Theory and Design of Reinforced Concrete Tanks

By

DIPL. ING. M. HILAL DR. SC. TECHN.

PROFESSOR, FACULTY OF ENGINEERING

CAIRO UNIVERSITY, GIZA.

2005

شكر ودعاء

جزيل الشكر لكل من حرص على اقتناء هذا الكتاب

ودعاء إلى الله

أن ينتفع بما حواه من علم وأن يتغمد المولى برحمته

مؤلفه الأستاذ الدكتور / محمد هلال

سامح هلال

P R E F A C E

This new revised edition includes the following additions :

1) Determination of fixing and connecting moments in fixed and continuous surfaces of revolution. Chapter IV.

2) The application of the data given in the previous article appears in the design of the Inze tank given in Chapter VI.

3) The design of circular flat plates with overhanging cantilevers and eventually central holes under different load conditions is shown in Chapter VI in a numerical example of a circular water tower 300 m^3 capacity.

4) Bunkers and Silos being a natural continuation to tanks and containers are dealt with in Chapter IX.

5) The statistical behavior of pump rooms is similar to that of under ground tanks ; the discussion of some examples is given in Chapter X.

6) Temperature stresses in walls of tanks, bunkers, silos and pump rooms are shown in Chapter XII.

The author hopes that these additions open new scopes in the theory and design of reinforced concrete tanks and containers and may be of benefit to both graduate and undergraduate students in the faculties of engineering and practicing structural engineers.

January 1976   M. HILAL

CONTENTS

INTRODUCTION

Reinforced concrete and eventually prestressed concrete are generally the most convenient materials for liquid tanks and containers.

Due to the internal pressure of the liquid stored in such structures, the walls and floors are mainly subject to tensile forces, bending moments and eccentric tension which cause in most cases critical tensile stresses on the surface of the different elements facing the liquid. If such elements are designed according to the general principles adopted in ordinary reinforced concrete, cracks will be developed and the liquid contained in tne tank has the possibility to penetrate under its hydrostatic pressure through the cracks and cause rusting of the steel reinforcement. Therefore, special provisions must be taken to prevent the formation of such cracks. Such provisions generally lead to an increased thickness of the walls towards their foot and at their other corners. If the effect of this increase is not considered, it may lead to serious defects so that a thorough investigation is absolutely essential.

Porous concrete or concretes containing honey combing or badly executed joints lead to the same possibility of rusting with all its ill effects to the structure. Therefore, dense, water-tight concrete is one of the essential requirements of liquid tanksr and containers, the necessary provisions required in the careful design of the mix and in the execution of the structure must be taken.

The protection of the finished concrete structure by convenient plastering, painting or casing as well as its thorough curing must be carefully studied.

The previous investigation gives some points showing that liquid containers are delicate structures and need, due to their intensive use in structural engineering, special care and knowledge in the design, execution and protection.

The present work is concerned with the theory and design of liquid tanks and containers giving the necessary provisions aiming to satisfy the final goal of structural engineering by designing safely, economically and efficiently. It is designed to adapt the teaching requirements in our universities and higher schools for undergraduate and advanced studies. It gives the practicing designer a simple, scientific and basic reference not only in liquid containers but also in many other fields such as surfaces of revolution, high towers, circular beams, rectangular and cicular flat plates, deep beams, beams on elastic foundation, pyramid roofs, etc., as they are needed in design of tanks and containers. The statistical behavior of bunkers and silos is similar to that of tanks; for this reason, it has been decided to show their design in a separate chapter of this edition.

In preparing this book, it was taken for granted that the reader has a thorough knowledge of the strength of materials and fundamentals of reinforced concrete that is usually covered by our faculties of engineering. It lays thereon the foundation of a thorough understanding of the basic principles of reinforced concrete and eventually prestressed concrete tanks presented and developed in a simple systematic clear manner.

One of the main aims of this book is to show that careful scientific designs based on sound theoretical basis do not need neceassarily complicated calculations and, in most cases an attempt has been made to simplify complicated derivations as much as possible and to give simple practical relations carefully derived and easy to remember for otherwise complicated problems. In complicated cases, for which solutions could not be derived without going beyond the limit of the usual standard in structural engineering studies or in cases which need tedious or lengthy calcutions, the final results only were given and where possible simplified by design tables and curves.

Using these methods of simplifying the presentation, the author was able to discuss a big number of problems in a detailed manner facing and proposing solutions for the defferent difficulties that may arise in the design.

In this manner, a complete treatment of the design fundamentals is given and fully discussed and illustrated with numerical examples and full constructional details showing the engineer how to attack structural problems with confidence.

The book includes the following main chapters :
Chapter I gives a short account on the production of dense, watertight concrete.

Chapter II deals with the design of sections under different kinds of stresses. The effect of shrinkage on symmetrical sections is also included. In order to have sufficient safety against cracking, the final stresses in the different elements are generally low, so that the elastic theory which is taken as a basis for the design is justified.

The design of circular tanks with sliding, hinged, fixed and continuous base are shown in Chapter III. In this chapter, the theory of Lamè for thick cylinders and the theory of Reissner for determining the internal forces in circular tank walls fixed at the base and sbject to hydrostatic pressure are given. The design of such tanks can be made by a very quick and simple manner if the simplified methods proposed by the author are used. The tables published by the American Portland Cement Association for circular tanks and circular plates are also included.

Chapter IV is devoted to the roofs and floors of circular tanks : It includes the membrane theory of thin surfaces of revolution and the internal forces in circular flat plates.

Prestressed concrete may give a convenient economic solution for big circular tanks ; for this purpose, the fundamentals of circumferential, longitudinal and dome prestressing as may be used for liquid containers are given in Chapter V.

In order to show the application of the theories of circular tanks circular plates domes and cones, three detailed examples are illustrated in Chapter VI. The first example shows the design of a paste container and the other two show different types of water towers.

Chapter VII gives a thorough investigation of rectangular tanks calculated according to the approximate strip method and according to the methematical theory of flat plates. for which purpose, the internal forces in flat rectangular plates supported on three and four sides subject to uniform and triangular loads as determined by Czerny for different conditions at the supports. are given.

Tanks directly built on the ground are dealt with in Chapter VIII. The theory of beams on elastic foundations and its application to some tank problems are shown in two numerical examples.

