_c = 39$ kg/cm^2 and $k_2 = 1850$

$A_s = 43500 / 1850 \times 0.75 = 31.5$ cm^2 12 ϕ 19

The diagonal tensile stress is given by

$\tau = 77900 / 0.87 \times 180 \times 75 = 6.6$ kg/cm^2 accepted

The punching shear stress is

$\tau_p = 230000 / 2 (155 + 115) 75 = 5.4$ kg/cm^2 safe

Bending moments at section A - A :

due to pile loads = 0.35 x 240/2 = 42 mt

due to own weight of cap = 2.5 x 0.85 x 2.9 x 0.7^2 / 2 = 1.5 mt

due to distributed column load 129 x 1.55 x 0.375^2 / 2 = 14.-mt

So that the bending moment at section A - A is given by

42 - (1.5 + 14) = 26.5 mt

Shearing force at inner surface of piles

Q = 240 / 2 - 2.9 x 0.55 x 0.85 x 2.5 - 129 x 1.55 x 0.225 = 71.6

Using normal mild steel for the cross reinforcements, then

$A_s = 26500 / 1300 \times 0.73 = 28$ cm^2 14 ϕ 16

The diagonal tensile stresses are safe .

CIRCULAGE METHOD

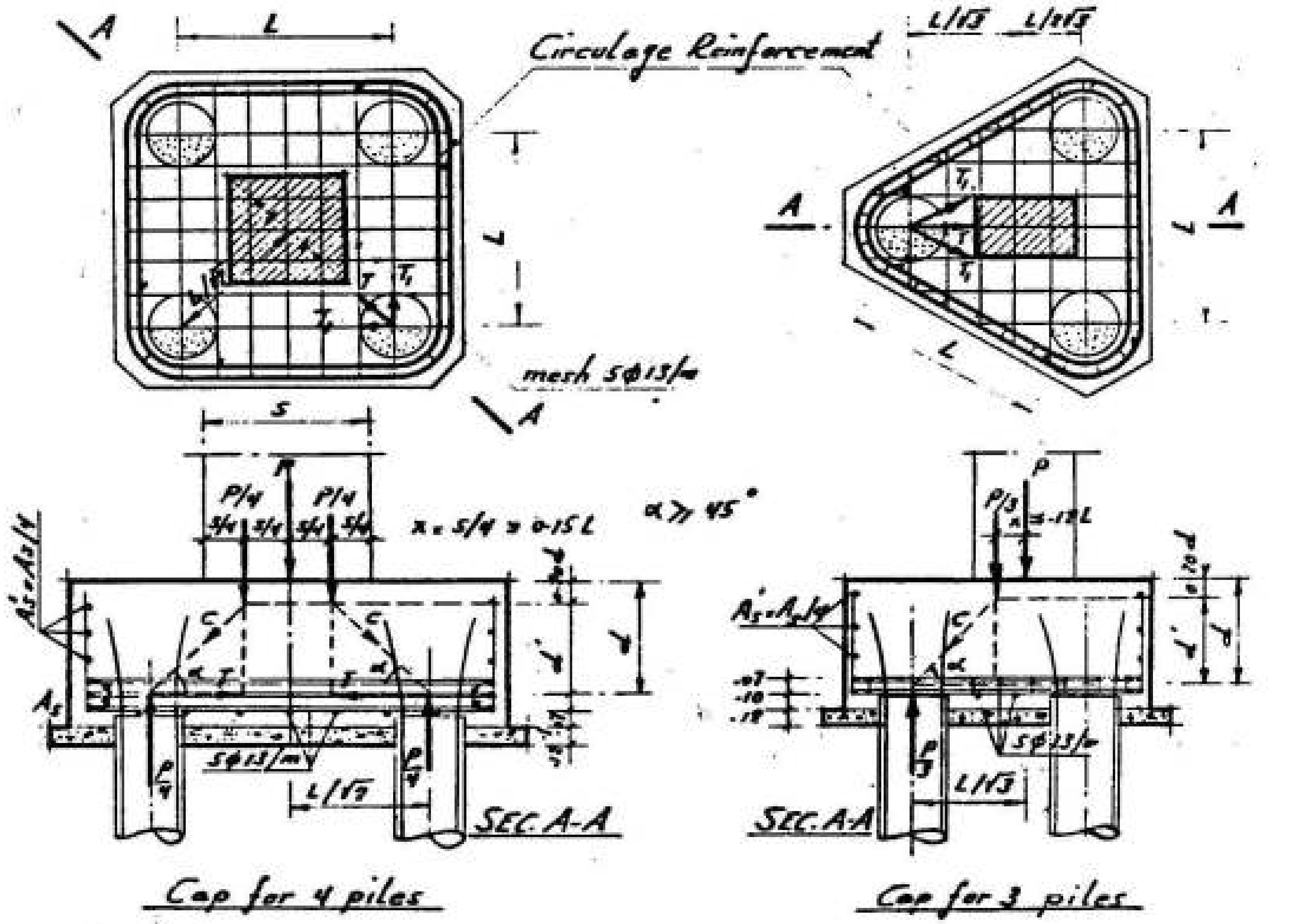


Fig. 9-39 Fig. 9-40

Proceeding in the same way as for caps on two piles, we get

For Cap on Three Piles (Fig. 9-39)

$\frac{P}{3} / T = d' / \frac{L}{\sqrt{3}} - 0.12\ L$ and $T_1 = A_s/\sigma_s = T/\sqrt{3}$, $d' = 0.8\ d$ or

$A_s = P\ L\ /\ 9\ \sigma_s\ d$

Example :

A column 50 x 50 cms carrying 140 tons on 3 piles φ 45 cms spacing 1.20 ms. Assume $\sigma_s = 1.4\ t/cm^2$.

For $\alpha = 45°$, $L = 1.20$ ms, $x = 0.12L = 0.12 \times 120 = 14.4$ cms

$d' = 0.8d = L/\sqrt{3} - 14.4 = 120/\sqrt{3} - 14.4 = 69.4 - 14.4 = 55$ cms, then

$d = 55 / 0.8 = 69$ cms and $t = 69 + 11 = 80$ cms

Long. reinforcement $A_s = P\ L\ /\ 9\ \sigma_s . d$ or

$A_s = 140 \times 1.20 / 9 \times 1.4 \times 0.69 = 19.3 \ cm^2$ chosen 4 ϕ 25

cross reinforcement $A'_s = A_s / 4$ or

$A'_s = 19.3 / 4 = 4.80 \ cm^2$ chosen 3 closed horizontal stirrups ϕ 10 mm

Punching shear $\tau_p = P / 4 \ (a+d) \ d$ or

$\tau_p = 140\,000 / 4 \ (50+69) \ 69 = 4.3 \ kg/cm^2$ safe.

<u>For a Cap on Four Piles</u> (Fig. 9-40)

$\frac{P}{4} / T = d' / \frac{L}{\sqrt{2}} - \frac{s}{4}$, $T_1 = A_s \ \sigma_s = T / \sqrt{2}$, $d' = 0.8 \ d$

Assuming $s / 4 = 0.15 \ L$, we get

$$A_s = P \ L / 8 \ \sigma_s \ d$$

For $L = 1.10$ m and $\alpha = 45°$,

then $d = 77$ cms and $t = 90$ cms

If $P = 180$ tons and $\sigma_s = 1.4 \ t/cm^2$

then

$A_s = 180 \times 1.1/8 \times 1.4 \times 0.77 = 23 \ cm^2$

chosen 5 ϕ 25

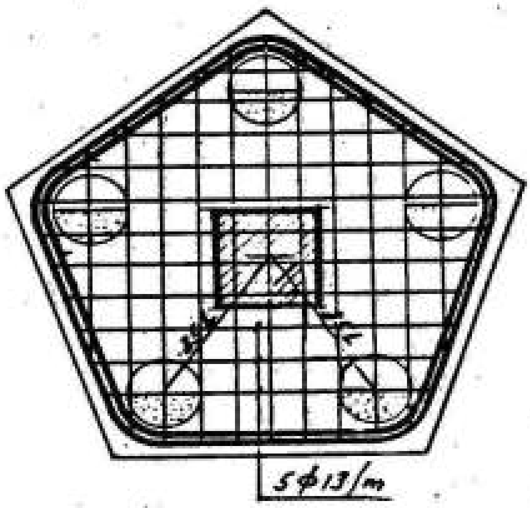


Fig. 9-41

<u>For a Cap on Five Piles</u> (Fig. 9-41)

$\frac{P}{5} / T = d'/.85 \ L - x$, $T_1 = A_s \ \sigma_s = 0.85 \ T$

Assuming $d' = 0.8 \ d$ and $x = 0.17 \ L$,

we get :

$$A_s = P \ L / 7 \ \sigma_s \ d$$

<u>For a Cap on Six Piles</u> (Fig. 9-42)

$\frac{P}{6} / T = d'/L - x$, $T_1 = A_s \ \sigma_s = T$

Assuming $d' = 0.8 \ d$ and $x = 0.2 \ L$

We get :

$$A_s = P \ L / 6 \ \sigma_s \ d$$

<u>For a Cap on Seven Piles</u>

$$A_s = P \ L / 7 \ \sigma_s \ d$$

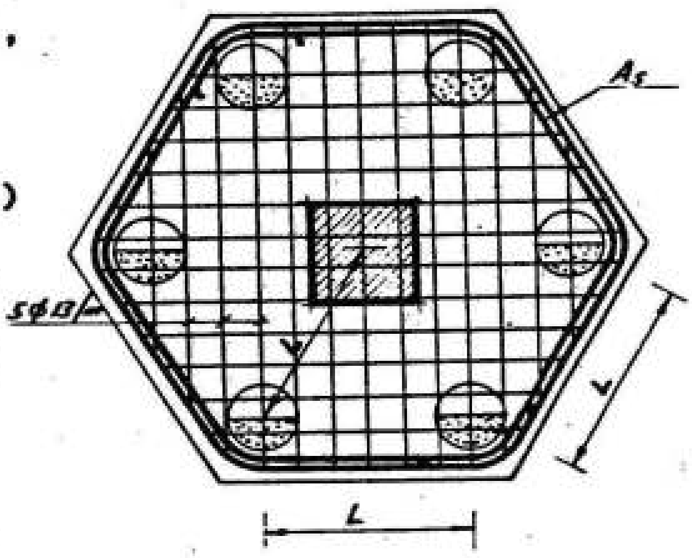


Fig. 9-42

Hence the minimum depths and the corresponding reinforcements according to this method are as follows :

Number of Piles	Diam. of Pile cms	Load per Pile tons	Total load on Cap tons	Spacing between Piles cms	Total depth t cms	Reinforcement A_s in cm2 σ_s=1.4	Reinforcement A_s in cm2 σ_s=2.0
2 Piles	25 30 35 40 45	15 20 30 40 50	30 40 60 80 100	75 90 100 110 120	50 60 65 70 75	10.7 14.3 21.4 29.0 36.0	7.5 10.0 15.0 20.0 25.0
3 Piles	25 30 35 40 45	15 20 30 40 50	45 60 90 120 150	75 90 100 110 120	55 65 70 75 80	6.3 8.4 12.8 16.7 20.8	4.4 5.9 8.9 11.7 14.6
4 Piles	25 30 35 40 45	15 20 30 40 50	60 80 120 160 200	75 90 100 110 120	65 75 85 90 95	7.6 10.3 15.3 20.7 26.0	5.8 7.2 10.7 14.5 18.2
5 Piles	25 30 35 40 45	15 20 30 40 50	75 100 150 200 250	75 90 100 110 120	80 90 100 110 120	9.0 12.1 18.0 24 29.4	6.3 8.5 12.6 16.8 20.6
6 Piles	25 30 35 40 45	15 20 30 40 50	90 120 180 240 300	75 90 100 110 120	90 105 115 125 135	10.7 14.3 21.4 29.0 35.7	7.5 10.0 15.0 20.2 25.0
7 Piles	25 30 35 40 45	15 20 30 40 50	105 140 210 280 350	75 90 100 110 120	90 105 115 125 135	10.7 14.3 21.4 29.0 35.7	7.5 10.0 15.0 20.2 25.0

Effect of Horizontal Forces on Piles

It is recommended to resist horizontal forces acting on a group of piles by semelles at the top of level of the piles, because the lateral displacements that are liable to take place due to these forces are relatively big. Further, tension cracks in the piles due to bending moments may cause undesirable attacks on both concrete and steel created by ground water.

Bending moments and lateral displacements of piles due to horizontal forces can be estimated as follows : (Fig. 9-43).

The foundation modulus k_o due to horizontal farces may be assumed equal to 1.00 kg/cm^3 . If the diameter (or side length) of the pile is d, then

$$k = k_o \, d$$

For all pile diameters one may assume k = 30 kg/cm^2
The characteristic length is given by

$$\ell = 1/n$$

where n is the characteristic value

$$n = \sqrt[4]{\frac{k}{4EI}}$$

So that

$$\ell = \sqrt[4]{\frac{4EI}{k}}$$

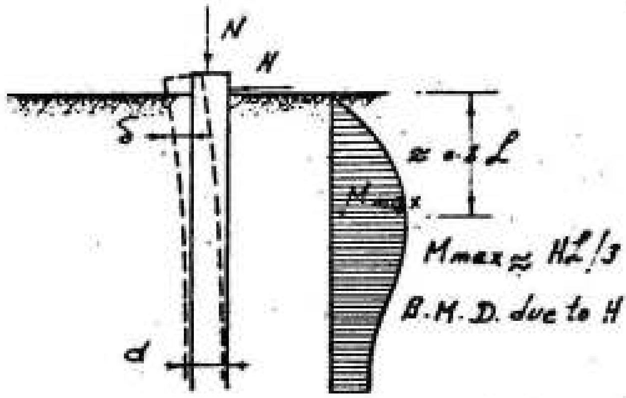


Fig. 9-43 -

E = 200 000 kg/cm^2 is the modulus of elasticity of the concrete in the pile, and
I = moment of inertia of pile section.

If it is required to have no tensile stresses in the pile, then the eccentricity e from N and M_{max} must be smaller than the core distance c of the pile section i.e. $e \leqslant c$.

$$e = M_{max} / N \approx H.\, \ell \,/\, 3\,N$$

For a square pile-section $c = d/6$, $I = d^4 / 12$, and

$$H/N \leqslant d/2\ell = \frac{1}{2} \sqrt{3\,K/E} = \frac{1}{2} \sqrt[4]{3 \times 30/200\,000} = 0.073$$

For a circular pile-section $c = d/8$., $I = \pi d^4 / 64$,and

$$H/N \leqslant 3d/8\ell = \frac{3}{8} \sqrt[4]{16\,k/\pi E} = \frac{3}{8} \sqrt[4]{16 x 30/3.14 x 200000} = 0.062$$

For $e = c$, the stresses at the extreme fibers of the pile are

$$\sigma_{max} = 2\,N/A \quad \text{and} \quad \sigma_{min} = 0$$

Where

A = area of cross-section of pile.

• G. Franz " Konstruktionslehre des Stahlbetons." Zweiter Band Tragwerke, Springer Verlag.

This means that for a horizontal force $H \approx 0.06\ N$, the compressive stress in the pile will be doubled and the tensile stress is equal to zero.

The horizontal displacement $= 2\ H / k\ell$.

Accordingly, the maximum allowed lateral force H on piles must be smaller than @ 5% of the normal force N; otherwise, use inclined piles or arrange semelles capable of resisting these forces, in which case, the horizontal displacement is much reduced as shown in Fig. 9-44.

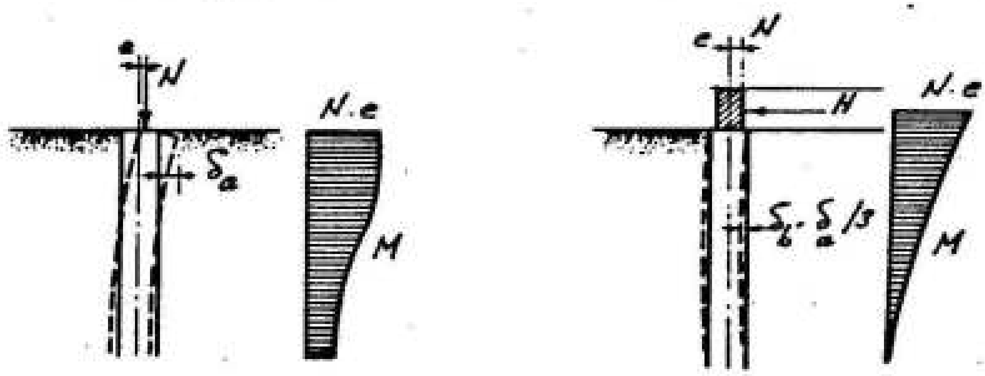


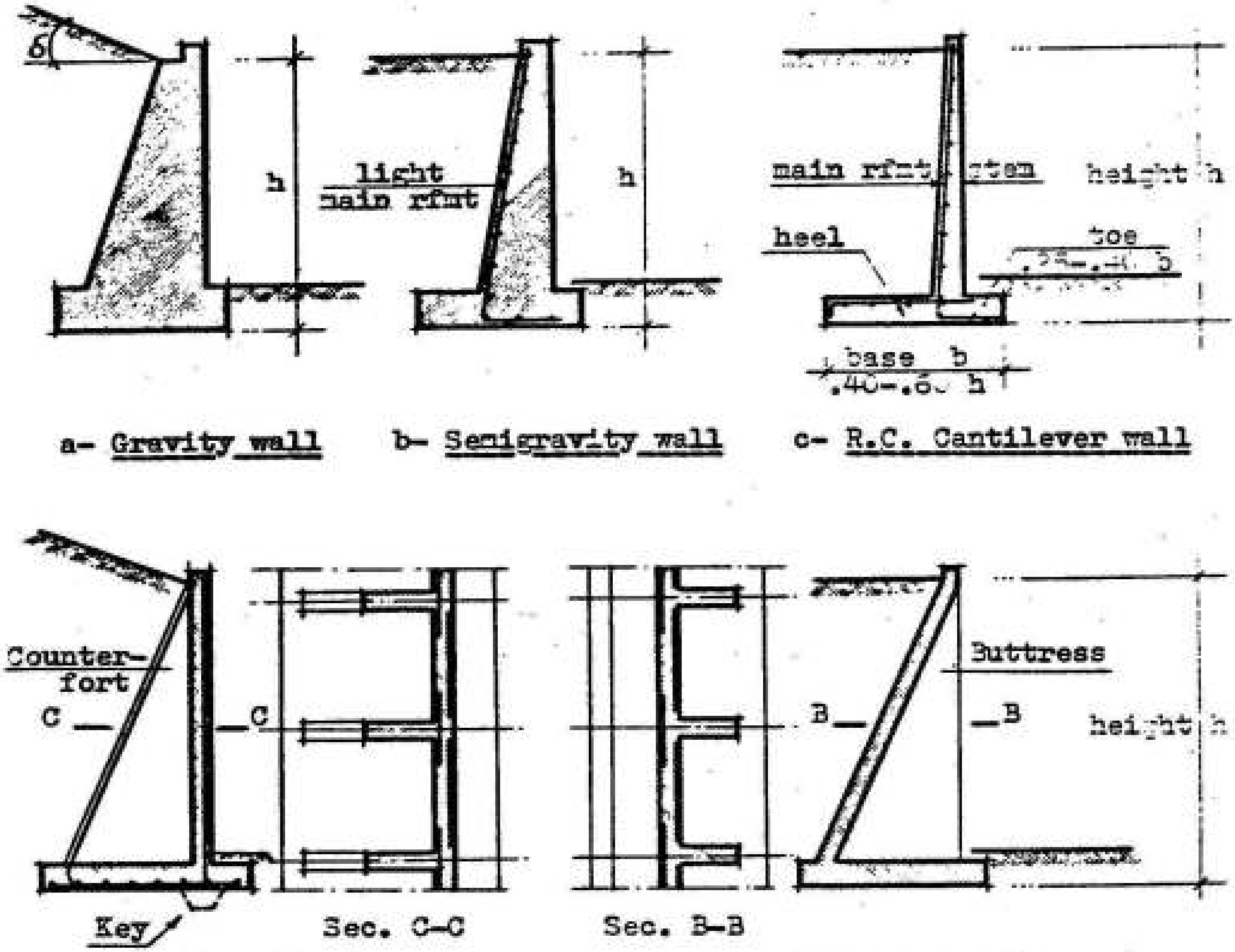
Fig. 9-44

CHAPTER 10

RETAINING WALLS

10.1 Functions and Types of Retaining Walls

Retaining walls are used to hold back masses of earth or other loose material, the top of which is at a higher elevation than the earth in front of the wall and where conditions make it impossible to let



a- Gravity wall b- Semigravity wall c- R.C. Cantilever wall

d- Counterforted wall e- Buttressed wall

Fig. 10-1

those masses assume their natural slopes.

Free standing retaining walls are of various types, the most common of which are shown in Fig. 10-1.

The gravity wall (Fig. 10-1 a) retains the earth entirely by its own weight. Plain concrete or even masonry constitutes an adequate material. Design is then concerned chiefly by keeping the thrust line within the middle third of the cross-section, so that no tensile stresses are developed in the section of the wall on its back.

The reinforced concrete cantilever wall (Fig. 10-1 c) is a reinforced concrete wall that utilizes the weight of the soil itself, on the heel of the wall, to provide the desired stability. Usually, stem, toe and heel are each designed as cantilever slabs.

The breadth of the base b should be 0.40-0.60 times the height h of the stem. In case of heavy surcharge or weak soils b may be bigger than 0.60 h. The length of the toe may be from 0.25-0.40 of the base.

Property rights or other restrictions sometimes make it necessary to omit the toe of the wall as shown in case C_1 of Fig. 10-2 or to omit the heel as in case C_2 of the same figure.

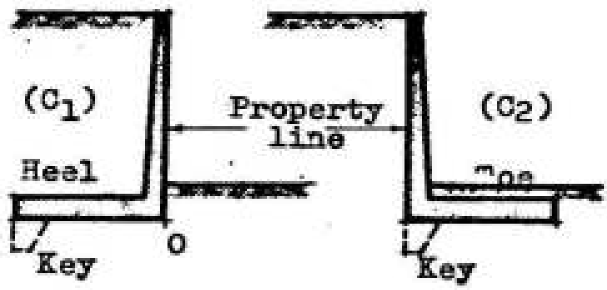


Fig. 10-2

The disadvantage of case C_1 is that, if the length of the heel is not sufficiently big (> 0.50 h) the stress on the soil at O may be excessive.

It is usually difficult to attain the stability of a wall as that shown in Fig. C_2 and to keep it from sliding if the height is great, because of the fact that the stabilizing dead load is small.

The semigravity wall (Fig. 10-1 b) uses very light reinforcement and is intermediate between the gravity and the cantilever types.

The counterforted wall (Fig. 10-1 d) has intermediate vertical ribs on the earth side called counterforts. This is advantageous for the relatively high walls because the counterforts can be heavily reinforced so as to act as supports for the stem and the heel, really transforming the last two parts from simple cantilevers into continuous slabs which are supported by the counterforts. Although the stem and the heel can be relatively thin, the extra formwork and details may offset the economy in materials.

The buttressed wall (Fig. 10-1 e) is like the counterforted wall with ribs on the front side of the wall serving the same general functions as the corresponding parts in the counterforted wall except that their ribs are mainly in compression and not in tension as in the previous case. Such a wall may be built with an inclined slab as shown in Fig. 10-1 e).

Cantilever walls are economical for heights $\leqslant$ @ 5 ms, while counterforts or buttresses are preferred for greater heights.

A retaining wall must be safe against sliding which requires that the frictional force between the lower surface of the base and the soil must be @ 1.5 times the horizontal component of the earth pressure. In critical cases, the use of a key as that shown in Figs. 10-1 d and 10-2 may be recommended.

10.2 Earth Pressure*

In their physical behavior, soils and other granular masses occupy a position intermediate between liquids and soils. if a pit is dug in sand, its sides maintain themselves in a stable position subtending with the horizontal an angle of repose whose tangent is roughly equal to to the coefficient of intergranular friction. If the pit is dug in clay soil, its sides can usually be made vertical over considerable

* 'Design of Concrete Structures' by Winter and others.Published by Mc Graw-Hill Book Company.

depths without support; i.e., the clay will behave like a solid and will retain the shape it is given. If, however, the pit is flooded, the sides will give way, and, in many cases, the saturated clay will be converted nearly into a true liquid. The clay is capable of maintaining its shape by means of internal cohesion, but flooding reduces that cohesion greatly, often to zero.

If a wall is built in contact with a solid, such as rock face, no pressure is exerted on it. If, on the other hand, a wall retains a liquid, as in a reservoir, it is subject at any level h to the hydrostatic pressure w h, where w is the unit weight of the liquid, and h the distance from the surface. If a vertical wall retains soil, the earth pressure p similarly increases proportionally to the depth h, and its magnitude is given by:

$$p = k\,w\,h \qquad 10\text{-}1$$

where w is the unit weight of the soil, and k is a constant which depends on the physical characteristics of the soil.

The pressure which soil exerts on an immovable wall, known as the rest pressure is not easily determined. The measurements that have been made seem to indicate that in this case k = 0.45 to 0.50 for uncompacted, noncohesive soils, such as sand, while it may be as high as 0.80 for the same soils in highly compacted state.

For the triangular pressure distribution to be possible, there must be some sliding of the soil towards the wall. Usually, however, walls move slightly under the action of the earth pressure. Since walls are constructed of elastic material, they deflect under the action of the pressure, and since they generally rest on compressible soils, they tilt and shift away from the soil. Even if this movement at the top of the wall is of the order of h/1000, the rest pressure is materially decreased by it. The magnitude of this pressure against slightly yielding walls is much better known than the rest pressure.

If the wall moves away from the fill, a sliding plane a b (Fig. 10-3) forms in the soil mass, and the wedge a b c, sliding along that plane, exerts pressure on the wall.

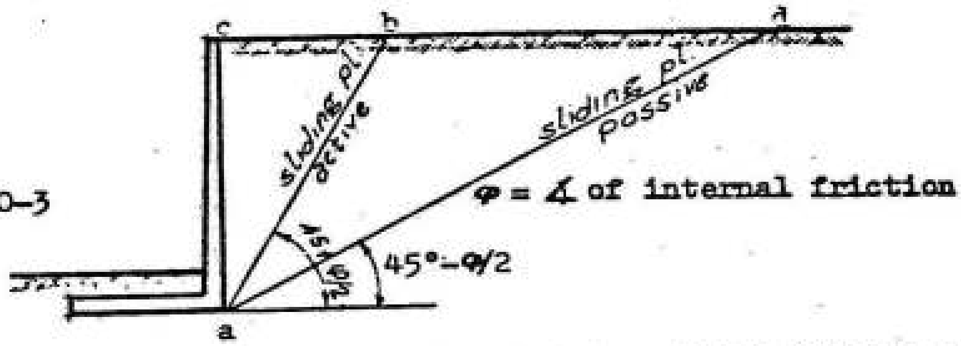


Fig. 10-3

Here, the angle φ is known as the angle of internal friction; i.e., its tangent is equal to the coefficient of intergranular friction, which can be determined by laboratory tests. The corresponding pressure is known as the active earth pressure.

If, on the other hand, the wall is pushed against the fill, a sliding plane a d is formed, and the wedge a c d is pushed upward by the wall along that plane. The pressure which this larger wedge exerts against the wall is known as the passive earth pressure.

The magnitude of these pressures has been analyzed by Rankine, Coulomb and others. If the soil surface subtends an angle δ with the horizontal then, according to Rankine, the coefficient for active earth pressure is:

$$k = \cos\delta \frac{\cos\delta - \sqrt{\cos^2\delta - \cos^2\varphi}}{\cos\delta + \sqrt{\cos^2\delta - \cos^2\varphi}} \qquad 10\text{-}2$$

and for passive earth pressure is:

$$k' = \cos\delta \frac{\cos\delta + \sqrt{\cos^2\delta - \cos^2\varphi}}{\cos\delta - \sqrt{\cos^2\delta - \cos^2\varphi}} \qquad 10\text{-}3$$

For $\delta = \varphi$, the coefficients for active and passive earth pressures are the same and equal to

$$k = k' = \cos\varphi \qquad 10\text{-}4$$

In the frequent case of horizontal surface with $\delta = 0$, we get:

for active earth pressure

$$k = \frac{1 - \sin\varphi}{1 + \sin\varphi} \qquad 10\text{-}5$$

and for passive earth pressure

$$k' = \frac{1 + \sin\varphi}{1 - \sin\varphi} \qquad 10\text{-}6$$

Rankine's theory is valid only for noncohesive soils such as sand and gravel but, with corresponding adjustments, can also be used for cohesive clay soils.

Unfortunately, the true failure due to passive earth pressure is generally not along a d (Fig. 10-3) but is on a curved plane, and the actual passive earth pressure developed is significantly less than this solution indicates.

It should be noted that many practical constructions do not satisfy the displacement requirement, assumed as a necessity for the possible application of the previous equations, for example, basement walls when supported at or near the ground level by the first floor framing. Another case is the usual braced trench construction, where excavation starts with a brace placed near the top of the trench. In both these cases, the total rest pressure* is roughly the same (say 10% more, for the ideal cohesionless soil, than discussed above, or in extreme cases on an individual strut in loose sand possibly as much as 45%), but the resultant acts nearer the mid-depth than at the lower third point. the distribution may vary considerably, but it may be thought of as somewhat parabolic as shown in Fig. 10-4.

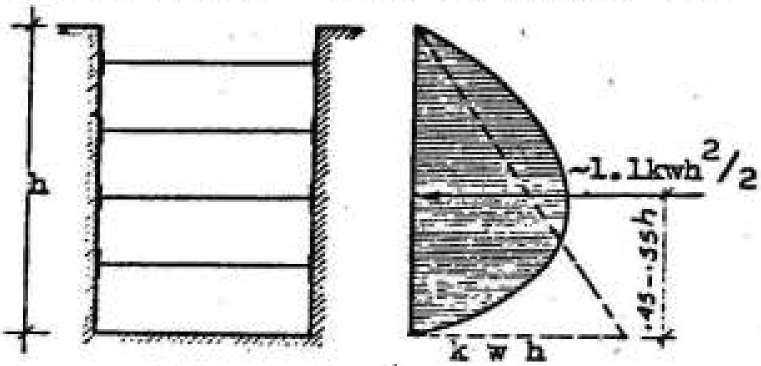


Fig. 10-4

* 'Reinforced Concrete Fundamentals' by Ferguson. Published by John Wiley and son. New York, London.

It might be again emphasized here that this entire discussion is related to cohesionless soils and has ignored the complications brought about by cohesion and probably swelling actions of clay. Materials which expand under increasing moisture content should not be used as backfill behind retaining walls.

Table 10-1 gives the average values for w, φ and the coefficient of friction μ between concrete and the various soils.

Table 10-1. Unit weights w, angles of internal friction φ and coefficients of friction with concrete μ.

Soil	Unit weight in kg/ m^3	φ	μ
1. Sand or gravel without fine particles, highly permeable	1750 - 1900	33 - 40°	0.50 - 0.60
2. Sand or gravel with silt mixture, low permeability	1900 - 2100	25 - 35°	0.40 - 0.50
3. Silty sand, sand and gravel with high clay content	1750 - 1900	23 - 30°	0.30 - 0.40
4. Medium or stiff clay	1600 - 1900	25 - 35°	0.25 - 0.40
5. Soft clay, silt	1500 - 1700	20 - 25°	0.20 - 0.30

The low values of φ for soils 3-5 and the resulting high soil pressure indicate that soils of type 1 or 2 should preferably be used for backfill of retaining walls wherever possiple.

10.3 Earth Pressure for Common Conditions Of Loading

When computing earth pressures on walls four common conditions of loading are most often met:

1) horizontal surface of dry fill at the top of the wall, Fig. 10-5,
2) inclined surface of dry fill sloping up and back from the top of the wall, Fig. 10-6,

3) horizontal surface of dry fill carrying a uniformly distributed additional load (surcharge) such as from goods in a storage yard or traffic on a road. The increase in pressure caused by a uniform surcharge s is computed by converting its load into an equivalent, imaginary height of earth h' above the top of the wall such that

$$h' = s/w \qquad 10\text{-}7$$

and measuring the depth to a given point on the wall from this imaginary surface. This amounts to replacing h with (h + h'). Fig. 10-7

Surcharge far enough removed from the wall causes no pressure on the wall. The presence of a surcharge well to the right of point b in Figs. 10-3 and 10-8 cannot influence the sliding plane a b or the earth pressure. A surcharge within the area c b influences the earth pressure in the manner shown in Fig. 10-8. A slope of 40 or 45° may be assumed.

4) existance of ground water at a level above the base of the wall either permanently or seasonably. In this case, the pressure of the soil above the ground water is determined as usual. The part of the wall below ground water is subject to the sum of the water pressure and the earth pressure. The former is equal to the full hydrostatic pressure $p_w = h_2 \cdot 1$, where h_2 is the distance from the ground-water-level to the point on the wall. The additional pressure of the soil below ground water level is computed from equation 10-1, where, however, for the portion of the soil below water, w is replaced with (w - 1) and the weight of the earth above the ground water can be assumed as a surcharge with height $h' = w\,h_1/(w - 1)$. That is, for submerged soil, buoyancy reduces the effective weight in the indicated manner. Pressures of this magnitude are considerably larger than those of dry soil and must not be overlooked. Fig. 10-9.

10.4 Safety and External Stability

A retaining wall must be safe against failure, sliding, excessive

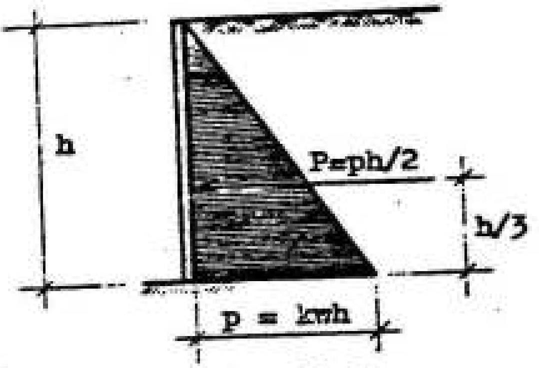


Fig. 10-5

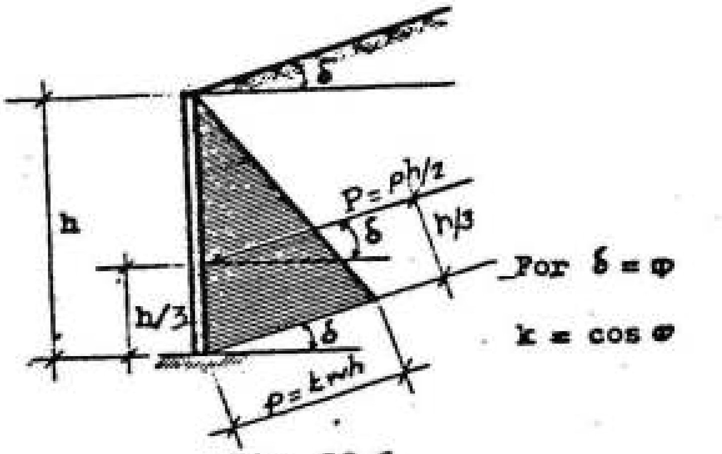


Fig. 10-6

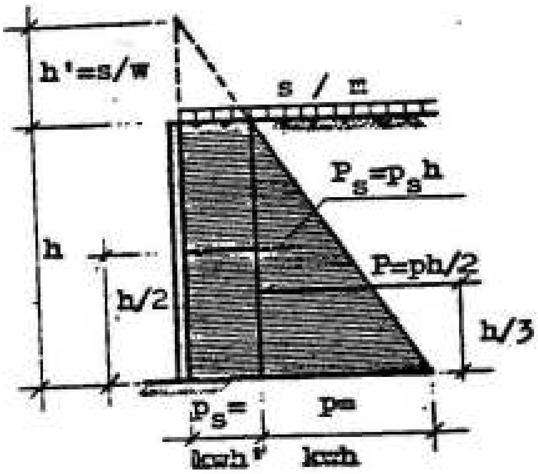


Fig. 10-7

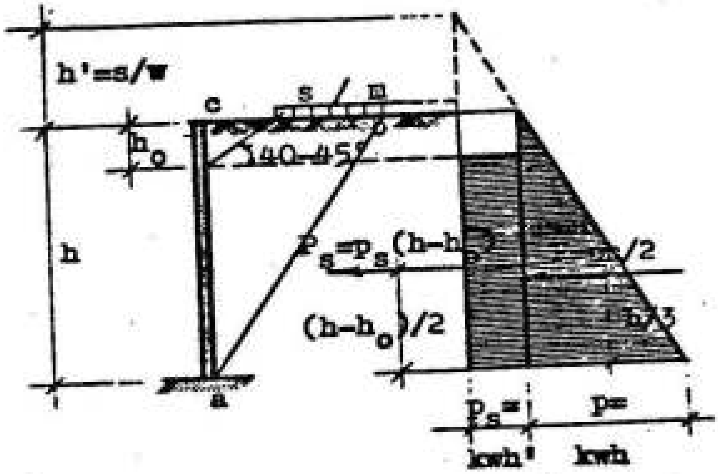


Fig. 10-8

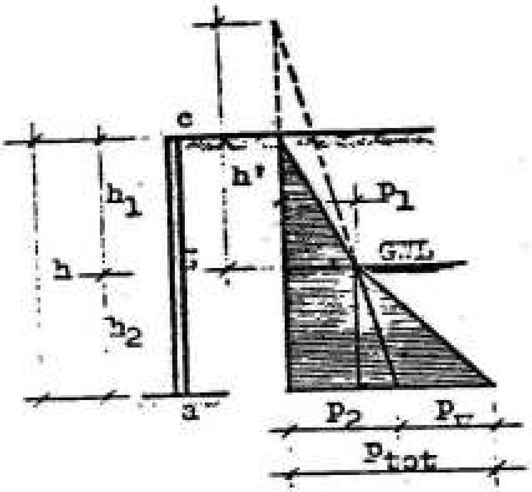


Fig. 10-9

$p_1 = k_1 w h_1$ as usual for dry soil

$p_2 = k_2 (w-1)(h_2 + h')$ where

$h' = w\, h_1 / (w-1)$

$p_w = h_2 \cdot 1$

$p_{tot} = p_2 + p_w$

settlement or tilting and overturning in the following manner:

1) Safety against failure

A reinforced concrete retaining wall must be safe against failure of any of its individual parts: the stem, the heel, the toe and eventually the counterforts or buttresses, if any, i.e., the dimensions, thickness and reinforcement of any of these elements must be so chosen that the maximum moments and shears can be resisted safely. This is in no way different from that used in any other type of reinforced concrete structures.

2) Safety against sliding

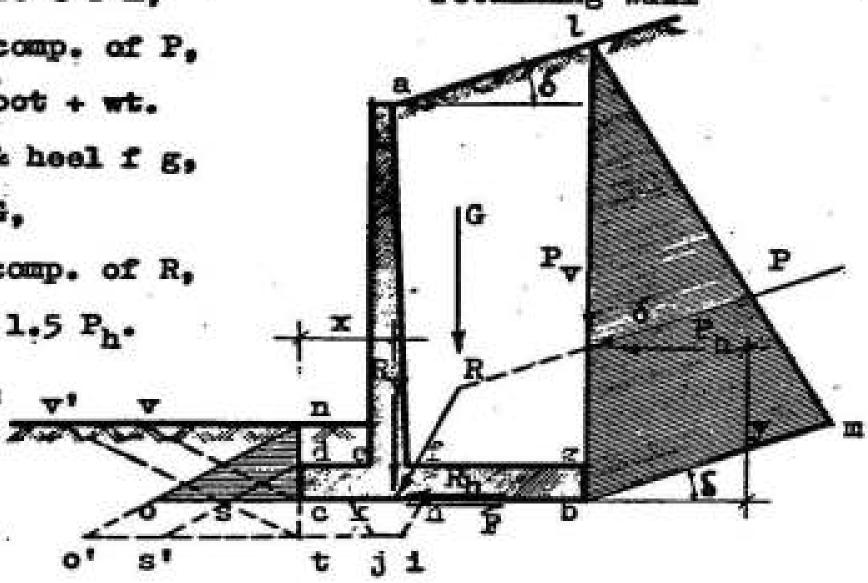
The wall must be safe against sliding i.e. bodily displacement in the direction of the earth pressure.

A wall, such as that of Fig. 10-10, together with the soil on the base slab, may be bodily displaced towards the left by the earth pressure P which acts on plane b l by sliding along the plane b c. Such sliding is resisted mainly by the friction between soil and base along the

P = active earth pressure l b m,
P_v & P_h = vert. & horiz. comp. of P,
G = wt. of wall + its foot + wt. of soil on toe d e & heel f g,
R = resultant of P and G,
R_v & R_h = vert. & horiz. comp. of R,
$F = \mu R_v$ = fric. force > 1.5 P_h.

Passive earth pressures:
Δ n c o on nc or
Δ d c s on dc.
when a key is used:
Δ n t o' on nt or
Δ d t s' on dt.

Forces acting on a rfd. conc. retaining wall



Passive earth pressure   Active earth pres.

Fig. 10-10

same plane.

To prevent sliding, the frictional force $F = \mu R_v$ must exceed the force P_h which tends to produce sliding; a factor of safety of 1.5 is generally assumed satisfactory in this connection, i.e.

$$\mu R_v \geq 1.5\, P_h \qquad 10\text{-}8$$

Actually, for the wall to move to the left, it must push with it the earth n c v, which causes the passive earth pressure indicated by the triangle n c o to act. This passive earth pressure represents a further resisting force which could be added to the left side of equation 10-8. However, this should be allowed only if the earth on the left side is secured against scour or removal. If this condition is not met, it is better not to count on the additional resistance of the passive earth pressure.

If the required sliding resistance cannot be developed by these means, a key - h i j k - can be arranged, preferably below the stem of the wall, to increase the horizontal resistance. In this case sliding, if it occurs, takes place along the planes b h and i j. While along these two planes the friction coefficient μ applies, sliding along t j occurs within the soil mass. The coefficient of friction that applies in this portion is consequently tan φ, where the value of φ may be taken from table 10-1. In this situation, sliding of the front soil occurs upward along t v' so that, if the front fill is secure, the corresponding resistance from passive soil pressure is represented by the pressure triangle n t o'. If doubt exists as to the reliability of the fill above the toe, the free surface should more conservatively be assumed at the top level of the footing, in which case, the passive pressure is represented by the triangle d t s'.

3) Safety against excessive settlement or tilting

In order to prevent excessive settlement, the pressure f_1 under the footing should not exeed the permissible bearing pressure for the particular soil.

To prevent excessive tilting, the smaller bearing pressure f_2 should be bigger than or equal to half the bigger pressure f_1 in clay soils. In rectangular footings, this is the case if the resultant R lies in the middle sixth of the base (Fig. 10-11 a). In sandy soils, f_2 is to be chosen bigger than or equal to zero, a case which takes place if R lies in the middle third of the base (Fig. 10-11 b). If the resultant were located outside the middle third, then f_2 will be tension at and near point b. Obviously, tension cannot be developed between soil and a concrete footing which merely rests on it. Hence, in this case, the pressure distribution of Fig. 10-11 c will develop, which means a slight lifting off the soil of the base at b. Equilibrium requires that R_v pass through the centroid of the pressure distribution triangle.