The design of bunkers and silos according to the classic theories and the new researches is dealt with in Chapter IX.

The behavior of pump stations is similar to empty underground tanks. Some examples are shown in Chapter X.

Some complementary designs. required for tank problems. showing stress distribution, internal forces and design of deep beams, pyramid roofs and temperature stresses in walls are given in Chapter XI.

I. PRODUCTION OF WATER-TIGHT CONCRETE

Dense concrete, free from cracks or honey combing is the main requirement for water-tightness. Porous concrete having cracks on the liquid side allow the liquid in the tank, under its hydrostatic pressure to penetrate through the concrete and cause rusting of the steel reinforcement leading to all its serious effects on the structure. Dense water-tight concrete can be achieved through careful selection of aggregates, suitable granular composition, use of low water cement ratio, sufficient cement content and thorough mixing, compaction and curing.

We give, in the following, a short account about the main factors affecting the density and water-tightness of concrete.

I.1. COMPOSITION, MIXING AND COMPACTION

Dense concrete can be produced if the voids are reduced to a minimum, such a provision can be attained through the choice of a convenient mix composed of fine aggregates (smaller than 5 mms), medium aggregates (between 5 mms and 10 mms) and coarse aggregates (over 10 mms). The maximum grain size is to be according to the thickness of the element in which it is used and preferably not more than 30 mms in reinforced concrete water structures.

In normal cases, the cement content in the mix is generally 350 kg. per cubic meter finished concrete, in small tanks and in cases of low stresses, the cement dose may be reduced to 300 kgs/m^3 . Richer doses with a maximum of 400 kgs/m^3 may be used for big under-ground tanks in wet medium. The use of high cement doses in dry weather under normal conditions is not recommended because the shrinkage tensile stresses causing cracks in the concrete increase with the increase of the cement content.

It is recommended to use the least possible amount of mixing water giving good plastic concrete. The water-cement ratio to be specified depends on the method of compaction - by hand or by mechanical vibration - and on the nature of the concrete constituents, in this respect figures based on a slump test are recommended. Excess of mixing water is to be avoided as it leads to porous concrete due to the evaporation of the surplus water not needed for the chemical action, and increases the shrinkage strains.

In big tanks, the use of mechanical mixers with automatic water control is essential.

To produce dense concrete, good compaction is necessary as it compensates for the possible gaps in the granulometric composition of the aggregates. The use of surface and immersion vibrators gives satisfactory results.

I.2. ADMIXTURES

Some admixures have a mechanical effect on concrete while others have a chemical effect. Admixures having a mechanical lubricating effect increase the workability of concrete mixes, thus allowing a reduction in the water content which in turn results in an increase in the strength and water-tightness of concrete (e.g. baraplast and air entraining agents). Admixures having chemical effects on the concrete mix are to be used only when tests prove that they have no ill effect on the concrete or the steel throughout their lifetime.

Other admixures help to seal the pores in the concrete, their presence is to be considered only as an addition to the water - tightness attained by the above mentioned steps and not in any way as a replacement.

I.3. CURING

Concrete undergoes a volume change during hardening, it shrinks in dry weather and swells under water. Shrinkage causes tensile stresses in the concrete. If such stresses are developed and act on fresh concrete of low strength, they cause shrinkage cracks. It is absolutely essential to prevent such stresses from being developed until the concrete has gained sufficient strength to resist them. This can be done by intensive curing of fresh concrete (keeping it continously wet) starting immediately after the final setting of the concrete and for a minimum period of 15 days.

I.4. SURFACE TREATMENT, PAINTS AND CASINGS

The most effective surface treatment is cement mortar plaster composed of 600 to 650 kgs cement per cubic meter sand and applied by the cement gun. The thickness may be chosen 1.5 to 2.0 cms. It is recommended to apply such a plaster on side facing the liquid after filling the tank with water for 7 days. The surface should be thoroughly cleaned by wire brushes before the application of the cement gun. In this manner, the preliminary cracks which may appear after the first filling of the tank will be sealed by the plaster. Moreover, the plaster will not be subject to a big part of the plastic strains due to water pressure.

Paints (e.g. barafluate, baranormal, bituminus paints... etc) may also be used either directly on the concrete surface or on the cement plaster. The use of special paints whose object is to close the surface pores (such as glass paints, plastic paints, watertight casings, lining with metallic sheets - e.g. stainless steel or water tight tiling) may be of advantage.

Any material for water-tightness either as admixure or surface treatment must not be used unless it is proved by experiments to be suitable for the purpose.

II. DESIGN OF SECTIONS

II.1. NOTATIONS :

σ_{c28} = Compressive strength of standard concrete cube after 28 days.

σ_t = Axial tensile strength of concrete.

σ_{tb} = Bending strength of concrete.

σ_{co} = Allowable axial compressive stress of concrete.

σ_{cb} = Eventually σ_c = allowable compressive bending stress of concrete.

σ_t = Tensile stress on concrete.

σ_{tb} = Max. tensile concrete stress in bending.

σ_s = Allowable stress in steel.

N = Normal force acting on a section.

T = Tensile force acting on a section.

A = Area of a section (general).

A_c = Area of concrete section

A_s = Area of steel reinforcement.

E = Modulus of elasticity -(general).

E_c = Modulus of elasticity of concrete.

E_s = Modulus of elasticity of steel.

n = E_s / E_c = modular ratio.

ε = Strain (general).

ε_c = Strain of concrete.

ε_s = Strain of steel.

ε_{sh} = Strain due to shrinkage of concrete.

ε_{cr} = Strain due to creep of concrete

b = Breadth of a rectangular section.

t = Total depth of a section.

d = Theoretical depth of a section = distance between center of gravity of tension steel and outside fiber of compression zone.

μ = A_s / A_c = Ratio of tension steel reinforcement in a section
= $A_s / b\,t$ in case of axial forces acting on a rectangular section.
= $A_s / b\,d$ in case of bending moments acting on a rectangular section.

y_o = Distance of c.g. axis from putside fiber in tension zone.

A_v = $A_c + n\,A_s$ = virtual area of a reinforced concrete section.

Z = Section modulus, for a rectangular section $Z = b\,t^2 / 6$.