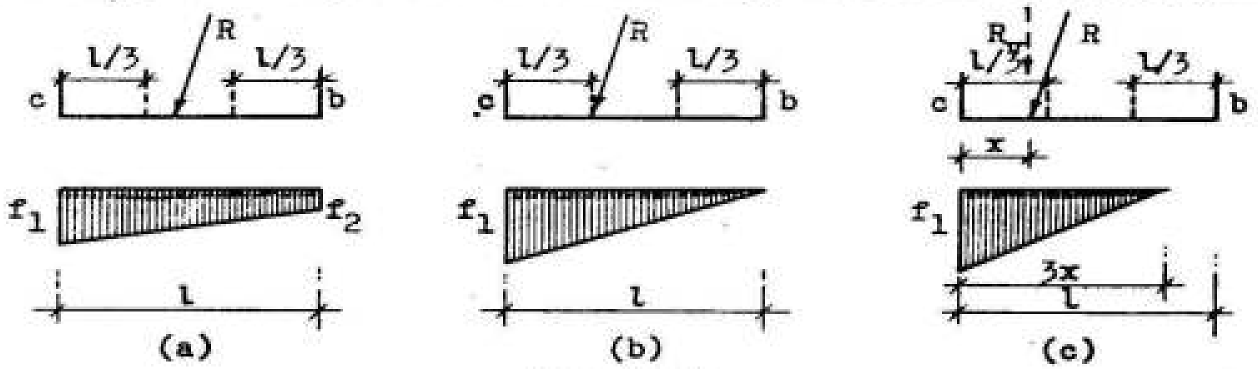


Fig. 10-11

This last case, may be allowed, if the base of the wall is on very incompressible soil, such as well compacted gravel or rock.

It is however good practice to have the resultant located within the middle third.

4) <u>Safety against overturning</u>

If the resultant R (Fig. 10-11) strikes the base b c outside point c, the wall overturns bodily around the same point. If, as is mostly the case, the resultant strikes within the middle third, adequate safety against overturning exists, and no special check need be made. If the resultant is located outside the middle third, a factor of safety of at least 1.5 should be maintained against overturning, i.e.

$$R_v \cdot x > 1.5\ P_h \cdot y \qquad 10\text{-}9$$

10.5 Illustrative Example

It is required to design the reinforced concrete retaining wall shown in Fig. 10-12 and to check its stability.

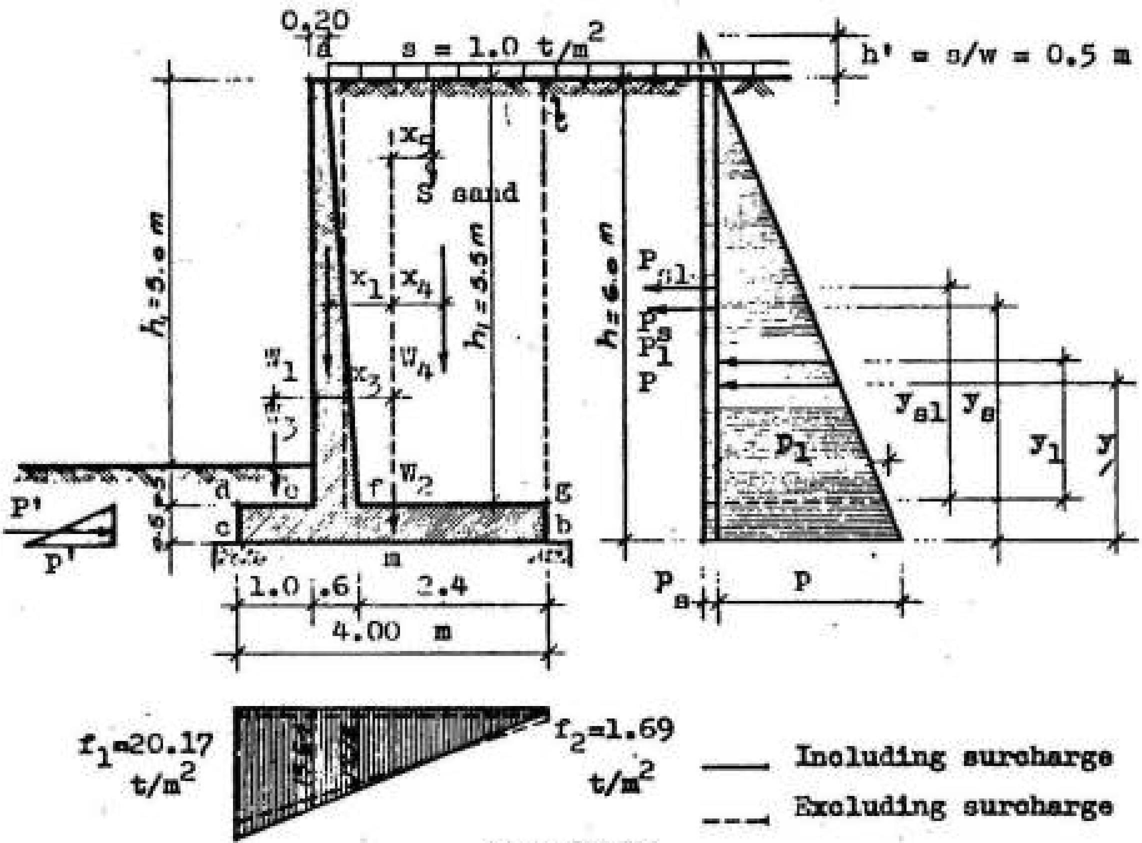


Fig. 10-12

Data: Surcharge s = 1.00 t/m²; for the existing sandy soil assume $w = 2.00$ t/m³, $\varphi = 30°$, $\mu = 0.5$ and max. $f_1 = 2.00$ kg/cm² at 1.00 m from lower ground level.

Lateral pressures

a) Active earth pressures

$$k = \frac{1 - \sin\varphi}{1 + \sin\varphi} = \frac{1 - 0.5}{1 + 0.5} = \frac{0.5}{1.5} = 1/3$$

Surcharge $\quad p_s = k\ s = \frac{1}{3} \cdot 1.0 = 1/3 \quad$ t/m²

Total $P_s \quad P_s = p_s\ h = \frac{1}{3} \times 6 = 2.0$ t/m $\quad$ at $\quad y_s = 6/2 = 3.0$ m

On height h_1: $P_{s1} = \frac{1}{3} \times 5.5 = 1.867$ t/m acting at $y_{s1} = \frac{5.5}{2} = 2.75$ ms

Earth pressure $p = k\ w\ h = \frac{1}{3} \times 2.00 \times 6.0 = 4.00\ t/m^2$

Total $P = p\frac{h}{2} = 4.00 \times \frac{6.0}{2} = 12.00$ t/m acting at $y = 6.0/3 = 2.00$

At height h_1: $p_1 = k\ w\ h_1 = \frac{1}{3} \times 2.0 \times 5.5 = 3.667\ t/m^2$

Total $P_1 = p_1\frac{h_1}{2} = 3.667 \times \frac{5.5}{2} = 10.084$ t/m acting at $y_1 = 5.5/3 = 1.867$

b) <u>Passive earth pressure</u>

The passive earth pressure will be considered on the depth c d = h_f of the footing only:

$$k' = \frac{1 + \sin\varphi}{1 - \sin\varphi} = \frac{1 + 0.5}{1 - 0.5} = 3$$

$$p' = k'\ w\ h_f = 3 \times 2.00 \times 0.5 = 3\ t/m^2$$

Total $$P' = \frac{p'\ h_f}{2} = \frac{3 \times 0.5}{2} = 0.75\ t/m$$

It has to be noted that the value of P' is small and may be neglected.

<u>Design of wall</u>

In order to compute the weight of the wall W_1 exactly, one has first to check the thickness of 60 cms chosen at its base.

Max. moment M at base of wall: $M_{max} = P_{s1}\ y_{s1} + P_1\ y_1$ or

$M_{max} = 1.867 \times 2.75 + 10.084 \times 1.867 = 5.13 + 18.83 = 23.96$ mt

Using the W.S.D-method and assuming concrete C200 & high grade steel with $f_y = 3600\ kg/cm^2$, then $\sigma_c = 70\ kg/cm^2$ and $\sigma_s = 2000\ kg/cm^2$.

In case of concrete deposited against ground, a protective covering of minimum 5 cms is required. Accordingly: d = t - cover = 60 - 5 = 55 cms

so that $d = 55 = k_1\sqrt{M}$ or $55 = k_1\sqrt{23960}$ giving $k_1 = 0.355$

According to sheet 11, $\sigma_c = 58\ kg/cm^2 < 70\ kg/cm^2$ and $k_2 = 1800$

Therefore $A_s = \frac{M}{k_2\ d} = \frac{23960}{1800 \times 0.55} = 24.0\ cm^2$ chosen 12 Φ16 mm/m

The thickness of the wall at its top edge is assumed 20 cms.

In order to design the base of the wall, one has first to check whether its length assures sufficient safety against tilting and sliding. For this purpose, we shall assume preliminarily that the thickness $\overline{d\ c} = \overline{g\ b} = 50$ cms.

Stresses on the soil and safety against excessive settlement and tilting

$$W_1 = \frac{0.2 + 0.6}{2} \times 5.5 \times 2.5 = 5.50 \text{ t} \qquad x_1 = 0.80 \text{ ms}$$

$$W_2 = 4.0 \times 0.5 \times 2.5 = 5.00 \text{ t} \qquad x_2 = 0.00 \text{ ms}$$

$$W_3 = 1.0 \times 0.5 \times 2.0 = 1.00 \text{ t} \qquad x_3 = 1.50 \text{ ms}$$

$$W_4 = 2.6 \times 5.5 \times 2.0 = 28.60 \text{ t} \qquad x_4 = 0.70 \text{ ms}$$

$$40.10 \text{ t}$$

$$S = 2.8 \times 1.0 = 2.80 \text{ t} \qquad x_5 = 0.60 \text{ ms}$$

$$42.90 \text{ t}$$

Stresses including surcharge on $\overline{a\ l}$

Moment about point m at center of base:

$$M = P_s\, y_s + P\, y + W_3\, x_3 + W_1\, x_1 - W_4\, x_4 - S\, x_5$$

$$= 2.0 \times 3.0 + 12.0 \times 2.0 + 1.0 \times 1.5 + 5.5 \times 2.8 - 28.6 \times 0.6 - 2.8 \times 0.6 = 25.2 \text{ mt}$$

Excentricity: $e = M / R_v = 25.2 / 42.9 = 0.588$ ms from center m.

1/6 base = $4.00/6 = 0.667$ ms > 0.558 ms i.e. R lies inside middle $\frac{1}{3}$.

The stresses on the soil are given by:

$$f_{1,2} = \frac{R_v}{A} \pm \frac{M}{Z} = \frac{42.90}{4.0 \times 1.0} \pm \frac{25.2 \times 6}{1.0 \times 4.0^2} = 10.73 \pm 9.44 \qquad \text{giving}$$

$$f_1 = 20.17 \text{ t/m}^2 \qquad \text{and} \qquad f_2 = 1.69 \text{ t/m}^2$$

Stresses excluding surcharge on $\overline{a\ l}$

In this case, the surcharge S, the corresponding horizontal pressure P_s and their moments are to be excluded; i.e. :

$$M = P\, y + W_3\, x_3 + W_1\, x_1 - W_4\, x_4 \qquad \text{or}$$

$M = 12.0 \times 2.0 + 1.0 \times 1.5 + 5.5 \times 2.8 - 28.6 \times 0.7 = 20.9$ mt

In this case $R_v = 40.1$ t so that

excentricity $e = M/R_v = 20.9/40.1 = 0.52$ ms i.e. R lies inside middle 1/3 and

$$f_{1,2} = \frac{40.10}{4.0 \times 1.0} \pm \frac{20.9 \times 6}{1.0 \times 4^2} = 10.0 \pm 7.9 \quad \text{giving}$$

$$f_1 = 17.9 \ t/m^2 \quad \text{and} \quad f_2 = 2.1 \ t/m^2$$

In both cases: max. $f_1 = 2.017$ kg/cm^2 which is approximately equal to the allowed value of 2.00 kg/cm^2 and may be accepted; min. $f_2 = 0.169$ kg/cm^2 is bigger than zero, i.e. there is sufficient safety against excessive stresses or tilting.

Mreover, the resultant R lies inside the middle third of the base, i.e., there is sufficient safety against overturning.

Safety against sliding

Condition 10-8 specifies that μR_v must be bigger than $1.5 P_h$, i.e., including surcharge: $\mu(W_1 + W_2 + W_3 + W_4 + S) > 1.5 (P + P_s - P')$

We have $0.5 \times 42.9 = 21.45 > 1.5 (12.0 + 2.0 - 0.75) = 19.88$ tons.

Excluding surcharge, we have:

$$0.5 \times 40.1 = 20.05 > 1.5 (12.0 - 0.75) = 16.88 \text{ tons.}$$

This means that condition 10-8 is satisfied and the wall has sufficient safety against sliding.

Design of base

For the design of base, the maximum moments at f and e (in heel and toe respectively), take place for the case of surcharge on $\overline{a\,1}$, i.e.,

$M_f = S \times 1.0 + W_4 \times 1.1 + W_{fg} \times 1.2 - f_2 \times 2.4 \times 1.2 - [(12.78 - 1.69) \times 2.4 \times 0.8] = 2.8 \times 1.0 + 28.6 \times 1.1 + 2.4 \times 0.5 \times 2.5 \times 1.2 - 11.09 \times 2.4 \times 0.8 = 10.9$ mt

Note: The tensile stresses due to M_f are at the upper fiber of the heel.

$M_e = W_3 \times 0.5 + W_{de} \times 0.5 - 15.54 \times 1.0 \times 0.5 - [(20.17 - 15.54)\ 1.0 \times 0.67] = 1.0 \times 0.5 + 1.0 \times 0.5 \times 2.5 \times 0.5 - 7.77 - 3.1 = - 9.74$ mt

The tensile stresses due to M_s are at the lower fiber of the toe.

For a thickness of base = 50 cms, we have:
$0.45 = k_1 \sqrt{10900}$ giving $k_1 = 0.43$ i.e. $\sigma_c = 45$ kg/cm^2 & $k_2 =$ 1850, so that $A_s = \frac{10900}{1850 \times 0.45} = 13.25$ cm^2 at the upper fiber of the heel; the same order of reinforcement may be used at the lower fiber of the toe.

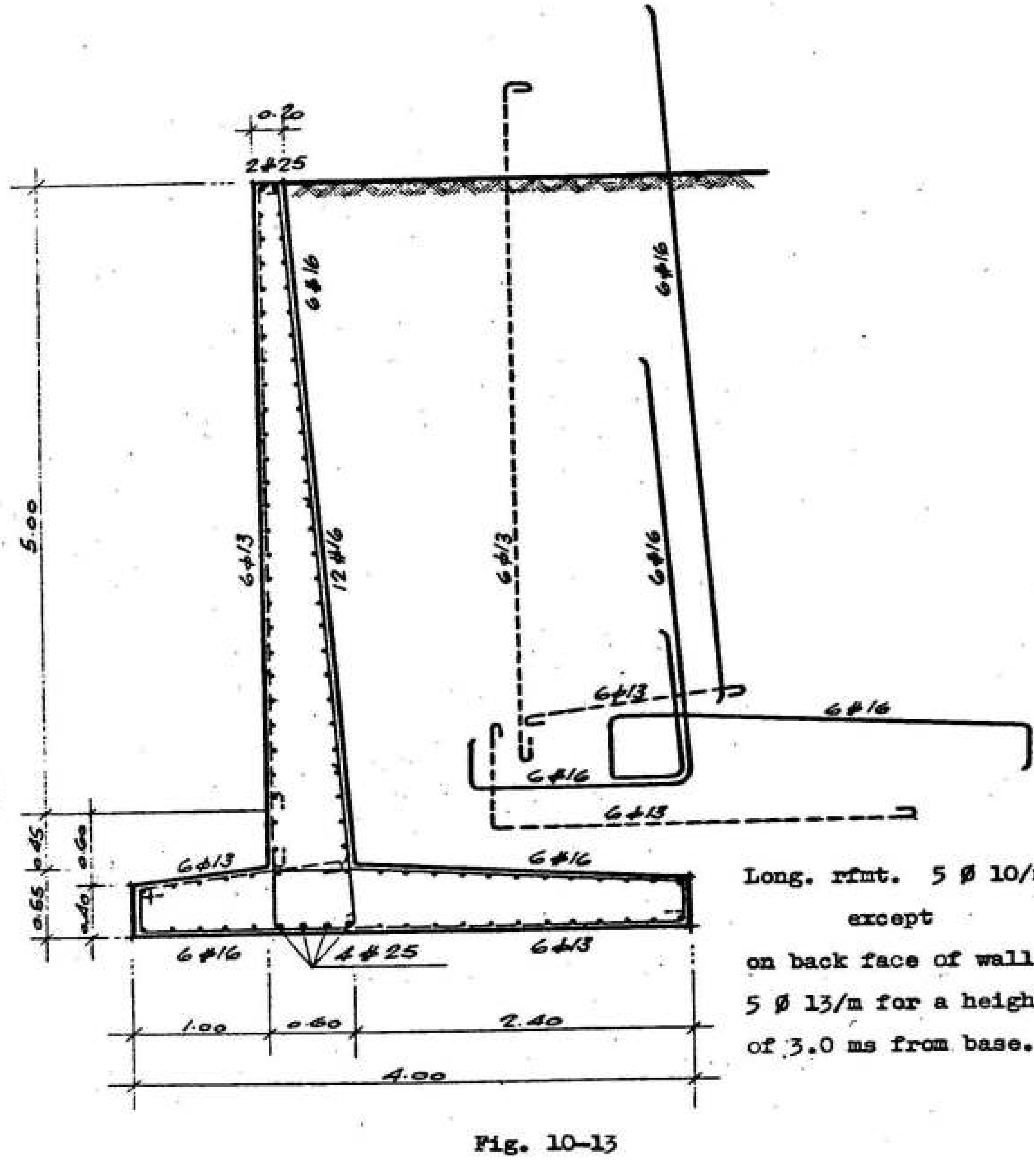
In order to have a convenient choice of reinforcement of 6 Φ 16/m (A_s = 12 cm^2), the thickness of both the heel and toe may be chosen 40 cms at their free ends and 55 cms at the stem of the wall, in which case

$$A_s = \frac{10900}{1850 \times 0.50} = 11.8 \text{ cm}^2 \qquad \text{chosen} \quad 6 \, \Phi \, 16 \text{ mm/m}$$

The compression reinforcement at the outer face of the wall, the upper face of the toe and the lower face of the heel is chosen 5 Φ 13/m; the longitudinal distributing reinforcements are to be > 20% of main reinforcements; they are chosen 5 Φ 10/m except on back face of wall, where they are chosen 5 Φ 13/m for a height of 3.0 ms from base.

In order to count for the eventual settlement of the wall in the longitudinal direction, it is advisable to arrange expansion joints every 25 to 40 ms and, to introduce some heavy longitudinal reinforcement at the bottom and top surfaces of the stem of the wall (@ 1% of the section of the wall at bottom and @ 0.5% at top). In our case, such reinforcement is chosen 4 Φ 25 mm at bottom and 2 Φ 25 mm at top. The details of reinforcements are shown in Fig. 10-13.

For the design of counterforted walls refer to "Theory and Design of Reinforced Concrete Tanks" Chapter VII-4. By M. Hilal.



Long. rfmt. 5 Ø 10/m
except
on back face of wall:
5 Ø 13/m for a height
of 3.0 ms from base.

Fig. 10-13

CHAPTER 11

REINFORCED CONCRETE SUBJECT TO TORSION

11.1 Fundamentals

If a homogeneous element of circular cross section and length l is subject to a torsional moment M_t (Fig. 11-1), torsional shear stresses will be developed.

The two end sections twist, relative to each other, an angle φ. The twisting angle per unit length ϑ is given by:

$$\vartheta = \frac{\varphi}{l} \quad \text{.............. 11-1}$$

In homogeneous materials, the torsional shear stresses τ are proportional to the strains.

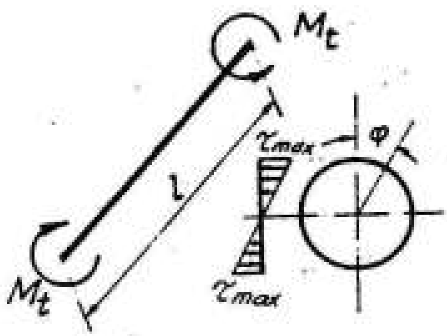


Fig. 11-1

Assuming that the torsional shear stress at a radius r = 1 is τ_1, then we get for a circular section: $\tau = \tau_1 \, r$ 11-2

i.e. the torsional shear stresses increase linearly from the center of a circular section to its outside surface.

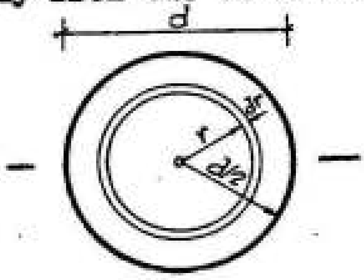


Fig. 11-2

If an elemental area $dA = 2\pi \, r \, dr$, (Fig. 11-2), is subject to a torsional shear stress τ, it shall cause a torsional moment dM_t about the center equal to:

$$dM_t = \tau \, dA \, r = \tau_1 \, r^2 \, dA$$

and

$$M_t = \tau_1 \int r^2 \, dA = \tau_1 \, I_t \quad \text{............... 11-3}$$

where I_t is the torsional moment of inertia of the section; it is given by:

$$I_t = 2\ I_x = \frac{\pi\ d^4}{32} = \frac{A\ d^2}{8}$$

The torsional section modulus Z_t is therefore equal to:

$$Z_t = \frac{2\ I_t}{d} = \frac{\pi\ d^3}{16} = \frac{A\ d}{4}$$

At the surface of a circular section: $\tau_{max} = \tau_1 \frac{d}{2}$ or

$$\tau_{max} = \frac{d}{2} \cdot \frac{M_t}{I_t} = \frac{M_t}{Z_t} = \frac{16\ M_t}{\pi\ d^3} = \frac{4\ M_t}{A\ d} < \text{allowable } \tau. \quad \text{.........} \quad 11\text{-}4$$

If the relative angular displacement of two sections at a unit distance is γ; where $\gamma = \frac{\tau}{G}$ 11-5

and $G = \frac{E}{2(1 + \nu)}$ = bulk modulus of elasticity in which

ν = Poisson's ratio, then

the relative displacement in the corresponding points on the outside surface of two cross sections at a distance l (or dx), Fig. 11-3, is given by $\gamma\ l = \varphi \frac{d}{2}$. Hence

$$\varphi = \frac{2\ \gamma\ l}{d} = \frac{M_t\ l}{I_t\ G} \quad \text{......................} \quad 11\text{-}6$$

and

$$d\varphi = \frac{M_t\ dx}{I_t\ G}$$

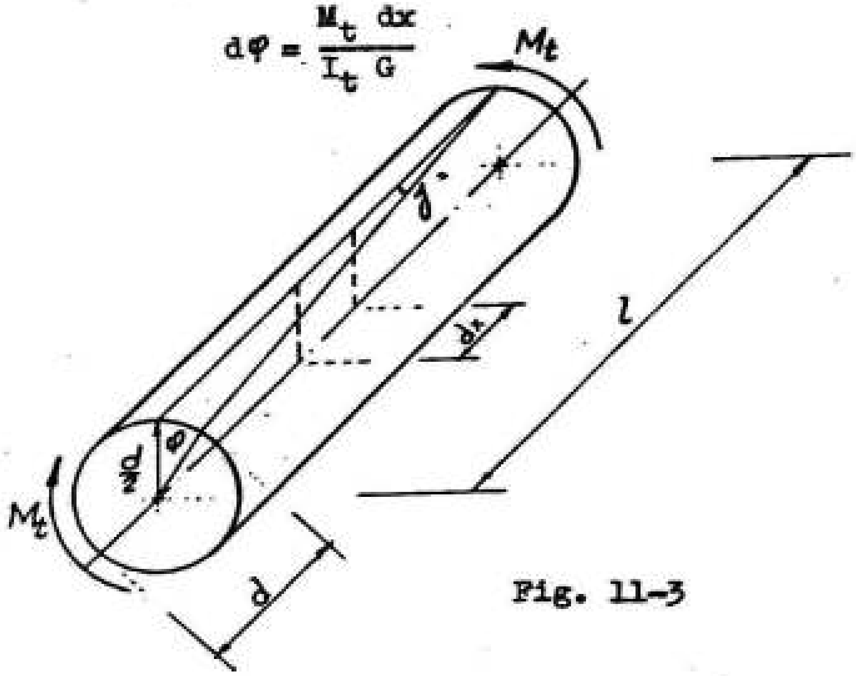


Fig. 11-3

The angle of twist ϑ is therefore given by:

$$\vartheta = \frac{M_t}{I_t\ G} \quad \text{.....................} \quad 11\text{-}7$$

In rectangular cross-sections subject to torsion, (Fig. 11-4), the torsional shear stress at the center is equal to zero. They have their bigger values at the outside surface with maximum magnitude at the middle of the longer side of the section and zero at the corners.

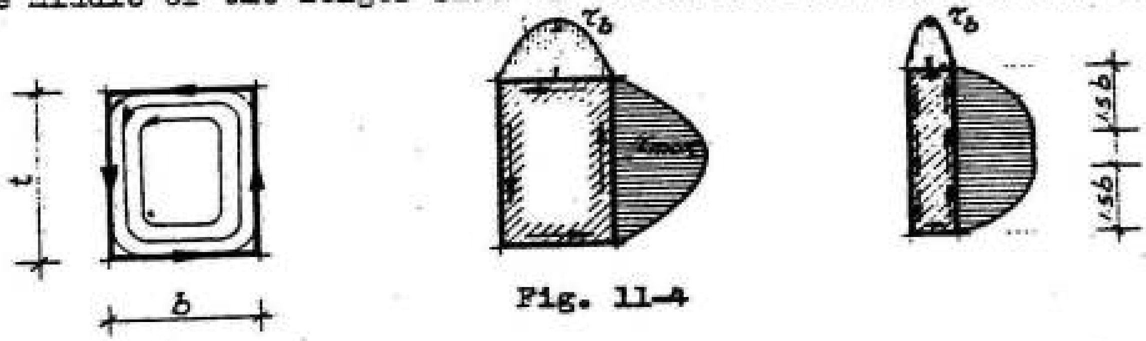


Fig. 11-4

The torsional shear stresses τ and the angle of twist ϑ can however be expressed by relations similar to those of circular sections, in the form:

$$\tau_b = \psi_1 \tau_{max} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 11\text{-}8$$

$$\tau_{max} = \frac{M_t}{Z_t} = \psi_2 \frac{M_t}{b^2\, t} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 11\text{-}9$$

in which

$$\psi_2 = 3 + \frac{2.6}{t/b + 0.45} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 11\text{-}10$$

$$\vartheta = \frac{M_t}{G\, I_t} = \psi_3 \frac{M_t}{G\, b^3 t} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 11\text{-}11$$

The values of ψ are given in table 11-1.

Table 11-1: Values of ψ_1, ψ_2 and ψ_3 for rectangular sections

t/b	1	1.5	2	3	4	6	8	10	∞
ψ_1	1.000	0.858	0.796	0.753	0.745	0.743	0.743	0.743	0.743
ψ_2	4.81	4.33	4.07	3.74	3.55	3.40	3.25	3.20	3.00
ψ_3	7.15	5.10	4.37	3.81	3.56	3.40	3.26	3.20	3.00

For a circular ring with diameters D and d, we have:

$$A = \frac{\pi}{4} (D^2 - d^2)$$

and

$$\left.\begin{aligned} \tau_{max} &= \frac{16\, M_t\, D}{\pi (D^4 - d^4)} = \frac{4\, M_t \cdot D}{A\, (D^2 + d^2)} \\ \vartheta &= \frac{32\, M_t}{G\, \pi\, (D^4 - d^4)} = \frac{8\, M_t}{G\, A\, (D^2 + d^2)} \end{aligned}\right\} \quad \ldots\ldots \quad 11\text{-}12$$

For T, I and L-sections common to reinforced concrete constructions, torsional shear at the face of the web can be approximated by the expression:

$$\tau_{max} = \frac{3\ M_t\ b_w}{\Sigma\ b^3\ t} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 11\text{-}13$$

in which b_w = width of web,

b = smaller width of each component rectangle of section,

t = longer dimension of each component rectangle of sec.

$\Sigma\ b^3\ t$ = sum of $b^3\ t$ terms of the component rectangles.

For a thin circular ring of thickness s and average diameter d_m where : $d_m = D - s$, we have

$$\tau_{max} = \frac{2\ M_t}{\pi\ d_m^2\ s} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 11\text{-}14$$

$$\vartheta = \frac{4\ M_t}{G\ \pi\ d_m^3\ s} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 11\text{-}15$$

In thin hollow sections, the torsional shear τ s may be assumed constant at all points of the section, Fig. 11-5. Taking moments about any point O, we get

$M_t = \int \tau\ s\ du\ a = \tau\ s \int a\ du$

According to Fig. 11-5, we have:

$\overline{dA} = \frac{1}{2}\ a\ du$ so that

$M_t = \tau\ s \int 2\ \overline{dA} = \tau s\ 2\ \bar{A}$ or

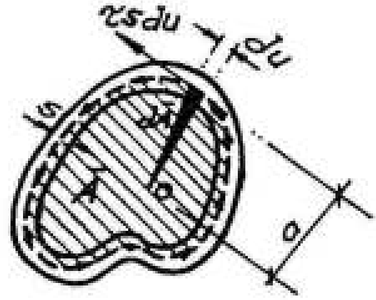


Fig. 11-5

$$\tau = \frac{M_t}{2\ \bar{A}\ s} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 11\text{-}16$$

in which $\bar{A}$ is the shaded area inside the dashed line shown in Fig. 11-5. The maximum torsional shear stress takes place at points of minimum thickness.

Example (Fig. 11-6) $\bar{A} = b\ t$

$$\tau_1\ s_1 = \tau_2\ s_2 = \tau_3\ s_3 = \frac{P\ e}{2\ b\ t} \quad 11\text{-}17$$

$$\tau_1 = \frac{Pe}{2bt\ s_1}\ ,\ \tau_2 = \frac{Pe}{2bt\ s_2}\ ,\ \tau_3 = \frac{Pe}{2bt\ s_3}$$

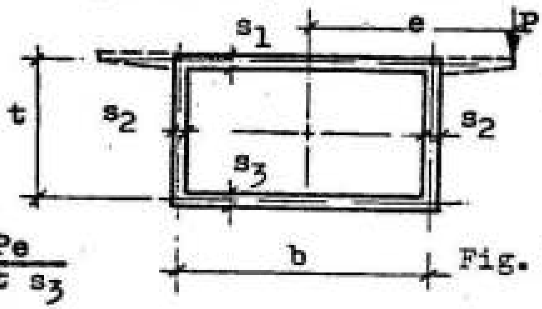


Fig. 11-6

11-2 Torsional Moments and Angles of Twist

The torsional moment in a simple beam or cantilever subject to M_t is the shearing force for M_t as a load, while the twisting angle φ in beams is represented by the bending moment for M_t as a load (and in cantilevers as shown in Fig.) divided by I_t G. Hence (Fig. 11-7)

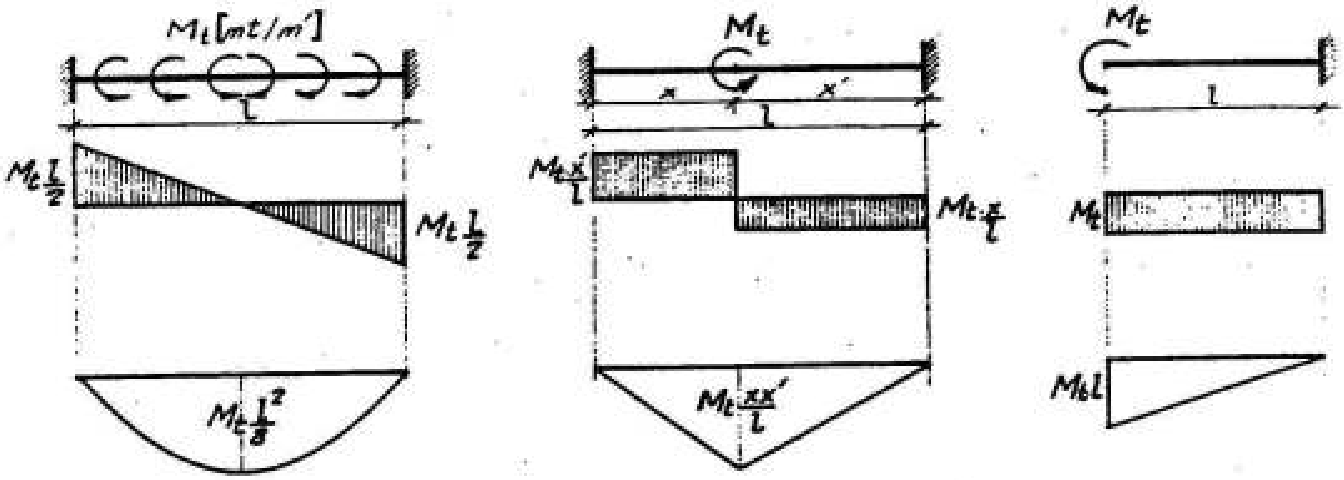


Fig. 11-7

In statically indeterminate structures, the statically indeterminate torsional moments may be determined by the theory of virtual work; in which:

$$1.\delta_{12} = \int \frac{M_{t1}\ M_{t2}}{I_t\ \ G} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 11\text{-}18$$

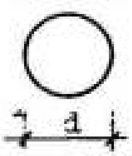
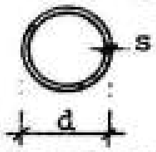
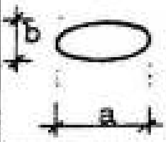
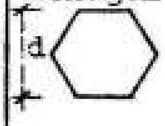
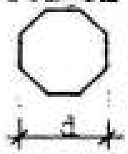
The torsional moment of inertia I_t and the section modulus Z_t for different sections are given in table 11-2.

11-3 Allowable Stresses

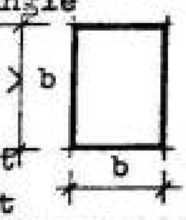
Elements subject to torsion are generally calculated according to the working stress design method. According to the Egyptian Code of Practice, the allowable and limit of shear torsional stresses (τ_1 & τ_2) are the same as those for shear. (Refer to table 3-2).

In case torsion is combined with flexural shear, the allowable and limit of shear stresses (τ_{t1} and τ_{t2}) may be chosen as given in table 11-3.

Table 11-2 I_t and Z_t for Different Sections

Section	Circle	Cir. ring	Thin ring	Ellipse	Hexagon	Octagon
						
I_t	$\frac{\pi}{32} \cdot d^4$	$\frac{\pi}{32}(D^4-d^4)$	$\frac{\pi}{4} \cdot d^3 \cdot s$	$\frac{\pi}{16} \frac{a^3 b^3}{a^2+b^2}$	$0.133\ d^4$	$0.130\ d^4$
Z_t	$\frac{\pi}{16} \cdot d^3$	$\frac{\pi}{16}(\frac{D^4-d^4}{D})$	$\frac{\pi}{2} \cdot d^2 \cdot t$	$\frac{\pi}{16}\ a \cdot b^2$	$0.188\ d^3$	$0.185\ d^3$

Rectangle



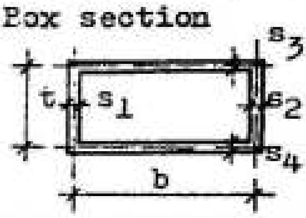
$t > b$

$I = \alpha b^3 t$

$Z = \beta b^2 t$

t/b	1.00	1.25	1.50	2.00	3.00	4.00	5.00	10.00	∞
α	.140	.171	.196	.229	.263	.281	.299	.313	.333
β	.280	.221	.231	.246	.267	.282	.299	.313	.333

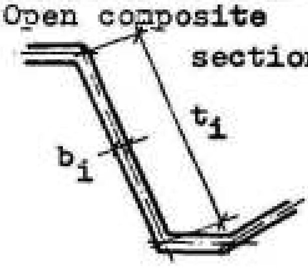
Box section



$$I_t = \frac{4\ b\ t}{\frac{1}{b}\left(\frac{1}{s_1} + \frac{1}{s_2}\right) + \frac{1}{t}\left(\frac{1}{s_3} + \frac{1}{s_4}\right)} \qquad Z_t = 2\ b\ t\ s_{min}$$

N.B. $s_1, s_2 \ll b$ and $s_3, s_4 < t$

Open composite section



$$I_t \simeq \Sigma\ b_i^3\ t_i / 3$$

* The distribution of the torsional moment on the different elements of the section can be done according to the relation: $M_{ti} = M_t\ I_{ti}/\Sigma I_{ti}$

Closed thin-wall sec. s = constant: $I_t = 4\ A_m^2\ s\ /\ U$ $\qquad Z_t = 2\ A_m\ t_{min}$

Table 11-3 Maximum allowable Torsional + Shear Stresses in kg/cm^2

Conc. quality	C120	C160	C180	C200	C225	C250	C275	C300
Without rfmt τ_{t1}	5	7	8	8	9	9	10	10
with rfmt τ_{t2}	17	20	21	22	23	24	25	26

11-4 Reinforcement for Torsion

Failure of a concrete member subject to torsion can be delayed by arranging longitudinal reinforcement to cross the potential cracks. The most efficient form of torsional reinforcement would consist of 45° spirals arranged in the direction of the tensile stresses. Such spirals are difficult to execute and are generally not used.

Diagonal tensile stresses due to torsion differ from those due to transverse shear in that they exist on all four faces of a rectangular section. It is evident that U-shaped stirrups or bent up bars are not suitable as torsional reinforcement; closed loops must be used. These are placed perpendicular to the axis of the member. Additional longitudinal bars are placed at the corners of the stirrups and along the perimeter of the section to accomodate the longitudinal component of the stress and form a closed cage, (Fig. 11-8 a). Stirrup loops

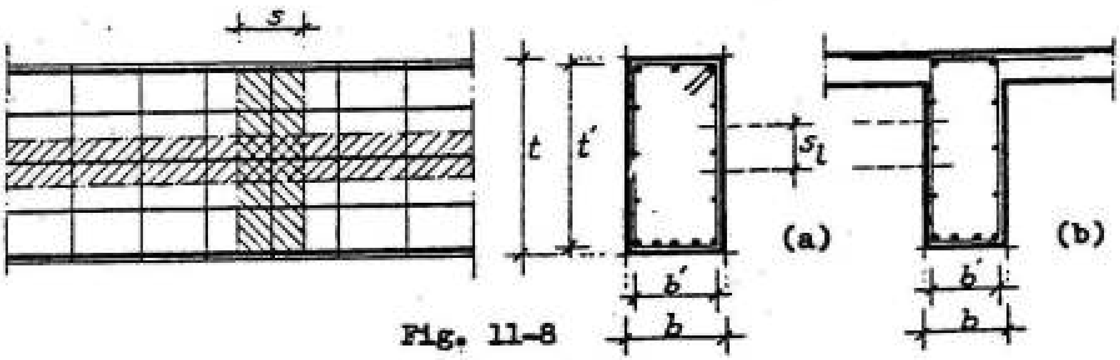


Fig. 11-8

must have a sufficient length of embedment at the end of bars to ensure that allowable stress can be developed. In flanged sections, the ends of the stirrups may project into the flange, (Fig. 11-8 b).

It is obvious that all such web reinforcement is in addition to that placed for flexural shear; the two areas of stirrups can however be combined in one set.

The area required for a longitudinal bar A_{sl} resisting torsion is given by:

$$A_{sl} = \frac{M_t \, s_l}{2 \, A_k \, \sigma_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 11\text{-}19$$

and the area of one branch of vertical stirrups resisting torsion is given by:

$$A_{st} = \frac{M_t \; s}{2 \; A_k \; \sigma_s} \quad \text{..................} \quad 11\text{-}20$$

in which

A_k = area of core = b' t'

s = spacing of vertical stirrups } resisting torsion

s_l = spacing of longitudinal bars }

Experimental evidence has indicated that the resistance of reinforced concrete to torsion is low that relatively big sections are required for low values of torsional moments, and that if torsional reinforcement is provided, it need be designed only for the excess shear above that permitted on the plain concrete. In cases where torsional and flexural shear combine, the shear resisted by the concrete (3 - 4 kg/cm^2) should be substracted only once, either from the torsional or flexural-shear requirement, but not from both.

It is therefore advisable to have relatively small spans for elements subject to appreciable values of torsion.

11-5 <u>Illustrative Examples</u>

1) It is required to determine the minimum depth and the necessary reinforcement for a beam of rectangular cross-section subject to a torsional moment M_t = 3000 kg m, if normal mild steel and concrete C160 are used.

Assume ratio of depth t to breadth b of cross-section is 3. Then According to equation 11-9, we have: $\tau_{max} = \psi_2 \frac{M_t}{b^2 \; t}$

Table 11-1 gives ψ_2 = 3.74, and table 11-3 gives τ_{max} = 20 kg/cm^2.

Therefore: $20 = 3.75 \frac{300000}{b^2 \times 3 \; b}$ giving b^3 = 18700 i.e. b = 26.5 cms

and t = 3 x 26.5 = 80 cms chosen b = 30 cms and t = 80 cms.

Accordingly: b' = 25 cms and t' = 75 cms

The torsional moment that can be resisted by the concrete core, M_{tc}, may be calculated from equation 11-9 in which b' and t' of core replace b and t, and, τ_c = 3.0 kg/cm^2 replace τ_{max}. Hence

$$M_{tc} = \frac{b'^2\ t'\ \tau_c}{\psi_2} = \frac{25^2 \times 75 \times 3.0}{3.74} = 377\ 00 \text{ kg cm}$$

The torsional moment to be resisted by steel reinforcement M_{ts} is:

$$M_{ts} = M_t - M_{tc} = 3000\ 00 - 377\ 00 = 2623\ 00 \text{ kg cm}$$

Area of vertical stirrups per meter in each face is given by:

$$A_{st} = \frac{M_{ts} \times 100}{2\ A_k\ \sigma_s} = \frac{2623\ 00 \times 100}{2 \times 25 \times 75 \times 1400} = 5.0 \text{ cm}^2/\text{m} \quad \text{chosen } \phi\ 10 \text{ mm @ } 15 \text{ cm.}$$

The longitudinal reinforcement is to be chosen also ∅ 10 mm at a minimum spacing of 15 cms.

2) A reinforced concrete T-beam has an overall depth of 80 cms, web width of 30 cms, flange width of 90 cms and flange thickness of 12 cms. Calculate the amount of torsional reinforcement required if the beam is subject to a twisting moment of 4000 kg m using normal mild steel, and concrete C160.

According to equation 11-13, the maximum torsional shear stress is:

$$\tau_{max} = \frac{3\ M_t\ b_w}{\Sigma\ b^3\ t} \qquad \text{in which}$$

b_w = 30 cms , $\Sigma b^3 t = 30^3 \times 80 + 10^3 \times 90 = 2\ 250\ 000$ cm^4 so that

$$\tau_{max} = \frac{3 \times 4000\ 00 \times 30}{2\ 250\ 000} = 16.0 \text{ kg/cm}^2 < 20 \text{ kg/cm}^2$$

Assuming τ_c = 3.0 kg/cm^2, The torsional moment that can be resisted by the concrete core is given by:

$$M_{tc} = \frac{\tau_c}{3\ b'_w}\ \Sigma b'^3\ t' \qquad \text{in which}$$

b'_w = 25 cms , $\Sigma b'^3 t' = 25^3 \times 75 + 7^3 \times 87 = 1\ 202\ 000$ cm^4 i.e.

Torsional moment to be resisted by steel reinforcement is given by:

$$M_{ts} = M_t - M_{tc} = 4000\ 00 - 480\ 00 = 3520\ 00 \text{ kg cm}$$

Area of vertical stirrups or longitudinal bars per m in each face is:

$$A_{st} = \frac{M_{ts} \times 100}{2\ \sigma_s\ \Sigma A_k} \quad \text{in which} \quad \Sigma A_k = 25 \times 75 + 7 \times 87 = 2448 \text{ cm}^2 \quad \text{or}$$

$$A_{st} = \frac{3520\ 00 \times 100}{2 \times 1400 \times 2448} = 5.135 \text{ cm}^2/\text{m} \quad \text{chosen} \quad \phi\ 10 \text{ mm @ } 15 \text{ cms.}$$

PART IV

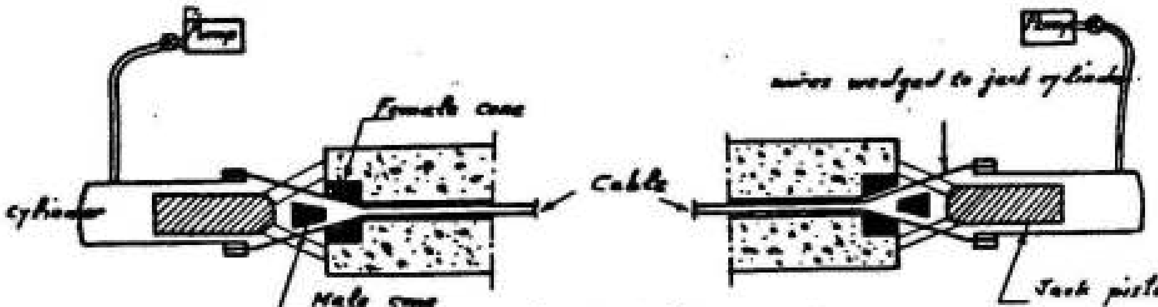
FUNDAMENTALS OF PRESTRESSED CONCRETE

I - INTRODUCTION

I-1 Basic Idea

Concrete can resist high compressive stresses and very low tensile stresses. Due to this fact, cracks generally appear in the tension zone of reinforced concrete elements under working loads; for which reason, it is generally assumed in reinforced concrete design that concrete in tension does not statically act and steel reinforcements resist all the tensile stresses. The tension zone in reinforced concrete elements subject to axial tension covers the whole section , and in elements subject to simple bending or eccentric forces it generally covers a big part of the cross-section which means that a big amount of the concrete used in such elements is cracked and statically not acting although it adds to the dead weight of the structure and concequently to the loads on the columns and foundations.

Prestressing of concrete has been introduced to counterbalance any tensile stresses due to dead and live loads, the prestress is generally created by tensioning the steel reinforcements in the manner shown diagramatically in figure I-1.

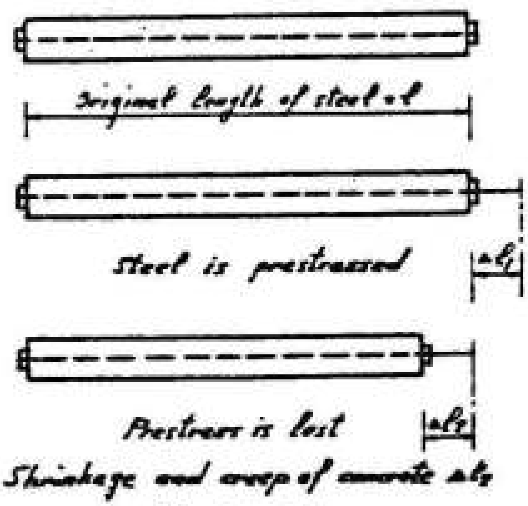


(FIG I-1)

The definition of prestressed concrete as given by the U.A.R. provisional Code of Practice is :

"Prestressed Concrete" denotes concrete in which internal stresses are induced, generally by the use of high tensile steel. This operation is done before or during the action of external loads and in such a way as to completely eliminate or at least to effectively reduce tensile stresses under the action of working loads together with the provision of an ample factor of safety against collapse.

The first patented methods were not successful because the low prestress then produced in the steel was soon lost as a result of the shrinkage and creep of the concrete. This can be explained if we assume that an ordinary structural steel bar of length l is prestressed to a working stress of $f_s = 1400$ kg/cm^2 as shown in figure I-2. The modulus of elasticity of steel being equal to $E_s = 2000\ 000$ kg/cm^2, then the unit lengthening of the bar ϵ_s is given by:



(FIG I-2)

$$\epsilon_s = f_s/E_s = 1400/2000\ 000$$
$$= 0.7/1000 \text{ which}$$

means that $\epsilon_s = 0.7$ mm/m

The elongation of the steel due to this prestress is given by:

$$\Delta l_1 = \epsilon_s \cdot l = \frac{0.7}{1000} l$$

It is further known that the unit shortening due to shrinkage ϵ_{sh} may be considered equal to : $\epsilon_{sh} = 0.25$ to 0.30 mm/m
and the unit shortening due to creep ϵ_{cr} equal to :

$$\epsilon_{cr} = 0.40 \text{ to } 0.50 \text{ mm/m}$$

so that $$\epsilon_{sh} + \epsilon_{cr} = 0.65 \text{ to } 0.80 \text{ mm/m}$$

and, the shortening of the concrete with its prestressing steel is given by :

$$\Delta l_2 = (\epsilon_{sh} + \epsilon_{cr})l \approx \frac{0.7}{1000} l$$

This investigation shows that the initial lengthening of the bar which causes the prestress could be entirely lost in the course of time.

Modern development of prestressed concrete is credited to E. Freyssinet of France, who in 1928 started using high strength steel wires for prestressing. Such wires with an ultimate strength as high as

16 t/cm^2 and a yield point around 14 t/cm^2 , are prestressed to about 10 t/cm^2 creating a unit strain of

$$\epsilon_s = f_s / E_s = 10000/2000\ 000 = 5/1000$$

Assuming a total loss of 0.8/1000 due to shrinkage and creep of concrete and other causes, a net strain of

$$\epsilon_s = 5/1000 - 0.8/1000 \qquad = 4.2/1000$$

would still be left in the wires; this strain is equivalent to a stress of

$$f_s = E_s\ \epsilon_s = 2000\ 000 \times 4.2/1000 = 8400\ kg/cm^2$$

This use of high tensile steel is one of the essential fundamentals of prestressed concrete techique; it is this which has enabled prestressing to develop.

The high tensile steel generally used in prestressed concrete is hard drawn wires of an ultimate strength 15 – 20 t/cm^2 . Its proof stress (tensile stress producing a permanent elongation of 0.2%) must not be less than 80% of the ultimate strength. Its modulus of elasticity E_s = 2000 t/cm^2.

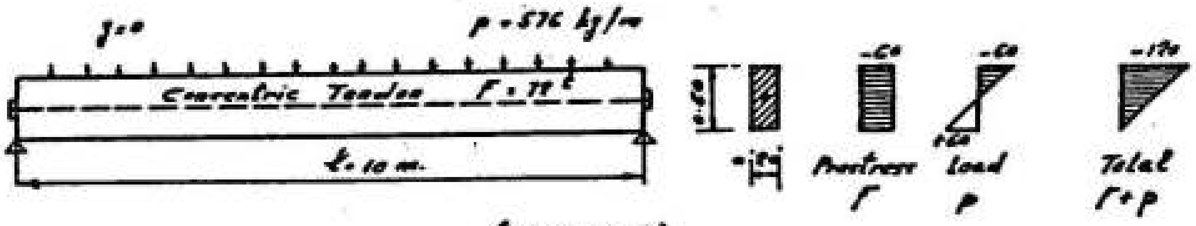
In reinforced concrete sections subject to simple bending, it is generally more economic to use small ratios of tension reinforcements; in which cases, the bearing capacity of the beam is governed by the moment of resistance of the tension steel and the use of high grade concrete cannot be fully utilised. For this reason, concretes having an ultimate strength f_{c28} varying between 160 and 250 kg/cm^2 are generally used in reinforced concrete elements subject to simple bending and eccentric forces. For prestressed elements, stronger concrete with f_{c28} = 300 to 600 kg/cm^2 is required, becasue, the higher the strength, the bigger is the working stress and the bearing capacity.

I-2 <u>General Principles of Prestressed Concrete</u>

In the following examples, concrete is assumed as being subject to two systems of forces: prestress and external load, with the tensile stresses due to the external load counteracted by the compressive stresses due to the prestress. Similarly, the cracking of concrete due to load is prevented or delayed by the precompression produced by the tendons. So long as there are no cracks, the stresses, strains and deflections of the concrete due to the two systems of forces can be considered separately and superimposed.

In order to explain the basic behaviour of prestressed concrete beams, let us discuss the following cases :

1) A prestressed-concrete rectangular beam 20 x 60 cms has a simple span of 10 ms is loaded by a uniform load p = 576 kg/m. The prestressing tendon (cable) is located along the centroidal axis of the beam and produces a prestress of 72 tons. The maximum allowable concrete stress is 120 kg/cm^2. (Fig I-3).



(FIG I-3)

Neglecting the own weight of the beam, we get:

Prestressing force	$F = 72$ tons
Area of cross-section	$A = 20 \times 60 = 1200$ cm^2
Max. B.M. due to load	$M_{max} = pl^2/8 = 576 \times 10^2/8 = 7200$ kgm
Section modulus	$Z = bd^2/6 = 20 \times 60^2/6 = 12000$ cm^3

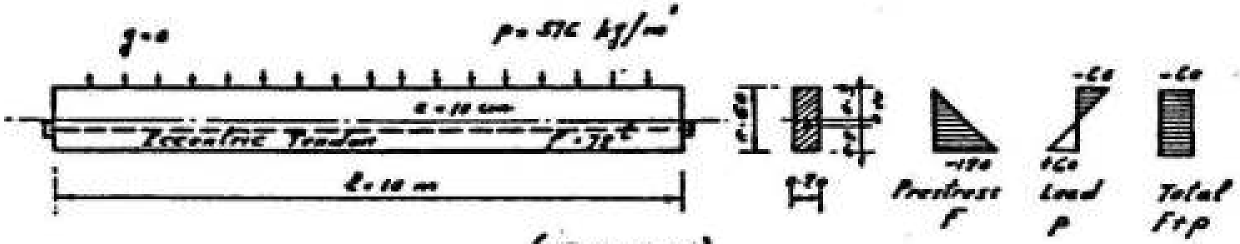
Stresses at middle section due to :

prestressing force F	$f = -F/A = 72000/1200 = -60$ kg/cm^2
load p	$f = \pm M/Z = \pm 720000/12000 = \pm 60$ " "

Total stresses:

at extreme upper fiber	$f_{top} = -60 - 60 = -120$ kg/cm^2
" " lower "	$f_{bot} = -60 + 60 = 0$ " "

2) If the prestressing tendon is located at the lower third point of the section (i.e. the eccentricity e of the prestressing force F is equal to 10 cms) as shown in figure I-4, the stresses at the middle section are as follows:



(FIG I-4)

Stresses due to:

prestressing force F $\quad f = -\frac{F}{A} \pm \frac{F_e}{Z}$ i.e

$$f = -\frac{72000}{1200} \pm \frac{72000 \times 10}{12000} = -60 \pm 60 \ \text{kg/cm}^2$$

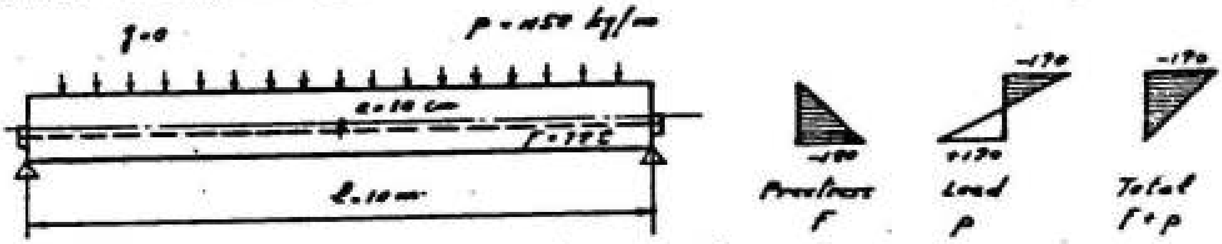
load p $\quad f = \pm M/Z$ i.e

$$f = \pm 720000/12000 \qquad = \pm 60$$

So that $\quad f_{top} = 0 - 60 = -60 \ \text{kg/cm}^2$

$$f_{bot} = -120 + 60 = -60 \ \text{kg/cm}^2$$

3) If the load p is doubled i.e p = 1152 kg/m then the maximum moment M = 2 x 7200 = 14400 kgm and the stresses due to the load p will be ± 60 x 2 = 120 kg/cm² ; the corresponding final stresses will be as shown in figure I-5.



(FIG I-5)

Thus by keeping the same prestressing force as in the first case but giving it an eccentricity we have made the beam capable of supporting a bending moment twice as much.

4) Assuming that the same beam is subject further to a dead load: then

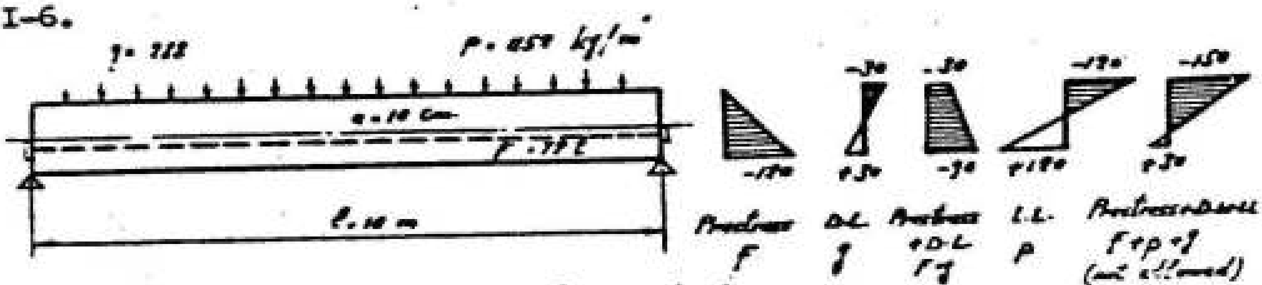
$$g = 0.2 \times 0.6 \times 2400 = 288 \quad \text{kg/m}$$

$$M_g = 288 \times 10^2/8 \quad = 3600 \quad \text{kg/m}$$

and the coresponding stress will be :

$$f = \pm 3600\,00/12000 = \pm 30 \qquad \text{kg/cm}^2$$

The stresses for the different cases of loading are shown in figure I-6.



(FIG I-6)

5) But if we give the prestressing force F = 72 tons an extra eccentricity $e' = M_g/F$ at the middle section, we get

$e' = M_g/F = 3600 / 72000 = 0.05$ ms below the lower third point, i.e.

$e = 10 + 5 = 15$ cms at middle, and

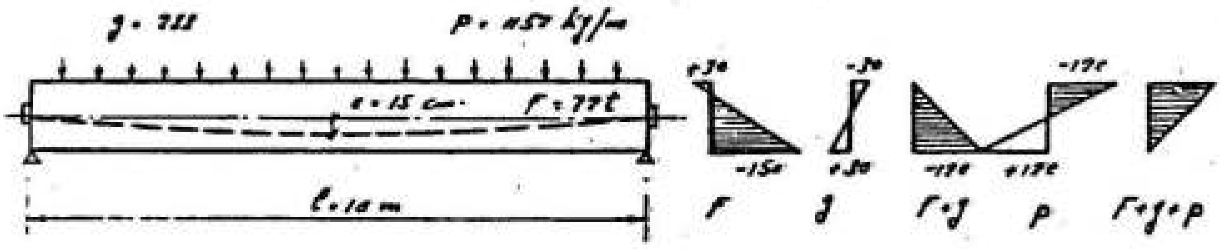
the stresses for the different cases are:

Due to F = 72t $\quad f = \frac{72000}{1200} \pm \frac{72000 \times 15}{12000} = -60 \pm 90\ kg/cm^2$

" " $M_g = 3{,}6$ mt $\quad f = \pm\ 3600\ 00/12000 \quad = \pm\ 30$ " "

" " $M_p = 14{,}4$ mt $\quad f = \pm\ 14400\ 00/12000 \quad = \pm\ 120$ " "

as shown in figure I-7.



(FIG I-7)

This means that the bending moments due to dead loads could be resisted without exceeding the maximum stresses, by the same prestressing force if it is located at a distance $e' = M_g/F$ below the lower third point (lower core point).

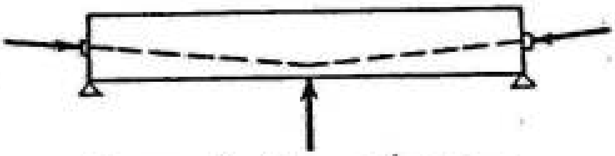
This method of approach becomes a little more complicated when the axis of the member or the tendons are bent or curved. Then it will be more convenient to take the concrete as a free body, cut loose from the tendons.

In the case shown in figure I-8, we have due to the bent in the tendon a vertical upward force applied at the middle of the span in addition to the prestress applied at the end of the beam. Thus, the tendon supplies not only a direct prestress but also an upward force at mid-span which can help to balance the external loads, very effectively at times.

It is also evident that, if the tendon has a parabolic shape , a uniform upward force will be supplied instead of a concentrated force. (Fig. I-9).

Example

A concrete beam 10 ms span, 20 x 60 cms cross-section is prest-



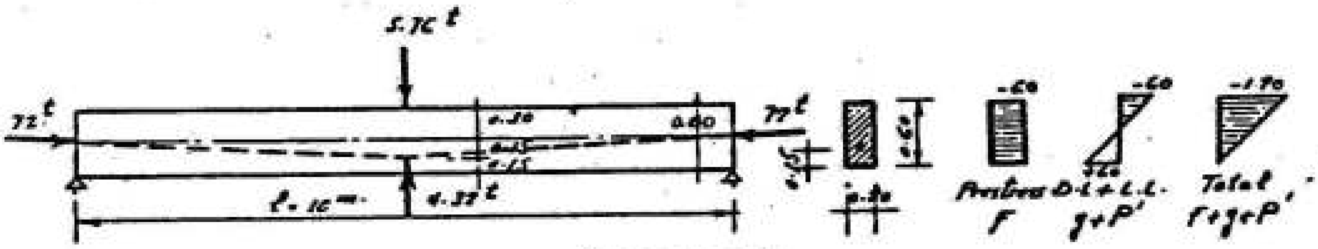
Forces due to prestress in a beam with bent tendon

(FIG I-8)

Forces due to prestress in a Beam with parabolic tendon

(FIG I-9)

ressed by a bent tendon as shown in figure I-10. The prestressing force is 72 tons. It is required to determine the stresses in concrete at mid-span due to own weight g = 288 kg/m and a concentrated load P = 5.65 tons.



(FIG I-10)

The tendon can be replaced by an inclined force of 72 tons at each end and an upward force at the center equal to:

$$2 \times \frac{15}{500} \times 72 = 4.32 \text{ tons}$$

i.e. net downward force P' is given by :

$$P' = 5.76 - 4.32 = 1.44 \text{ tons}$$

Bending moment due to P' is :

$$M_{P'} = 1440 \times 10/4 = 3600 \text{ kgm}$$

Bending moment due to dead load g is :

$$M_g = 288 \times 10^2/8 = 3600 \text{ kgm}$$

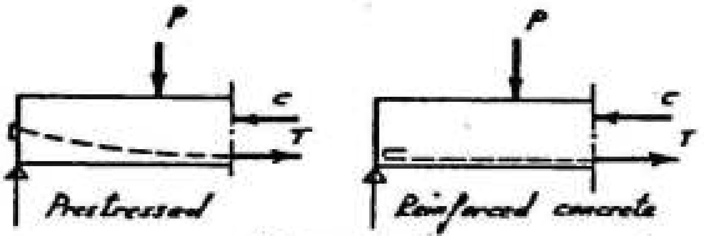
Total bending moment M = 3600 + 3600 = 7200 kgm

Stresses due to M = 7200 kgm $\quad f = \pm 7200\,00/12000 = \pm 60 \text{ kg/cm}^2$

" " " F = 72000 kgs $\quad f = -72000/1200 = -60$ " "

So that the final stresses will be as shown in figure I-10.

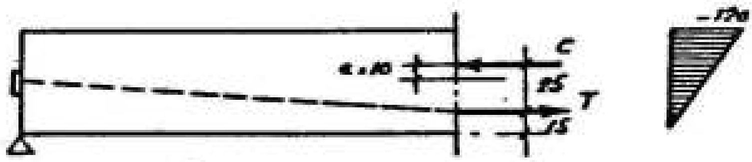
The concrete stresses in a prestressed beam can however be determined by considering the steel and concrete acting together, with steel taking tension and concrete taking compression so that the 2 materials form a resisting couple against the external moment.(Fig.I-11).



(FIG I-11)

It has to be noted that in prestressed concrete, high tensile steel is used which will have to be elongated a great deal before its strength is fully utilized. If the high tensile steel is simply buried in the concrete, as in ordinary concrete reinforcement, the serrounding concrete will crack very seriously before the full strength of the steel is developed. Hence it is necessary to prestrech the steel with respect to concrete. By prestreching and anchoring the steel against the concrete, we produce desirable stresses and strains in both materials: compressive stresses and strains in concrete, and tensile stresses and strains in steel.

In the following , the last example will be solved using the principle of the internal resisting couple . (Fig. I-12).



(FIG I-12)

External moment due to a dead load g = 288 kg/m and a concentrated load P = 5.76 t acting at the center.is given by:

$M = 288 \times 10^2/8 + 5760 \times 10/4 = 3600 + 14400 = 18000$ kgm

The internal couple is furnished by the forces

$$C = T = 72 \text{ tons}$$

which must act at a lever arm :

$$y_{CT} = 18000/72000 = 0.25 \text{ ms}$$

Since the cable is located at 15 cms from the bottom, then

$$e = 0.10 \text{ m}$$

and the stresses are given by :

$$f = \frac{72000}{1200} \pm \frac{72000 \times 10}{12000} = -60 \pm 60 \ kg/cm^2$$

i.e $f_{top} = -120$ and $f_{bot} = 0 \ kg/cm^2$ same answers as before.

I-3 Classification and Types

Prestressed concrete structures can be classified in a number of ways depending upon their features of design and construction :

1) Pre-Tensioning & Post-Tensioning. The term pre-tensioning is used to describe any method of prestressing in which the tendons are tensioned before the concrete is placed. It is evident that the tendons must be temporarily anchored against some abutments or stressing beds when tensioned and the prestress transferred to the concrete after it has set. This proceedure is employed in precasting plants or laboratories where permanent beds are provided for such tensioning; it is also applied in the field where abutments can be economically constructed. In contrast to pre-tensioning, post-tensioning is a method of pre-stressing which the tendon is tensioned after the concrete has hardened. Thus the prestressing is almost always performed against the hardened concrete, and the tendons are anchored against it immediately after prestressing. This method can be applied to members either precast or cast in place.

2) End Anchored or Non-End. Anchored Tendons. When post-tensioned , the tendons are anchored at their ends by means of mechanical devices to transmit the prestress to the concrete. Such a member is termed end-anchored. In pretensioning, the tendons generally have their prestress transmitted to the concrete by their bond action near the ends. The effectiveness of such stress transmission is limited to wires of small size.

3) Bonded or Unbonded Tendons. Bonded tendons denote those bonded throughout their length to the serrounding concrete. Nonend-anchored tendons are necessarily bonded ones; end-anchored tendons may be either bonded or unbonded to the concrete. In general, the bonding of post-tensioned tendons is accomplished by subsequent grouting; if unbonded, protection of the tendons from corrosion must be provided by galvanising , greasing, or some other means.

4) Partial or Full Prestressing: When a member is designed so that under the working load there are no tensile stresses in it, then the concrete is said to be fully prestressed. If some tensile stresses will be produced in the member under working load, then it is termed

partially prestressed. For partial prestressing additional mild steel bars are frequently provided to re-inforce the portion under tension.

5) Precast, Cast-in-place, Composite Construction.

Precasting permits better control in mass production and is often economical. Cast-in-place requires more form and falsework per unit of product but saves the cost of transportation and erections; it is a necessity for large and heavy members. Often-times, it is economical to precast part of a member, erect it, and then cast the remaining portion in place. This proceedure is called composite construction. The precast elements in a composite construction can be more easily jointed together than those in a totally precast structure. By this type of construction it is possible to save much of the form work required for cast in place construction.

I-4 Advantages of Prestressed Concrete as a Building Material

The most outstanding advantage of prestressed concrete is the posibility of the effective use of high strength concrete and steel producing concrete structural elements free from cracks.

In reinforced concrete, the concrete is effectively used only in the compression zone of the sections and high strength concrete cannot be fully utilized so long as we use low percentages of tension reinforcements. Even if it is possible to use high strength concrete, the sections will be smaller and the required reinforcement will be correspodingly bigger giving a more costly design. Furthermore, the use of high grade steels for the tension reinforcements is limited by the allowed width of cracks.

In prestressed concrete, the high tensile steel is freely prestretched to any possible desirable amount and anchored against the concrete creating desirable strains and stresses which serve to eliminate or reduce cracks in concrete. Thus the entire section of the concrete becomes effective and the higher the strength of the concrete, the smaller is the section. Stronger concrete is also necessary to resist high stresses at the anchorages and to give strength to the thinner sections frequently employed for prestressed concrete.

Prestressed concretes free from cracks possess bigger elasticity bigger stiffness, higher resistance to external and climatic actions and bigger gas and water tightness.

By convenient prestressing, it is possible to eliminate or reduce inconvenient undesirable strains and stresses as will be shown in the following cases in addition to what has been explained.

1) By prestressing the tie of an arch one can eliminate its elongation causing undesirable bending moments in the arch.

2) The prestressing of the footing of a concrete surface of revolution (a dome or a cone) or the edge beam of a cylindrical shell, eliminates or reduces undesirable stresses and strains at the edges.

3) Prestressing reduces the elastic and plastic deformations caused by shrinkage and creep of beams due to the negative deflections which take place at transfer under the action of dead loads.

4) The use of curved or inclined tendons create upward forces which counterbalance a part of the downward dead and working loads and hence reduce the shearing forces in the beam. In addition, precompression in the concrete reduces the diagonal tension. Thus it is possible in prestressed concrete to use smaller webs and in many cases to dispence with most of the web reinforcements to carry the same amount of external shear in a beam.

Prestressed-concrete design is more suitable for structures of long span and those carrying heavy loads, principally because of the higher strengths of materials employed.

Prestressed structures are more slender due to smaller dimensions. They do not crack under working loads, and what ever cracks may be developed under over loads will be closed up as soon as the load is removed, unless the load is excessive. Under dead load the deflection is reduced, owing to the cambering effect of prestress. This becomes an important consideration for such structures as long cantilevers. Under live load, the deflection is also smaller becasue of the effectiveness of the entire uncracked concrete section, which has a much bigger moment of inertia than that of the cracked section. Prestressed elements are more adaptable to precasting because of the lighter weight.

In spite of the previous advantages, it has to be noted that stronger materials will have a higher unit cost. More auxiliary materials are required for prestressing , such as end anchorages, conduits and grouts. The price of the end anchorage per cable is fixed, hence,its cost per m decreases for longer cables. Prestressed concrete projects require more elaborate design and continual engineering supervision. Such over-head charges will decrease if the same typical design is repeated many times. The use of high strength wires is more economic than the use of normal mild and high grade steels because the rate of increase of the price is smaller than that of the strength.

From the above discussion, it can be concluded that prestressed concrete design is more likely to be economical when the same unit is repeated many times or when heavy loads are acting on long spans. Its application may also be suitable for pre-cast and semi-precast composite elements. The economy of each case must be considered individally.

Prestressed-concrete can however only be used by specialized designers and contractors having special pre-tensioning factories and experienced executing crew.

II - MATERIALS

The most effective use of prestressing can only be obtained if the concrete and the steel are of a very high quality.

II-1 Concrete

Special care shall be given to the properties of individual materials used for the production of concrete for structural members in prestressed concrete and their effect on such properties of concrete as compressive strength, modulus of elasticity, shrinkage, creep and bond. The aim shall be produce high quality concrete not only for the sake of obtaining higher crushing strength but also in order to avoid serious reductions in the prestressing force. High quality concrete can be achieved through careful selection of aggregates, suitable granular composition, use of low water cement ratio, the eventual addition of convenient admixures, sufficient cement content and through proper mixing compaction and curing.

The minimum cement content for prestressed concrete work shall be 350 kg. per m^3 of finished concrete. The cement content shall preferably be within 500 kg/m^3. In no case shall it exeed 600 kg/m^3.

The expected crushing strength of the different mixes may be :

Cement dose	350	400	450	500	kg/m^3
Strength f_{C28}	300	400	500	600	kg/cm^2

For pre-tensioned concrete min.

f_{C28} = 400 kg/cm^2

For post-tensioned concrete min.

f_{C28} = 300 kg/cm^2

The stress-strain relation for a C400 concrete (i.e concrete with a minimum crushing strength of 400 kg/cm^2 at 28 days) is shown in figure II-1.

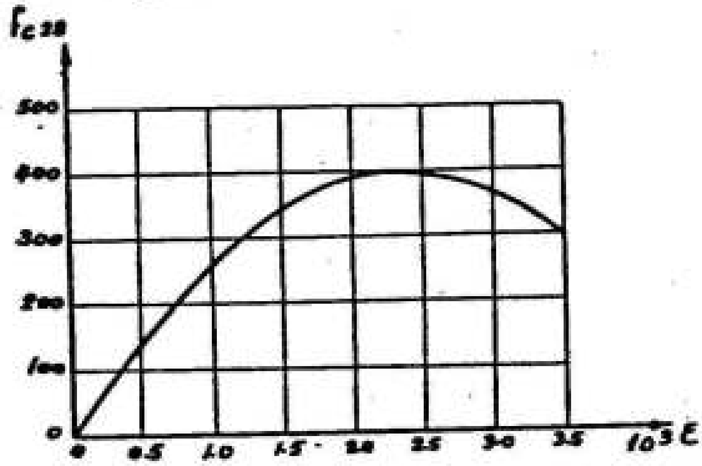


FIG. II-1

The modulus of elasticity

E_c, and the modular ratio $n = E_s/E_c$ for the different concrete qualities may be assumed as follows :

Cube strength	f_{C28}	300	400	500	600	kg/cm^2
Prism "	f_{cp}	240	310	380	450	" "
Mod. of elasticity	E_c	300	340	370	400	t/cm^2
Modular ratio n		6.5	6	5.5	5	

Elastic strain of concrete ε_c

$$\varepsilon_c = f_c / E_c$$

The given values of E_c may be used for $f_c \leq 0.3$ to $0.4\ f_{C28}$

Shrinkage strain ε_{sh}

For purposes of design, the shrinkage strain ε_{sh} may be assumed :

$$\varepsilon_{sh} = 0.20 \text{ to } 0.30 \text{ mm/m}$$

Creep ε_{cr} and coefficient of creep C_{cr}

The term coefficient of creep C_{cr} is employed to indicate the ratio of the strain ε_t after a lengthy period of constant stress to the instantanious strain ε_i immediately obtained at the application of the stress. Hence

$$C_{cr} = \varepsilon_t / \varepsilon_i$$

For purposes of design it is considered safe to take C_{cr} around 3. For post-tensioned members, where the prestress is applied late, the coefficient could be a little less, for pre-tensioned members, where the prestress is applied at an early age, the coefficient could be a little more.

For a creep coefficient of 3, the amount of creep strain is two times the instantaneous elastic strain. Of this two, it can be roughly estimated that about 1/4 takes place within the first two weeks, after application of prestress, another 1/4 within 2 to 3 months,another 1/4 within a year or two, and last 1/4 within the course of many years.

II-2 Expansive Cement

Types of cement that expand chemically after setting and during hardening are known as expansive or self-stressing cements. When these cements are used to make concrete with embedded steel, the steel is elongated by the expansion of the concrete. Thus the steel is prestressed in tension, which in turn produces compressive prestress in the concrete, resulting in what is known as chemical prestressing or self stressed concrete.

When concrete made with expanding cement is unstrained, the amount of expansion produced by the chemical reaction between the cement and water amount to 30-50 mm/m and the concrete would then disintigrate by itself. When restrained either internally or externally with steel or other means, the amount of expansion can be controlled.

When high-tensile steel is used to produce the prestress, say corresponding to tensile stress at 10 t/cm^2 and an E_s = 2000 t/cm^2, an expansion of

$$\frac{10}{2000} = \frac{5}{1000} = 5 \text{ mm/m}$$

is required. For other stress levels, varying amounts of expansion will be required.

Because of the expansion in all three directions, it seems difficult to use this cement for complicated structures cast in place, such as buildings.

Expanding cement has been successfully applied for many interesting projects, especially in France. However, many problems concerning the use of expanding cement for self-stressing such as the exact control of the stresses and strains are still unsolved but its extensive application in the near future is expected.

II-3 <u>Steel</u>

The steel to be used for the prestressing members shall be one of the 2 types described below :

a) Hard drawn wire of an ultimate strength not less than 15 t/cm^2 supplied in coils as given in the following table :

Ø of wire m/m	Ultimate Strength t/cm^2	Min. diam. of Coil cm.
7 - 5	15 - 16	180
4 - 3	18 - 20	150
2 - 1	22 - 24	90

If diam. of coil is smaller, the wires must be mechanically straightened.

b) Silicon chrome or other alloy steel of an ultimate strength of at least 10 t/cm^2 .

Proof stress : If the steel has no definite yield point as is generally the case for hard drawn wires, the proof stress shall be assumed as the tensile stress producing a permanent elongation of 0.2%. The proof stress shall not be less than 80% of the ultimate tensile strength.

The min. elongation at rupture: For hard drawn wires 4% in a 25 cm gauge and for high strength steel alloy bars 4% in a gauge of 20 diameters. Wires and bars used for prestressing shall not be spliced by welding. The stress-strain relation for hard drawn wires is shown in figure II-2.

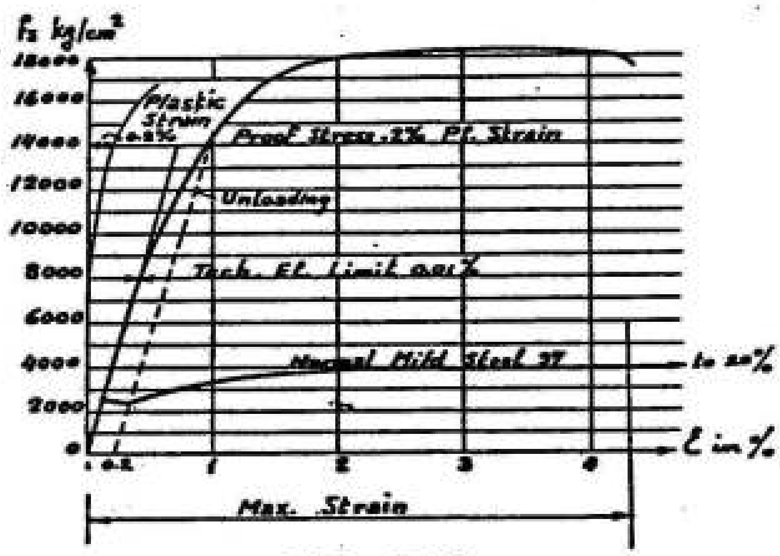


FIG. II-2

<u>Creep of Steel</u>

Creep of steel due to normal prestressing forces may be assumed for design purposes between 3 & 4% . While creep in steel is a function of time, there is evidence to show that under the ordinary working stress for high-tensile steel, creep takes place mostly during the first few days. Under constant strain, creep ceases entirely after about 2 weeks. If the steel is stressed to a few per cent above its initial prestress and the overstress is maintained for a few minutes, the eventual creep can be greatly lessened, and it practically stops in about 3 days.

II-4 <u>Fiberglass Tendons</u>

Fiberglass is manufactured by drawing fluid glass into fine filaments. Although this new material has not yet been commercially applied in prestressed-concrete construction, it has been proved by tests that it possesses certain superior qualities that indicate high promise for prestressing. An Ultimate tensile strength of 70 t/cm^2 is quite commonly obtained. Values as high as 350 t/cm^2 have been reached for individual silica fibers 0.0003 mm. in diameter, it being known that the strength varies approximately inversily as the diameter of the fiber.

Fiberglass can be made in three formes: parallel chords, twisted strands and parallel fibers embedded in plastic. The last form in the shape of fiberglass rods is considered most suitable for prestressing because of its relative simplicity for handling, gripping and anchoring. As bonding agent in the manufacture of fiberglass rods, epoxy resin has proved to be superior. Their tensile strength is generally > 16 t/cm^2 based on the gross area of the rod.

Another advantage of fiberglass is its low modulus of elasticity which ranges between 450 and 750 t/cm^2. With its high strength and low modulus, the percentage of loss of prestress would be quite small. Other advantages claimed for this material are high resistance to acids andalkalies and the ability to withstand high temperature. However some major problems must be solved before its commercial use in practice such as : its long-time ulitimate strength,its dynamic fatigue, the best method of fabricating chords giving an even distribution of stress and the design of suitable end anchorages protecting the brittle fiberglass rods from failing in the grip under the effect of stress concenterations and combined stresses.

II-5 Grout

Steel installed in holes or flexible metal tubes cast in the concrete shall be bonded, in which case the annular space between the perimeter of the hole or tube and the steel shall be pressure grouted after the prestressing process has been completed. The grout shall be made to the consistency of thick paint and shall be mixed in the proportions, by volume, of one part portland cement to 0.75 part (max.) of sand and 0.75 part (max.) of water. It may be necessary to eliminate the sand from the mix and use neat cement grout.

II-6 Sheathing.

Sheathing is used when the cables are put in the form and concrete is cast around them. The sheathing shall be metallic and completely water tight specially at joints so that fine concrete mortar cannot penetrate and hinder the free movement of the wires. Sheaths shall be strong enough to maintain their shape against the forces due to handling, placing and compaction of concrete and eventual rust effect. The dimensions of the sheath must permit the easy flow of grout around the wires inside it.

III - PRESTRESSING SYSTEMS

III - 1 Pre-tensioning System

A simple way of stressing a pre-tensioned member is to pull the tendons between two bulkheads anchored against the ends of a stres - sing bed. The forms are there erected around the tensioned tendons and the concrete is poured and well compacted, effectively by the use of vibrators. After the concrete hardens, the tendons are cut loose from the bulkheads and the prestress is transferred to the concrete by bond. Such stressing beds are often used in laboratories and prestressing factories.

The dependence on bond to transmit prestress between steel and concrete necessitates the use of small wires to ensure good anchorages. Wires greater than about 3mm. diameter are to be used only if they are waved along their length or if they are corrugated. In any case, a certain length of transfer is required to develop the bond. Should there be insufficient length of transfer, for example, when cracks occur near the end of a beam, the bond may be broken and the wires may slip. A more reliable method is to add mechanical end anchorages to the pre-tensioned wires.

III - 2 Post-tensioning System

The methods used for post-tensioning can be classified under three main groups: 1) mechanical prestressing by means of jacks ; 2) electrical prestressing by application of heat ; 3) chemical prestressing by the use of expanding cement.

1) Mechanical Prestressing :

In both pre-tensioning and post-tensioning, the most common method for stressing the tendons is jacking. In post-tensioning, jacks are used to pull the steel against the hardened concrete as shown in figure I-1, in pre-tensioning, to pull it against some bulkheads or molds. Hydraulic jacks are often used, because of their high capacity and the relatively small force required to apply the pressure.

Pressure guages for jacks are calibrated either to read the pressure on the piston or to read directly the amount of tension applied to the tendon. It is usual practice to measure the elongation of steel to be checked against the guage indications.

In order to minimize creep in steel and also to reduce frictional loss of prestress, tendons are sometimes jacked a few per cent above their specified initial prestress. Over-jacking is also necessary to compensate for slippage and take-up in the anchorage at the release of jacking pressure. When tendons are long or appreciably curved, jacking should be done from two ends.

2) Electrical Prestressing :

The electrical method of prestressing dispenses with the use of jacks altogether. The steel is lengthened by heating with electricity. This electrical process is a post-tensioning method where the concrete is allowed to harden fully before the applications of prestress. It employs smooth reinforcing bars coated with thermoplastic material such as sulfur or low - melting alloys and buried in the concrete like ordinary reinforcing bars but with protruding threaded ends. After the concrete has set, an electric current of low voltage but high amperage is passed through the bars. When the steel bars heat and elongate, the nuts or the protruding ends are tightened against heavy washers. When the bars cool, the prestress is developed and the bond is restored by the resolidification of the coating.

However, this method has been found to be uneconomical in competition with prestressing using high tensile steel although it has found wide usage in the U.S.S.R. in pretensioning.

3) Chemical Prestressing :

As described previously, the chemical reactions which take place in expensive cements can stress the embedded steel which in turn compresses the concrete. This is often termed self-stressing , but can also be termed chemical prestressing (Refer to section II-2).

IV - LOSS OF PRESTRESS

IV - 1 Loss due to Elastic Shortening of Concrete

a) Pre-tensioned Concrete

As the prestress is transfered to the concrete, the member shortens and the prestressed steel shortens with it, hence there is a loss of prestress in the steel, it can be calculated as follows :

The concrete strain ε_c due to an axial prestress F_o is given by

$$\varepsilon_c = f_c / E_c = F_o / A_c E_c$$

The steel and concrete being subjected to the same strain, then

$$\varepsilon_s = \varepsilon_c = F_o / A_c E_c$$

where F_o is the final prestress after the shortening of the concrete has taken place.

The loss of the prestress in the steel is therefore given by

$$\Delta f_s = \varepsilon_s E_s = F_o E_s / A_c E_c$$

$$\underline{\Delta f_s = n F_o / A_c = n f_{co}}$$

assuming $n = 6$ and average $f_{co} \simeq ½ \max f_{co} \sim 60$ kg/cm^2, then

$$\Delta f_s = 6 \times 60 = 360 \text{ kg/cm}^2, \text{ and for } f_s \max = 10\,000 \text{ kg/cm}^2$$

$$\Delta f_s = 3.6 \%$$

b) Post-tensioned Concrete

For post-tensioning, the problem is different. If we have only a single tendon, the concrete shortens as that tendon is jacked against the concrete. Since the force in the cable is measured after the elastic shortening of the concrete has taken place, no loss in prestress due to that shortening, need be accounted for.

If we have more than one tendon and the tendons are tensioned in succession, then the prestress is gradually applied to the concrete, the shortening of concrete increases as each cable is tightened against it, and the loss of prestress due to elastic shortening differs in the tendons. The tendon that is first tensioned would suffer the max amount of loss due to the shortening of concrete by the subsequent application of prestress from all the other tendons. The tendon that is tensioned last will not suffer any loss due to the elastic concrete shortening, since all that shortening will have already taken place when the prestress in the last tendon is being measured. For practical purposes, it is accurate enough to determine the loss for the 1st. cable and use half of that value for the average loss of all the cables. This is shown in the following example. Consider that the prestressing steel stress in a post-tensioned member is 10 t/cm^2 and carried by 4 cables which are tensioned one after another.