Z_v = Section modulus of the virtual section.

I = Moment of inertia of a section, for a rectangular section $I = b\,t^3 / 12$.

I_v = Moment of inertia of the virtual section.

M = Bending moment acting on a section.

M_s = Bending moment about tension steel = $N.\ e_s$

M_f = Fixing moment.

e = M / N = eccentricity of normal force N from c.g. axis.

e_s = eccentricity of normal force N from tension steel.

k_1 = Coefficient for determining the theoretical depth d of a reinforced concrete section subject to bending moment $d = k_1 \sqrt{M / b}$ or eccentric forces $d = k_1 \sqrt{M_s / b}$.

k_2 = Coefficient for determining the area of the tension steel A_s in a reinforced concrete section subject to bending moment $A_s = M / k_2\,d$ or eccentric forces $A_s = \frac{M_s}{k_2 d} \pm \frac{N}{\sigma_s}$

(+) for eccentric tension and (−) for eccentric compression.

α = A_s' / A_s = ratio of compression steel to tension steel.

II.2. REQUIREMENTS AND ALLOWABLE STRESSES

Sections of liquid containers must be so designed that no cracks in concrete are allowed in the fibers facing the liquid, because if such cracks are allowed, the liquid in the container will penetrate through these cracks & cause rusting of the steel reinforcement which must be prevented by all possible means.

In order to satisfy this requirement, the concrete dimensions must be chosen so that the tensile stresses in concrete- if they take place on the liquid side - are smaller than its tensile strength.

But as the tensile strength of concrete cannot be garanteed and because the above mentioned provision does not include any factor of safety for the tensile strength of concrete, the tension steel reinforcement must be designed to carry all the tensile stresses i. e. the concrete in tension is neglected - Stage II -

The axial tensile strength of concrete $\grave{\sigma}_t$ may be assumed a factor of the prism strength according to the relation :

$$\grave{\sigma}_t = 0.7 \text{ to } 1\sqrt{\sigma_{cp}}$$

The building Research Institute in Egypt gives for σ_t the following relation :

$$\grave{\sigma}_t = 0.03\ \sigma_{c28} + 8 \text{ kg/cm}^2$$

The tensile bending strength of concrete, $\grave{\sigma}_{tb}$ is double as much as its tensile strength $\grave{\sigma}_t$ i.e. $\grave{\sigma}_{tb} = 2\sigma_t = 1.4 \text{ to } 2\sqrt{\sigma_{cp}}$. The average value to be used in the design is generally not bigger than 3/4 this value.

$$\grave{\sigma}_{tb} = 1.0 \text{ to } 1.5\sqrt{\sigma_{cp}}$$

The code of practice for the use of reinforced concrete in buildings gives :

" In structures where no cracks in tension are allowed, the permissible tensile stress of concrete may be taken

$\tilde{\sigma}_t = \sigma_{co/4}$ for axial tension and $\tilde{\sigma}_{tb} = \sigma_{cb/4}$ for tension in bending".
For normal water structures, the values of the design stresses in concrete and steel can accordingly be determined.

Values of $\tilde{\sigma}_t$ & $\tilde{\sigma}_{tb}$ recommended for design purposes are given in the following table :

Values of $\tilde{\sigma}_t$ & $\tilde{\sigma}_{tb}$ for different concrete mixes

Case of Design	cement dose kg/m^3	cube strength kg/cm^2	tensile stress of concrete kg/cm^2	
			Axial tension $\tilde{\sigma}_t$	Bending $\tilde{\sigma}_{tb}$
Shrinkage is not taken into consideration	300	160	10	15
	350	200	12	18
	400	250	15	20
Shrinkage is taken into consideration	300	160	14	18
	350	200	16	20
	400	250	20	25

The allowable stresses in steel subject to tensile stresses can be chosen as follows :

For normal mild steel $\sigma_s = 1400$ kg/cm^2

For deformed high grade steel $\sigma_s = 2000$ kg/cm^3

For deformed cold twisted steel $\sigma_s = 2200$ kg/cm^2

On condition that the cube strength of concrete $\sigma_{c28} > 200$ kg/cm^2

In case $\sigma_{c28} < 200$ kg/cm^2 $\sigma_s = 1200$ kg/cm^2

II.3 SECTIONS SUBJECT TO AXIAL TENSION

II.3.1. Stability Design

The steel alone must be sufficient to resist all the tensile force T acting on the section

$$\text{i.e. } A_s = T/\sigma_s \qquad (1)$$

II.3.2. Safety Against Cracking

The tensile stress in concrete σ_t must be smaller than its tensile strength $\grave{\sigma}_t$

$$\text{i.e. } \sigma_t = \frac{T}{A_c + n\,A_s} < \grave{\sigma}_t \qquad (2)$$

A_c = The area of the required concrete section

A_s = Area of steel reinforcement determined from equation (1)

n = Modular ratio = 10 (no cracks in concrete Stage I)

II.3.3. Effect of Shrinkage

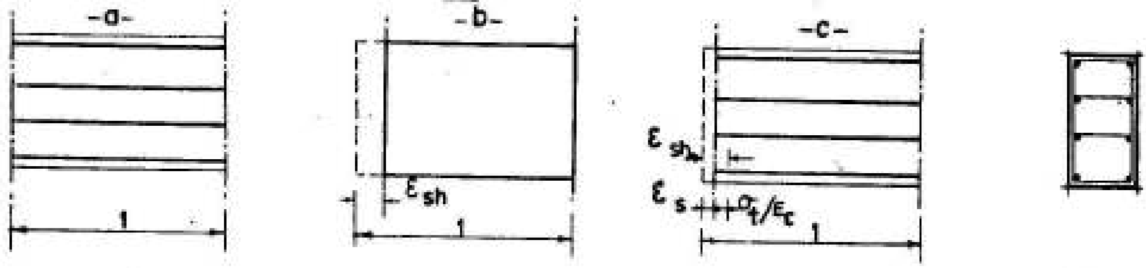


Fig. II-1

The effect of shrinkage in a symetrically reinforced element can be determined as follows : (Fig. II.1)

Figure (a) represents a symetrically reinforced concrete block of unit length. If the bars are left out as in figure (b), shrinkage will shorten the concrete block a distance = ε_{sh} . The presence of the steel bars prevents some of the shortening of the concrete and the block shortens $\varepsilon_s < \varepsilon_{sh}$ only; i.e. the steel shortens a distance ε_s and accordingly is subject to compressive stresses σ_s , while the concrete will elongate a distance = $\varepsilon_{sh} - \varepsilon_s$ and accordingly will be subject to tensile stresses σ_t .

The element being not subject to any external forces, the sum of the stresses on the section must be equal to zero.

The stresses in steel & concrete can be calculated in the following manner :

Strain due to shrinkage $= \varepsilon_{sh}$

Final strain in steel $= \varepsilon_s = \sigma_s / E_s$

Final strain in concrete $= \varepsilon_c = \varepsilon_{sh} - \dfrac{\sigma_t}{E_c}$

Both materials are subject to the same final strain : $\varepsilon_s = \varepsilon_c$

or $$\frac{\sigma_s}{E_s} = \varepsilon_{sh} - \frac{\sigma_t}{E_c}$$

or $$\sigma_s = \varepsilon_{sh} E_s - n\,\sigma_t \qquad (3)$$

As no external forces act on the element, then

$$\sigma_t A_c - \sigma_s A_s = 0 \qquad (4)$$

Equations (3) & (4) give :

$$\sigma_t A_c - (\varepsilon_{sh} E_s - n\,\sigma_t)\, A_s = 0$$

or $$\sigma_t = \frac{\varepsilon_{sh} E_s A_s}{A_c + n A_s} \quad \text{tension} \qquad (5a)$$

Equation (4) gives :

$$\sigma_s = \sigma_t \frac{A_c}{A_s} \qquad \text{i.e.}$$

$$\sigma_s = \frac{\varepsilon_{sh} E_s A_c}{A_c + n A_s} \quad \text{compression} \qquad (5b)$$

In this manner, the tensile stress in concrete σ_t due to a tensile force T and shrinkage is given by :

$$\sigma_t = \frac{T + \varepsilon_{sh} E_s A_s}{A_c + n A_s} \qquad (6)$$

in which

ε_{sh} = Strain due to shrinkage = 0.0002 to 0.0003 (0.2 to 0.3 mm/m)

E_s = Modulus of elasticity of steel = 2100 t/cm^2

n = Modular ratio in stage I = 10

The area of steel A_s and the thickness t of a rectangular section of breadth b = 1 m. can be determined as follows :

$A_s = T/\sigma_s$ and $A_c = b \cdot t = 100\, t$

Introducing these values in equation (6), we get :

$$\sigma_t = \frac{T + \varepsilon_{sh} E_s T/\sigma_s}{100\, t + n\, T/\sigma_s} \quad \text{or} \quad t = \frac{\varepsilon_{sh} E_s + \sigma_s - n\, \sigma_t}{100\, \sigma_s\, \sigma_t} \cdot T$$

Assuming further that :

$\varepsilon_{sh} = 0.00025$ $\quad \sigma_s = 1400$ kg/cm^2 $\quad \sigma_t = 16$ kg/cm^2 ,

then

$$\underline{t = 0.8\, T} \qquad (7)$$

t in cms for T in tons/m

For this thickness, the area of steel can be given as :

$A_s = \frac{T}{\sigma_s} = \frac{t}{0.8\, \sigma_s}$ for $\sigma_s = 1.4$ t/cm^2 , we get

$$\underline{A_s = 0.9\, t} \qquad (8)$$

A_s in cm^2 for t in cms i.e. $\mu = 0.9$ % of section

Example 1 :

To illustrate the effect of shrinkage assume :

$T = 40$ t/m , $\varepsilon_{sh} = 0.00025$ $\quad E_s = 2100$ t/cm^2 $n = 10$ $\quad b = 100$ cm

& $\sigma_s = 1400$ kg/cm^2 , then

the thickness $t = 0.8\ T = 0.8 \times 40 = 32$ cms

reinforcement $A_s = 0.9\ t = 0.9 \times 32 = 28.8$ cm^2 or

$A_s = T/\sigma_s = 40/1.4 = 28.5$ cm^2

chosen 14 Φ 16 mm/m ($A_s = 28$ cm^2)

7 Φ 16 mm/m on each side

$$\text{and } \sigma_t = \frac{T + \varepsilon_{sh} E_s A_s}{A_c + n A_s} = \frac{40000 + 0.00025 \times 2100000 \times 28}{100 \times 32 + 10 \times 28}$$

$$= \frac{40000 + 14700}{3480} = \frac{54700}{3480} = 15.7 \text{ kg/cm}^2 \text{ tension} ! < 16 \text{ kg/cm}^2$$

This example shows that the effect of shrinkage on the tensile stresses of concrete is about 37% of the effect of the direct tensile force T.

Taking the effect of shrinkage into consideration, it is possible to prove that the concrete tensile stress σ_t and respectively the possibility of cracking increases with decreasing steel stress as follows:

It has been proved that $\sigma_t = \dfrac{T + \varepsilon_{sh} E_s A_s}{A_c + n A_s}$ but $A_s = T/\sigma_s$

$$\text{then} \quad \sigma_t = \frac{\sigma_s + \varepsilon_{sh} E_s}{A_c \sigma_s + n T} \cdot T$$

Assuming for example $T = 40$ t/m then $t = 0.8 \times 40 = 32$ cms and $A_c = 100 \times 32 = 3200\ cm^2$; with $\varepsilon_{sh} = 0.00025$ & $E_s = 2100\ t/cm^2$

we get :

$$\sigma_t = \frac{\sigma_s + 0.00025 \times 2100000}{3200\ \sigma_s + 10 \times 40000} \times 40000 = \frac{40\ \ \sigma_s + 21000}{3.2\ \sigma_s + 400}$$

Taking $\sigma_s = 700$ to ∞ kg/cm^2 , the corresponding σ_t will be as follows :

σ_s	700	800	1000	1200	1400	1600	1800	2000	∞ kg/cm^2
σ_t	18.6	17.9	17.0	16.3	15.6	15.4	15.1	14.8	13.33 kg/cm^2

N.B. for $\sigma_s = \infty$, $A_s = 0$ i.e. plain concrete wall.