The loss of prestress in the 1st. tendon will be due to the shortening of concrete as caused by the prestress in the other 3 tendons where as the average loss will be assumed one half that of the 1st. tendon i.e.

$$\Delta f_s = n\, F_o' \,/\, A_c \qquad \text{where} \quad F_o' = \frac{1}{2} \times \frac{3}{4} \times F_o = \frac{3}{8}\, F_o$$

The loss of prestress in our case is therefore given by :

$$\Delta f_s = 3.6\ \% \times \frac{3}{8} = 1.3\ \%$$

In actual practice, either of the following 2 methods is used:

1) We stress all tendons to the specified initial prestress (e.g. 10 000 kg/cm^2) and allow for the average loss in the design (e.g. 130 kg/cm^2).

2) We stress all tendons to a value above the specified initial prestress by the magnitude of the average loss (e.g. 10 000 + 130 = 10 130 kg/cm^2). Then, the loss due to the elastic shortening is not to be considered.

IV - 2 <u>Loss due to Creep & Shrinkage in Concrete</u>

The amount of creep is 1-2 times the elastic shortening, but the loss due to elastic shortening can be counterbalanced for post-tensioned members, whereas the loss due to creep cannot be easily compensated for.

Assuming that the coeff. of creep $C_{cr} = \epsilon_t / \epsilon_i$, the loss of prestress is given by the relation :

$$\Delta f_s = (C_{cr} - 1)\, \frac{f_c}{E_c} \,.\, E_s = (C_{cr} - 1)\, n\, f_c$$

for $C_{cr} = 2.5$ $\qquad n = 6 \qquad f_c = 60$

$$\Delta f_{s\,cr} = 1.5 \times 6 \times 60 = 540 \text{ kg/cm}^2$$

For a prestress of 10 000 kg/cm^2 , the loss is $\approx$ 5 - 6%

Assuming the shortening strain due to shrinkage = ϵ_{sh}
Then the loss can be directly given by

$$\Delta f_{s\,sh} = \epsilon_{sh} \times E_s$$

For ϵ_{sh} = 0.3 mm/m & E_s = 2000 000 kg/cm^2

$$f_{s\,sh} = \frac{0.3}{1000} \times 2000\,000 = 600 \text{ kg/cm}^2$$

For a prestress of 10 000 kg/cm^2 the loss is $\approx$ 6 %

IV - 3 Losses due to Creep in Steel

Creep of steel may be assumed 3 to 4% ; these values can be effectively reduced if the cables are overtensioned by 5 to 10 % and held there for 2-3 minutes.

IV - 4 Losses due to Anchorage Take-Up

For most systems of post-tensioning, when a tendon is tensioned to its full value, the jack is released and the prestress is transferred to the anchorage. The anchorage fixtures that are subject to stresses at this transfer will tend to deform, thus allowing the tendon to slacken slightly. Friction wedges employed to hold the wires will slip a little distance before the wires can be firmly gripped. The amount of slippage varies between 2 & 3 mms. and depends upon the type of wedge and the stress in the wires ... etc.

A general formula for computing the loss of prestress due to anchorage deformation Δ_a is

$$\Delta f_s = \frac{\Delta_a E_s}{l}$$

for Δ_a = 0.5 cm. & l = 40 ms. $\Delta f_s = \frac{0.5 \times 2000 \times 000}{4000} = 250$ kg/cm^2

IV - 5 Losses due to Friction

a) General . In post-tensioning systems there will be movement of the greater part of the tendon relative to the serrounding duct during the tensioning operation, and if the tendon is in contact with either the duct or any spacers provided, friction will cause a reduction in the prestressing force as the distance from the jack increases. In addition, a certain amount of friction will be developed in the jack itself and in the anchorage through which the tendon passes. The frictional loss can be conveniently considered in two parts : the length effect and the curvature effect. The length effect is the amount of friction that would be encountered if the tendon is a straight one i.e. one that is not purposely bent or curved. Since in practice the duct for the tendon cannot be perfectly straight, some friction will exist between the tendon and its serrounding material even though the tendon is meant to be straight. This is sometimes described as the wobbling effect of the duct and is dependent upon the length and stress of the tendon, the coefficient of friction betxeen the contact materials, and the workmanship and method used in aligning and obtaining the duct. The loss of prestress due to curvature effect results from friction and intended curvature of the tendons in addition to the wobble of the duct. This loss is again dependent on the coefficient of friction between the contact materials and the pressure exerted by the tendon on the concrete. The coefficient of friction, in turn, depends on the smoothness and nature of the surfaces in contact, the amount and nautre of lubricants, and sometimes the length of contact. The pressure between the tendon and concrete is dependent on the stress in the tendon and the total change in angle.

There are several methods of overcoming the frictional loss in tendons. One method is to overtension them. Where friction is not excessive, the amount of overtension is usually made to equal the max. frictional loss. The amount of wire lengthening corresponding to that overtension and the estimated friction can also be computed to serve as a check. This amount of overtension required for overcoming friction is not to be added over that required for overcoming anchorage losses or for minimizing creep in steel. It is sufficient to take the greatest of three required values and overtension for that amount. This is because in all three cases the overtensioning consists of an overstreching and a subsequent release back. It must be noted that, if most of the friction exists near the jacking end, overtensioning to balance that friction will not produce any overstretching of the main portion of the tendon and hence will not serve to minimize creep to any extent.

The effect of overtensioning with a subsequent release back is to put the frictional differemce in the reverse direction.

When the frictional loss is a high percentage of the prestress it cannot be totally overcome by overtnesioning, since the max. amount of tensioning is limited by the strength or yield point of the tendon. The portion of the loss that has not been overcome must then be allowed for in the design. Jacking from both ends, of course is another means for reducing frictional loss. It involves more work in the field but is often used when the tendons are long or when the angles of bending are large.

b) <u>Theory of Frictional Losses</u>

The basic theory of frictional loss of a cable around a curve. can be derived as follows :

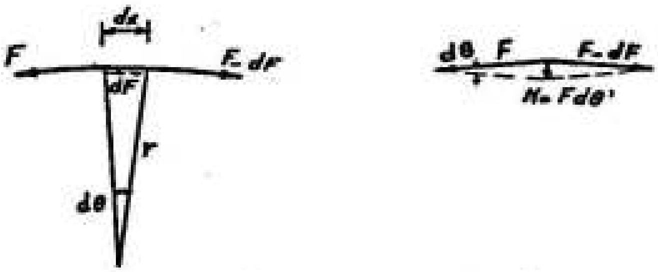


FIG. IV-1

Consider an infinitismal length dx of a prestressing circular part of a tendon of radius r, then the change in angle of the tendon as it goes around the length dx is $d\Theta = dx / r$

For this infinitismal length dx, the stress in the tendon may be considered constant and equal to F; then the normal component of pressure produced by the stress F bending around an angle $d\Theta$ is given by

$$N = F\, d\Theta = F\, dx / r$$

The amount of frictional loss dF around the length dx is given by the pressure times a coefficient of friction μ , thus,

$$dF = -\mu N$$

$$= -\mu F\, dx/r = -\mu F\, d\Theta \qquad \text{i.e.}$$

$$dF / F = -\mu\, d\Theta$$

Intigrating on both sides, we get

$$\log_e F = -\mu\, \Theta$$

Using the limits F_1 and F_2 we have the conventional friction formula $\underline{F_2 = F_1 e^{-\mu\theta} = F_1 e^{-\mu l/r}}$

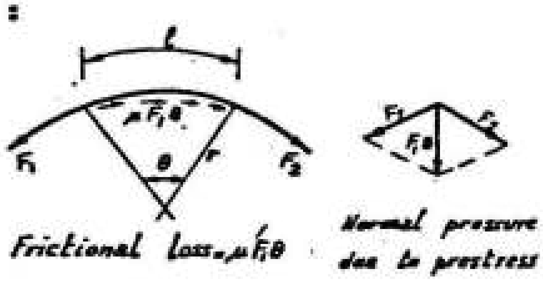
since $\theta = l/r$ for a section of length l and constant radius = r
For tendons with a succession of curves of varying radii, it is necessary to apply this formula to the different sections in order to obtain the total loss.

The above formula can also be applied to compute frictional loss due to wobble or length effect. Substituting the loss kl for $\mu\theta$ in the previous formulea we get

$$\log_e F = -kl \quad \text{or} \quad \underline{F_2 = F_1 e^{-kl}}$$

The above formulea are theoretically correct ones which take into account the decrease in tension and hence the decrease in the pressure as the tendon bends around the curve and gradually loses its stress due to friction. If, however, the total difference between the tension in the tendon at the start and that at the end of the curve is not excessive (<20%), an approx. formula using the initial tension for the enitre curve will be close enough. On this assumption, a simpler formula can be derived in place of the above exponential form. If the normal pressure is assumed to be constant, the total frictional loss around a curve with angle θ and length l according to fig. IV-2, is given by :

FIG. IV-2



$$F_2 - F_1 = -\mu F_1\theta = -\mu F_1 l/r$$

For length or wobble effect, we can again substitute kl for $\mu\theta$, thus $F_2 - F_1 = -kl\ F_1$

To compute the total loss due to both curvature and length effect, the above 2 formulas can be combined, giving

$$F_2 - F_1 = -\mu F_1\theta - kl\ F_1 \quad \text{or}$$

$$\underline{\frac{F_2 - F_1}{F_1} = -kl - \mu\theta = (k + \frac{\mu}{r})\,l}$$

The loss of prestress for the entire length of a tendon can be considered from section to section, with each section consisting of either a straight line or a simple circular curve. The reduced stress at the end of a segment can be used to compute the frictional loss in the next segment etc..

Since, for practically all prestressed concrete members, the depth is small compared with the length, the projected length of tendon measured along the axis of the member can be used when computing frictional losses. Similarly, the angular change θ is given by the transverse deviation of the tendon divided by its projected length, both referred to the axis of the member.

Coefficients of Friction μ & k

Values of μ may be taken as :

0.55 for steel moving on concrete
0.30 " " " " steel
0.25 " " " " lead.

In circular construction where circumferential tendons are tensioned by means of jacks, the losses due to friction may be calculated from the formulea given before, but the values of μ may be taken as :

0.45 for steel moving on smooth concrete
0.25 " " " " steel bearers fixed to concrete
0.10 " " " " " rollers.

The value of k per meter length should generally be taken as not less than 32×10^{-4}/m length but where strong sheaths or duct formers are used closely supported so that they are not displaced during the concreting operation, the value of k may be taken as 16×10^{-4}

c) Example :

A prestressed-concrete beam, Fig. IV-3, is continuous over 2 spans and its curved tendon is to be tensioned from both ends. Compute the percentage loss of prestress due to friction from one end to the centre of the beam (A to E). The coefficient of friction between the cable and the duct is taken as 0,4, and the average wobble or length effect is represented by

$$k = 32 \times 10^{-4}/m$$

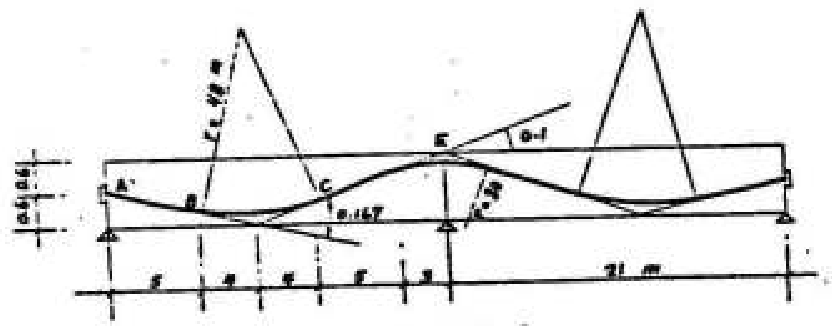


FIG. IV-3

Solution 1 : A simple approx. solution will 1st. be presented :

$$\frac{F_A - F_E}{F_A} = - kl - \mu\theta$$

$$= - 32 \times 10^{-4} \times 21 - 0.4 \; (0.167 + 0.1)$$

$$= - 0.067 - 0.107 = 0.174$$ i.e. loss = 17.4 %

Solution 2 : The above solution does not take into account the gradual reduction of stress from A toward E. A more exact solution would be to divide the tendon into 4 portions from A to E and sonsider each portion after the loss has been reduced from the preceeding portions. Thus, for stress at $A = F_1$,

AB, length effect $kl = 0.0032 \times 5 = 0.016$

Stress at B $= 1 - 0.016 = 0.984 \; F_1$

BC, length effect $kl = 0.0032 \times 8 = 0.026$

Curvature effect $\mu\theta = 0.4 \times 0.167 = 0.067$

total $0.026 + 0.067 = 0.093$

Using the reduced stress at B of 0.984,

the loss is $0.093 \times 0.984 = 0.091$

Stress at C $= 0.984 - 0.091 = 0.893 \; F_1$

CD, length effect $kl = 0.0032 \times 5 = 0.016$

Using the reduced stress of 0.893 at C,

the loss is $0.016 \times 0.893 = 0.014$

Stress at D $= 0.893 - 0.014 = 0.879 \; F_1$

DE, length effect $kl = 0.0032 \times 3 = 0.010$

Curvature effect $\mu\theta = 0.4 \times 0.1 = 0.040$

total $= 0.01 + 0.04 = 0.050$

loss $= 0.05 \times 0.879 = 0.044$

Stress at E $= 0.879 - 0.044 = 0.835 \; F_1$

Total loss from A to E $= 1 - 0.835 = 0.165$ i.e. 16.5 %

The loss according to this method is slightly less than the first approx. method.

IV - 6 Total Amount of Losses

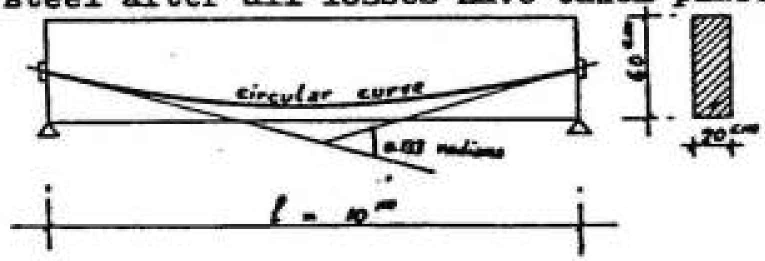
From the previous investigations we may conclude that for average steel and concrete properties, cured under average air conditions, the following tabulated percentages may be taken as representative of the average losses :

	Pre-tensioning	Post-tensioning
Elastic shortening	3	1
Creep of concrete	6	5
Shrinkage of concrete	7	6
Creep in steel	2	3
Total loss	18 %	15 %

The table assumes that proper overtensioning has been applied to reduce creep in steel and to overcome friction and anchorage losses.

Example :

A post-tensioned concrete beam (fig. IV-4) with a cable of 24 parallel wires (total steel area 7,2 cm^2) is tensioned with 2 wires at a time. The jacking stress is to be measured by jack gage pressure. The wires are to be stressed from one end to a vaule of f_1 to overcome frictional loss, then released to a value of f_2 so that immediately after anchorage an initial prestress of 10 t/cm^2 would be obtained. Compute f_1 & f_2 . Then compute the final design stress in the steel after all losses have taken place.



Assume the following : *FIG. IV-4*

1) Coeff. of friction $\mu = 0.55$ between steel and concrete $k = 32 \times 10^{-4}$ for length effect.
2) Deformation of anchorage and slippage of wires estimated at 1.5 mm. $E_s = 2000\ 000$ kg/cm^2
3) Elastic shortening of concrete is to be computed for $E_c = 300\ 000$ kg/cm^2

4) Creep coeff. of concrete C_{cr} = 2.5

5) Shrinkage of concrete ϵ_{sh} = 0.2 mm/m

6) Creep of steel ≈ 3% of initial steel stress

Solution :

1) Loss due to friction :

Length effect = kl = 0.032 x 10 = 0.032
Curvature effect = $\mu\theta$ = 0.55 x 0.133 = 0.072
Total = 0.032 + 0.072 = 0.104

Hence it is necessary to tension the steel to f_1 = 10 000/(1-0.104)
= 11 160 kg/cm^2

at one end in order to overcome the friction and obtain a stress of 10 000 kg/cm^2 at the unjacked end.

2) The anchorage slippage of Δ = 1.5 mm. occurs only on one end, since the unjacked end would have its slippage taking place before the release of jack.

The loss is given by $\Delta f_s = \frac{a}{l} E_s = \frac{0.15 \times 2000\,000}{1000} = 300 kg/cm^2$

Hence, by tensioning the steel to f_1 = 11 160 kg/cm^2 and then releasing it to f_2 = 10 300 kg/cm^2 for anchoring, the min. stress after anchorage will be 10 000 kg/cm^2. This min. stress will occur at both ends of the beam.

3) Since the wires are tensioned two by two, the first pair will loose some stress due to the elastic shortening of concrete under the action of the subsequent 11 pairs, and the average amount of loss will be approx.

$$\frac{1}{2} \times \frac{11}{12} \times \frac{10\,000 \times 7.2}{20 \times 60} \times \frac{2000\,000}{300\,000} = 183 \text{ kg/cm}^2$$

4) The force F_o in the steel is : 10 x 7.2 = 72 tons
the corresponding elastic shortening is

$$\frac{72\,000}{20 \times 60 \times 300\,000} = 0.2 \times 10^{-3}$$

Creep of concrete (2.5 - 1) 0.2 x 10^{-3} = 0.3 x 10^{-3}

Loss due to creep = 0.3 x 10^{-3} x 2000 000 = 600 kg/cm^2

5) For ε_{sh} = 0.2 mm/m

Loss due to shrinkage = $0.2 \times 10^{-3} \times 2000\,000$ = 400 kg/cm^2

6) Creep of steel 3%

Loss due to creep of steel = 10 000 3/100 = 300 kg/cm^2

The total loss :

Elastic shortening	183
Creep of conc.	600
Shrinkage of conc.	400
Creep of steel	300
	1483 kg/cm^2

Eventual loss of prestress ⇏ 15

V. ANALYSIS OF PRESTRESSED CONCRETE SECTIONS

The U.A.R. code of practice for the use of prestressed concrete given in the appendix shows the main outlines for the analysis and dimensioning of prestressed sections. It includes in its third part :

The fundamentals of design, the assumptions, the stages of loading, the allowable stresses in concrete, safety against cracking and failure, allowable stresses in steel .... etc.

V-1 Stresses in Concrete and Steel

a) Stresses due to the prestress :

If the prestress F is applied at the centroid of the concrete section, the concrete compressive stress is given by :

$$f_c = F / A$$

in which

F = prestressing force (initial or final)

A = area of net or transformed section according to case under consideration, generally assumed equal to A_c

Induced compressive stress in steel is :

$$f_s = n \ f_c$$

which corresponds to the loss of prestress due to elastic shortening given before in the form

$$f_s = n F_o / A_c$$

If the prestress is applied at an eccentricity e from the c.g. axis, then the concrete stress at any fiber at a distance y from the same axis is given by

$$f_c = \frac{F}{A} \pm \frac{F e y}{I}$$

in which

F = initial prestress, at transfer

or = final prestress , under working loads.

The extreme fiber stresses are therefore given by

$$f_c = \frac{F}{A} \pm \frac{F\,e}{Z}$$

where Z is the section modulus.

b) Stresses due to loads

The extreme fiber concrete stress due to external loads causing a bending moment M is :

$$f_c = \pm\, M\,y / I = \pm\, M / Z$$

One has to determine the extreme fiber stresses under the different stages of loading, generally :

1) under the initial condition, transfer, with full prestress and no live load, and

2) under the final condition, after losses have taken place and with full live loads.

In prestressed concrete, prestress in the steel is measured during tensioning operations, then the losses are computed or estimated as given in chapter IV. When dead and live loads are applied to the member, minor changes in stress will be induced in the steel. In a reinforced concrete beam, the steel stresses are assumed to be directly proportional to the external bending moment. When there is no moment, there is no stress. When the moment increases, the steel stresses increase in direct proportion. This is not true for a prestressed concrete beam, whose resistance to external moment is furnished by lengthening of the lever arm between the resisting forces C and T which remain relatively unchanged in magnitude.

The stress in steel of a bonded beam can be computed by the usual elastic theory as n times the corresponding concrete stress, i.e.

$$f_s = n\,M\,y / I = n\,f_c$$

Since the maximum change in concrete stress at the level of steel is not more than 150 kg/cm^2 in most cases, the corresponding change of stress in steel is limited to 150 n or 900 - 1000 kg/ cm^2 even though, the prestress is as high as 8000 to 10000 kg/cm^2 .

Example : a post-tensioned simple beam on a span of 10 ms. carries a superimposed load of 1152 kg/m in addition to its own weight of

288 kg/m'. The initial prestress is 10 t/cm^2, reducing to 8.5 t/cm^2 after deducing all losses. The parabolic bonded tendon has an area of 7.2 cm^2, n = 6. (Fig I-7) compute the stresses in the steel at mid-span.

Total dead and live load on beam	$w = 1152 + 288 =$	$= 1440$ kg/m
Max. bending moment	$M_w = 1440 \times 10^2/8$	$=18000$ kg/m
Max. moment due to prestress	$M_p = 7.2 \times 8500 \times .15$	$= 9150$ kg/m
Net moment at midspan	$M = 18000 - 9150$	$= 8850$ kgm
Stress in concrete at level of steel	$f_c = \frac{8850\ 00 \times 15}{3600\ 00}$	$= 37$ kg/cm^2
Corresponding stress in steel	$f_s = 6 \times 37$	$= 222$ kg/cm^2
Resulting stress in steel	$f_s = 8500 + 222$	$= 8722$ kg/cm^2

V-2 Cracking Moment

The moment producing first hair cracks in a prestressed concrete beam is computed by the elastic theory, assuming that craking starts when the tensile stress in the extreme fiber of concrete reaches its max. tensile strength f_t ; thus (Fig V-I)

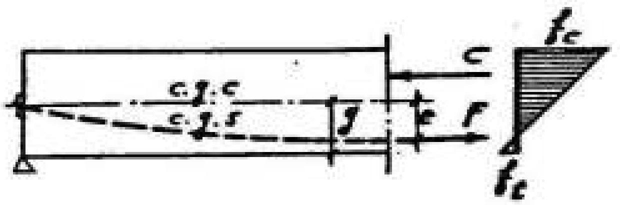


FIG V-1

$$-\frac{F}{A} - \frac{F\ e\ y}{I} + \frac{M\ y}{I} = f_t$$

from which, one can determine the cracking moment M.

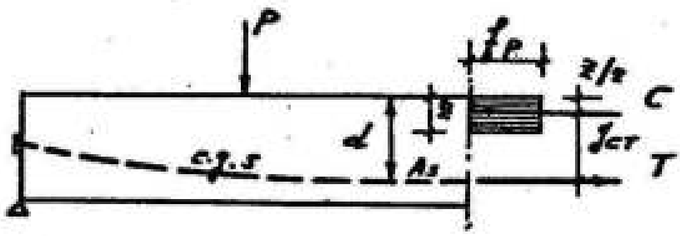
V-3 Ultimate Moment

As in reinforced concrete, the failure of a section may be due to the excessive elongation of the tension steel witn a corresponding failure of the concrete in compression - case of under - reinforced sections - or due to the failure of the compression zone before yielding of the steel - case of over-reinforced beams..

There is no sharp line of demarcation between the percentage of reinforcement for an over-reinforced beam and that of an under-reinforced one. The transition from one type to another takes place gradually as the percentage of steel is varied. For the materials presently used in prestressed work, the normal reinforcement ranges between 0.3% and 0.8% . Such ratios of reinforcement can be termed as under-reinforced ratios. If the ratio is over 1%, sudden crushing of concrete without substantial elongation of steel will be likely to take place. If it is less than about 0.15%, breaking of the wire following cracking of concrete may occur.

a) Under-Reinforced Bonded Beams

In under-reinforced bonded beams, rupture is assumed to take place when the stress in the prestressing tendons reaches the ultimate value and in the ordinary steel the yield value. The compressive stress in concrete in the compression side may be assumed in the shape of a rectangular block with an average stress $f_p = 0.85\ f_{cp}$. (Fig V-2).



$z = \xi d$

$y_{cr} = d(1 - \frac{\xi}{2})$

FIG V-2

The average prism strength f_p may be assumed as follows :

Cube strength	f_{c28}	300	400	500	600	kg/cm^2
Prism "	f_{cp}	240	310	380	450	" "
Aver. prism "	f_p	200	260	320	380	" "

The depth of the neutral axis z in the case of simple bending on a rectangular section of breadth b can be determined from the condition :

$$C = T$$

or

$$f_p\ z\ b = A_s\ f_{su}$$

so that $z = \zeta\, d = \frac{A_s}{b} \cdot \frac{f_{su}}{f_p}$

and $\zeta = \frac{A_s}{b\,d} \cdot \frac{f_{su}}{f_p} = \mu \frac{f_{su}}{f_p}$

in which $\mu = A_s / b\,d$ = ratio of tension steel

The ultimate moment M_u can be calculated in the following manner :

$$M_u = C\, y_{CT} = T\, y_{CT}$$

or

$$M_u = f_p \cdot b \cdot z \cdot y_{CT} = f_{su} \cdot A_s \cdot y_{CT}$$

Assuming $y_{CT} = d - \frac{z}{2} = d - \zeta \frac{d}{2} = d\,(1 - \frac{\zeta}{2}) = d(1 - \mu \frac{f_{su}}{2 f_p})$

Then $\underline{M_u = f_{su} \cdot A_s \cdot d\,(1 - \frac{0.5\,\mu\, f_{su}}{f_p})}$

b) Over - Reinforced Bonded Beams

The above method assumes that the ultimate strength of the steel can be developed at the rupture of the beam. But when a section is over - reinforced, the compressive failure will take place in the concrete before the ultimate strength in the steel is developed, the ultimate load in this case is increased by prestressing. For design purposes, the ultimate moment in a rectangular section or a flanged section in which the neutral axis lies in the flange may approximately be assumed as follows :

$$\underline{M_u = 0.3\ f_p\, b\, d^2}$$

Failure by compression occurs suddenly without warning signs. It is advisable to avoid this condition of failure at the ultimate load.

V-4 Composite Sections

In prestressed concrete construction, it is often advantageous to precast part of a section , lift it to position, and cast the remainder of the section in place. The precast and cast in place portions thus act together with proper keys if necessary and form a composite section.

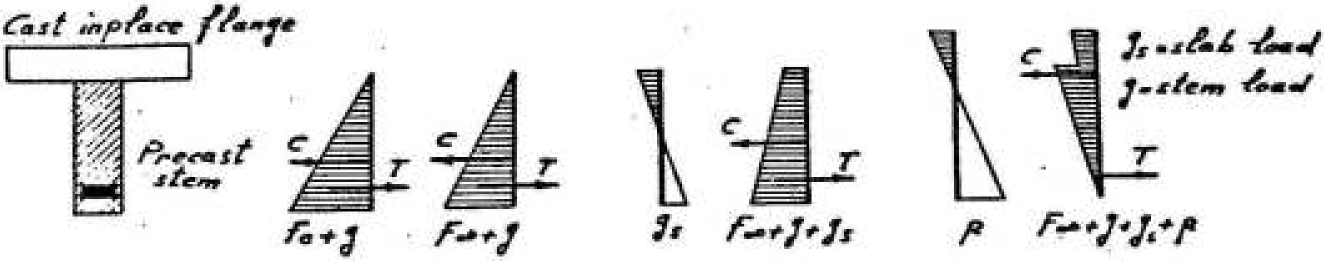


FIG V-3

The figure (V-3) shows a composite section at the midspan of a simply supported beam, whose lower stem is precast and lifted into position with the top slab cast in place resting directly on the stem. The weight of both the slab and the stem will be carried by the stem acting alone. After the slab concrete has hardened, the composite section will carry out live or dead load that may be added on to it. The fig. (V-3) gives the stress distribution for various stages of working load conditions. For the load producing first cracks, it is assumed that the lower fibers reach a tensile stress equal to the tensile strength of concrete. Further, if failure in bond and shear is prevented the ultimate strength of a composite sec. can be computed by a method similar to that previously described for a simple prestressed section. (Fig. V-4).

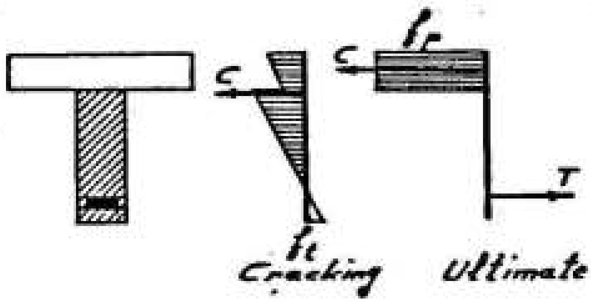
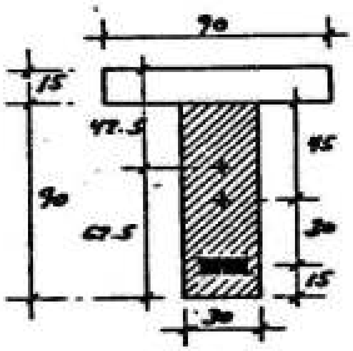


FIG V-4

V-5 Example

The midspan section of a composite beam is shown in figure V-5. The precast stem 30 x 90 cms is post-tensioned with an initial force of 175 t. The effective prestress after losses is taken as 148 t.

FIG V-5



Moment due to the weight of that precast section is 26 mt at midspan. After it is erected in place, the top slab of 15 cms. by 90 cms wide is to be cast in place producing a moment of 13 mt. After the slab concrete has hardened, the composite section is to carry a max live load moment of 48 mt. Compute stresses in the section at various stages and determine the factors of safety against cracking s_1 and failure s_2 .

$A_s = 17.5\ cm^2$ $\quad f_{su} = 16\ t/cm^2$ $\quad f_{c28} = 300\ kg/cm^2$ $\quad f_p = 200\ kg/cm^2$

	Rect. Prestressed Sec.	Composite Sec.
Area	30x90 = 2700	2700+15x90 = 4050 cm^2
Moment of inertia	$30x90^3/12$ =1822500	4328437 cm^4
c.g. from bottom at	45	62.5 cms

Stresses in concrete due to :

Initial prestress F_o: $-\frac{175\ 000}{2700} \pm \frac{175\ 000 \times 30 \times 45}{1822500} = -65 \pm 130$

$f_{top} = -65 + 130 = +65\ kg/cm^2$ $\quad f_{bot} = -65 - 130 = -195\ kg/cm^2$

Final prestress F_∞: $-\frac{148\ 000}{2700} \pm \frac{148\ 000 \times 30 \times 45}{1822500} = -55 \pm 110$

$f_{top} = -55 + 110 = +55\ kg/cm^2$ $\quad f_{bot} = -55 - 110 = -165\ kg/cm^2$

Dead load moment $M_g = 26$ mt. $\quad f = \pm \frac{26 x 10^5 x 45}{1822500} = \pm 65\ kg/cm^2$

Slab moment $M_s = 13$ mt. $\quad f = \pm \frac{13 x 10^5 x 45}{1822500} = \pm 32\ kg/cm^2$

The previous cases act on the rectangular section of the precast stem. The live load acts on the composite section, producing the stresses:

$$f_{top} = \frac{48 \times 10^5 \times 42.5}{4328437} = -47 \ \ kg/cm^2$$

$$f_{bot} = \frac{48 \times 10^5 \times 62.5}{4328437} = +69 \ \ kg/cm^2$$

So that the stress at the bottom of the flange is given by

$$f = - 47 \times \frac{27.5}{42.5} = -30 \ \ kg/cm^2$$

The stresses for the different cases are shown in figure V-6a and for the different stages in figure V-6 b

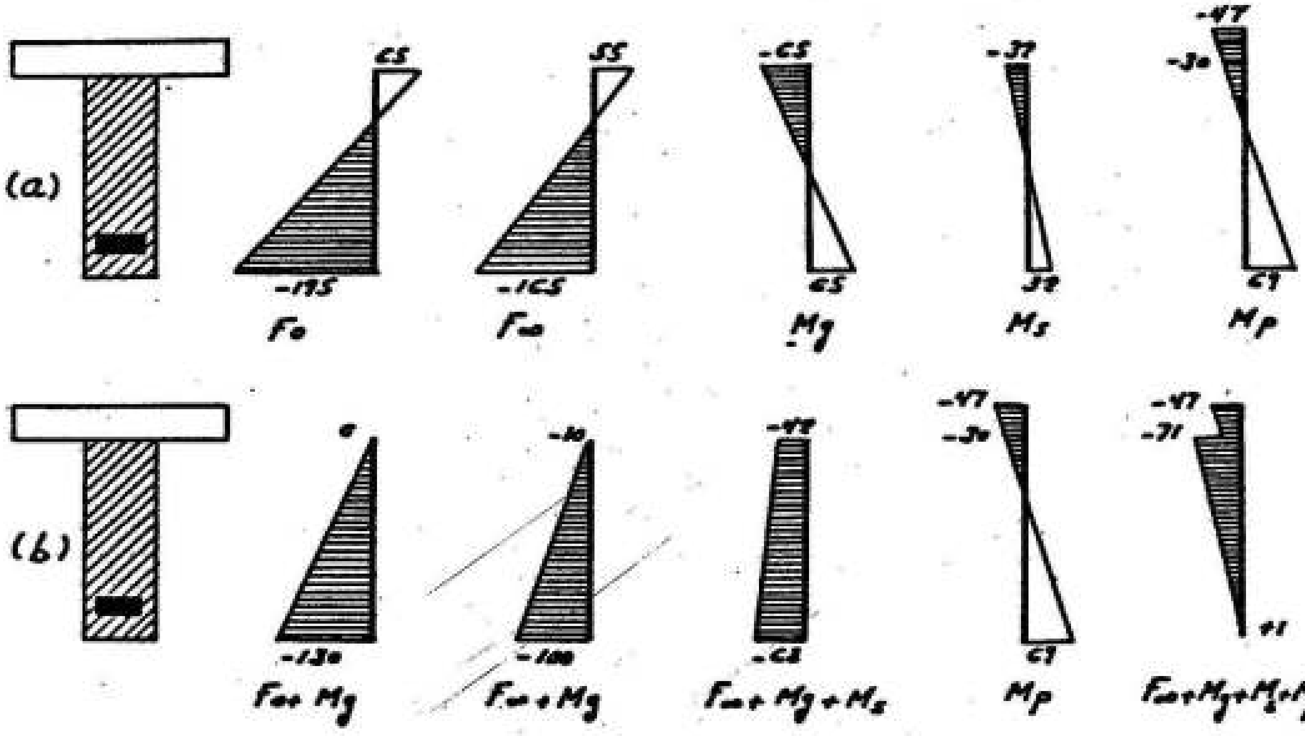


FIG V-6

Assuming the tensile strength of concrete in bending $f_t = 30 kg/cm^2$, the cracking live load-moment can be calculated as follows : In order to have the final value of $f_{bot} = 30 \ kg/cm^2$, the tensile stress due to live load must be equal to $69 + 29 = 98 \ kg/cm^2$ so that

$$M_{crack.} \times 10^6 \times \frac{62.5}{4328437} = 98 \quad \text{or} \quad M_{crack} = 68 \ \ mt.$$

The factor of safety against cracking is therefore

$$s_1 = 68/48 = \underline{1.4}$$

The ultimate strength of steel $T_u = A_s f_{su} = 17.5 \times 16 = 280$ t

Average compressive stress at failure $f_p = 200$ kg/cm^2

Area of compression zone at failure $= 280\ 000/200 = 1400$ cm^2

Height of compression zone $= 1\ 400/90 = 15.6$ cm

Center of compression lies at $\sim$ 7.5 cms from top. So that

Lever arm at M_u $y_{CT} = 15 + 90 - 15 - 7.5 = 82.5$ cm

Ultimate moment $M_u = 280 \times 0.825 = 231$ mt

Total dead + live load moments $M_g + M_s + M_p = 26 + 13 + 48 = 87$ mt

Factor of safety against failure is therefore

$$s_2 = 231/87 = \underline{2.66}$$

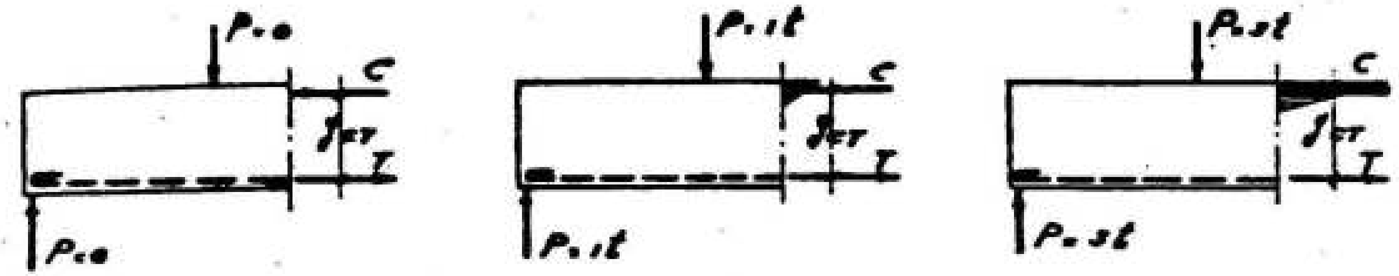
V-6 General Remarks

The elastic design of prestressed sections is based on the fact that the ~~section is governed by two controlling~~ values of external bending moment: the total moment M which controls the stresses under the action of the working loads; and the girder load moment M_g, which determines the location of the c.g.s. and the stresses at transfer.

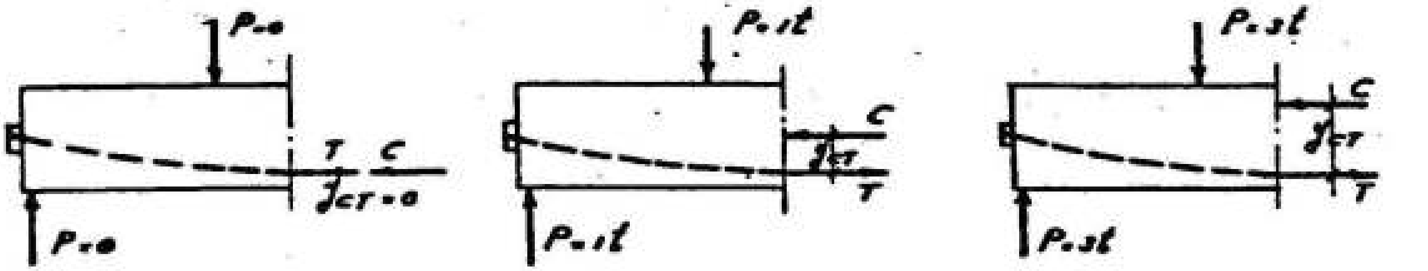
It is desirable to revise here the basic concept of a resisting couple in a prestressed-concrete beam section. From the laws of statics, the internal resisting moment in a prestressed beam, as in a r.c. beam, must equal the external moment. That internal moment can be represented by a couple, C-T, for either the prestressed or the r.c. beam section. T is the centroid of the of the prestress or tensile force in the steel; and C is the center of pressure or the center of compression on the concrete.

There is, however, an essential difference between the behaviour of a prestressed or a reinforced concrete beam section. The difference is explained as follows :

1) In a reinforced concrete beam section, as the external bending moment increases, the magnitude of the forces C & T is assumed to increase in direct proportion while the lever arm y_{CT} between the two forces remains unchanged. (Fig V-7a)



a) Constant y_{CT} in a Reinforced Concrete Beam



b) Variation of y_{CT} in a Prestressed Beam

FIG V-7

2) In aprestressed-concrete beam section under working load, as the external bending moment increases, the magnitude of C & T remains practically constant while the lever arm y_{CT} lengthens almost proportionally. Since the location of T remains fixed, we get a variable location of C in a prestressed section as the bending - moment changes. For a given moment M, C can be easily located, since

$$C\, y_{CT} = T\, y_{CT} = M \qquad \text{and} \qquad y_{CT} = M/C = M/T$$

Thus, when M = 0 , $y_{CT} = 0$, and C must coinside with T. When M is small, y_{CT} is also small. When M is large, y_{CT} is also large . (Fig V-7 b).

In a prestressed-concrete beam, the amount of initial prestress F_o is measured and is rather accurately known. At the time of transfer of prestress, $T = F_o$. After all losses have taken place, $T = F_\infty$. Although the value of T does change as the beam bends under loading, the change is small within the working range and can be neglected in design.

Once the magnitude of T is known, the value of y_{CT} can be computed for any value of M. The location of C can thus be determined. With the position and magnitude of C known, stress distribution across the concrete section can be obtained either by elastic or the plastic theory, although only the elastic theory is usually followed.

It may be convenient in some cases to determine the stress from the moments about the core points and c_b shown in figure V-8, that is

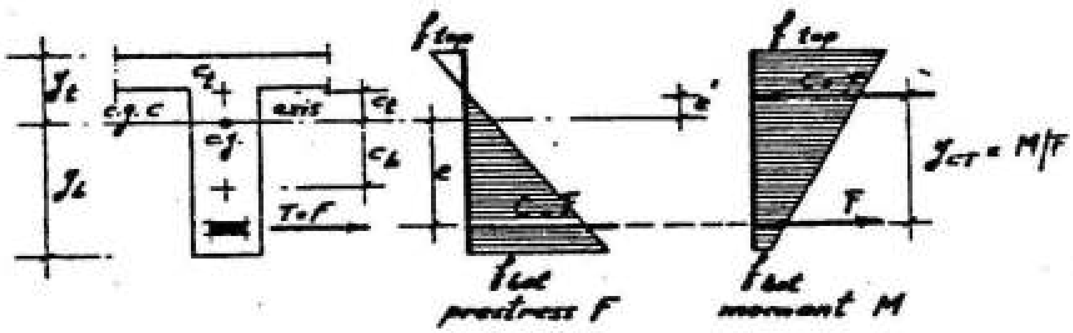


FIG V-8

The stresses at the extreme outside fibers of the section can be given in the form :

$f_{top} = M_b / Z_t$ and $f_{bot} = M_t / Z_b$

in which

M_b , M_t = moment of the internal compression C about the bottom and top core points respectively

For a prestress F acting on a section : C = F and

$M_b = F (e - c_b)$ $M_t = F (e + c_t)$

For any moment M acting on a section : $C = F = M/y_{CT}$

and $e' = y_{CT} - e$

so that

$M_b = F (e' + c_b)$ $M_t = F (c_t - e')$

We have further

$Z_t = I/y_t$ and $Z_b = I/y_b$

$c_t = I/A\ y_b = r^2/y_b$ and $c_b = I/A\ y_t = r^2/y_t$

in which

I, Z, A and r are the moment of inertia, section modulus, area and radius of gyration of cross-section respectively.

In the previous equations, the section values A, I, y_t, y_b, c_t, c_b, e and e' are generally calculated for the full concrete section, although it is supposed, in case of pretensioning, to use the transformed sections and in case of post tensioning to determine the stresses on the net section due to the initial prestress and dead loads and on the transformed section due to the final prestress and external live loads. Judgment should be exercised in deciding whether refinement is necessary or whether approximation is permissible for each particular case.

Figure V-9 shows some of the simple relations between the stress distribution and the location of C, according to the elastic theory.

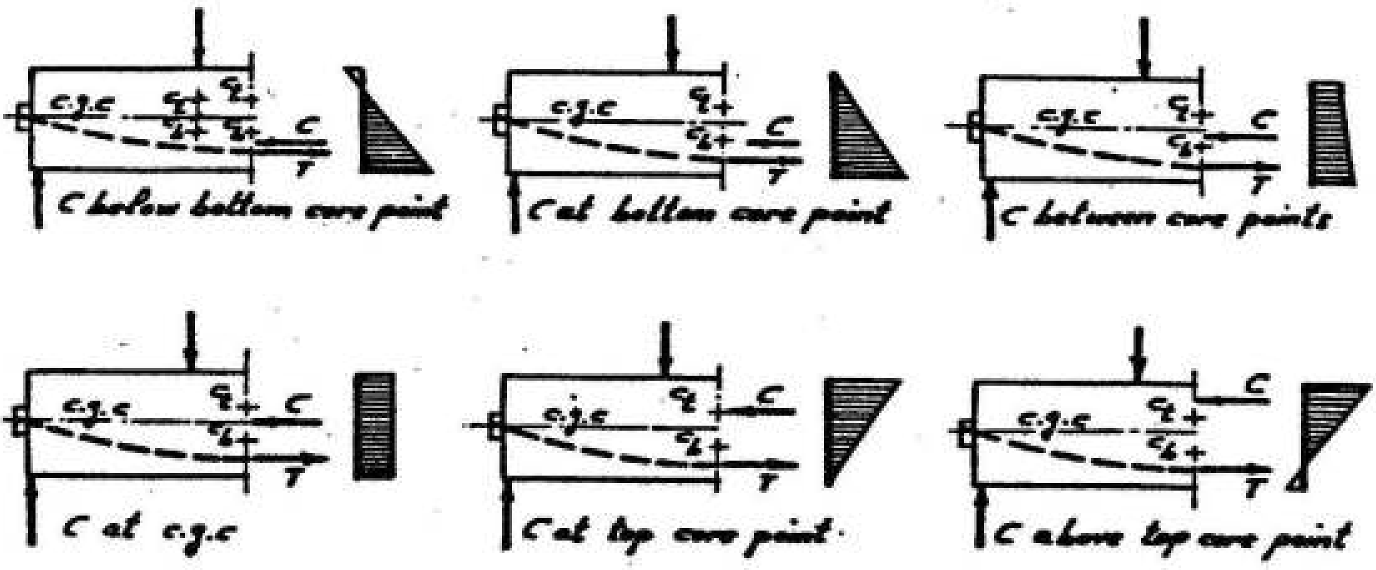


FIG V-9

VI. DIMENSIONING OF PRESTRESSED SECTIONS FOR BENDING

VI-I Factors Affecting the Design of Prestressed Sections

The statical system of a prestressed element is to be chosen according to the available constructional means, the aesthetic requirements & the necessary economic studies .

Having chosen the system, the loads and the corresponding internal forces can be estimated based on previous experience.

The following shows some simple methods for choosing the convenient concrete section and the necessary prestressing steel.

The design of prestressed sections is generally governed by the max. moments. In prestressed girders of not relatively big spans and with inclined tendons, the principal diagonal tensile stresses are generally low and do not need any special provisions.

The prestressing force is to be chosen such that it counteracts the tensile stresses due to the max. internal forces. Due to the min. internal forces, very high compressive stresses exist in the prestressed zone, accompanied, in some cases, by tensile stresses in the compression zone of the section.

The design of the section is mainly affected by the compressive stresses, and eventually the tensile stresses, which are liable to take place due to maximum and minimum internal forces. The ratio of the live load to the total load is one of the main factors which affect the design of a prestressed section.

In case of relatively small live loads, we get high compressive stresses only at the extreme upper fiber. Because of the small possible variations in the stresses due to the different cases of loading, it is always possible in this case to choose the prestressing such that no tensile stresses are created and the section of the prestressed tension zone can be kept small. For such cases, a T-section is most convenient.

In case of relatively big live loads and especially in cases where the bending moments in a section change their sign, it may be necessary to increase the area of cross-section of the prestressed tension zone, which remains generally smaller than the compression zone of the section. This means that one may use a T-section with strengthened tension zone or, in the limiting case, an I or a box - section.

Rectangular sections do not give a convenient form for prestressed elements because they possess an inconvenient relation between the area and the moment of inertia.

The section must be so chosen that the maximum allowable stresses are not exceded in the different stages of loading and with sufficient safety against cracking or failure. The safety against cracking or failure do not generally, in normal cases, affect the design of prestressed sections so that a design based on the maximum bending moment is generally possible especially if some factors which do not affect the dimensioning of the compression and tension zones are neglected.

VI-2 Choice of Prestressed Concrete Sections

The section of the compression zone can be determined from the condition that due to the maximum dead plus live load and final prestressing F_{∞}, the extreme upper fiber of the compression zone is stressed to the maximum allowable compressive stress and the stress in the prestressed tension zone is equal to zero at the level of the prestressing steel as shown in fig VI-I. The area of the prestressed tension zone, being subject to low stresses, does not affect the design of the section and may be neglected without appreciable error. Assuming first that the section has a T form and

$$\delta = t / h_s < 1 \quad , \quad \psi' = b'/b < 1$$

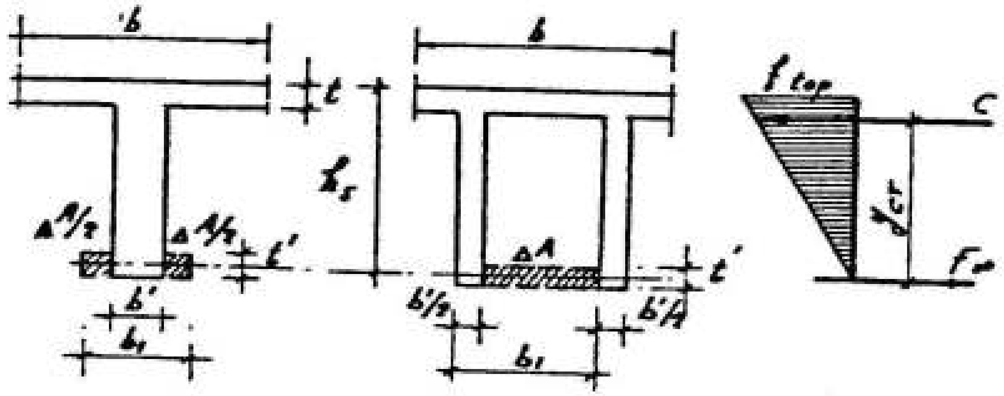


FIG VI-1

then the maximum working moment that can be resisted by the section can be given in the form :

$$M_{max} = k\, f_{top}\ \frac{b\, h_s^2}{3}$$

in which

$$k = 1 - (1-\delta)^3\,(1-\psi')$$

The depth h_s can therefore be given in the form :

$$h_s = c\sqrt{\frac{M_{max}}{b\ f_{top}}}$$

in which

$$c = \sqrt{\frac{3}{k}} = \sqrt{\frac{3}{1-(1-\delta)^3(1-\psi')}}$$

1. Values of c

$\delta = \frac{t}{h_s}$	$\psi' = b'/b$: 0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50	0.75	1.00
0.00	-	7.75	5.48	4.47	3.87	3.46	3.16	2.74	2.45	2.00	1.73
0.05	4.57	4.02	3.53	3.33	3.10	2.90	2.74	2.48	2.29	1.96	1.73
0.10	3.33	3.13	2.96	2.81	2.68	2.57	2.48	2.31	2.18	1.92	1.73
0.15	2.79	2.68	2.51	2.50	2.44	2.36	2.30	2.18	2.08	1.88	1.73
0.20	2.48	2.42	2.36	2.30	2.26	2.21	2.16	2.08	2.01	1.86	1.73
0.25	2.28	2.24	2.20	2.16	2.13	2.10	2.07	2.01	1.95	1.84	1.73
0.30	2.14	2.11	2.08	2.06	2.03	2.01	1.99	1.94	1.91	1.82	1.73
0.40	1.96	1.94	1.93	1.92	1.91	1.89	1.88	1.86	1.84	1.78	1.73
0.50	1.85	1.84	1.84	1.83	1.83	1.82	1.82	1.80	1.79	1.76	1.73
0.75	1.75	1.75	1.75	1.75	1.74	1.74	1.74	1.74	1.74	1.73	1.73
1.00	1.73	1.73	1.73	1.73	1.73	1.73	1.73	1.73	1.73	1.73	1.73

The lever arm y_{CT} of the internal forces can be given in the form

$$y_{CT} = \eta\, h_s$$

in which

$$\eta = \frac{2}{3}\cdot\frac{1-(1-\delta)^3\,(1-\psi')}{1-(1-\delta)^2\,(1-\psi')}$$

2. Values of η = y_{CT} / h_s

$\delta = \frac{t}{h_s}$	$\psi' = b'/b$ 0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.40	0.50	0.75	1.00
0.00	-	.667	.667	.667	.667	.667	.667	.667	.667	.667	.667
0.05	.975	.867	.811	.769	.754	.733	.724	.706	.694	.676	.667
0.10	.951	.889	.847	.814	.789	.770	.754	.730	.712	.684	.667
0.15	.928	.885	.854	.825	.802	.784	.769	.744	.724	.689	.667
0.20	.904	.874	.848	.826	.807	.791	.775	.750	.730	.692	.667
0.25	.882	.856	.836	.819	.802	.788	.774	.751	.731	.693	.667
0.30	.859	.841	.825	.809	.796	.784	.772	.750	.732	.695	.667
0.40	.817	.806	.795	.784	.775	.766	.757	.741	.726	.693	.667
0.50	.778	.771	.764	.756	.750	.744	.738	.726	.715	.689	.667
0.75	.700	.698	.697	.694	.693	.691	.690	.687	.683	.675	.667
1.00	.667	.667	.667	.667	.667	.667	.667	.667	.667	.667	.667

The maximum compressive stresses in the lower fiber of the prestressed tension zone take place due to minimum moment and initial prestressing. The maximum tensile stresses at the same fiber take place due to maximum moment and final prestressing. It may be approximately assumed that the tensile stress at lower edge due to $F - F_o$ (i.e; due to shrinkage and creep) is equal to the allowed tensile stress in case of partial prestressing. Its value is generally chosen 20% of the allowable compressive stress. The required section modulus is therefore

$$Z_b = \frac{\max M - \min M}{f_{bot}} \quad \text{In case of partial prestressing}$$

and

$$Z_b = \frac{\max M - \min M}{f_{bot}} \times 1.25 \quad \text{In case of full prestressing}$$

In case of simple beams max M - min M is equal to the bending moment due to live loads.

If the section modulus Z_b of the chosen section is smaller than the required values, the prestressed tension zone is to be increased (by increasing b' as shown in figure VI-I). Assuming that the area to be increased is $\Delta A = (b_1 - b')t'$

and ΔZ_b = required Z_b - chosen Z_b

then

$$\Delta A = \frac{\Delta Z_b}{y_{CT} - t'/2}$$

The value of y_{CT} can be determined from table 2.

The required area of the prestressing steel can be determined from the relation

$$A_s = \frac{\max.\ M}{y_{CT}\ f_{s\infty}}$$

in which

$f_{s\infty}$ = the final prestress in steel after losses

$\simeq$ 0.8 ÷ 0.85 the initial prestress = 0.8 ÷ 0.85 f_{so}

$y_{CT} \simeq 0.67\ h_s$ for rectangular sections

$\simeq 0.80\ h_s$ for normal T and box sections

$\simeq h_s - 0.5t$ for T and box sections with big b and small b'

The required area of the prestressing steel giving a factor of safety s against failure is given by :

$$A_s = s.\frac{\max.\ M}{y_{CT}\ f_{su}}$$

f_{su} = Ultimate tensile strength of prestressing steel

s = factor of safety against failure > 1.75

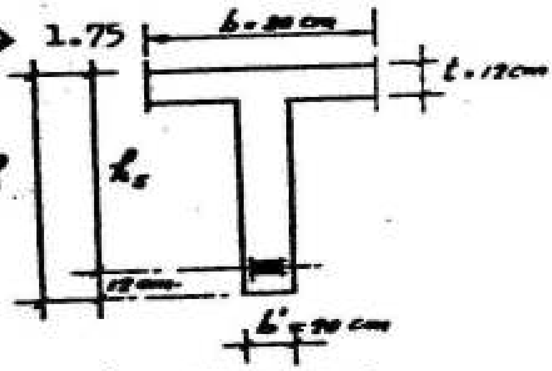


FIG VI-2

Example :

Determine the depth and prestressing steel for the T-section shown in figure VI-2 to carry a max. moment of 80 mt.

Assume :

max. allowable concrete stress f_{top} = 90 kg/cm^2

allowable steel stresses f_{so} = 10 t/cm^2 $f_{s\infty}$ = 8.5 t/cm^2

For $\psi' = b'/b = 20/80 = 0.25$ and assuming $\delta = t/h_s = 0.15$

table 1 gives $c = 2.36$, then

$$h_s = c\sqrt{\frac{M_{max}}{f_{top}\ b}} = 2.36\sqrt{\frac{80}{900 \times .80}} = 79 \text{ cms}$$

$$h = h_s + 12 = 79 + 12 \simeq 90 \text{ cms}$$

According to table 2

$y_{CT} = \eta\, h_s = 0.784 \times 79 \qquad = 62 \text{ cms.}$

Therefore

$$A_s = \frac{M_{max}}{y_{CT}\; f_{s\infty}} = \frac{80}{0.62 \times 8.5} = 15.2 \text{ cm}^2$$

Initial prestressing force $F_o = 10 \times 15.2 \qquad = 152$ ton

VI - 3 Method of Herberg

For known forms of sections, Herberg proposes the following steps for the design of prestressed sections.

1) The cross-section will be chosen according to practical requirements, previous experience and similar structures. In estimating the depth of the section, an approximate rule is to use 70% of the corresponding depth of conventional reinforced concrete structures.

2) In order to determine the prestressing force F and the corres - ponding area of the cables A_s , one can proceed as follows :

a) The stresses due to dead plus live loads are first determined.

b) The stresses in the section due to a prestressing force F=1 acting at a convenient distance from the edge of the section is then calculated.

c) In order to have $f_b = o$ under working loads, we should have:

$$F_\infty = f_{b\,(g+p)} \;/\; f_{b\,(F=1)}$$

d) The final stress can be determined by superposition of stresses due to dead and live loads and stresses due to F_∞ . One has to see that the stress in the upper fiber f_{top} is smaller than the allowable values. (Fig. VI-3)

e) F_∞ being the final prestressing force after the losses, the initial prestressing force at transfer F_o can be determined from the relation $F_\infty = F_o\,\omega$ or $F_o = F_\infty / \omega$ where ω lies between 0.85 and 0.9 according to amount of losses.

Accordingly, the area A_s of the prestressing steel is given by

$$A_s = F_o \;/\; f_{so}$$

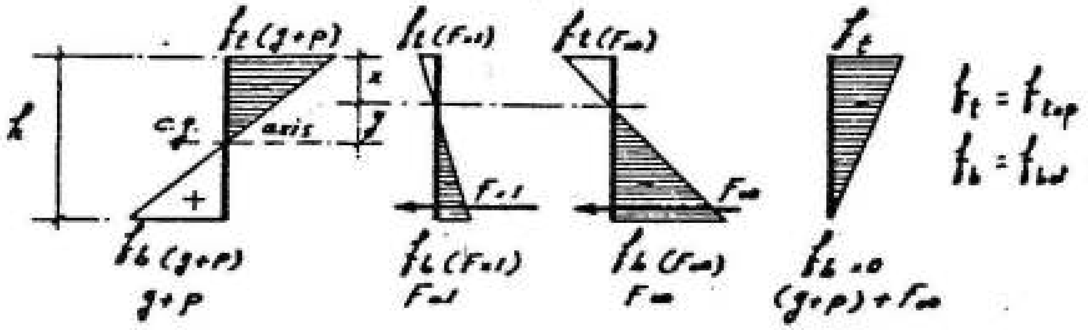
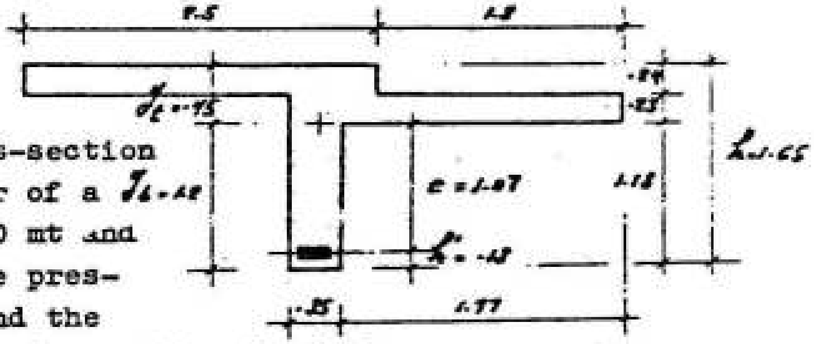


FIG VI-3

f) The stresses at transfer are to be determined for dead loads and the initial prestress F_o . The max. values must be within the allowable limits.



Example :

Fig VI-4 shows the cross-section of the outside main girder of a bridge subject to M_g = 700 mt and M_p = 150 mt, determine the prestressing steel required and the stresses at transfer and under working loads.

FIG VI-4

Data : Initial prestress f_{s0} = 10 t/cm^2 losses 15% max. allowable concrete stress f_c = 140 kg/cm^2

Concrete Section : h = 1.65 m h' = 0.13 m

A = 1.547 m^2 I = 0.2774 m^4

y_t = 0.45 m y_b = 1.20 m e = 1.07 m

$Z_t = I/y_t$ = 0.616 m^3 $Z_b = I/y_b$ = 0.231 m^3

Determination of prestressing force Fig. VI-5

Bending moment due to dead and live loads M_{g+p} = 700 +150 = 850 mt.

Stresses due to dead and live loads :

$f_b = M_{g+p} / Z_b$ = 850 / 0.231 = 3680 t/m^2 = 368 kg/cm^2

$f_t = M_{g+p} / Z_t$ = 850 / 0.616 = 1380 t/m^2 = 138 "

Stresses due to F = 1 t

$$f_b = -\frac{1}{A} - \frac{1.e}{Z_b} = -\frac{1}{1.547} - \frac{1x1.07}{0.231} = -5.29 \quad t/m^2 = -0.529 \ kg/cm^2$$

$$f_t = -\frac{1}{A} + \frac{1.e}{Z_t} = -\frac{1}{1.547} + \frac{1x1.07}{0.616} = +1.09 \quad t/m^2 = 0.109 \quad "$$

Prestressing force $F_\infty = f_{b(g+p)} / f_{b(F=1)} = 3680/5.29 = 700$ t

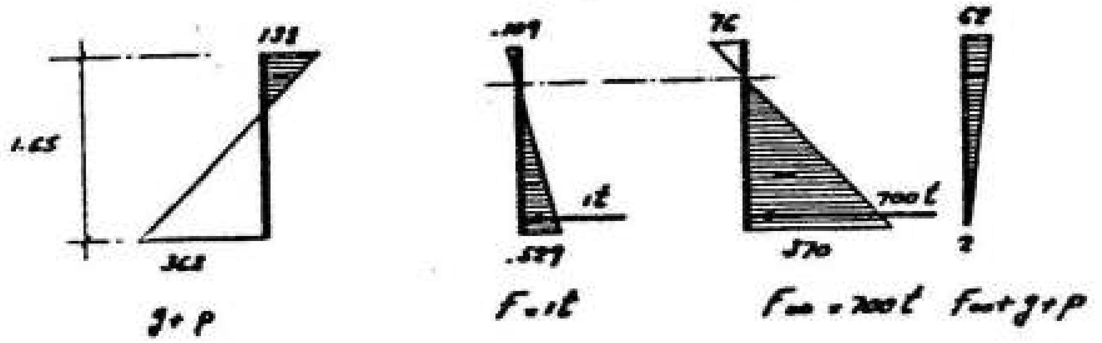


FIG VI-5

Prestressing steel A_s = F_∞ / $f_{s\infty}$ = 700/8.5 = 82.5 cm^2

Initial prestressing force $F_o = F_\infty /0.85$ = 700/0.85 = 825 t

Stresses due to dead or live loads = stresses due to (g + p) multiplied by M_g/M_{g+p} = 700/850 = 0.824 or M_p / M_{g+p} = 150/850=.176 respectively

Stresses due to initial prestressing F_o = Stresses due to F_∞ multiplied by F_o / F_∞ = 825/700 = 1.18

So that the stresses at the different stages are as shown in figure VI-6

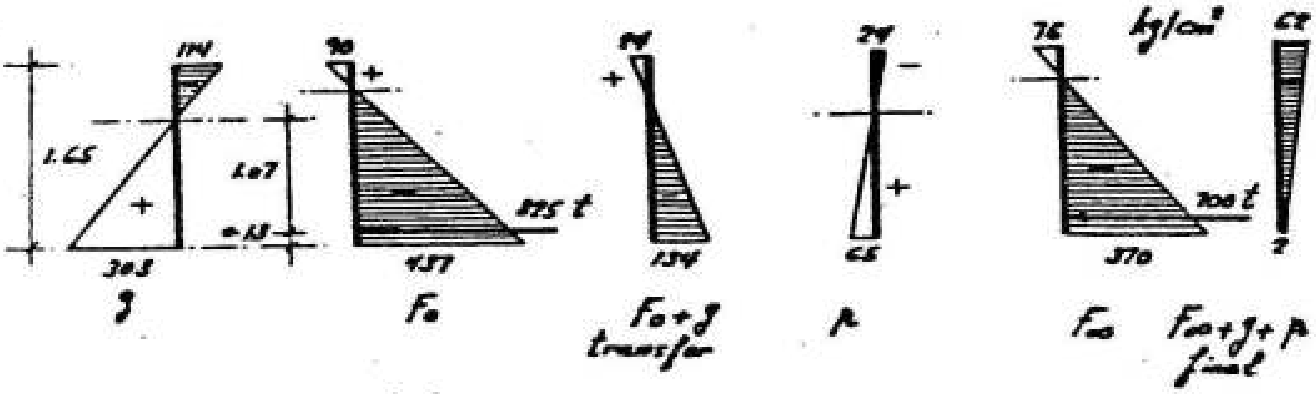


FIG VI-6

Additional tensile stress in steel due to bending can be calculated as follows :

Bending moment due to dead + live loads = 850 mt

Bending moment due to final prestressing = $F_\infty . e$ = 700x1.07= 749 "

net moment = 850 - 749 = 101 "

Assuming n = 6.5 , the additional stress will be :

$f_s = n M y / I = 6.5 \times 101 \times 1.07/0.2774 = 2530 \ t/m^2 = 253 kg/cm^2$

So that the final stress in prestressing steel = 8500 + 253=8753 "

VI-4 Method of Leonhardt

a) General Relations

The following general conditions must be satisfied when designing a section subject to positive bending moments :

1) For maximum bending moment (max.M = M_{g+p}) and minimum prestressing force F_∞ , the compressive stress in the upper fiber f_{top} must be smaller than or equal to the allowable value. Hence

$$f_{top} \text{ for max.M} + f_{top} \text{ for } F_\infty \leq f_{top} \text{ allowed (comp)}$$

Further, the stress in the lower fiber f_{bot} must be bigger than or equal to zero by full prestressing and smaller than or equal to the allowable tensile stress $f_{t\ bot}$ by partial prestressing. Hence

$$f_{bot} \text{ for max.M} + f_{bot} \text{ for } F_\infty \geq 0 \text{ or } \leq f_{t\ bot} \text{ allowed (tension)}$$

2) For minimum bending moment (M_g or $M_g - M_p$) and initial prestressing force F_o , the compressive stress in the lower fiber f_{bot} must be smaller than or equal to the allowable value. Hence

$$f_{bot} \text{ for min.M} + f_{bot} \text{ for } F_o \leq f_{bot} \text{ allowed (comp)}$$

Further, the stress in the upper fiber f_{top} must be bigger than or equal to zero by full prestressing and smaller than or equal to the allowable tensile stress $f_{t\ top}$ by partial prestressing . Hence

$$f_{top} \text{ for min.M} + f_{top} \text{ for } F_o \geq 0 \text{ or } \leq f_{t\ top} \text{ allowed (tension)}$$

b) Application to Rectangular Sections

Applying these general relations to rectangular sections (fig. VI-7) we get the following :

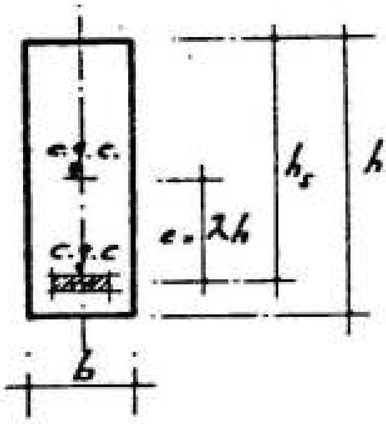


FIG VI-7

Assume : $e = \lambda h$ and $F_\omega = \omega F_o$

then the max. stresses due to a bending moment M are :

$$f = \pm \frac{6\ M}{b\ h^2}$$

and the max. stresses due to a prestressing force F acting at the c.g.s. are :

$$f = \frac{F}{b\ h} \pm \frac{6\ F\ e}{b\ h^2} = \frac{F}{b\ h} \pm \frac{6\ F\ \lambda\ h}{b\ h^2}$$

or

$$f = \frac{F}{b\ h}\ (1 \pm 6\ \lambda)$$

The max. stresses for the limiting cases of full prestressing are therefore :

1.) $$f_{top} = -\frac{6\ M_{max}}{b\ h^2} - \frac{\omega\ F_o}{b\ h}\ (1 - 6\ \lambda) < f_{top} \text{ allowed (comp)}$$

2.) $$f_{t\ bot} = +\frac{6\ M_{max}}{b\ h^2} - \frac{\omega\ F_o}{b\ h}\ (1 + 6\ \lambda) = 0$$

3.) $$f_{bot} = +\frac{6\ M_{min}}{b\ h^2} - \frac{F_o}{b\ h}\ (1 + 6\ \lambda) = f_{bot} \text{ allowed (comp)}$$

4.) $$f_{t\ top} = -\frac{6\ M_{min}}{b\ h^2} - \frac{F_o}{b\ h}\ (1 - 6\ \lambda) = 0$$

This means that the dimensioning depends on 10 values while we have only four equations so that 6 values must be given and four values are to be determined. The dimensioning equations in both cases of full and partial prestressing are as follows :

Full Prestressing

From the first two equations, we get

$$h = k_1 \sqrt{\frac{M_{max.}}{b\ f_{top}}}$$

in which

$$k_1 = \sqrt{\frac{12}{1 + 6\lambda}}$$

so that

$$b\ h^2\ f_{c\ top} = \frac{12\ M_{max}}{1 + 6\lambda}$$

$$\frac{6\ M_{max}}{b\ h^2} = \tfrac{1}{2}\ f_{top}\ (1 + 6\ \lambda)$$

Substituting this value in equation 2, we get

$$F_{\omega} = \omega F_o = \frac{1}{2} b\, h\, f_{top}$$

Equation 4 gives :

$$M_{min} = k_3 F_o h$$

in which

$$k_3 = \lambda - \frac{1}{6}$$

Therefore

$$f_{bot} = 2 F_o / bh$$

The relation between M_{min} and M_{max} can be calculated from equations 2 and 4, thus

$$M_{min} = \frac{M_{max}}{\omega} \cdot \frac{6\lambda - 1}{6\lambda + 1}$$

So long as M_{min} is bigger than the value given by this equation, then h and F are correct. If M_{min} is smaller, one has to decrease the value of λ and thus increase h or F. Assuming $\omega = 0.9$, the limiting value of M_{min} / M_{max} can ~~be given as~~ a factor of λ as follows :

λ	0.475	0.45	0.40	0.35	0.30	0.25	0.20
M_{min}/M_{max}	0.53	0.51	0.46	0.39	0.32	0.22	0.10

For practical values of $\lambda = 0.45$ to 0.20 corresponding to h_s/h = 0.95 to 0.7, the values of k_1 and k_3 are as follows:(Fig.VI-8)

h_s/h	0.50	0.60	0.70	0.80	0.90	1.00
k_1	3.47	2.74	2.34	2.07	1.88	1.73
k_3	-0.167	-0.067	+0.033	+0.133	+0.233	+0.333

In normal cases, the c.g.s.- axis is located as low as possible so that one can assume :

$\lambda = 0.40$ i.e. $h_s / h = 0.9$ giving $k_1 = 1.9$ and $k_3 = 0.25$

Assuming further that $\omega = F_{\omega} / F_o = 0.9$ and the concrete used is C300 with $f_{top} = 100$ kg/cm^2 allowable stress , then

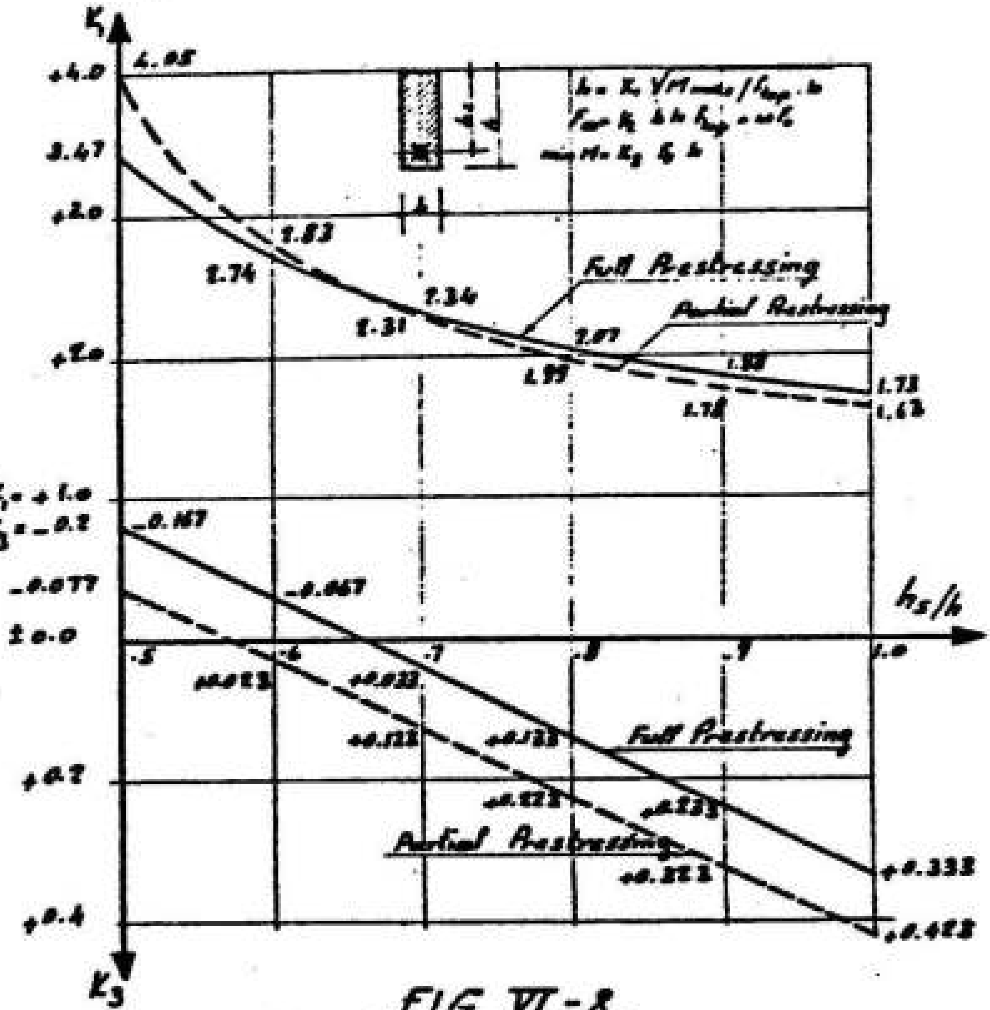


FIG VI-8

$h = 0.06 \sqrt{\frac{M_{max}}{b}}$, $F_{\infty} = 500\ bh$, $M_{min} = 130\ bh^2$ and $f_{bot} = 1.1\ f_{top}$

Dimensions are in tons and meters.

Partial Prestressing

In case of partial prestressing, the fundamental stress relations 2 and 4 will be :

$$2)\quad 6\,\frac{M_{max}}{b\,h^2} - \frac{\omega\,F_o}{b\,h}\,(\,1+6\lambda\,) = f_{t\ bot} = (\frac{30}{110}\,f_{top}) \qquad \text{(tension)}$$

$$4)'\quad -\,6\,\frac{M_{min}}{b\,h^2} - \frac{F_o}{b\,h}\,(\,1-6\lambda\,) = f_{t\ top} = (\frac{30}{140}\,f_{bot}) \qquad \text{(tension)}$$

Assuming $f_{top} = 110\ kg/cm^2$, $f_{bot} = 140\ kg/cm^2$ and $f_t = 30\ kg/cm^2$ (according to DIN) , equations 1 and 2' give

$$h = k_1 \sqrt{\frac{M_{max}}{b\ f_{top}}}$$

in which

$$k_1 = \sqrt{\frac{12}{4.36\lambda + 1.27}}$$

The corresponding prestressing force is :

$$F_\infty = \omega F_o = 0.364\ b\ h\ f_{top}$$

Equation 4' related to f_{top} can be given in the form :

$$4)'' \quad - \frac{6\ M_{min}}{b\ h^2} - \frac{F_o}{b\ h} (1 - 6\lambda) = \frac{30}{110} f_{top}$$

So that

$$M_{min} = F_o h \left(\lambda - \frac{1}{6}\right) - \frac{b\ h^2}{22} f_{top}$$

$$= \frac{M_{max}}{\omega} \cdot \frac{4(6\lambda - 1) - 3\omega}{4(6\lambda + 1)}$$

Assuming $\omega = 0.9$, the limiting value of M_{min} / M_{max} can be given as a factor of λ as follows :

λ	0.475	0.45	0.40	0.35	0.30	0.25	0.20
M_{min}/M_{max}	0.34	0.31	0.24	0.15	0.05	-0.08	-0.24

The dimensioning formulae under the same previous assumptions will be :

$$h = 0.063 \sqrt{\frac{M_{max}}{b}} \qquad F_\infty = 365\ b\ h \qquad M_{min} = 50\ b\ h^2$$

which mean that the depth is approximately the same as in case of full prestressing, while the prestressing force is only 75% ; the concrete tensile stresses are further not exceeded by small M_{min}.

Example :

Design a rectangular section to carry a max. moment of 80 mt and a min. moment of 40 mt.

a) <u>Full Prestressing</u>

Assume $b = 25$ cms , $f_{top} = 100$ kg/cm^2 , $\lambda = e/h = 0.4$, $\omega = 0.9$

$h = 0.06\sqrt{\frac{M}{b}} = 0.06\sqrt{\frac{80}{0.25}} = 1.07$ m

$F_{\infty} = 500\ b\ h = 500 \times 0.25 \times 1.07 = 134$ t

$F_o = F_{\infty} / \omega = 134 / 0.9 = 149$ t chosen 150^t

$M_{min} = 130\ b\ h^2 = 130 \times 0.25 \times 1.07^2 = 37.4$ mt < 40 m t

<u>Final design and check of stresses</u>

$M_g = 40$ mt $M_p = 40$ mt $F_o = 150^t$ $f_{so} = 10$ t/cm^2 losses=15%

Section : 25 x 107 cms , $f_{top} = 100$ kg/cm^2 $f_{bot} = 130$ kg/cm^2

$A_c = 25 \times 107 = 2675$ cm^2 , $y_t = y_b = h/2 = 107/2 = 53.5$ cms

$I = 25 \times 107^3/12 = 255.22 \times 10^4$ cm^4, $c_t = c_b = h/6 = 107/6 = 17.8$ cms

$A_s = F_o / f_{so} = 150 / 10 = 15$ cm^2 $F_{\infty} = 0.85\ F_o = 0.85 \times 150 = 127.5$t

location of c.g.s. $e - c_b = M_g / F_o = 40/150 = 26.7$cms

$e = 26.7 + 17.8 = 44.5$cms

Stresses due to initial prestress F_o :

$$f = -\frac{150\ 000}{2675} \pm \frac{150\ 000 \times 44.5 \times 53.5}{255.22 \times 10^4} = -56 \pm 140$$

so that $f_{top} = +84$ kg/cm^2 & $f_{bot} = -196$ kg/cm^2

Stresses due to final prestress F_{∞} :

$$f = -\frac{127\ 500}{2675} \pm \frac{127\ 500 \times 44.5 \times 53.5}{255.22 \times 10^4} = -48 \pm 119$$

so that $f_{top} = +71$ kg/cm^2 & $f_{bot} = -167$ kg/cm^2

Stresses due to dead and live loads

$$f = \pm \frac{40 \times 10^5 \times 53.5}{255.22 \times 10^4} = \pm 84 \text{ kg/cm}^2$$

Accordingly, the stresses at transfer (F_o + g) and at working conditions (F + g + p) are as shown in figure VI-9

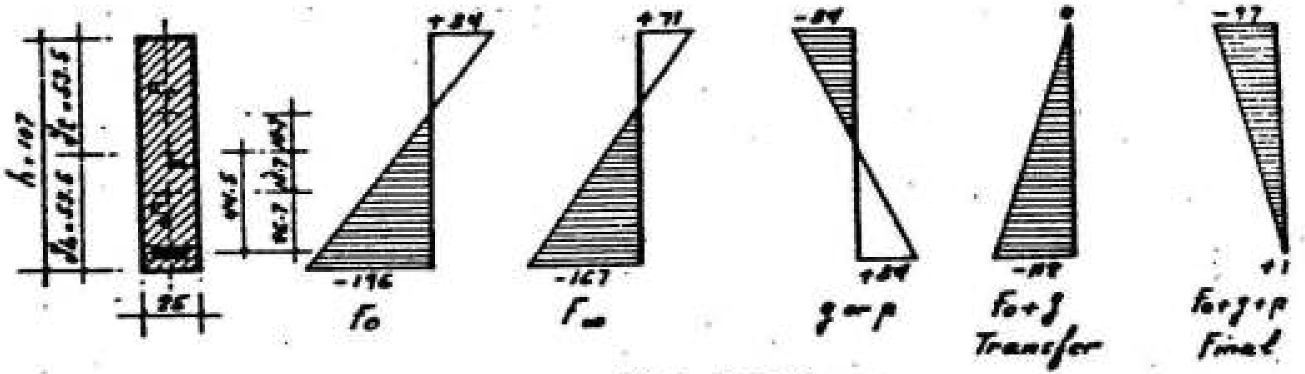


FIG VI-9

b) Partial Prestressing

If partial prestressing with an allowable tensile stress of 25 kg/cm^2 is allowed, then the compressive stress due to F_∞ must be 167 - 25 = 142 kg/cm^2 . i.e.

F_∞ = 127.5 x 142/167 = 108 ton and

F_o = 108/0.85 = 128 ton

The solution for this case will be as follows :

Section 25 x 107 cms , A_c = 2675 cm^2 , I = 255.22 x 10^4 cm^4

$y_t = y_b$ = 53.5 cms $c_t = c_b$ = 17.8 cms

A_s = 128/10 = 12.8 cm^2 $e - c_b$ = 40/128 = 31.2 cm

The c.g.s. will be chosen at e = 31.2 + 17.8 = 49.0 cm

The corresponding cover will be 53.5 - 49.0 = 4.5 cms is small an therefore, chosen e = 44.5 cms as before !