This means that if the steel stress is reduced from 1400 to 700 kg/cm^2, the concrete tensile stress is increased from 15.6 to 18.6 kg/cm^2 which is about 20 % increase.

From the previous investigation, one can see that the lower the allowable stress in steel, the bigger the amount of the reinforcement and the sooner the concrete will crack. From this point of view it is desirable to use higher allowable steel stress, and it is recommended to choose $\sigma_s > 1400$ kg/cm^2 .

The use of lower steel stress σ_s in order to reduce the tensile strains in concrete has now no meaning so long as the section is designed such that $\sigma_t \leqslant \grave{\sigma}_t$

II.4. SECTIONS SUBJECT TO SIMPLE BENDING

If the tension side of the section is not facing the liquid, the section is designed as ordinary reinforced concrete without any special precautions. If the tension is on the liquid side, it must have :

a) Adequate resistance against cracking and

b) Adequate strength.

In order to satisfy condition (a) the section may be designed as plain concrete with the stress $\sigma_{tb} = M/Z$

In case of rectangular sections $Z = b\, t^2 / 6$ & $\sigma_{tb} = 6\, M / b\, t^2$

For normal conditions $\sigma_{tb} = 18\ kg/cm^2$ & $b = 1\ m$, thus,

$$t = \sqrt{M / 3} \qquad (9) \quad t \text{ in cms for } M \text{ in kgm}$$

In order to satisfy condition (b) proceed according to normal principles of reinforced concrete design as follows :

For the value of t determined according to equation (9), calculate the value of k_1 from the relation

$$d = k_1 \sqrt{M} \qquad \text{where } d = t - 2.5 \text{ to } 4 \text{ cms.}$$

For this value of k_1^* and the corresponding stress in steel (e.g. $\sigma_s = 1400\ kg/cm^2$ & $\alpha = 0$), determine k_2^* ; then

$$A_s = M / k_2\, d$$

The tension steel A_s can however be taken as a factor of the concrete section determined according to equation (9) as follows :

$$t = \sqrt{M / 3} \qquad \text{or} \qquad t^2 = M / 3 \qquad \text{or} \qquad M = 3\, t^2$$

The tension steel :

$A_s = M / k_2\ d$ where $k_2 = 1300\ kg/cm^2$ for $\sigma_s = 1400\ kg/cm^2$

Assuming further that $d = 0.9\ t$ we get :

* Values of k_1 and k_2 for different σ_c , σ_s and α are given in text-books of reinforced concrete design.

$$A_s = \frac{3\ t^2 \times 100}{1300 \times 0.9\ t} = 0.26\ t \qquad A_s \text{ in } cm^2 \text{ for } t \text{ in cms}$$

therefore

$$\underline{A_s = 0.26\ t} \qquad \text{or} \qquad \underline{A_s = 0.26\ \%\ \text{of}\ A_c} \qquad (10)$$

Example 2 : Simple Bending

Rectangular section subject to M = 6000 kgm with tension on liquid side. Determine t and A_s . Assume $b = 1$ m, $\sigma_{tb} = 18$ kg/cm^2 and $\sigma_s = 1400$ kg/cm^2 .

Solution :

For safety against cracking (condition a) the depth of the section t is first to be determined according to equation (9)

thus, $t = \sqrt{M / 3} = \sqrt{6000 / 3} = 45$ cms.

Having determined the thickness t, the steel reinforcement can be calculated to satisfy condition b (adequate strength) according to stage II as follows :

$t = 45$ cms then $d = 41$ cms and

$41 = k_1 \sqrt{6000}$ i.e. $k_1 = 0.53$

for $\sigma_s = 1400$ kg/cm^2 , $n = 15$, $\alpha = 0$ we get

$\sigma_c = 31.5$ kg/cm^2 & $k_2 = 1282$ kg/cm^2 so that

$$A_s = \frac{6000}{1282 \times 0.41} = 11.4 \text{ cm}^2 \text{ chosen } 6\ \phi\ 16 \text{ mm/m}$$

According to equation (10) :

$$A_s = 0.26 \times 45 = 11.7 \text{ cm}^2 \quad \text{chosen } 6\ \phi\ 16 \text{ mm/m}$$

II.5. SECTIONS SUBJECT TO ECCENTRIC TENSION OR COMPRESSION

If the resultant stress on the liquid side is compression, the section is to be designed as ordinary reinforced concrete. But if the resultant stress on the liquid side is tension, the section must have (a) adequate resistance to cracking and (b) adequate strength.

To satisfy condition (a), the section may be designed as plain concrete such that :

$$\frac{M}{Z} \pm \frac{N}{A} \leqslant \tilde{\sigma}_{tb} \qquad (11)$$

For rectangular sections

$$\frac{6\,M}{b\,t^2} \pm \frac{N}{b\,t} \leqslant \tilde{\sigma}_{tb} \qquad (12)$$

As a good approximation take

$$\underline{t = \sqrt{M / 3} \pm 1.5 \text{ to } 2.5 \text{ cms}} \qquad (13)$$

t in cms for M in kgm

Positive sign for eccentric tension and negative sign for eccentric compression. The amount of increase or decrease depends on the magnitude of N in proportion to M.

To satisfy condition (b) calculate the area of steel reinforcement required for the section as an ordinary reinforced concrete section with breadth $b = 1$ m and depth $d = t - 2.5$ to 4 cms.

<u>Example 3</u> : <u>Eccentric Tension</u>

Rectangular section subject to $M = 6000$ kgm and $N = 8000$ kgs (tension) with the tensile stresses on liquid side. determine t and A_s .