Check of Stresses

Stresses due to an initial prestress F_o = 128 t

f_{top} = + 84 x 128/150 = + 71 f_{bot} = - 196 x 128/150 = - 167kg/cm^2

Stresses due to a final prestress F_∞ = 108 t

f_{top} = + 71 x 108/127.5 = + 61 f_{bot} = - 167 x 108/127.5= - 142kg/cm^2

Stresses due to dead or live load moment M = 40 mt

$f_{top} = -\ 84\ kg/cm^2$ $\qquad$ $f_{bot} = +\ 84\ kg/cm^2$

Accordingly, the stresses at transfer and at working conditions are as shown in figure VI-10

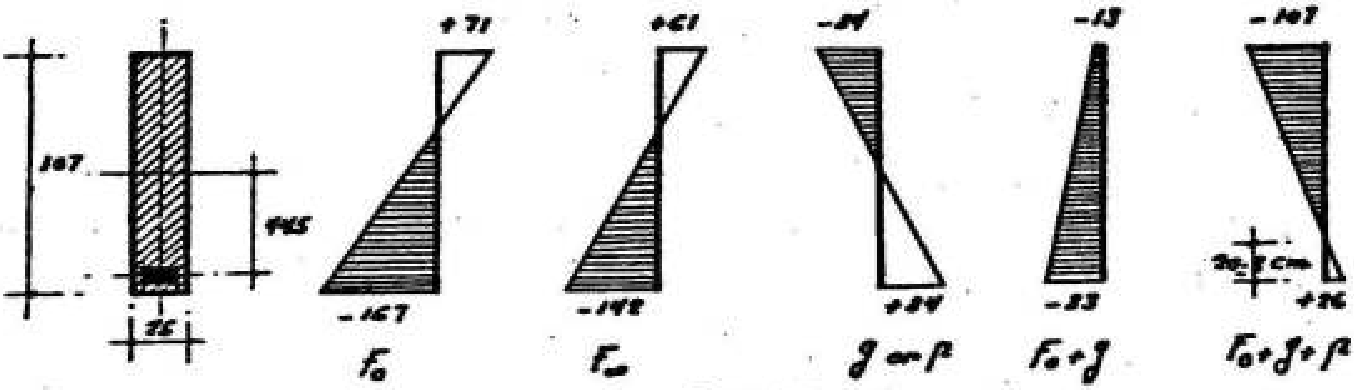


FIG VI-10

The tensile stresses in the section must be resisted by normal mild steel having an area :

$$A_s = \frac{26 \times 20.8 \times 25}{2 \times 1400} = 4.85\ cm^2$$

c) Application to T - I and Box - Sections

Leonhardt has given the series of curves shown in figures VI-12 to VI-17 for dimensioning of T - I and box sections based on the assumptions shown in figure VI-11

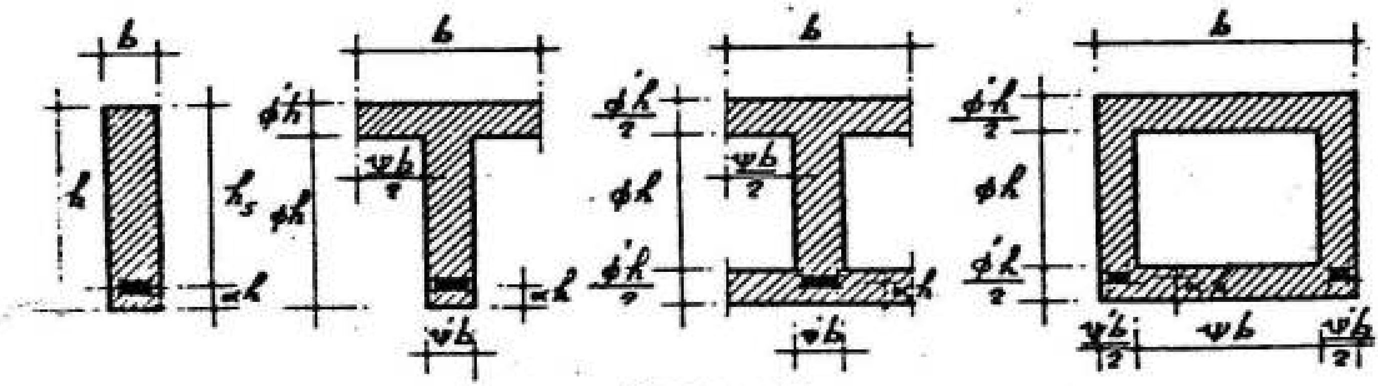
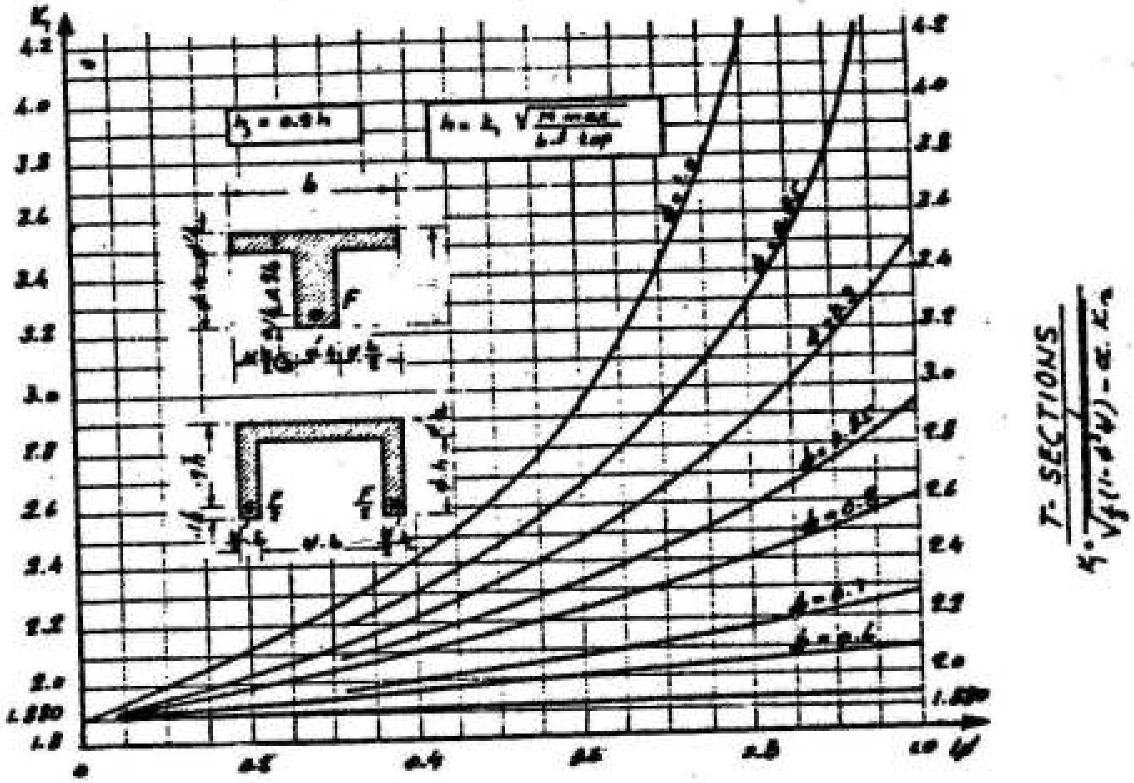


FIG VI-11

for $h_s = 0.9\ h$ or $h_s = 0.8\ h$ and $F_\infty = \omega F_o$ where $\omega = 0.9$ to 0.7

The depth is given by the relation

$$h = k_1 \sqrt{\frac{M_{max}}{b\ f_{top}}}$$



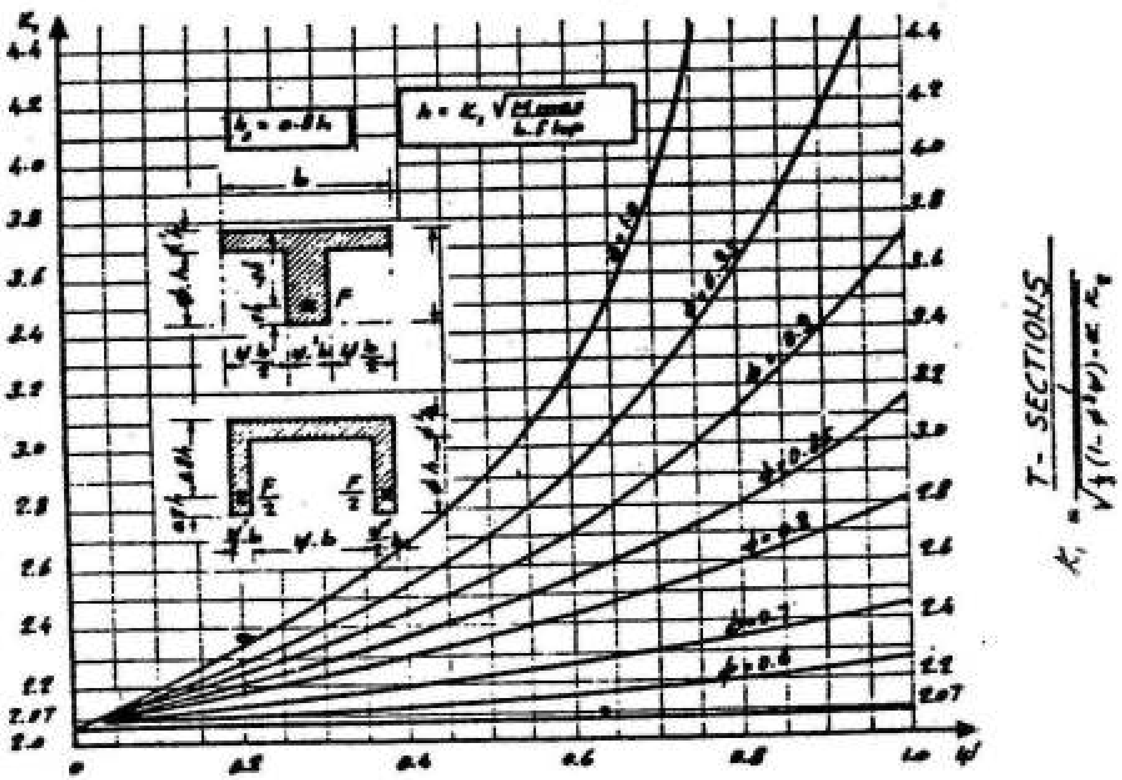
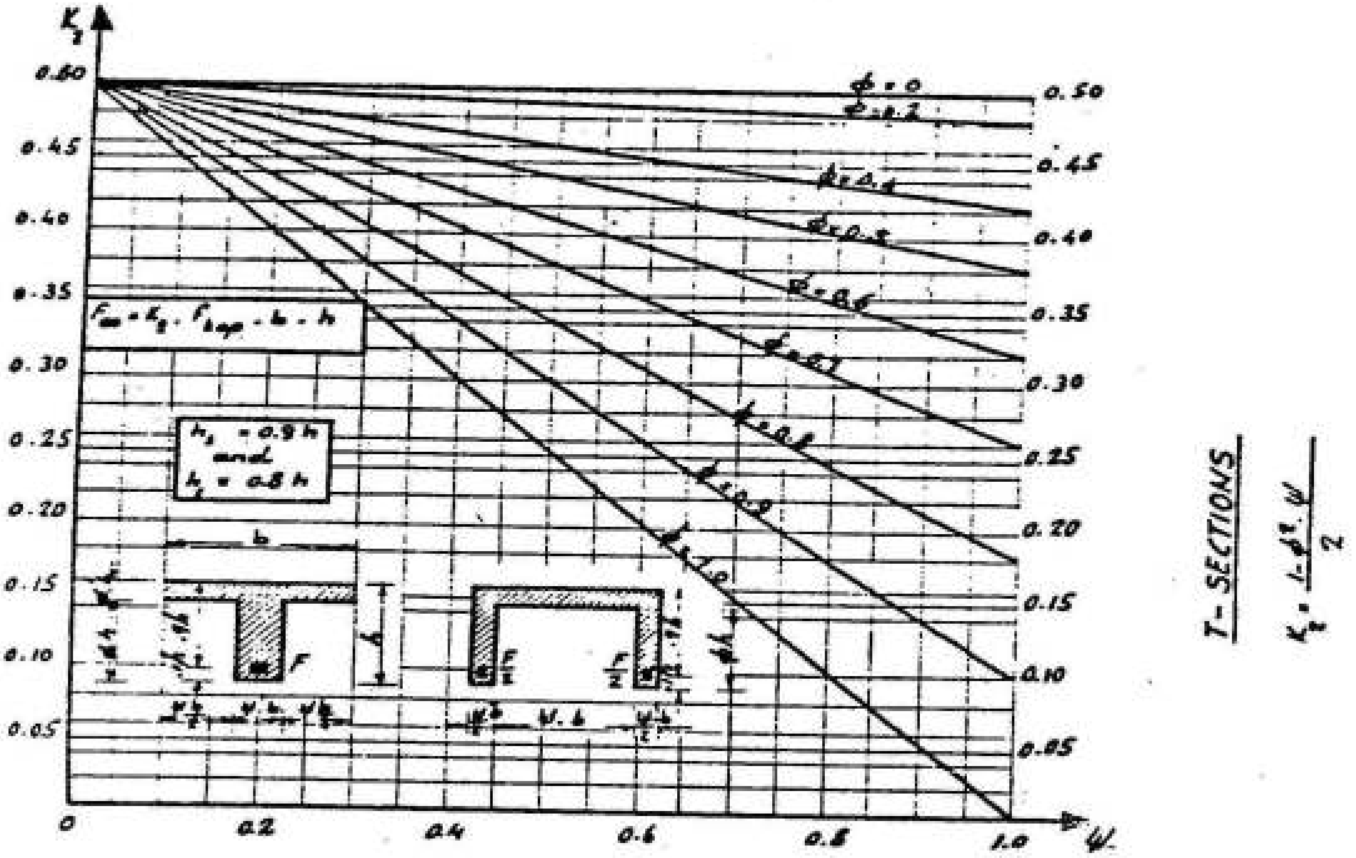


FIG VI-12


K2
0.50
0.45
0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0
0.2
0.4
0.6
0.8
1.0
ψ
φ = 0
h1 = 0.9 h
and
h2 = 0.8 h
b
F
T- SECTIONS

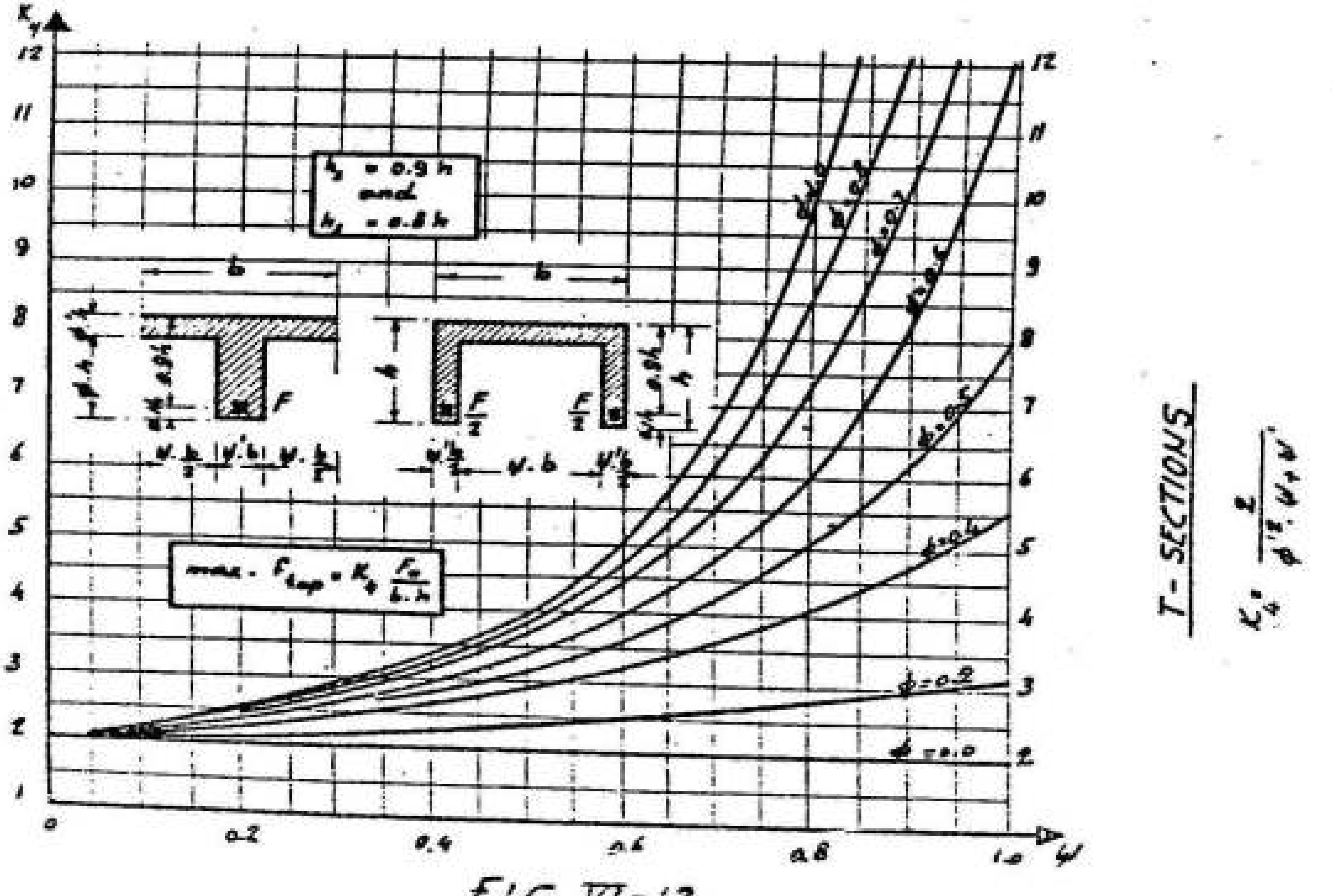
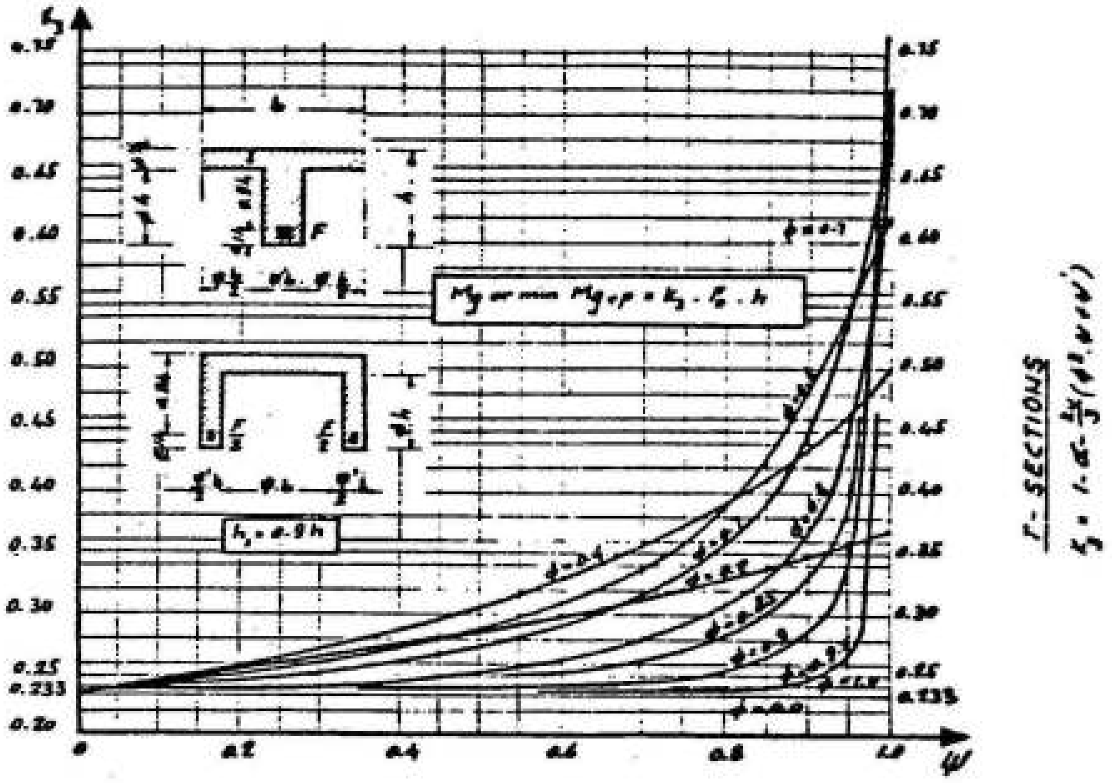

K4
12
11
10
9
8
7
6
5
4
3
2
1
0
0.2
0.4
0.6
0.8
1.0
ψ
h1 = 0.9 h
and
h2 = 0.8 h
b
F
φ = 0.0
T- SECTIONS

FIG VI-13



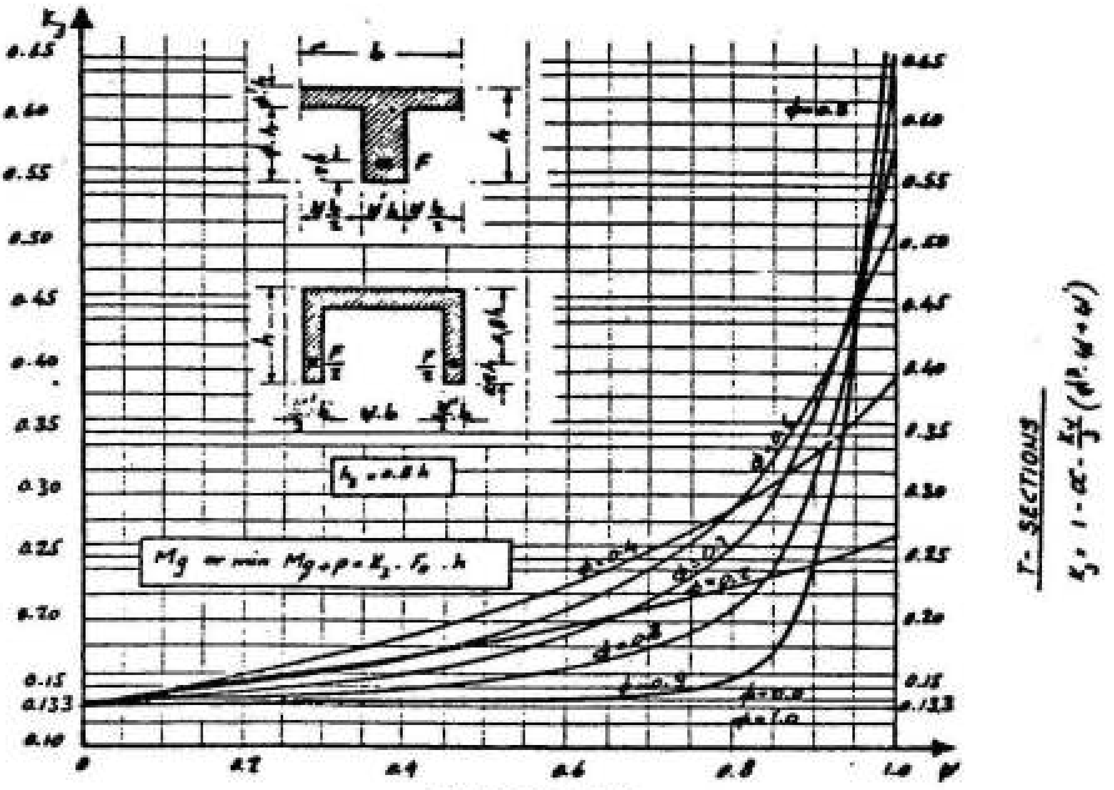
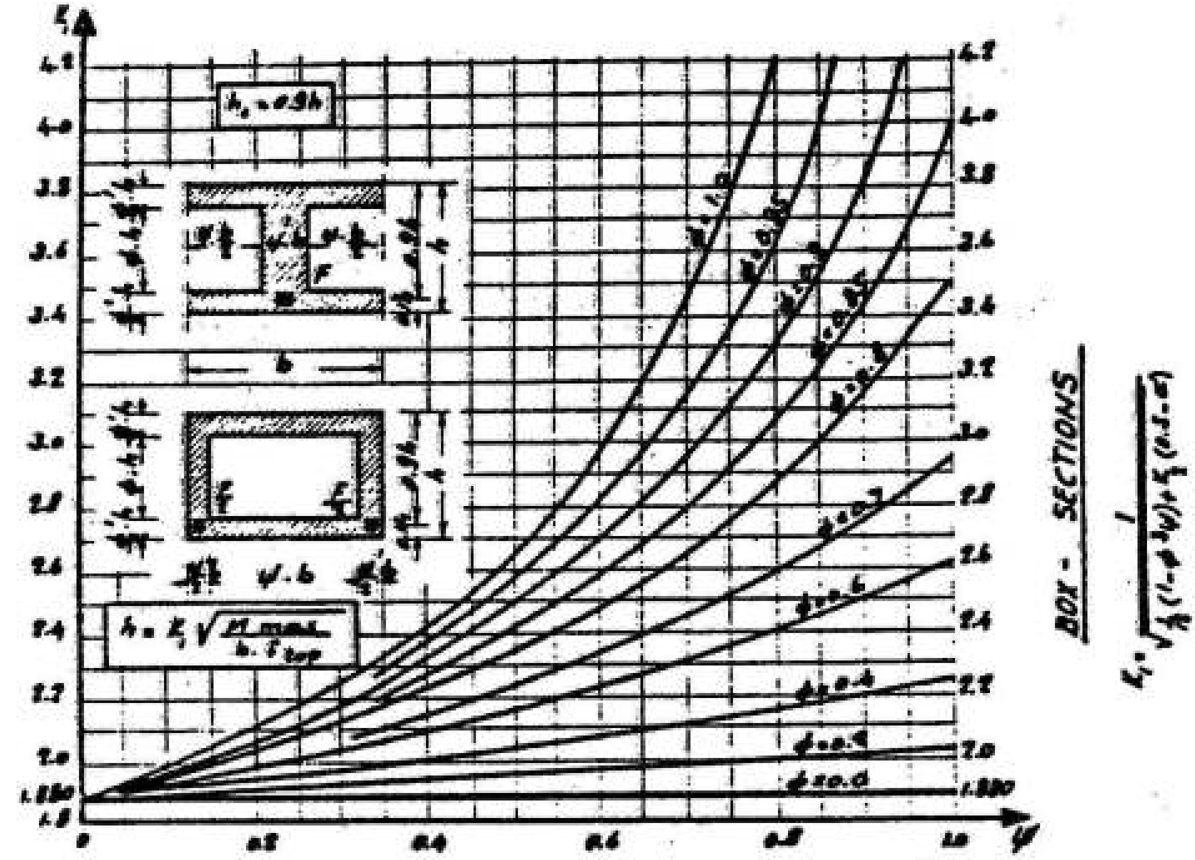


FIG VI-14


BOX - SECTIONS
$h = K_1 \sqrt{\frac{M_{max}}{b \cdot f_{top}}}$

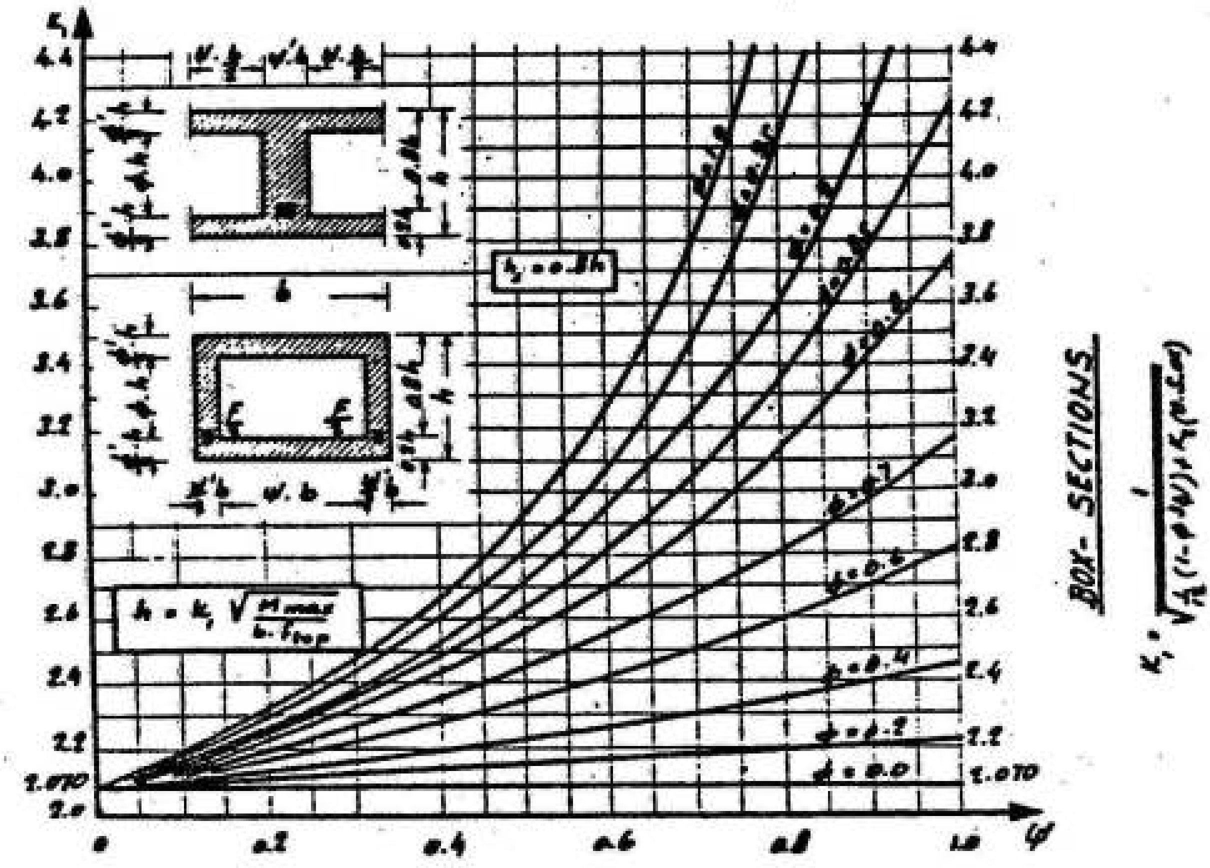
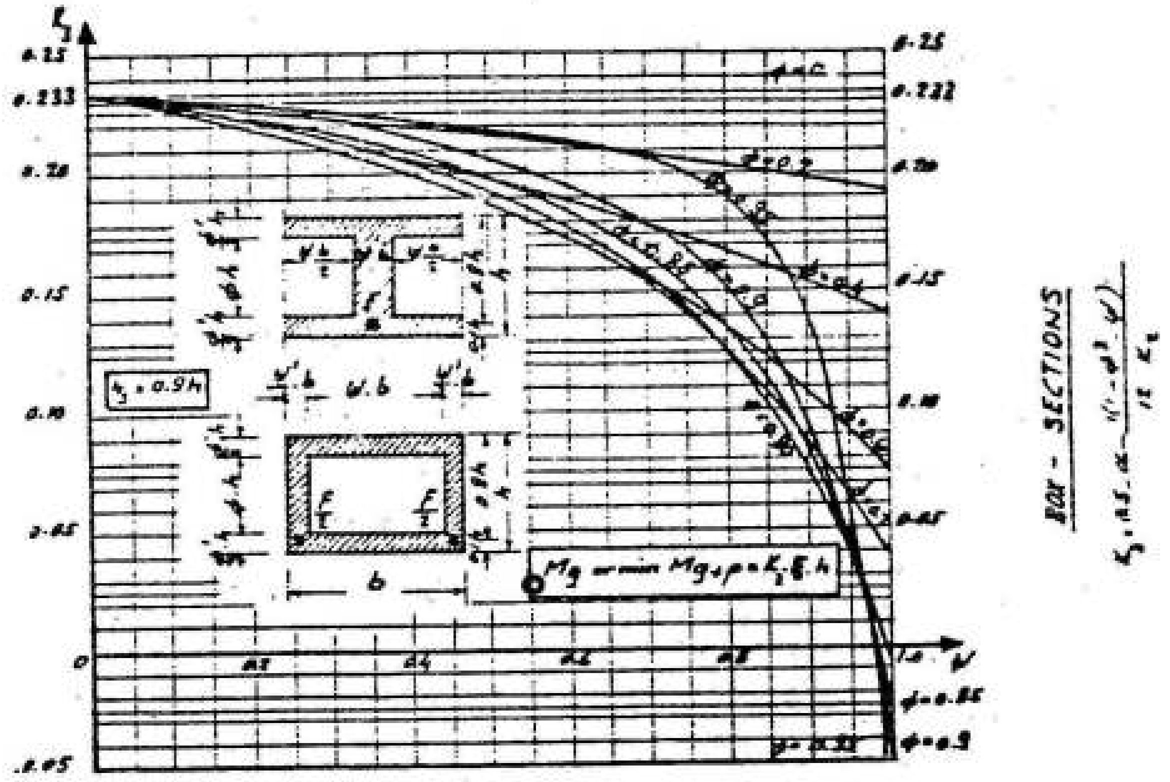

BOX- SECTIONS
$h = k_1 \sqrt{\frac{M_{max}}{b \cdot f_{top}}}$

FIG VI -15


0.25
0.233
0.20
0.15
0.10
0.05
0
-0.05
0.2
0.4
0.6
0.8
1.0
BOX - SECTIONS

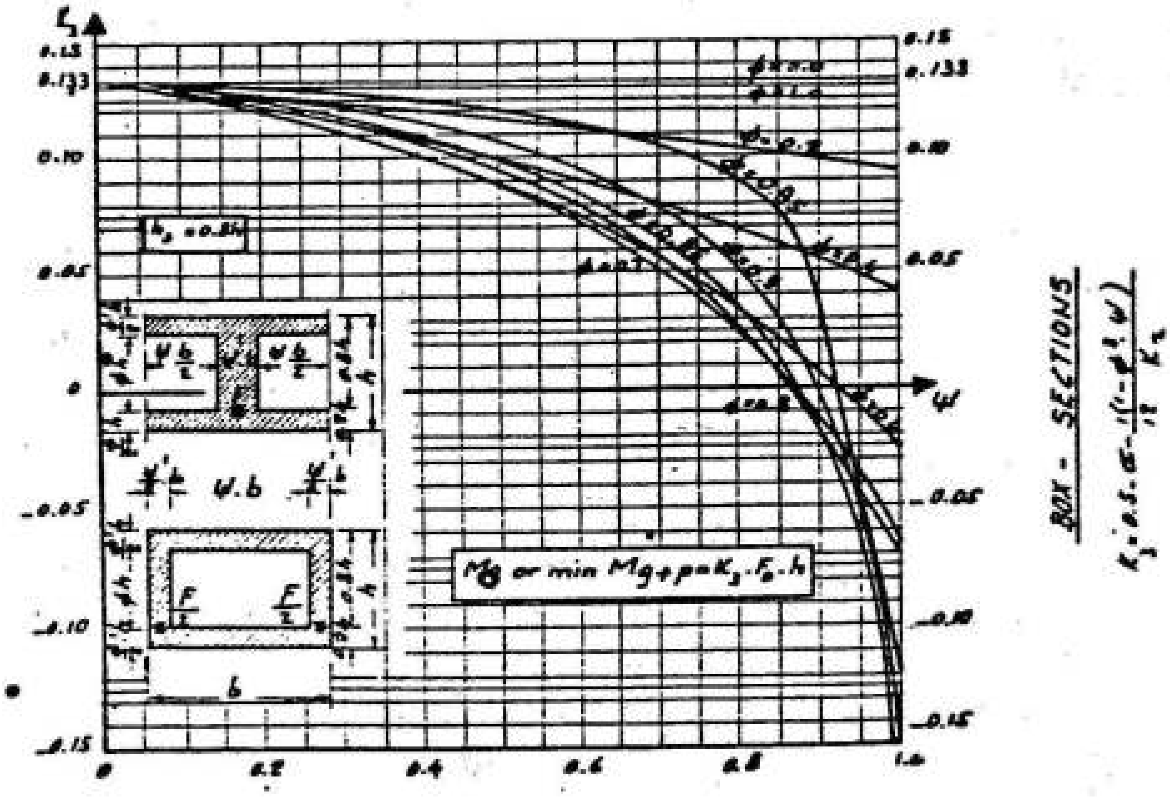

0.15
0.133
0.10
0.05
0
-0.05
-0.10
-0.15
0
0.2
0.4
0.6
0.8
1.0
BOX - SECTIONS

FIG VI-16

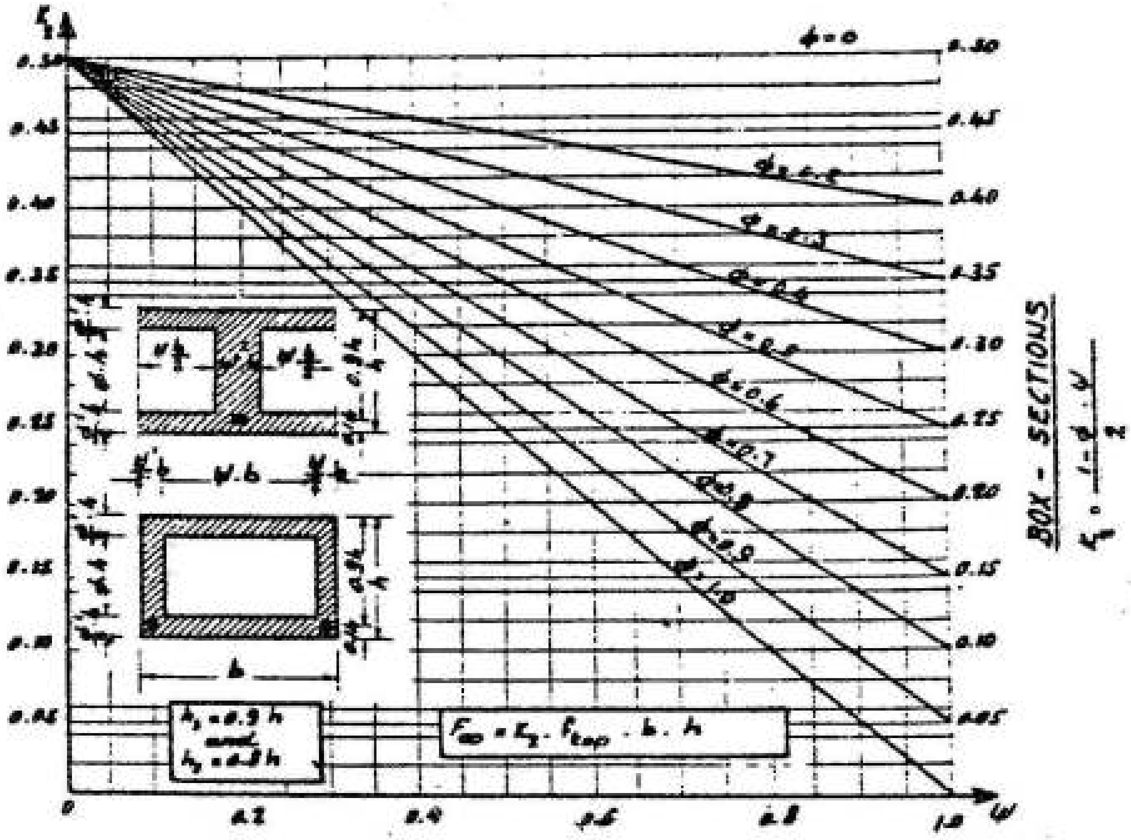


FIG VI-17

Note : It is generally more ecinomic to assume f_{top} smaller than the allowable values ; say 80 kg/cm^2 for a concrete C 300 .
The corresponding prestressing forces are

$$F_\infty = k_2 \; f_{top} \; b \; h$$

and

$$F_o = F_\infty / \omega$$

The dead load moment M_g or min. M_{g+p} is given by

$$M_g \text{ or min. } M_{g+p} = k_3 \; F_o \; h$$

The max. stress at the lower fiber f_{bot} is given for a T-section by the relation

$$\max.\ f_{bot} = k_4 \; F_o / bh$$

Example of a T-section: Fig VI-18

Given M_g = 40 mt	,	M_p = 35 mt
f_{bot} = 120 kg/cm^2	,	f_{top}=90 kg/cm^2
f_s = 10 t/cm	:	Losses 15%

T-section :

b = 60 cm b' = 20 cm
Compression flange 15 cm

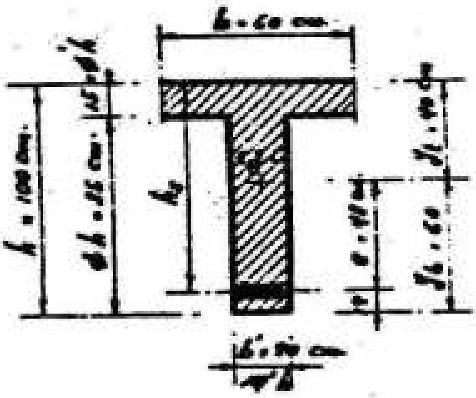


FIG VI-18

Solution

$b' = \psi' b = 20$ cm but $b = 60$ cm
therefore $\psi' = 0.33$ and $\psi = 1 - \psi' = 0.67$
Assuming $\phi' = 0.15$ then

$\phi = 1 - \phi' = 0.85$

Assuming $h_s = 0.9\,h$, then
Fig VI - 12a gives $k_1 = 2.42$,
Fig VI - 13a " $k_2 = 0.26$,
Fig VI - 14a " $k_3 = 0.26$ and
Fig VI - 13b " $k_4 = 0.57$.

As stated before, it is recommended to design T-sections for a reduced f_{top}. It will be chosen 75 kg/cm^2 . Hence : *

$$h = k_1 \sqrt{\frac{M_{max.}}{b\, f_{top}}} = 2.42\sqrt{\frac{75}{0.6 \times 750}} = \underline{1.00\ m}$$

The corresponding prestressing force F_∞ is given by :

$F_\infty = k_2\ f_{top}\ b\ h = 0.26 \times 750 \times 0.6 \times 1 = \underline{117\ ton}$

The losses being 15% then $\omega = 0.85$ and the initial prestressing force F_o is given by

$F_o = F_\infty / \omega = 117/0.85 = \underline{137\ ton}$

The min. dead load moment M_g can be calculated from the relation:
min. $M_g = k_3\ F_o\ h = 0.26 \times 137 \times 1 = 35.6$ mt
This value being smaller than $M_g = 40$ mt, then the c.g.s. may be shifted few centimeters upwards, otherwise, tensile stresses in the upper fiber and high compressive stresses in the lower fiber may be developed. Thus
For $h_s = 0.9\ h$ and min. $M_g = 35.6$ mt , we have

$F_{bot} = k_4\ F_o\ /\ b\ h = 0.57 \times 137/0.6 \times 1 = 1300\ t/m^2 = 130\ kg/cm^2$

If the c.g.s. is located at 12 cms from the bottom, then the real values of the stresses at transfer can be determined from the fundamental principles in the following manner :

$A_c = 20 \times 85 + 15 \times 60 = 1700 + 900 = 2600 \quad cm^2$

$\bar{y}_t = \frac{20 \times 100 \times 50 \times 40 \times 7.5}{2600} = 40 \quad cms$

$y_b = 100 - 40 = 60 \; cms$

$I = \frac{20x100^3}{12} + 20 \times 100 \times 10^2 + \frac{40x15^3}{12} + 40x15x32.5^2 = 2511800 \; cm^4$

$e = 60 - 12 = 48 \; cms$

$f_{bot} = -\frac{F_o}{A_c} - \frac{F_o \; e \; y_b}{I} + \frac{M_g \; y_b}{I} = -\frac{137\;000}{2600} - \frac{137000x48x60}{2511800} + \frac{40x10^5x60}{2511800}$

$= -52.5 - 157.5 + 95.5 = -\underline{114.5} \; kg/cm^2$

$f_{top} = -\frac{F_o}{A_c} + \frac{F_o \; e \; y_t}{I} - \frac{M_g \; y_t}{I} = -\frac{137\;000}{2600} + \frac{137000x48x40}{2511800} - \frac{40x10^5x40}{2511800}$

$= -52.5 + 105 - 63.5 = -\underline{11} \quad kg/cm^2$

The stresses under working loads are given by

$f_{bot} = -\frac{F}{Ac} - \frac{F \; e \; y_b}{I} + \frac{M_{g+p} \; y_b}{I} = -\frac{117\;000}{2600} - \frac{117000x48x60}{2511800} - \frac{75x10^5x40}{2511800}$

$= -45 - 134 + 179 = 0$

$f_{top} = -\frac{F}{A_c} + \frac{F \; e \; y_t}{I} - \frac{M_{b+p} \; y_t}{I} = -\frac{117\;000}{2600} + \frac{117000x48x40}{2511800} - \frac{75x10^5x40}{2511800}$

$= -45 + 89.5 - 119.5 = -75 \; kg/cm^2$

The stresses are shown in figure VI-19

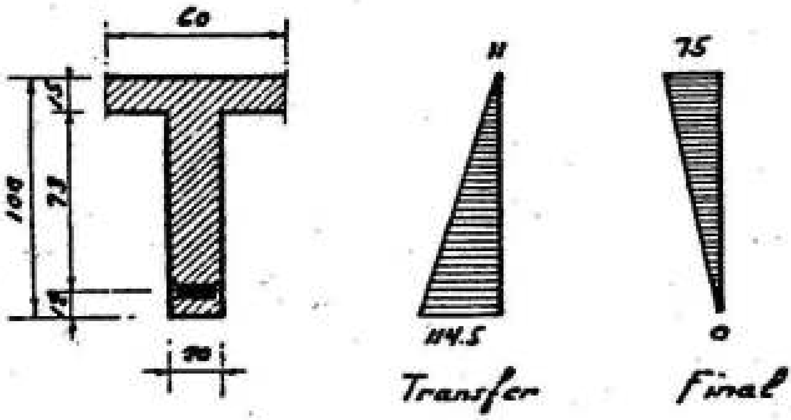


FIG VI-19

Fig. VI-20

Example of a box-section

Given : $M_g = 25$ mt $\qquad M_p = 75$ mt

$f_{bot} = 120 \; kg/cm^2 \qquad f_{top} = 100 \; kg/cm^2$

$f_s = 10 \; t/cm^2 \qquad$ losses 15 %

Box-section :

$b = 100$ cm $\psi b = 70$ cm $\psi = 0.7$

$h \approx 80$ cm $\phi h = 60$ cm $\phi = 0.75$

$h_3 \approx 0.9\ h$ $\omega = 0.85$

For this data, the curves shown in figs. VI-15a, VI-16 and VI-17a give :

$k_1 = 2.57$ $k_2 = 0.238$ $k_3 = 0.158$

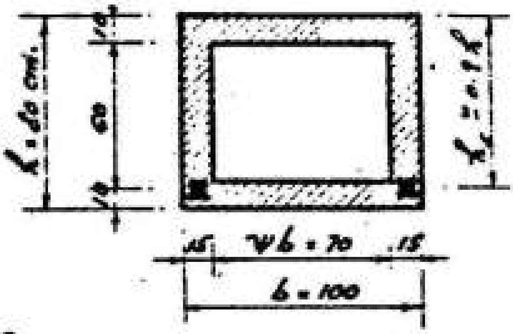


FIG VI-20

Therefore

$$h = k_1 \sqrt{\frac{M_{max}}{b\ f_{top}}} = 2.55 \sqrt{\frac{100}{1 \times 1000}} = \underline{81.5 \text{ cms}}$$

The corresponding prestressing force F_∞ is given by :

$$F_\infty = k_2\ f_{top}\ b\ h = 0.238 \times 1000 \times 1 \times 81.5 = 194 \text{ t}$$

The initial prestress F_o is therefore

$$F_o = F_\infty / \omega = 194/0.85 = 228 \text{ t}$$

Min. dead-load-moment :

min $M_g = k_3\ F_o h = 0.158 \times 228 \times 0.815$ $= 29.4$ mt > 25

<u>Check of Stresses</u>

The section will be chosen 100 cms wide and 80 cms deep, thickness of top & bottom slabs = 10 cms. and thickness of vertical webs 15 cms each. The final prestressing force is chosen 200 tons.

$A_c = 100 \times 10 \times 2 + 60 \times 15 \times 2 \qquad = 3800 \text{ cm}^2$

$I = 100 \times 80^3/12 - 70 \times 60^3/12 \qquad = 30.1 \times 10^5 \text{ cm}^4$

$r^2 = I/A_c = 30.1 \times 10^5 / 3800 \qquad = 793 \text{ cm}^2$

$c_t = c_b = r^2/y = 793 / 40 \qquad = 19.8$ cms

Initial prestressing force $F_o = F_\infty / \omega = 200/0.85 = 235$ t

Location of c.g.s : $e - c_b = M_g / F_o = 25/235 = 10.7$ cms

Therefore : $e = 10.7 + 19.8 = 30.5$ cms

chosen $e = 30$ cms.

Stresses at transfer :

$$f_{bot} = -\frac{F_o}{A_c} - \frac{F_o\, e\, y_b}{I} + \frac{M_g\, y_b}{I} = -\frac{235000}{3800} - \frac{235000 x 30 x 40}{30.1 \times 10^5} + \frac{25 x 10^5 x 40}{30.1 x 10^5}$$

$$= -62 - 93 + 33 = -122 \quad kg/cm^2$$

$$f_{top} = +\frac{F_o}{A_c} + \frac{F_o\, e\, y_t}{I} - \frac{M_g\, y_t}{I}$$

$$= -62 + 93 - 33 = -2 \quad kg/cm^2$$

Stresses under working loads :

$$f_{bot} = -\frac{F}{A_c} - \frac{F\, e\, y_b}{I} + \frac{M_{g+p}\, Y_b}{I} = \frac{200000}{3800} - \frac{200000 x 30 x 40}{30.1 \times 10^5} + \frac{100 x 10^5 x 40}{30.1 x 10^5}$$

$$= -53 - 80 + 133 = 0$$

$$f_{bot} = -\frac{F}{A_c} + \frac{F\, e\, y_t}{I} - \frac{M_{g+p}\, y_t}{I}$$

$$= -53 + 80 - 133 = -106 \quad kg/cm^2$$

The stresses are shown in fig VI-21

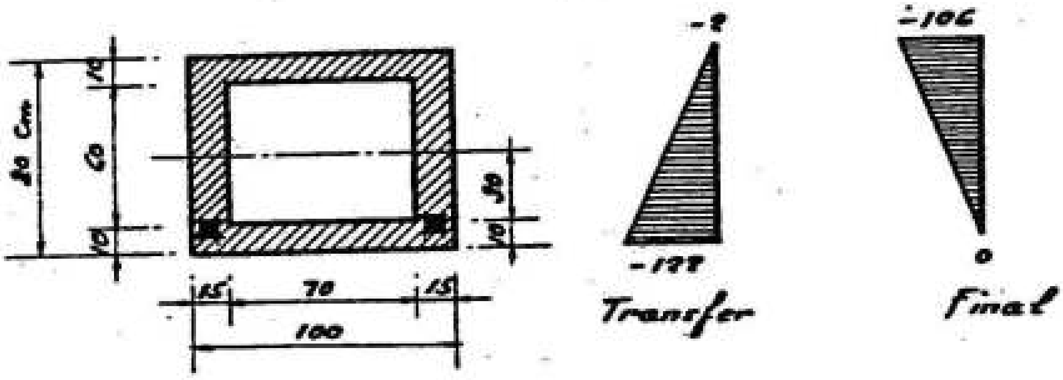


FIG VI-21

VII SHEAR, BOND AND BEARING STRESSES

VII-1 Shear Stresses

The shape of the prestressing cable affects the shearing force Q_c carried by the concrete; for example : (Fig. VII-1)

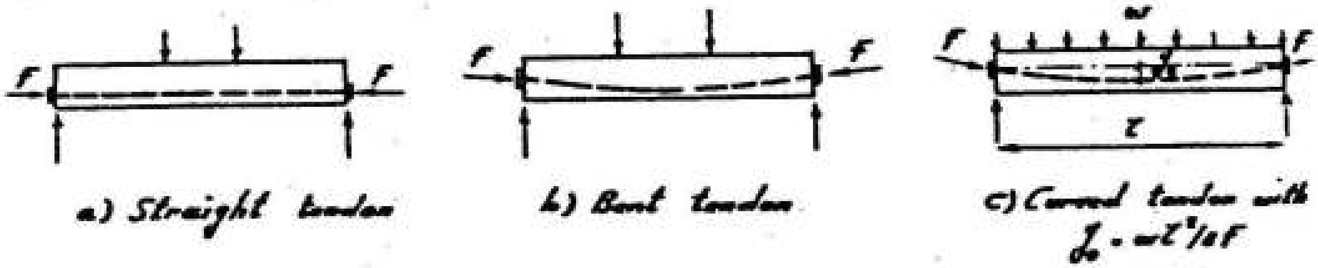


FIG VII-1

case a) The force in the cable has no effect on Q_c.

case b) The shearing force on the concrete Q_c is equal to the external shear Q minus the vertical component of the prestressing force Q_s.

case c) The shear due to w is reduced by the vertical component of the prestressing in the tendon. In prestressed concrete, it is sometimes possible to design a beam with no shear in concrete under a given condition of loading; for example, if the simple beam carrying a uniform load w is prestressed by a parabolic tendon with a sag equal to $y_o = w\,l^2/8\,F$. In this case, the transverse component of the cable equals the shear at any point, and there is no shear to be carried by the concrete.

The conventional design for shear in prestressed concrete beams is based on the computation of the principal tensile stress in the beam and the limitation of that stress to a certain specified value . The principal tensile stress f_1 can be computed if the shear stress q and the normal stress f are known. (Fig. VII-2)

It is known that :

$$q = \frac{Q\,S}{I\,b} \quad , \qquad f = \frac{F}{A} \pm \frac{F\,e\,y}{I} \pm \frac{M\,y}{I} \qquad \text{and}$$

$$f_1 = \sqrt{\frac{f^2}{4} + q^2} - \frac{f}{2}$$

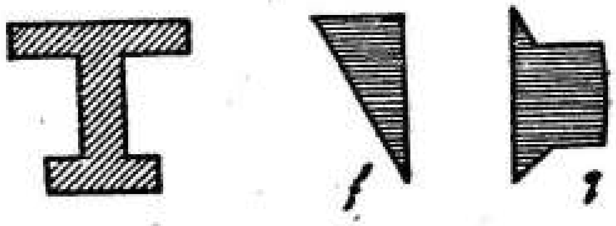


FIG VII-2

The inclination of the plane of the principal tensile stress with the horizontal is further given by

$$\tan 2\alpha = 2q / f$$

Theoretically speaking, if the principal tensile stress in concrete is kept within the allowable limit, no web reinforcement is required. But in practice nominal stirrups ϕ 8 mms at 20 cms or ϕ 10 at 25 cms are most commonly employed. Single stirrups are used for thin webs and double ones for thick webs.

According to the e l a s t i c t h e o r y , the web reinforcements can be calculated as follows :

1) For each section of the beam, compute the maximum principal tensile stress f_1 and the direction of the principal tensile plane making an angle α with the horizontal.

2) For vertical stirrups with spacing e, the force to be taken by each stirrup should be f_1 b e where b is the width of the beam.

3) Since the force supplied by each stirrup with area A_{st} at working stress f_s is $A_{st}\,f_s$, we can equate the resisting and working forces obtaining :

and

$$A_{st}\,f_s = f_1\,b\,e$$

$$e = A_{st}\,f_s / f_1\,b$$

To ensure safety under overloads, u l t i m a t e d e s i g n must be adopted. Because of the many factors involved in the problem , we may follow the following approximate solution : (Fig. VII-3)

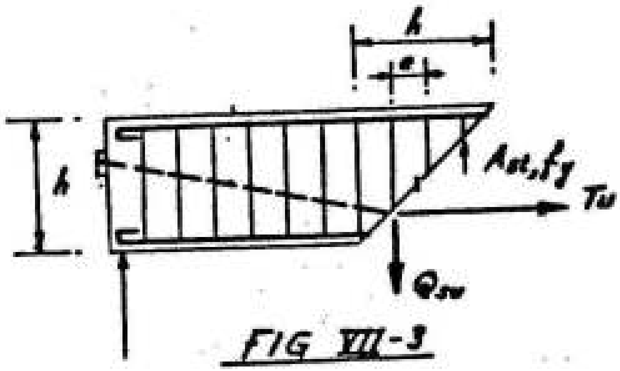


FIG VII-3

1) Assume that the potential shear cracks lie at 45° with the horizontal.

2) Compute the total ultimate shear Q_u at the section, using a proper load factor, say 2 (i.e. ultimate load = twice working load). Estimate the ultimate shear carried by the tendons Q_{su}. The ultimate shear to be carried by the concrete Q_{cu} is then :

$$Q_{cu} = Q_u - Q_{su}$$

3) Assume the shear to be carried entirely by the stirrups across a 45° potential crack, the total number of stirrups is h/ e .

4) Assume the stirrups to be stressed to their yield point f_y ; then the total resistance of intercepted stirrups is given by $h\ A_{st} \times f_y / e$. Equating this to the ultimate shear stress in the concrete, we get :

$$Q_{cu} = h\ A_{st}\ f_y / e \qquad \text{or}$$

$$\underline{e = h\ A_{st}\ f_y / Q_{cu}}$$

Example :

A prestressed concrete beam has a rectangular section 20x120 cms and is subject to a shear of 60 tons under working loads. The effective prestress in the tendons is 120 ton and is inclined 1/6. The fiber stress distribution under working load is 100 kg/cm^2 at top fiber and zero at bottom fiber. Allowable principal tensile stress is 7 kg/cm^2 . 12 mm U normal mild steel stirrups are to be used:

(A_{st} = 2.25 cm^2 , f_s = 1400 kg/cm^2 and f_y = 2400 kg/cm^2)

a) By the elastic design for working load conditions determine the stirrup spacing.

b) By ultimate design method with a load factor of 2, compute the stirrups spacing, assuming that the ultimate stress in the longitudinal steel is 1.8 times its effective prestress.

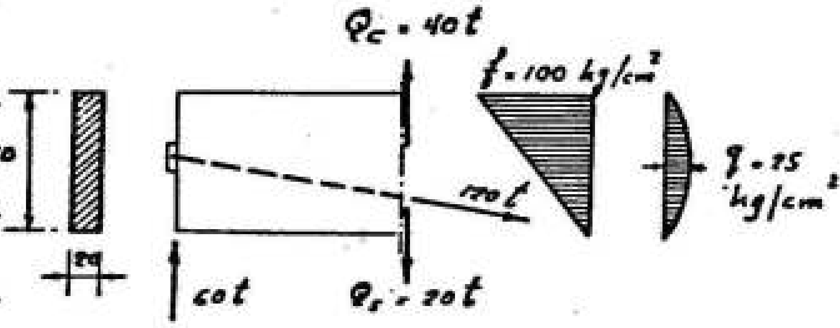


FIG VII-4

Solution : figure VII-4

a) Under working loads

$Q_s = 120/6 = 20$ ton $\qquad Q_c = Q - Q_s = 60 - 20 = 40$ ton

$q \approx 3\,Q_c / 2\,A_c = 3 \times 40\,000/2 \times 20 \times 120 = 25$ kg/cm², $\quad f = 50$ kg/cm²

$$f_1 = \sqrt{\frac{50^2}{4} + 25^2} - \frac{50}{2} = 34.2 - 25 = 9.2 \text{ kg/cm}^2$$

$\tan 2\alpha = 2\,q / f = 2 \times 25 / 50 = 1$, therefore

$2\alpha = 45°$ and $\alpha = 22.5°$ with the horizontal.

The spacing of the stirrups e is given by :

$e = A_{st}\, f_s / f_1\, b = 2.25 \times 1400 / 9.2 \times 20 =$ <u>17.0 cms</u>

b) For ultimate design with a load factor of 2

$Q_u = 2 \times 60 = 120$ t $\qquad Q_{su} = 1.8 \times 20 = 36$ t

$Q_{cu} = 120 - 36 = 84$ t

The spacing of the stirrups e is therefore given by :

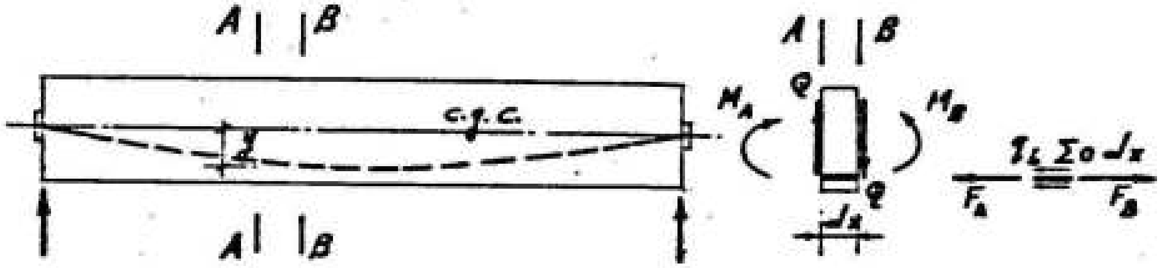
$e = h\, A_{st}\, f_y / Q_{cu} = 120 \times 2.25 \times 2400 / 84\,000 \approx$ <u>8.0 cms</u>

VII-2 Bond Stresses

To determine the bond stresses existing between concrete and tendons, two stages have to be considered : before and after cracking of concrete.

a) Before cracking :

Consider an elementary segment dx of a beam between sections A and B where the shear is Q . Fig. VII-5



It is known that :

FIG VII-5

$$Q\ dx = d\ M = M_A - M_B$$

But $M_B = f_B\ I\ /\ y$ and $M_A = f_A\ I\ /\ y$

Hence

$$Q\ dx = \frac{f_B\ I}{y} - \frac{f_A\ I}{y} = \frac{I}{n\ A_s\ y}\ (\ n\ A_s\ f_B - n\ A_s\ f_A\ )$$

Since $n\ A_s\ f_B\ -\ n\ A_s\ f_A\ =\ F_B - F_A$ then

$$Q\ dx = \frac{I}{n\ A_s\ y}\ (\ F_B - F_A\ )$$

But $F_B - F_A\ =\ q_b \sum O\ dx$ then

$$Q\ dx = \frac{I}{n\ A_s\ y}\ .\ \ q_b \sum O\ dx$$ or

$$q_b\ = Q\ \frac{n\ A_s\ y}{\sum O\ I}$$

For round wires $A_s\ /\ \sum O\ =\ \phi/4$ then

$$\underline{q_b = Q\ n\ y\ \phi\ /\ 4\ I}$$

In the previous equations

ϕ, O = diameter and circumference of wires

$n = E_s\ /\ E_c$ = modular ratio

q_b = bond stress

b) After cracking: The full shear is resisted by bond and we get a relation similar to that of reinforced concrete where

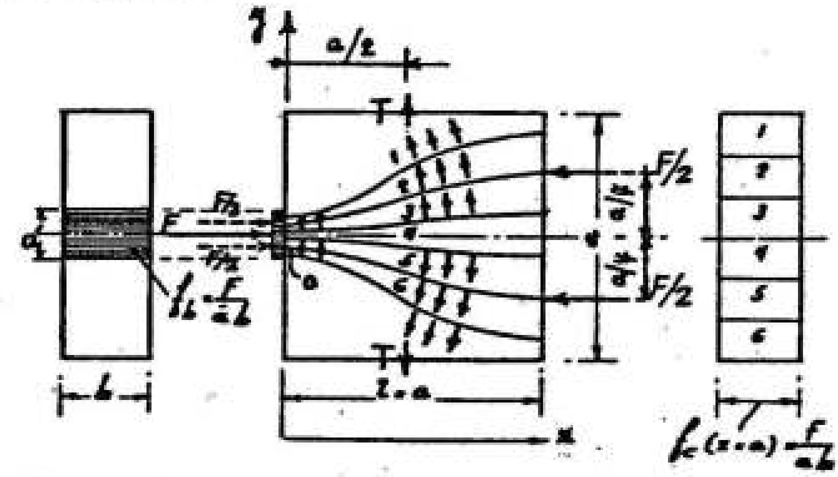
$$q_b = \frac{Q}{y_{CT} \cdot \Sigma o}$$

VII-3 Bearing Stresses due to End Anchorage

The portion of a prestressed member serrounding the anchorages of the tendons in a post-tensioned element is often termed the end block. Throughout the length of this block, prestress is transferred from one or more concentrated areas and distributed through the entire beam section. The theoretical length of the end block is the distance through which the change takes place and is not more than the height of the beam and often is much smaller. The stress distribution will be discussed for the following cases.

1) Single central cable Fig. VII-6

FIG VII-6



The prestressing force F acting on the area a'.b is to be transferred through the end block of length a to the full section of the beam depth a and breadth b. The lines of forces are shown in figure VII-6, convex at bearing and then concave as we move inwards. The change of direction of the forces causes transverse stresses, compressive in the convex region and tensile in the concave part These are generally called splitting tensile forces T; their magnitude can be determined according to Moersch in the following manner:

Assume that the resultant T of the splitting forces acts at the middle of the end block a. Considering the equilibrium of the forces acting on half the block we find that the two longitudinal forces F/2 are equal and opposite but not colinear. In order to keep the equilibrium, the forces T must exist.

Taking moments of the forces acting on the lower half of the block about point O,

we get :

$$\frac{T\,a}{2} = \frac{F}{2}\left(\frac{a - a'}{4}\right)$$

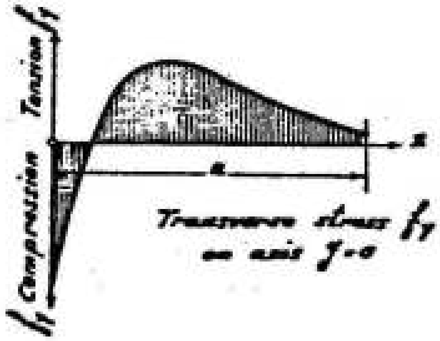


FIG VII-7

so that :

$$T = F\,(a - a')/4a$$

Assuming further a parabolic distribution of the stresses due to T, we get :

$$f_y = 1.5\ T/a\,b$$

The shape of the tensile stresses f_y depends on a'/a as can be seen from the curves of Guyon shown in figure VII-8.

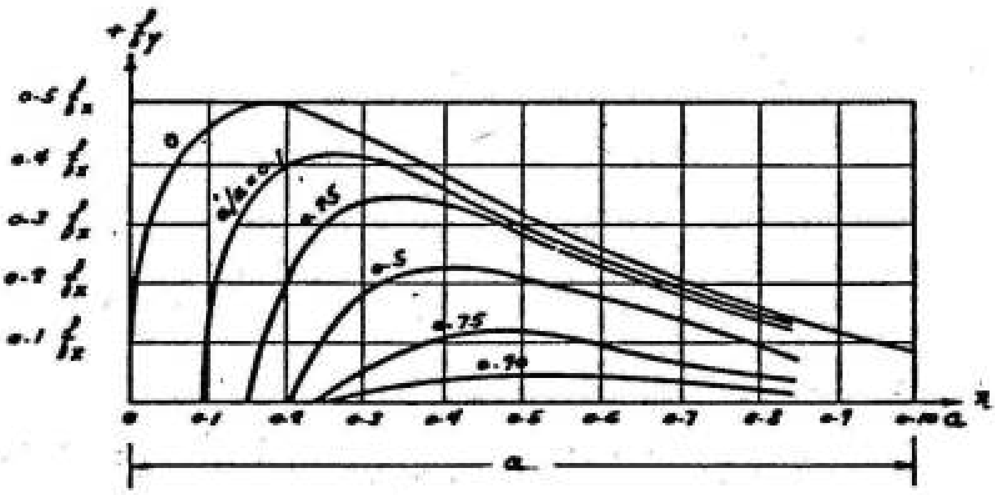


FIG VII-8

For $a'/a = 0$:

$f_y max = 0.50\ f_x = 0.50\ F/a\,b$ and lies at $a/6$ from end.

For $a'/a = 0.50$:

$f_y max = 0.25\ F/a\,b$ and lies at $0.4a$ from end.

If we draw the splitting force as a factor of the prestressing force, we get the values shown in figure VII-9 which gives further the position of the maximum transverse stresses.

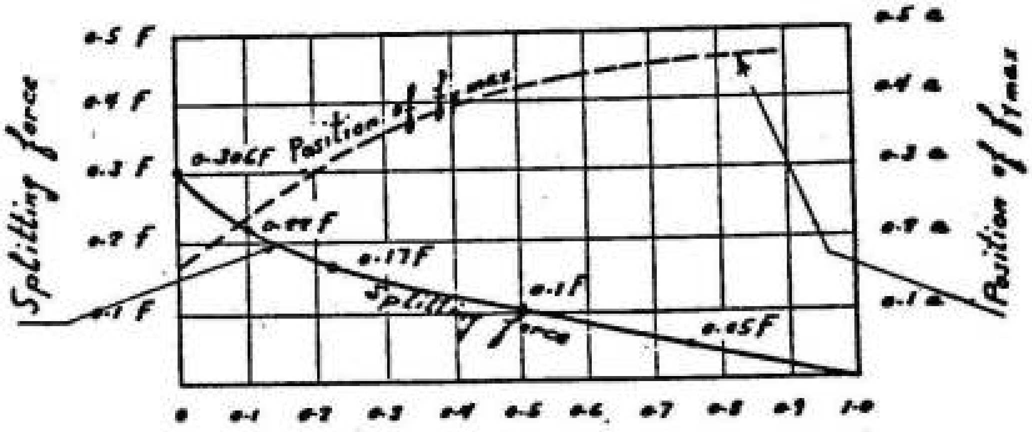
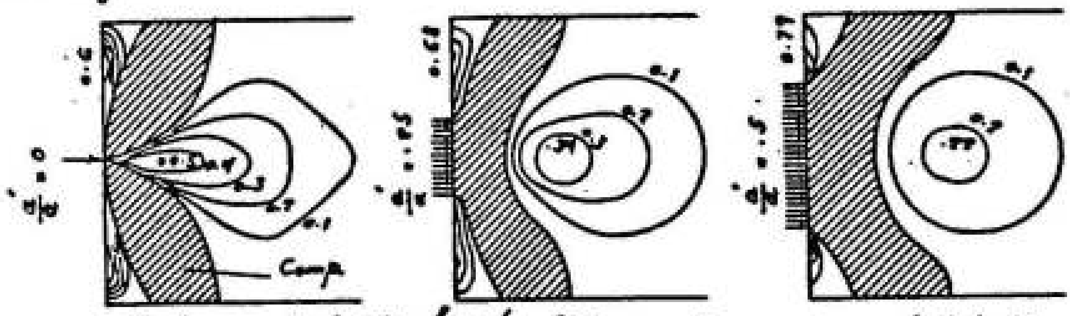


FIG VII-9

Based on photo-elastic studies, Guyon gave the Isobar-lines of equal f_y for different ratios of a'/a as follows. (Fig. VII-10)



Isobars of $f_y/f_x = \frac{f_y \cdot ab}{F}$ (Compression zones are hatched)

FIG VII-10

From these isobars, it can be observed that there are two general areas of tension. One area is in the center of the section ; it has a maximum tension along the line of the load and at some distance from it. Another area is on the sides of the load close to the end surface. This zone is subject to high tensile stresses but only over a small area. Accordingly it is recommended to construct the end block and its reinforcement according to one of the arrangements shown in figures VII-11 & VII-12.

2) Single eccentric cable

If the cable is eccentric, the compressive stresses at the end of the length of transfer will be trapezoidal. Guyon has proved that

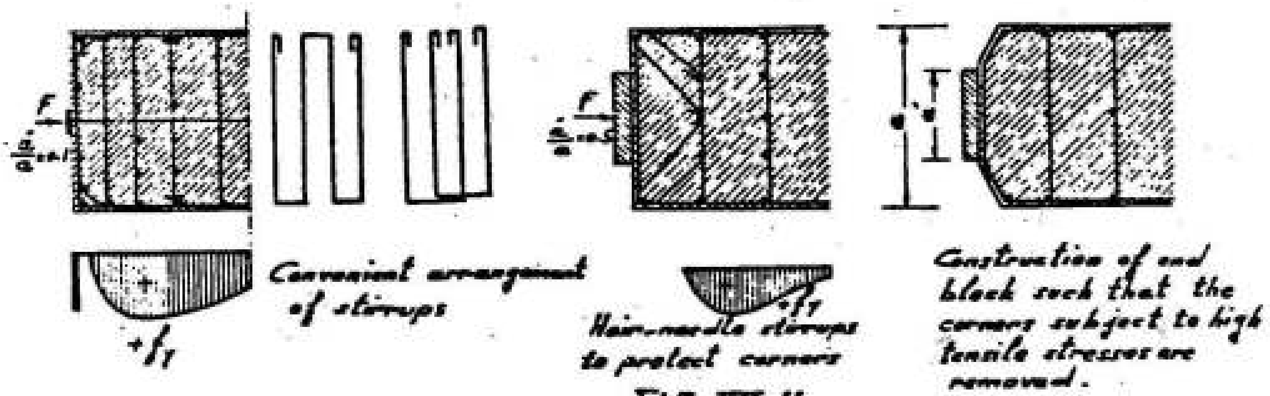


FIG VII-11

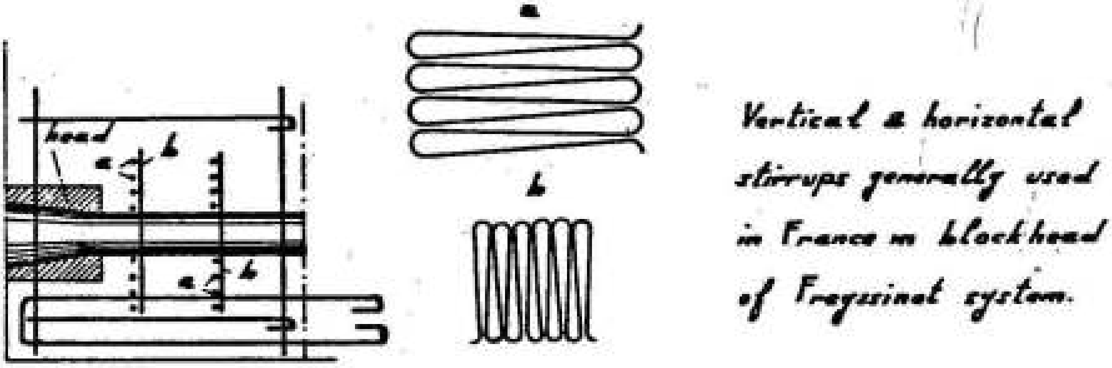


FIG VII-12

it is on the safer side to calculated the splitting force for an axially loaded prism with a x b side dimensions where a/2 is the smaller edge distance. In this case $f_x = F/ab$. (Fig VII-13)

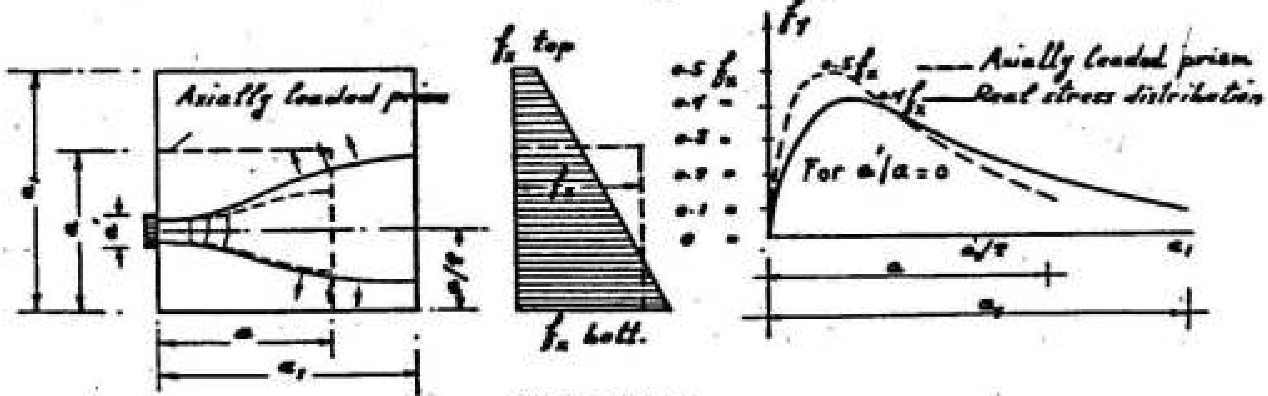


FIG VII-13

3) Series of cables arranged along the breadth of an element

Such a case is shown in figure VII-14 and can be treated in the same way as case 1, only a'/a is to be replaced by b'/b. It has to be noted here that we get tensile splitting forces along the lines of action of F, tensile stresses in the corners and on the surface between the anchor plates and compressive stresses in the zones lying between the outside lines of forces. Cross reinforcements are to be arranged

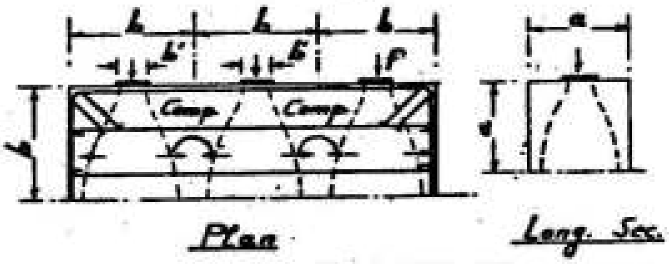


FIG VII-14

throughout the whole breadth of the element as well as hair-needle stirrups to protect the corners as shown in figure VII-14.

4) Series of cables arranged above each other

It might happen that the prestressing forces act inside or outside the c.g. axis of the equally divided areas of the f_x - diagram as shown in figure VII-15.

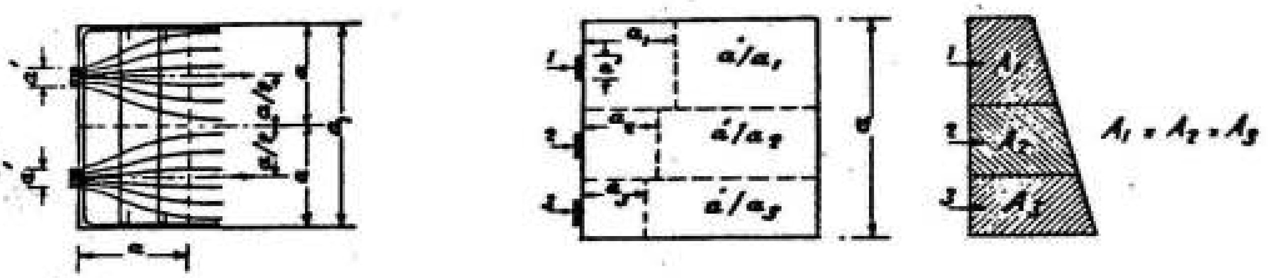


FIG VII-15

In case a where two cables are symmetrically arranged, the splitting force can be determined for prism $a = a_1/2$.

In case b where three cables are unsymmetrically arranged, divide the stress diagram into three equal areas putting each cable in the center of gravity of the corresponding area. In this manner, the splitting forces will be variable according to a'/a_1 , a'/a_2... It is recommended to arrange the maximum reinforcement obtained for a'/a_1 through the whole height.

In such cases, we might have different tension zones as can be seen from the following two examples :

Example 1 :

The direct splitting force behind each anchor plate of the case shown in figure VII-16 is to be calculated for the small prism with side length a/4 . Bigger tensile forces take place outside these prisms and can be calculated for the bigger prism a and a' = 3a / 8 .

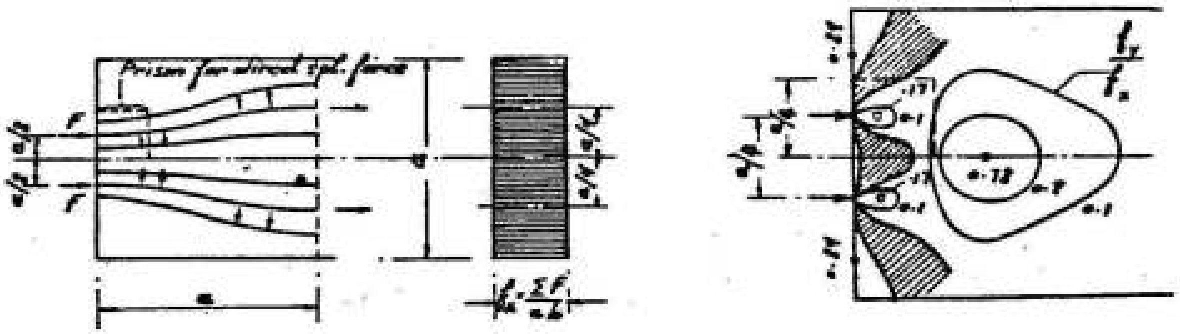


FIG VII-16

The magnitude and position of the splitting force can be given as a factor of the prestressing forces Σ F from the curves given in figure VII-9.

Example 2 :

In the case shown in figure VII-17, high tensile stresses at the outside surface are created.

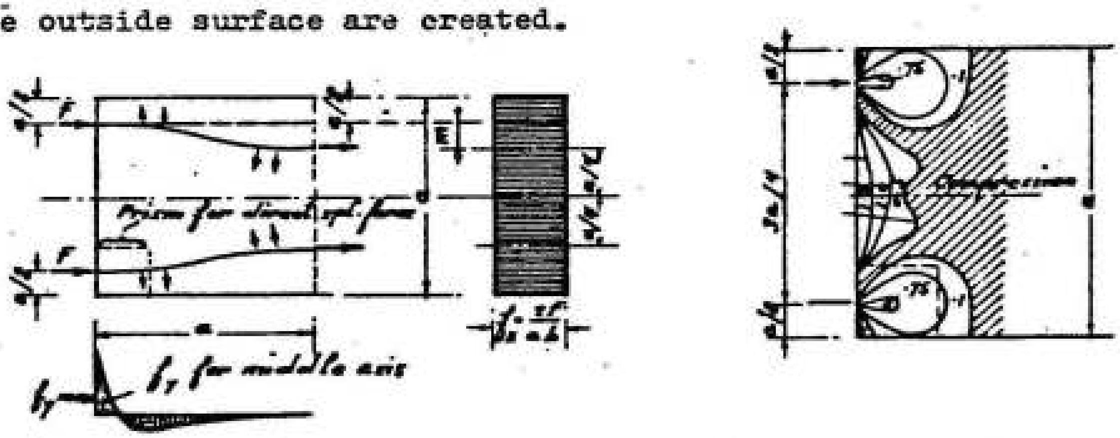


FIG VII-17

The maximum tensile stress can be calculated from the relation

$$\underline{\text{max. } f_y = 32\ F\ m\ /\ b\ a^2}$$

It has been further proved by photo-elastic tests that an inclination of a cable < 1/10 does not affect the magnitude and position of the splitting force.

VIII - DEFLECTION OF PRESTRESSED SIMPLE BEAMS

Deflections of prestressed beams differ from those of ordinary reinforced concrete beams in the effect of prestress. Deflection due to prestress can be computed by 2 methods :

The first method is to take the concrete as a free body, seperated from the tendons, which are replaced by a system of forces acting on the concrete. The component of the prestress along the beam axis is assumed constant, the component transverse to the beam is computed by the prestress times the tangent of the angle of bending. Where the tendons bend suddenly, the transverse componants may be assumed to be concentrated, where they form a flat curve, the transverse load may be assumed to be uniformly distributed along the bend.

In the second method, a moment diagram produced by the tendons is directly drawn from the c.g.s. profile. For statically determinate beams the moment diagram is similar to the eccentricity profile of the c.g.s. line, hence it is only necessary to plot the eccentricity profile to another scale to obtain the moment diagram. Then the computation of deflections from the moment diagram is performed by any method given in elementary strength of materials.

Example :

A concrete beam of 10 m span is post-tensioned with 7,2 cm^2 of high tensile steel to an initial prestress of 10 t/cm^2 immediately after prestressing. Compute the initial deflection at midspan due to prestress and the beam's own weight of 288 kg/m', assuming: E_c = 300 000 kg/cm^2 . Estimate the deflection after 3 months, assuming a creep coefficient of C_{cr} = 1.8 and an effective prestress of 8.5 t/cm^2 at that time due to a concentrated load of 5 tons acting at mid-span. (Fig VIII-1)

Solution :

Using the first method, take the concrete as a free body and replace the tendon by forces acting on the concrete. The parabolic tendon with 20 cms mid-ordinate is replaced by a uniform load acting upward along the beam and can be calculated from the relation :

$$F = w_F l^2 / 8\, y_o \qquad \text{or} \qquad w_F = 8\, F\, y_o / l^2$$

But $F = 7.2 \times 10 = 72$ tons and $y_o = 20$ cms.

Therefore

$$w_F = 8 \times 7200 \times 0.2 / 10^2 = 1152 \text{ kg/m' upwards.}$$

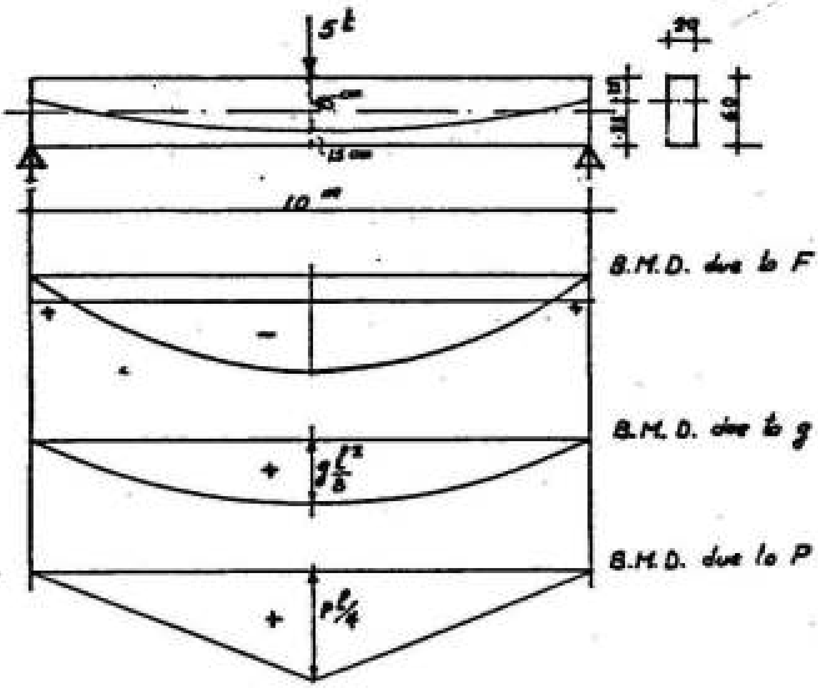


FIG VIII-1

In addition, there will be two eccentric loads acting at the ends of the beam, each producing a positive moment M_F given by :

$$M_F = +\ 72\,000 \times 0.05 = 3600 \text{ kgm}$$

Since the weight of the beam is 288 kg/m', the net uniform load on concrete is :

$$w = 1152 - 288 = 864 \text{ kg/m'}$$

which produces an upward deflection given by :

$\Delta = 5\, w\, l^4 / 384\, E\, I$ where $E = 30 \times 10^4$ kg/cm^2 and $I = 36 \times 10^4$ cm^4

Hence

$$\Delta = 5 \times 864 \times 10^{-2} \times 1000^4 / 384 \times 30 \times 10^4 \times 36 \times 10^4 = 1.04 \text{ cms}$$

The end moments produce a downward deflection given by :

$$\Delta = M\, l^2 / 8\, E\, I = 3600\ 00 \times 1000^2 / 8 \times 30 \times 10^4 \times 36 \times 10^4 = 0.416 \text{ cms}$$

i.e. the net deflection due to prestress and own weight is given by:
net Δ = 10.4 - 4.16 = 6.24 mms. upwards !

If we follow the second method, we draw the moment diagram from the eccentricity curve of the tendon and the deflection computed therefrom. For convenience in computation, the moment diagram can be divided into two parts, a parabola and a rectangle. By area moment principles or any similar method the upward deflection due to prestress can be computed, e.g.

$$\Delta = \frac{5\, M_m\, l^2}{48\, E\, I} - \frac{M_F\, l^2}{8\, E\, I}$$ in which

the first term gives the deflection due to a parabolic moment with maximum ordinate at the middle equal to M_m , and the second term gives the deflection due to a uniform moment M_F .

$M_m = 72\,000 \times 20 = 144 \times 10^4$ kg cm

$M_F = 72\,000 \times 5 = 36 \times 10^4$ kg cm

The deflection due to the prestress is therefore given by

$$\Delta_F = \frac{5 \times 144 \times 10^4 \times 1000^2}{48 \times 30 \times 10^4 \times 36 \times 10^4} - \frac{36 \times 10^4 \times 1000^2}{8 \times 30 \times 10^4 \times 36 \times 10^4} =$$

$$1.386 - 0.416 = 0.97 \text{ cms}$$

Downward deflection due to g = 288 kg/m'

$$\Delta_g = \frac{5\, g\, l^4}{384\, E\, I} = \frac{5 \times 288 \times 10^{-2} \times 1000^4}{384 \times 30 \times 10^4 \times 36 \times 10^4} = 0.346 \text{ cms}$$

So that the net upward deflection is

net Δ = 9.70 - 3.46 = <u>6.24 mms</u> same as before

While the above gives the initial deflection, the eventual deflection should be modified by 2 factors : first the loss of prestress, which tends to decrease the deflection and second the creep effect which tends to increase the deflection. Since the prestress is reduced from 10 to 8.5 t/m^2 the deflection due to prestress can be modified by the factor 8.5/10. Then for the creep effect, the net deflection should be increased by the coefficient 1.8. Thus if the beam is not subject to external loads the eventual deflection after 3 months can be estimated as

$$\Delta = \left(9.7 \times \frac{8.5}{10} - 3.46 \right) 1.8 = 8.6 \text{ mm} \quad \text{upwards.}$$

The calculation for deflections due to external loads is similar to that for non-prestressed beams. So long as the concrete has not cracked, the beam can be treated as a homogenious body and the usual elastic theory applied to it for deflection computations e.g. the deflection due to a concenterated load of 5t applied after 3 months is given by :

$$\Delta = \frac{P\,l^3}{48\,E\,I} = \frac{5000 \times 1000^3}{48 \times 30 \times 10^4 \times 36 \times 10^4} = 0.967 \text{ cms} \quad \text{downwards}$$

i.e. after 3 months, the net deflection will be

$$\Delta = 9.67 - 8.6 = 1.07 \text{ mm} \quad \text{downwards!}$$

IX - LAYOUT OF TENDONS IN STATICALLY DETERMINATE BEAMS

IX-1 Simple Beam Layout

The layout of a simple prestressed concrete beam is controlled by 2 critical sections: the maximum moment and the end sections. The maximum moment section is controlled by 2 loading stages, the initial stage at transfer with maximum prestressing force F_o and minimum moment M_g acting on the beam and the working load stage with minimum prestressing force F and maximum design moment M. The end sections are controlled by the area required for shear resistance, bearing plates, anchorage spacing, and jacking clearances. Convenient layouts for pre and post-tensioned beams are shown in the (Fig. IX-1).