Assume $b = 1$ m, $\sigma_{tb} = 18$ kg/cm^2 and $\sigma_s = 1400$ kg/cm^2

To satisfy condition (a) - safety against cracking :

$$t = \sqrt{M / 3} + 2 \text{ cms}$$

$$= \sqrt{6000 / 3} + 2 = 47 \text{ cms}$$

Check of the tensile stress in concrete

$$\sigma_{tb} = \frac{6\,M}{b\,t^2} + \frac{N}{b\,t} = \frac{6 \times 6000}{47^2} + \frac{8000}{100 \times 47} = 16.3 + 1.7 = 18 \text{ kg/cm}^2$$

To satisfy condition (b) adequate strength :

$$e = \frac{M}{N} = \frac{6000}{8000} = 0.75 \text{ m.}$$

$$e_s = e - \frac{t}{2} + 0.04 = 0.75 - 0.235 + 0.04 = 0.555 \text{ m} < e$$

$$M_s = N\, e_s = 8000 \times 0.555 = 4440 \text{ kg/m} < M$$

$$d = k_1 \sqrt{M_s} \quad \text{or} \quad 43 = k_1 \sqrt{4440} \quad \text{i.e.} \quad k_1 = 0.645$$

For $\sigma_s = 1400$ kg/cm^2 , $\alpha = 0$ and $n = 15$ we get :

$\sigma_c = 24$ kg/cm^2 and $k_2 = 1300$ kg/cm^2 so that

$$A_s = \frac{M_s}{k_2\, d} + \frac{N}{\sigma_s} = \frac{4440}{1300 \times 0.43} + \frac{8000}{1400} = 7.95 + 5.7 = 13.65 \text{ cm2}$$

7 ϕ16 mm/m

<u>Example 4</u> : <u>Eccentric Compression</u>

Same section subject to M = 6000 kgm and N = 8000 kg (comp.)

To satisfy condition (a) : safety against cracking

$$t = \sqrt{M / 3} - 2.0 \text{ cms} = \sqrt{6000 / 3} - 2 = 45.0 - 2 = 43 \text{ cms}$$

Check of tensile stress in concrete

$$\sigma_{tb} = \frac{6 \times 6000}{(43)^2} - \frac{8000}{100 \times 43} = 19.5 - 1.86 = 17.64 \text{ kg/cm}^2$$

To satisfy condition (b) : adequate strength :

$$e = \frac{M}{N} = \frac{6000}{8000} = 0.75 \text{ cms.}$$

$$e_s = e + \frac{t}{2} - 0.04 = 0.75 + 0.215 - 0.04 = 0.925 \text{ m} > e$$

$$M_s = N\, e_s = 8000 \times 0.925 = 7400 \text{ kg.m} \quad > M$$

$$d = k_1 \sqrt{M_s} \quad \text{or} \quad 39 = k_1 \sqrt{7400} \quad \text{i.e.} \quad k_1 = 0.455$$

For $\sigma_s = 1400$ kg/cm^2 , $\alpha = 0$ and $n = 15$ we get

$\sigma_c = 38$ kg/cm^2 and $k_2 = 1264$ kg/cm^2 so that

$$A_s = \frac{M_s}{k_2\, d} - \frac{N}{\sigma_s} = \frac{7400}{1264 \times 0.39} - \frac{8000}{1400} = 15 - 5.7 = 9.3 \text{ cm}^2$$

7 ϕ 13 mm/m

It is however recommended to begin with the choice of the wall thickness satisfying condition (a) (Stage I) and then to determine the steel reinforcements satisfying condition (b) (Stage II) as in pre - vious examples.

The effect of the steel reinforcements on the value of σ_t may be taken into consideration as can be seen from the following example:

<u>Example 5</u> : (<u>Fig. II.2</u>)

Assume that in example 3 given before for case of eccentric tension, the max. thickness available is 44 cms only, then :

$t = 44$ cms $\quad e = 0.75$ ms $\quad e_s = 0.75 - 0.22 + 0.04 = 0.57$ m $< e$

$M_s = 8000 \times 0.57 = 4560$ kgm $< M$

$d = k_1 \sqrt{M_s}$ or $40 = k_1 \sqrt{4560}$ i.e. $k_1 = 0.593$

For $\sigma_s = 1400$ kg/cm^2 $\quad \alpha = 0$ and $n = 15$ we get

$\sigma_c = 27.5$ kg/cm^2 and $k_2 = 1294$ kg/cm^2 so that

$$A_s = \frac{4560}{1294 \times 0.4} + \frac{8000}{1400} = 8.8 + 5.7 = 14.5 \text{ cm}^2 \quad 8\ \phi\ 16 \text{ mm/m}$$

i.e. actual $A_s = 16$ cm^2

Check of max. tensile stress in concrete σ_{tb}

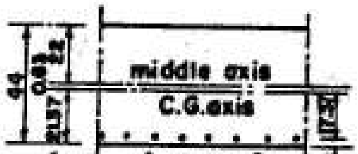


Fig. II-2

$$A_v = A_c + n\ A_s$$

$$= 100 \times 44 + 10 \times 16 = 4560 \text{ cm}^2$$

$$y_o = \frac{100 \times 44^2}{2} + 10 \times 16 \times 4 \ / \ 4560$$

$$= 21.37 \text{ cms}$$

$$I_v = \frac{100 \times 44^3}{12} + 4400 \times 0.63^2 + 10 \times 16 \times 17.37^2 = 743021 \text{ cm}^4$$

$$Z_v = \frac{I_v}{y_o} = \frac{743021}{21.37} = 34\,774 \text{ cm}^3$$

$$\sigma_{tb} = \frac{N}{A_v} + \frac{M}{Z_v} = \frac{8000}{4560} + \frac{6000 \times 1000}{34774} = 1.76 + 17.24$$

$$= 19 \text{ kg/cm}^2$$

III. DESIGN OF CIRCULAR TANKS

III.1 FUNDAMENTAL TYPES OF JOINTS OF WALLS

III.1.1. Introduction

In this section, our study will be limited to circular liquid containers, other forms will be dealt with later.

The structural behavior of circular cylinderical tank walls, subject to the action of hydrostatic pressure varies according to the type of joint between wall and other elements (base and roof if any). There are three main types of joints : free (or sliding), fixed and hinged. Any case in practice can be analysed by combination of these cases.

III.1.2. Free Joint (Sliding Joint) (Fig. III.1)

No restraint for motion of wall due to liquid pressure .