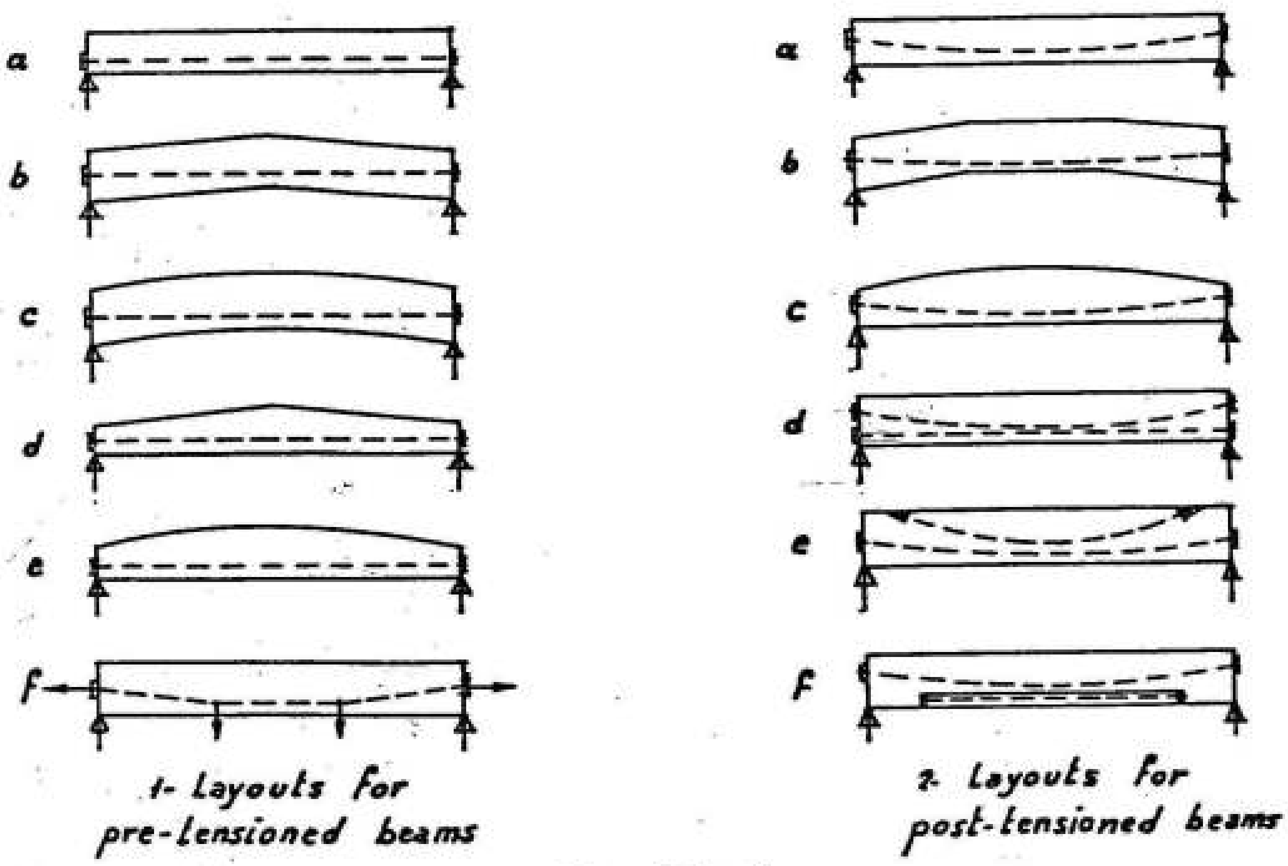


FIG IX-1

N o t e s :

. In pretensioned beams, straight cables are preferred, since they can be easily tensioned between two abutments. However in some modern pre-tensioning plants anchors along the prestressing beds have been burried so that the tendons for a pretensioned beam can be bent (system f).

. Most of the layouts for pretensioned beams can be used for post - tensioned as well. But for post-tensioned ones, it is not necessary to keep the tendons straight, since slightly bent or curved tendons can be easily tensioned as straight ones. Curving the tendons will permit favorable positions of c.g.s. to be obtained at both the end and midspan sections and other points as well.

IX-2 Cable Profiles (Fig. IX-2)

Location of limiting zone for c.g.s.

$a_1 = M/F$

$a_2 = M_g/F_o$

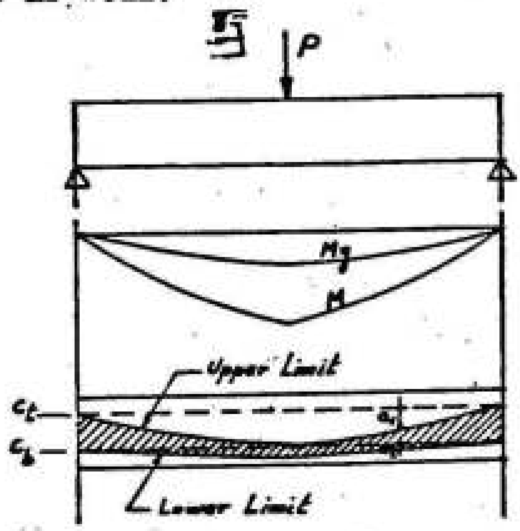
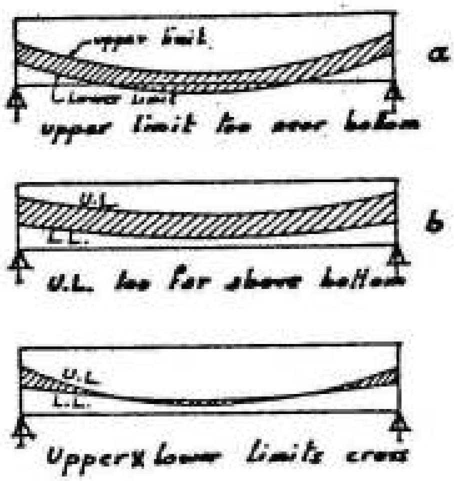


FIG. IX-2

The shown graphical method gives the limiting zone within which the c.g.s. must pass in order that no tensile stresses will be produced. In order that, under the working load, the center of pressure, the C-line, will not fall above the core line, it is evident that the c.g.s. must be located below the top core at least a distance : $a_1 = M / F$. If the c.g.s. falls above the upper limit at any point, then the C-line corresponding to maximum moment M and prestress F will fall above the top core, resulting in tension in the bottom fiber.

Similarly in order that the C-line will not fall below the bottom core line, the c.g.s. line must not fall below the bottom core by a distance greater than : $a_2 = M_g / F_o$ which gives the lower limit for the location of the c.g.s. If the c.g.s.

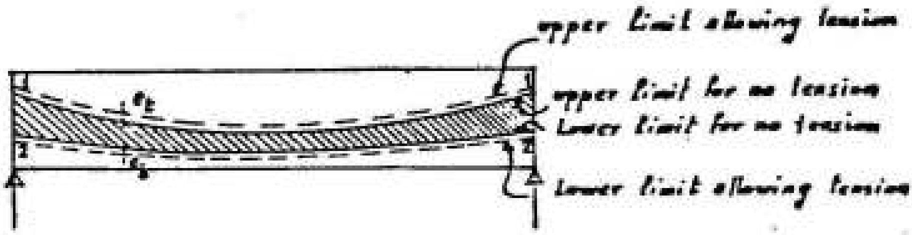
lies above that lower limit, it is seen that the C-line will be above the bottom core and there will be no tension in the top fiber under the girder load and initial stress F_o . Thus, it becomes clear that the limiting zone for c.g.s. is given by the shaded area in fig.IX-2, in order that no tension will exist both under the girder load and under the working load. The individual tendons, however, may be placed in any position so long as the c.g.s. of all the cables remains within the limiting zone.



Undesirable positions for c.g.s. zone limits.

FIG IX-3

The position and width of the limiting zone are often an indication of the adequacy and economy of design (FigIX-3) If some portion of the upper limit (U.L.) falls outside or too near the bottom fiber, in (a) either the prestress F or the depth of beam at that portion should be increased. On the other hand, if it falls too far above the bottom fiber, in (b), either the prestress or the beam depth can be reduced. If the lower limit crosses the upper limit, in (c), it means that no zone is available for the location of c.g.s. and either the prestress F or the beam depth must be increased or the girder moment must be increased to depress the lower limit if that can be done. The location of c.g.s. as described above is based on the elastic theory allowing no tension both at transfer and under the working load. If some tension is permitted, then it is possible to place the c.g.s. line slightly outside the previous limiting zone (Fig. IX-4)



Limiting zone for c.g.s. allowing tension in concrete.

FIG IX-4

For an allowable tensile stress f_t in the top fibers at transfer, we have:

$$f_t = \frac{M\, y_t}{I} = \frac{F_o\, e_b\, y_t}{I}$$

where e_b = the amount c.g.s. may fall below the lower limit.

For an allowable tensile stress f_t in the bottom fibers under the working load we have:

$$f_t = \frac{F\, e_t\, y_b}{I}$$

where e_t = the amount c.g.s. may fall above the upper limit.

Hence, we get:

$$e_b = \frac{f_t\, I}{F_o\, y_t} = \frac{f_t\, A\, c_b}{F_o}$$

and

$$e_t = \frac{f_t\, I}{F\, y_b} = \frac{f_t\, A\, c_t}{F}$$

Hence, the limiting zones for no tensile stresses can be extended to lines 1-1 and 2-2 if some tensile stresses are permitted.

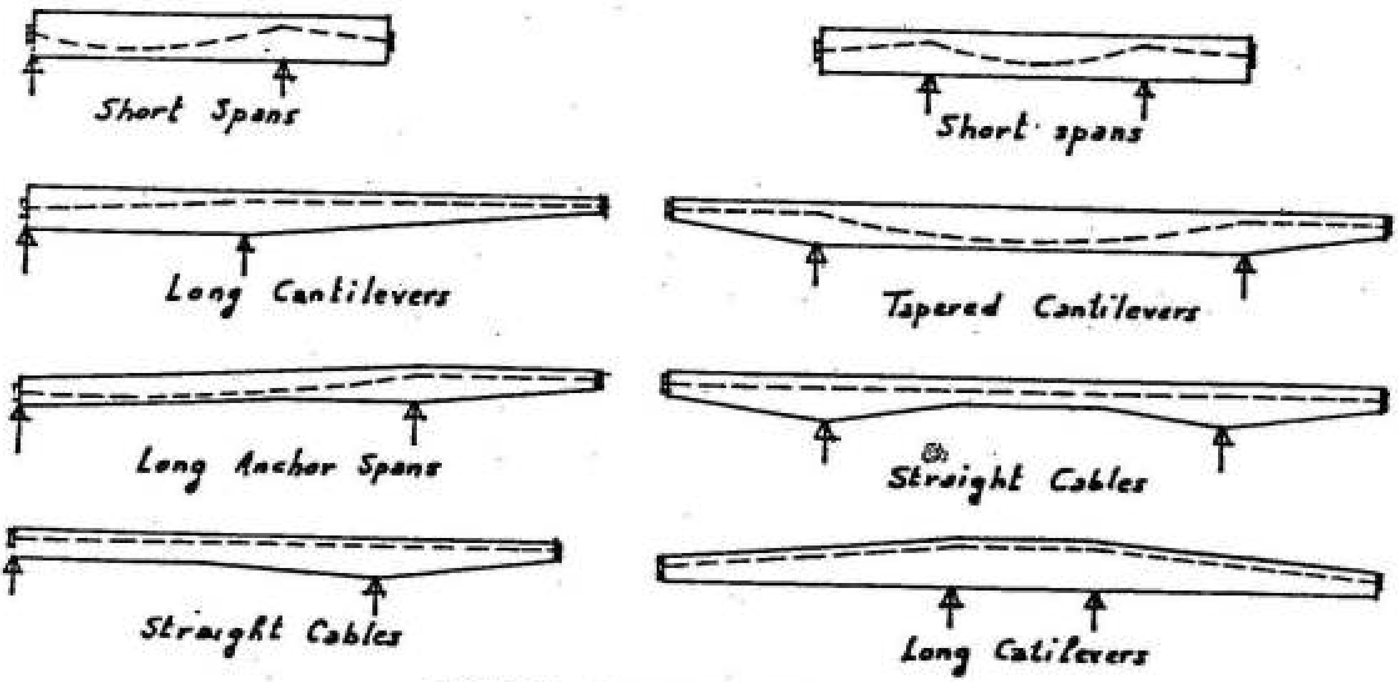
IX-3 Cantilever Beam Layouts

Two general layouts are possible for cantilevers: The single and the double cantilevers. Some typical layouts are shown in the following figures (Fig. IX-5).

Cable location for cantilevers can be obtained graphically as for simple beams, except that more thought should be given to the possibilities of partial live loads and the reversal of moments. The following figure shows a cantilever beam subject to dead load g and live load p. The bending moments due to dead load M_g, live load M_p and maximum or minimum moments M are shown. For convenience in discussion, maximum moment will signify the greatest positive or the smallest negative moment, while minimum moment will mean the smallest positive or the greatest negative moment.

In order to get the limiting zone for the c.g.s. line (Fig. IX-6) first plot the top and bottom core lines for the beam c_t and c_b - lines. If no tension is permitted in the concrete, one limiting line is obtained by plotting from each core line the permissible eccentricity e, with e = M / F. Note that e may be plotted from either the c_t or the c_b line whichever gives the more critical limit.

But e due to +M is always plotted downwards, since it tends to shift the c.g.s. line downwards. By similar reasoning, e due to -M is always plotted upward. In general the upper limit for the zone is plotted from the c_t line with a distance $e_1 = M_{max} / F$.



TYPICAL LAYOUTS FOR

SINGLE CANTILEVERS DOUBLE CATILEVERS

FIG IX-5

The lower limit for the zone is plotted from the c_b line with a distance $e_2 = M_{min} / F$. Consideration should also be given to the action of dead load alone, since in this case, we may have the initial prestress which is greater than the effective prestress and may impose a more critical situation. With the dead load acting alone another limit is obtained by plotting from the c_t line a distance $e_1' = M_g / F_o$

Again plotting the +M downward and the -M upward. In this figure it is not necessary to plot e_1' from the c_b line, because evidently it will not be controlling. When plotted from the c_t line, it is seen that, for certain portions of the beam, e_1' will be controlling rather than e_1. The resulting limiting zone is shown shaded.

For long cantilevers carrying heavy loads, it is sometimes economical to cut off some of the prestressing wires at intermediate points. The number and location of cut-offs can also be established

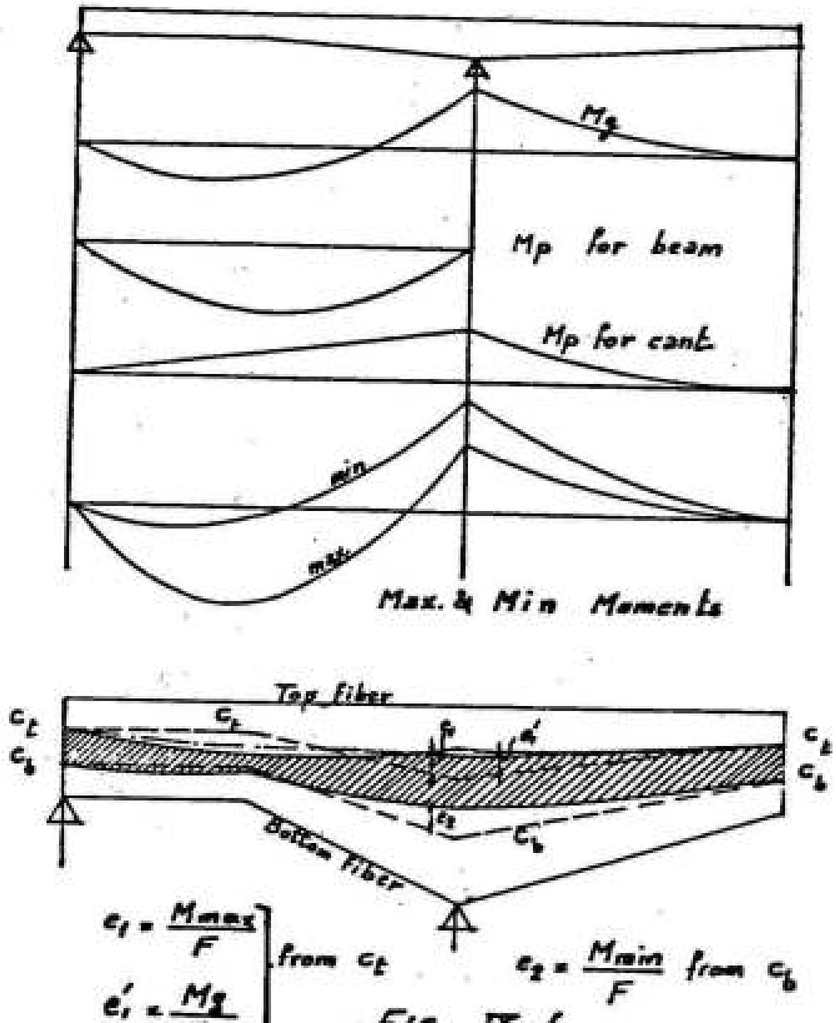


FIG. IX-6

by a graphical method, which is the reverse of the above procedure.

X PRESTRESSED TENSION AND COMPRESSION MEMBERS[a]

X-1 Elastic Design of Tension Members

Prestressed tension members combine the strength of high tensile steel with the rigidity of concrete and provide a unique resistance to tension consistent with small deformations that cannot be obtained by either steel or concrete acting alone. The rigidity of prestressed concrete serves well, especially for long tension members such as ties of arches. When prestressed concrete is given strength to resist any local bending and at the same time steel is stiffened and protected.

A prestressed tension member can be considered as a combined steel and concrete member whose strains and stresses before cracking can be evaluated, assuming elastic behaviour and taking into account the effect of plastic flow at the same time.

Assuming total initial prestress F_o and the total effective prestress F, then the stress in the concrete will be:

$$f_c = F_o / A_{co}$$

for the initial prestress and $f_c = F / A_{co}$ for the effective prestress in which A_{co} = the net concrete section.

For a load P applied externally, both concrete and steel will undergo the same strain. Hence the usual virtual area method as applied to reinforced concrete can be applied here.

Virtual area of the section $A_v = A_{co} + n\, A_s$ X-1

If the gross area of concrete A_c is used, the virtual area is given by:

$$A_v = A_c + (n - 1)A_s \qquad \text{X-2}$$

This formula is valid only when the section is grouted, otherwise use equation X-1.

The stress due to P:

for concrete $f_c = P / A_v$

and for steel $f_s = n\, P/A_v$

---

a) Lin " Design of prestressed concrete structures.

Thus the resultant stress due to the effective prestress plus the external load are :

for concrete $f_c = F/A_{co} + P/A_v$ X-3

and for steel $f_s = f_s + n\,P/A_v$ X-4

If it is desired to determine the load P producing zero stress in concrete, it is only necessary to put $f_c = 0$ in equation X-3, thus

$$F/A_{co} + P/A_v = 0 \quad \text{or}$$

$$P = -F\,A_v/A_{co} = -F\,(A_{co} + n\,A_s)A_{co} \quad \text{or}$$

$$P = -F\,(1 + n\mu) \qquad \text{X-5}$$

It can be easily seen that with no stress in the concrete, the load carried by the member is somewhat greater than the prestress F.

It is most important to investigate the strains in a prestressed concrete member, both those due to prestressing and those due to external loads. Under the initial prestress F_o, the stress in the concrete being F_o/A_{co}, the corresponding instantaneous unit strain will be

$$\varepsilon_c = F_o / E\,A_{co}$$

which will reduce to $\varepsilon_c = F / E\,A_v$ after the losses have taken place.

Under the action of external load P, the instantaneous strain is given by $\varepsilon_c = P / E\,A_v$

In all cases, the value of E must be chosen with regard to the level of stress and the age of concrete, and the effect of creep must be considered. Let us first compare the magnitude of strains in a prestressed concrete member with those in an ordinary steel member. For a structural steel member stressed to 1400 kg/cm^2 corresponding to a value of E_s = 2100 000 kg/cm^2, the unit elongation is :

$$\varepsilon_c = f_s/E_s = 1400/2100\,000 = 0.67/1000 = 0.67 \text{ mm/m}$$

For a prestressed concrete member, with the stress in concrete changing between 90 kg/cm^2 and 0, for an E_c = 300 000 kg/cm^2, the unit strain is :

$$\varepsilon_c = f_c/E_c = 90/300\,000 = 0.3/1000 = 0.3 \text{ mm/m}$$

which is less than half of the strain in structural steel.

High tensile steel alone cannot be used for long tension members where elongation must be limited. In order to be stressed to its working stress of 8500 kg/cm^2 , the unit elongation will be

$$\varepsilon_c = f_s/E_s = 8500/2100\ 000 = 4/1000 = 4\ mm/m$$

which is about 6 times that of structural steel and 13 times that of prestressed concrete in the above example.

Strains in prestressed concrete members are influenced by several factors. If the precompression in the concrete remains over a period of time, the shortening of that member due to creep could be considerable. Such creep strain, however, would be gradually recovered under the application of an external tension . Since a greater portion of the creep may be eventually recovered, the lengthening of the member under sustained external load may be greater than is indicated by the elastic calculations.

Example :

A straight concrete member 50 ms. long is prestressed with a high tensile steel cable through the centroid of the section. The cable is anchored to the concrete with end anchorages but separated from it by bond breaking agents along its length. A_{co} = 600 cm^2, A_s = 6 cm^2 , f_{cp} = 240 kg/cm^2, steel 16.5/14

Initial prestress f_{so} = 10 t/cm^2 final prestress $f_{s\infty}$ = 8.5 t/cm^2
E_c = 300 000 kg/cm^2 , E_s = 2100 000 kg/cm^2 .

a) Compute the allowable external load on the member, allowing no tension in concrete.
b) Compute the shortening of concrete due to prestress, assuming a creep coefficient of 2,-.
c) Compute the lengthening of the member due to the external load obtained in a neglecting creep.
d) If the member were designed of structural steel with an allowable stress of 1400 kg/cm^2 compute the lengthening under the load.
e) Compute the lengthening if the cable is used alone by itself with an allowable stress of 8.5 t/cm^2.

Solution :

a) From formula X-5
$P = F\ (1 + n\mu) = 8.5 \times 6.0\ (1 + 2100/300 \times 6/600) = 54.6$ t
b) Under the initial prestress, the shortening of the concrete will be :

$\Delta l = F_o\, l / E_c\, A_{co} = 10 \times 6 \times 50\ 00/300 \times 600 = 1.67$ cms.

If the effective prestress is considered, the shortening will be

$\Delta l_\infty = \Delta l \cdot f_{s\infty}/f_{so} = 1.67 \times .85 = 1.42$ cms

· If the creep coefficient is based on the effective prestress, the total elastic and creep shortening will be :

final $\Delta l = 1.42 \times 2 = 2.84$ cms

c) Under the external load of 54.6 tons, for a virtual area $A_v = A_{co} + n\, A_s = 600+7x6 = 642\ cm^2$, again using $E_c = 300\ 000\ kg/cm^2$ the lengthening of the member will be :

$\Delta l = P\, l / E_c\, A_v = 54.6 \times 50\ 00/300 \times 642 = 1.42$ cms

which checks closely with the shortening of the concrete computed in b.

d) For a structural steel stressed to 1400 kg/cm^2, the elongation will be

$\Delta l = f_s\, l / E_s = 1.4 \times 50\ 00/2100 = 3.33$ cms

e) For high tensile steel stressed to 8.5 t/cm^2, the elongation will be

$\Delta l = f_{s\infty}\, l / E_s = 8.5 \times 50\ 00/2100 \approx 20$ cms

X-2 Cracking and Ultimate Strength of Tension Members

The previous section discusses the computation of stresses in a prestressed concrete tension member, up to zero compression in the concrete. The design of such a member may or may not be made on this basis, depending upon the probable amount of overloading to which the member may be subjected. In order to get a sufficient factor of safety, it may be necessary to design the member so that, under working loads, there will always be some residual compression in the concrete. This will become evident after a study of the cracking and ultimate strengths of the member. Tension members are one of the typical cases in prestressed concrete where design by the allowable stress method may be very much on the dangerous side and may not yield consistent results.

Generally speaking, prestressed concrete tension members have a very low reserve strength above the point of zero stress. If the member is not cast in one piece, e.g. if it is made up of blocks, cracking may coincide with zero stress. Then any additional load on the member will be carried by the steel alone. Since the prestressing steel has a relatively small area of cross-section, excessive elongation will immediately start at the cracking of concrete, and failure of other parts of the structure may result. For such a member, then, it is evident that a considerable amount of residual compression is necessary in order to ensure safety, the amount being governed by the magnitude of the probable overloads.

If the member is cast in one piece and if shrinkage and other cracks have not occured, it will be able to take some tension before cracking. The direct tensile strength of concrete is variable and generally ranges from 1/16 to 1/12 f_{c28} . Thus for a concrete of 300 kg/cm^2 the tensile strength may be from 20 to 25 kg/cm^2, which may provide a good margin of safety if the strength exists and has not been destroyed. But, once the concrete has cracked, the margin of safety is gone. In fact, failure of the entire structure may result as soon as the concrete cracks, because at this moment the tensile load carried by the concrete in tension is suddenly transferred to the steel. Thus there may be a sudden elongation of steel which may have serious effects,.even though the ultimate strength of the steel is far from being reached.

The above discussion must not mean that such tension members are unsafe. They are just as safe as any other type of tension members and perhaps safer if properly designed. When heavy overloads are possible, they should not be designed on the basis of allowable stresses but rather on the basis of the cracking or ultimate strength with proper load factors. The choice of suitable load factors depends upon the possibilities of overloading.

<u>Example</u> :

For the tension member of the previous example, what working load can it carry, using a factor of safety of 1.8 against the cracking of concrete, assuming the direct tensile strength in concrete to be 20 kg/cm^2 . Compute the residual compression in concrete under that load.

For $f_c = f_t' = 20\ kg/cm^2$, equation X-3 gives

$$F/A_{co} + P/A_v = 20$$ in which

$F = 54.6$ tons $\quad A_{co} = 600\ cm^2 \quad A_v = 642\ cm^2$ then

$- 54600/600 + P/642 = 20$ or $P = 71.3$ tons which is cracking load. For a factor of safety of 1.8, the working load will be 71.3/1.8 = 39.60 tons.

Though the load factor of 1.8 is not always necessary, the great difference between this answer and the last one of 54.6 t should be noticed. The residual compression can be computed using the same formula for P = 39.60 tons.

$$f_c = F/A_{co} + P/A_v = -8500 \times 6/600 + 39600/642 = -85+63.5 = 21.5 kg/cm^2$$

X-3 Column Action due to Prestress

The question is often brought up whether a concrete member under prestress will have a tendency to buckle like an ordinary column under compression. The answer is that, if the prestressing element is in direct contact with concrete all along its length, there will be no "column action" in the member due to prestress.

Consider an ordinary column under an external load fig. (X-1a) When the column deflects, additional moment in section A-A is created by the deflection, because the external load now acts with a different eccentricity on that section. This additional moment is the cause of column action. Now consider a member internally prestressed but not externally loaded (b); so long as the steel and concrete deflect together, there is no change in the eccentricity of the prestress on the concrete, no matter how the member is deflected. Hence there is no change in moment due to any deflection of the member and no column action. When an external load is applied to a prestressed concrete column, any deflection of the column will change the moment, and column action will result.

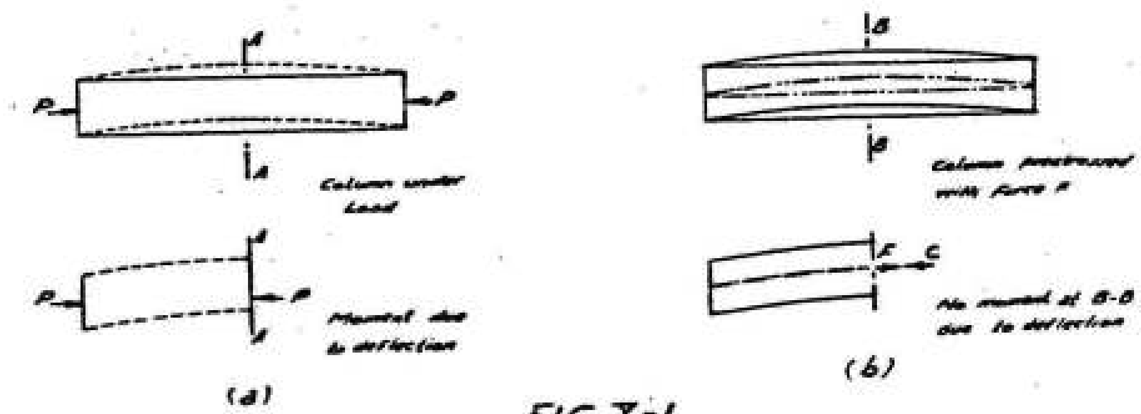


FIG X-1

If the steel and concrete are not in direct contact along the entire length, the problem will be different. The concrete under compression will have a tendency to deflect laterally. The deflection will not at first bring the steel to deflect together with it; hence the eccentricity of prestress on the concrete is actually changed, thus resulting in column action. After a certain amount of deflection, the steel is brought into contact with the concrete and the two will begin to deflect together. Hence the column action is limited to the differential deflection of the two materials.

If the steel is in contact with the concrete at several points, but not along the entire length, then the column action is limited to the length between the points of contact. If such length is short, column action will not be serious.

Next, consider a curved or a bent member subject only to internal prestress. If the prestress is concentric at all sections (the c.g.s. line coinciding with the c.g.s. line), then the concrete is behaving like an arch subject to axial force with the exception that the applied force from the steel will move with the deflection of the concrete and will always remain concentric. Hence there is no tendency to buckle.

If the prestress is eccentric as on sections G and H, fig. X-2 the compression in the concrete is still equal and opposite to the tension in the steel. Any deflection of the member will still displace both of them together and there will be no column action due to prestress. The effect of an eccentric prestress on the concrete, however, will produce deflection of the member. If the deflection is appreciable, the deflected axis of the member should be used in computing column effects due to external loads.

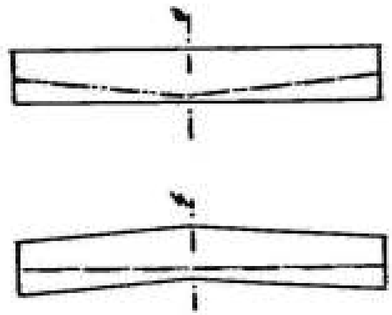


FIG X-2

X-4 <u>Compression Members.</u>

A prestressed concrete compression member is one that carries external compressive load. A member that is simply compressed by its prestress is not a compression member. As explained in the previous section, a prestressed member is not under column action due to its own prestress, but it is subject to column action under an external compressive load just like a column of any other material.

It is seldom that a prestressed-concrete member is utilized to stand compression and is prestressed for compression's sake. Evidently concrete can carry compressive load better without being precompressed by steel. And it is difficult to conceive of steel wires as adding any appreciable strength to a member carrying axial compression. However, many compression members, besides carrying direct compressive loads, are subject to transverse loads as well. Bending due to these transverse loads may more than offset the axial compressive stress at certain points so as to produce some resulting tension in the concrete. Then it will be advisable to reinforce such flanges for possible tension. In other words some compression members are actually flexural members, and all the advantages of prestressing a beam would apply to the prestressing of those members.

Consider an industrial building of one storey, for example; the columns or bearing walls may carry only light vertical loads. But they may be subject to bending during handling and erection if they are precast, or they may carry lateral force such as that due to wind and earthquake after the completion of the building. Similar conditions may exist in bridges. Then it is often feasible to precompress the member so that it can stand a certain amount of bending.

One benefitial effect of prestressing a compression member is the reduction of its deflection under transverse loads. One such pylon 30 ms. high was prestressed to resist an earthquake load of1200t applied horizontally along the pylon. Since the deflection of an uncracked section is about 40% that of a cracked section, a prestressed pylon could be about 2.5 times as stiff as an ordinary reinforced one. In this instance, the reduction of deflection at the top of the pylon minimizes the relative movements between the building floors and saves a tremendous amount of steel otherwise required to reinforce other parts of the building.

Within the working range, the stress in a prestressed compression member due to both prestress and external loads can be computed from the usual elastic theory. But the design of the member is another question, because the empirical methods for designing reinforced concrete columns cannot be directly applied to prestressed ones. The stresses ordinarily allowable for reinforced concrete are not applicable to prestressed concrete, partly because the stresses due to internal prestressing are of different nature from those due to external loads, the latter having column action, and the former not. For proper design of prestressed concrete members, one must go into basic theories of columns and prestress and choose a proper standard for the safety of the structure in each particular case.

If a section of a column is under an effective prestress F with an eccentricity e and loaded by a concentric load P plus an external moment M, the extreme fiber stress at the section can be computed by the formula:

$$f_c = F/A_{co} \pm F\ e\ y/I_{co} \pm P/A_v \pm M\ y/I_v \quad \ldots\ldots \quad X\text{-}6$$

If the column is a slender one, the deflections of the member due to both the prestress eccentricity and the external load ma y significantly affect the magnitude of the external moment M and must be included in it.

Few data are available on the strength of prestressed concrete columns. An approximate investigation of the effect of prestressing on the ultimate strength can be made however. Under the action of an external compressive load, the column will shorten and the prestress in the steel will be decreased. If, at the ultimate load, the unit compressive strain in the concrete is of the order of 0.003 (3 mm/m), then the pretensioned strain in the steel will be decreased by that same amount, and the remaining prestress at the moment of failure will be less than the original effective prestress.

If the effective prestress is 8.5 t/cm^2, the remaining prestress will be only:

$$f_s = 8.5 - 0.003\ E_s = 8.5 - 0.003 \times 2100 = 2.2\ t/cm^2$$

In other words, the major part of the prestress may be lost at the ultimate compressive strength of the concrete. This means that the ultimate load-carrying capacity of the column is not much decreased by prestressing. On the other hand, if the column fails on the tensile side as the result of bending or buckling, the steel on that side can be stressed to near its ultimate strength.

The buckling of the compression flange of prestressed beams is subject to the same reasoning. There is no danger of flange buckling produced by internal prestress in a beam. For external loads a tendency to buckle in the flange is governed by the usual theory of elasticity so long as there are no cracks in the concrete. After cracking or near the ultimate load, little is known about the buckling of the compressive flange in prestressed beams.

Example :

A concrete column 45 x 45 cms in cross section and 6 ms high, fig. X-3, is pre - tensioned with twelve 7 mm wires, which are end anchored to the concrete. The effective prestress is 8.5 t/cm^2 in the steel. For a concentric compressive load P = 50 tons and a horizontal load H = 5 tons at the mid height of the column, compute the maximum and minimum stresses in the column, assuming it to be hinged at the ends. Investigate the secondary moments in the column due to deflection. Discuss the safety of the column under such loads and also during handling. Assume that n = 7.

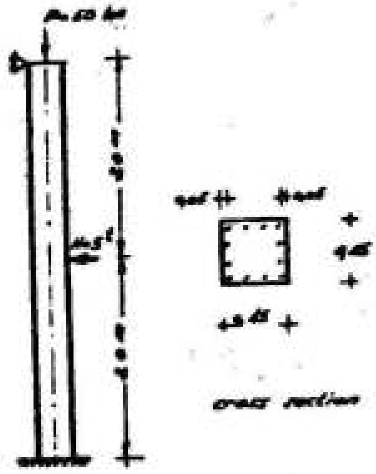


FIG X-3

f_{c28} = 300 kg/cm^2 . E_c = 300 t/cm^2 .

Steel 16.5/14 . E_s = 2100 t/cm^2

Solution :

Stress in concrete due to prestress

$f_c = F/A_{co}$ but $F = A_s f_s$ and $A_{co} = A_c - A_s$

A_s = 12 x 0.385 = 4.62 cm^2 F = 4.62 x 8.5 = 39.2 tons

A_{co} = 45 x 45 - 4.62 = 2020 cm^2

Therefore f_c = - 39200/2020 = - 19.4 kg/cm^2

Stress due to the axial load of 40 tons, disregarding deflection of column is

$f_c = P/A_v$ but $A_v = A_c + (n - 1) A_s$

A_v = 45 x 45 + (7 - 1) x 4.62 = 2025 + 28 = 2053 cm^2

Therefore f_c = - 50000/2053 = - 24.4 kg/cm^2

The maximum bending moment occurs at the mid height of column, and is

$M_{max} = M\ l/4 = 5 \times 6/4 = 7.5$ mt

The moment of inertia of the virtual area

$I_v = 45^4/12 + 0.385\ (8 \times 17.5^2 + 4 \times 5.5^2) = 343 \times 10^3\ cm^4$

The extreme fiber stresses due to bending

$f_c = M\ y\ /\ I_v = 7.5 \times 10^5 \times 22.5/343 \times 10^3 = \pm\ 49.2\ kg/cm^2$

The maximum and minimum stresses are hence

$- 19.4 - 24.4 - 49.2 = - 93.0\ kg/cm^2$
$- 19.4 - 24.4 + 49.2 = + 5.4\ kg/cm^2$

The maximum deflection of the column due to the horizontal load is

$= P\ l^3/48\ E_c\ I_v = 5000 \times 600^3/48 \times 300\ 000 \times 343 \times 10^3 = 0.32$ cms

This will increase the moment by the amount $50 \times .32/100 = 0.16$ mt

This moment will produce more deflection and further increase the eccentric moment in the column, but the magnitude is so small that it can be neglected. Hence the above computed stresses can be considered sufficiently correct. The maximin compressive stress of $93 kg/cm^2$ would appear high for a reinforced concrete column but is not excessive for a prestressed member which is more a beam than a column in this example.

The safety of the column can be determined only if we know the ultimate strength of the column under such combined axial and transverse loads and also if we know the possibilities of over loading, i.e. to what extent the axial or horizontal loads may be increased, and whether eccentricity of the applied axial load may be possible.

For the purpose of investigation, let us assume that both the horizontal and the axial load are increased by 20% while, in addition there will be an eccentricity of 4 cms for the axial load. Then the stresses will be

Due to axial load $1.2 \times 24.4 = - 29.3\ kg/cm^2$
Moment ΔM due to eccentricity of axial load

$\Delta M = 1.2 \times 50 \times .04 = 2.4$ mt.

the corresponding stresses are :

$49.2 \times 2.4/7.2 = \pm 15.8$ kg/cm^2

Stresses due to horizontal load will be

$1.2 \times 49.2 = \pm 59$ kg/cm^2

The resulting stresses will be

$- 19.4 - 29.3 - 15.8 - 59.0 = - 123.5$ kg/cm^2

$- 19.4 - 29.3 - 15.8 + 59.0 = + 26.1$ kg/cm^2

Note that the compressive stress of 123.5 kg/cm^2 is only 0.41 f_{c28} while the tensile stress is below the modulus of rupture of about $0.1 f_{c28} = 30$ kg/cm^2 . Hence the column would have not cracked, and the mid-height deflection can still be computed by the elastic theory as follows :

Due to a constant moment $\Delta M = 2.4$ mt the deflection, at mid-height will be :

$\delta_1 = \Delta M\ l^2/8\ E\ I = 2.4 \times 10^5 \times 600^2/8 \times 300\ 000 \times 343 \times 10^3 = 0.105$ cms

and due to a horizontal load of $5 \times 1.2 = 6.0$ tons the deflection δ_2 will be

$\delta_2 = 1.2 \times 0.32 = 0.385$ cms

so that, the total deflection $\delta = \delta_1 + \delta_2 = 0.49$ cms

which is not a significant value. Thus it can be concluded that the column is safe.

For investigating the handling stresses, let us assume that the column is picked up at the mid height.

Its own weight $g = 0.45 \times 0.45 \times 2500 = 500$ kg/m'

Moment M_g due to own weight $= g\ (l/2)^2/2 = 0.5 \times 3^2/2 = 2.25$ mt

which will produce a maximum tensile stress of

$M_g\ y / I_v = 2.25 \times 10^5 \times 22.5/343 \times 10^3 = 14.8$ kg/cm^2

which is less than the precompression of 19.4 kg/cm^2 and the column is safe during handling.

weight of the connecting beam to be 1.00 ton per meter, R_e is found by equating to zero the moments about the center line of the interior column; i.e.

$100 \times 5 + 1 \times 5.3^2/2 - R_e \times 4.15 = 0$

Hence $\underline{R_e = 123.5 \text{ t}}$ and

its intensity is 123.5/2.3 = 53.7 t/m.

The reaction R_i is found by equating to zero the sum of all vertical forces ; i.e.,

$R_i = 100 + 180 + 1 \times 5.3 - 123.5$

or $\underline{R_i = 161.8 \text{ t}}$

and its intensity is 161.8/2.65 = 61 t/m

The shearing force and bending moment diagrams of this beam are shown in figure 9-16 . The section of max. moment M = 65 mt is rectangular ; choosing its breadth b = 50 cms, and assuming σ_c = 70 kg/cm^2 and σ_s = 2000 kg/cm^2 , we get :

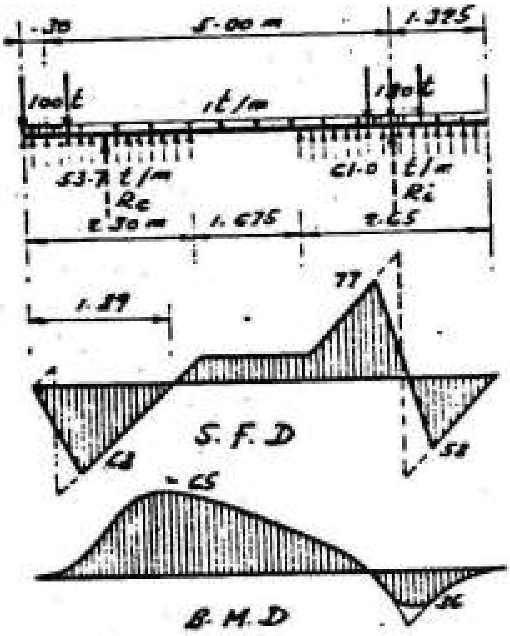


Fig. 9-16

$d = 0.31 \sqrt{65\,000 / 0.5} = 112$ cm t = 120 cm

$A_s = 65\,000 / 1785 \times 1.12 = 32.5$ 9 Φ 22

For section at center line of intermediate support M = 36 mt

$112 = k_1 \sqrt{36\,000 / 0.5}$ $k_1 = 0.417$

For σ_s = 2000 kg/cm^2 σ_c = 48 kg/cm^2 k_2 = 1825

$A_s = 36000/1825 \times 1.12 = 17.6$ cm^2 4 Φ 22 + 3 Φ 13

Check of shear, bond.... etc. are to be done according to the general principles of reinforced concrete design.

The footing slabs behave as double cantilevers and can be designed in the same way as wall footings. Hence

For the slab under the intermediate column :

Bending moment at face of strap beam $M = 28 \times 0.9^2/2 = 11.3$ mt/m

Assuming σ_c = 50 kg/cm^2 and σ_s = 2000 kg/cm^2, we get:

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