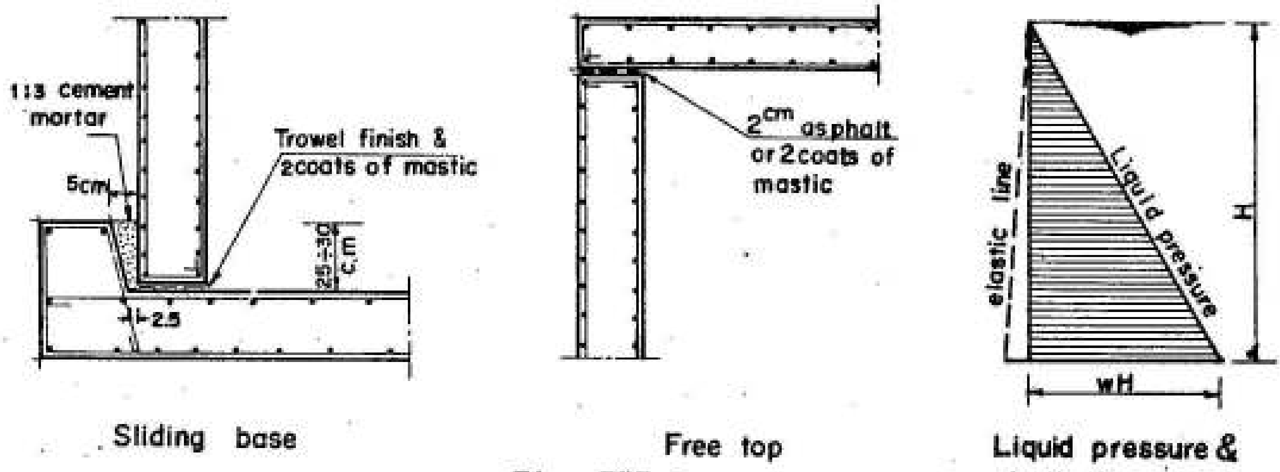
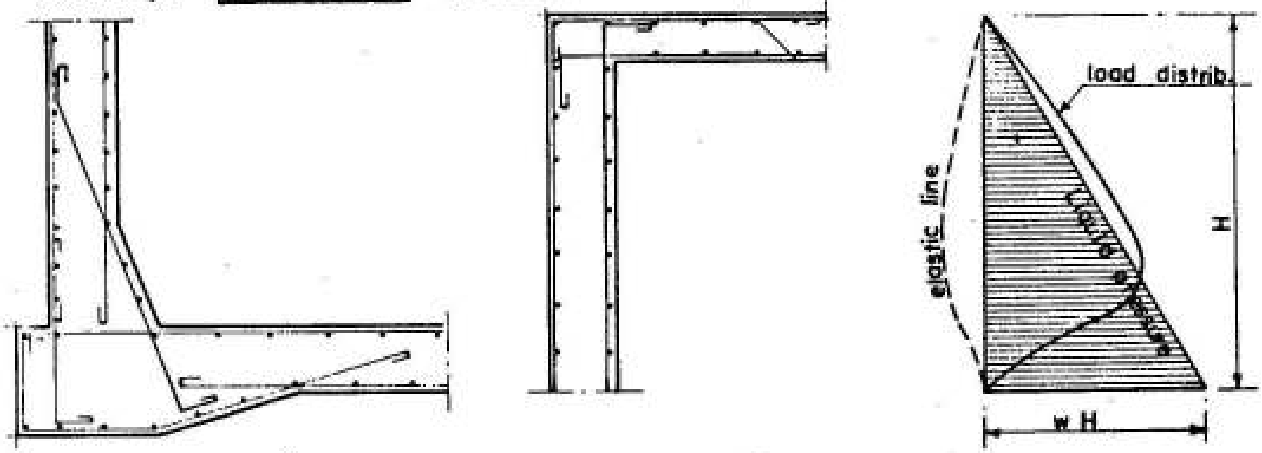


Fig. III-1

For this type of joint the elastic line of the wall is a straight line and the wall resists the liquid pressure by ring action.(i.e. by horizontal strips only). With respect to base, no indeterminate stresses are created.

To ensure water tightness in the joint a copper plate may be placed to join the wall and floor, neoprene plates may also be used.

III.1.3. <u>Fixed Joint</u> (continuous Joint)



Continuous base Continuous top Liquid pressure & elastic line

Fig. III-2

In this case, no allowance for motion or rotation is allowed for wall base (or top). Wall will carry liquid pressure partly by ring and partly by cantilever action (combined resistance by both vertical and horizontal strips). There is a connecting moment between wall and base. To obtain the required fixation, vertical reinforcement extends across the joint as shown. Good bond qualities are obtained by the following procedure :

After the concrete is placed and has stiffened sufficiently but is not thoroughly hardened - about 6 hours - clean the joint surface with a pressure water jet. Then cover the joint and keep it continuously wet. Just before new concrete is placed, flush the old surface with 1 : 2 portland cement mortar. Vibrate the new concrete and keep it moist for several days.

III.1.4 <u>Hinged Joint</u> (Fig. III.3)

It may sometimes be desirable to avoid transmitting moment between wall and base ; in such cases we use a hinged joint which allows for rotation as shown in (a) and (b).

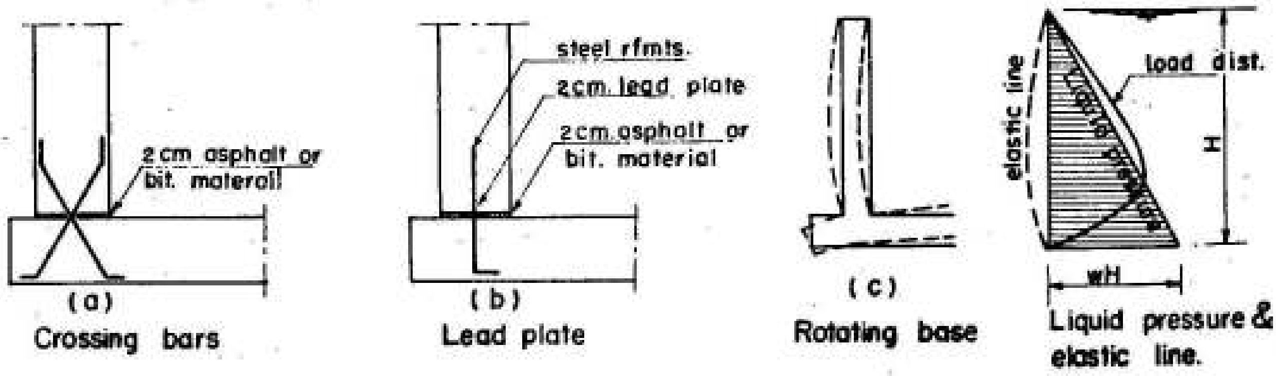


Fig. III-3

A wall rigidly connected to the base may however be conside-red hinged if the soil underneath is liable to rotate as shown in case c.

III.2 DESIGN OF WALLS WITH SLIDING BASE (Fig. III.4)

Liquid pressure p_x at any depth x is given by :

p_x = w x and p_{max} = w H in which

w = weight of liquid / m^3

For a wall with sliding base, the full water pressure will be resisted horizontally by ring action, thus :

$$2\,T_x = p_x\,D = 2\,p_x\,R \quad \text{or} \quad \underline{T_x = p_x\,R} \qquad (14)$$

If the thickness of the wall t is small in proportion to the radius R, the ring tension T_x may be assumed as uniformly distributed over the cross-section which can be designed as shown in II.2, i.e.

$T_x = p_x\,R = w\,x\,R$ $\qquad T_{max} = w\,H\,R$

$A_s = T / \sigma_s$ $\qquad \sigma_t = \dfrac{T + \varepsilon_{sh}\,E_s\,A_s}{A_c + n\,A_s}$

For σ_s = 1400 kg/cm^2 , σ_t = 16 kg/cm^2 ,

ε_{sh} = 0.00025 , E_s = 2100 t/cm^2 and

n = 10 , we get:

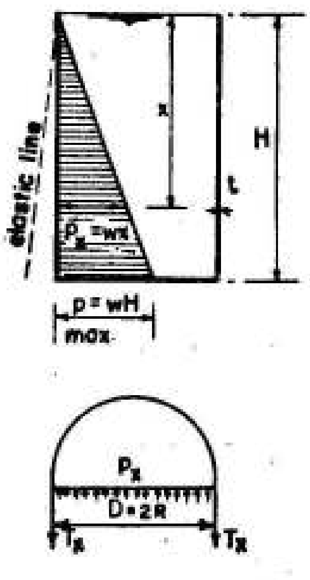


Fig. III-4

$$t \text{ cms} = 0.8\, T \quad \text{tons/m}$$
$$A_s \text{ cm}^2 = 0.9\, t \quad \text{cms}$$

In deep circular tanks, with the wall rigidly connected to the floor, the liquid pressure will be mainly resisted by ring action. Due to the fixation of the wall to the floor, the horizontal displacement of the wall at its foot cannot be fully developed and the pressure resisted by ring action will decrease to zero at the point of fixation. The small part of the liquid pressure at the foot of the wall that has not been resisted in the horizontal direction by ring action will be resisted in the vertical direction by cantilever action creating bending moments in the wall. In this case, the max. ring tension T_{max} takes place at 0.8 to 0.9 H and is given by :

$$T_{max.} = 0.80 \text{ to } 0.90 \text{ w H R}$$

The fixing moment at the base of the wall may be estimated from the relation :

$$M_f = \frac{w\, H\, D\, t_{max}}{7.5 \text{ to } 8}$$

which will be proved later.

In both equations, bigger values are to be chosen for deeper tanks. The mentioned bending moments affect only a small part, generally not exceeding H/10 at the foot of the wall.

Fig. III.5 shows the ring tension and cantilever moments of a deep circular tank subject to liquid pressure.

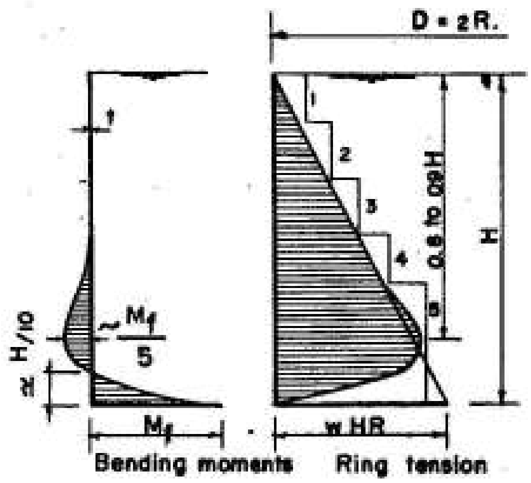


Fig. III-5

Example :

Deep water tank : H = 12 ms. and D = 8 ms. Wall fixed to floor. Max. ring tension at 0.9 H is given by :

T_{max} = 0.9 w H R = 0.9 x 12 x 4 = 43.2 tons

Required thickness t = 0.8 T = 0.8 x 43.2 = 35 cms.

As the wall is mainly subject to ring tension which increases with the depth, the section of the wall will be chosen trapezoidal with a minimum thickness at the top of 20 cms and a maximum thickness at the bottom t_{max} given by :

$t_{max.}$ = 0.8 w H R = 0.8 x 12 x 4 = 38 cms chosen 40 cms.

Thickness available at position of max. ring tension (0.9 H) is given by 20 + 0.9 x 20 = 38 cms > 35 cms.

Ring reinforcements at different depths is given in the following table :

Strip No.	1 0-2m	2 2-4m	3 4-6m	4 6-8m	5 8-12m	from top
p_{max}	2	4	6	8	10	t / m^2
$T = p_{max} R$	8	16	24	32	40	t / m
Total $A_s = T/1.4$	5.7	11.4	17.1	22.8	28.5	cm^2
Steel on each side = $A_s/2$	2.85	5.7	8.55	11.4	14.3	cm^2
Reinforcement	6Φ8	7Φ10	7Φ13	3.5Φ16 + 3.5Φ13	7Φ16	mm/m

The fixing moment at the base is given by : $M_f = \frac{1 \times 12 \times 8 \times 0.4}{7.5}$

$= 5.1$ m.t.

Max. thickness of wall required to resist this moment can be determined from equation (9) as follows :

$$t_{max} = \sqrt{M_f / 3} = \sqrt{5100 / 3} = 41 \text{ cm}$$

i.e. the maximum chosen thickness of 40 cms is convenient.

Vertical tension steel on the inside surface of the wall at its base can be determined from equation (10), thus :

$A_s = 0.26\ t = 0.26 \times 41 = 10.7\ cm^2$ chosen $6\ \phi\ 16$ ($12\ cm^2$)

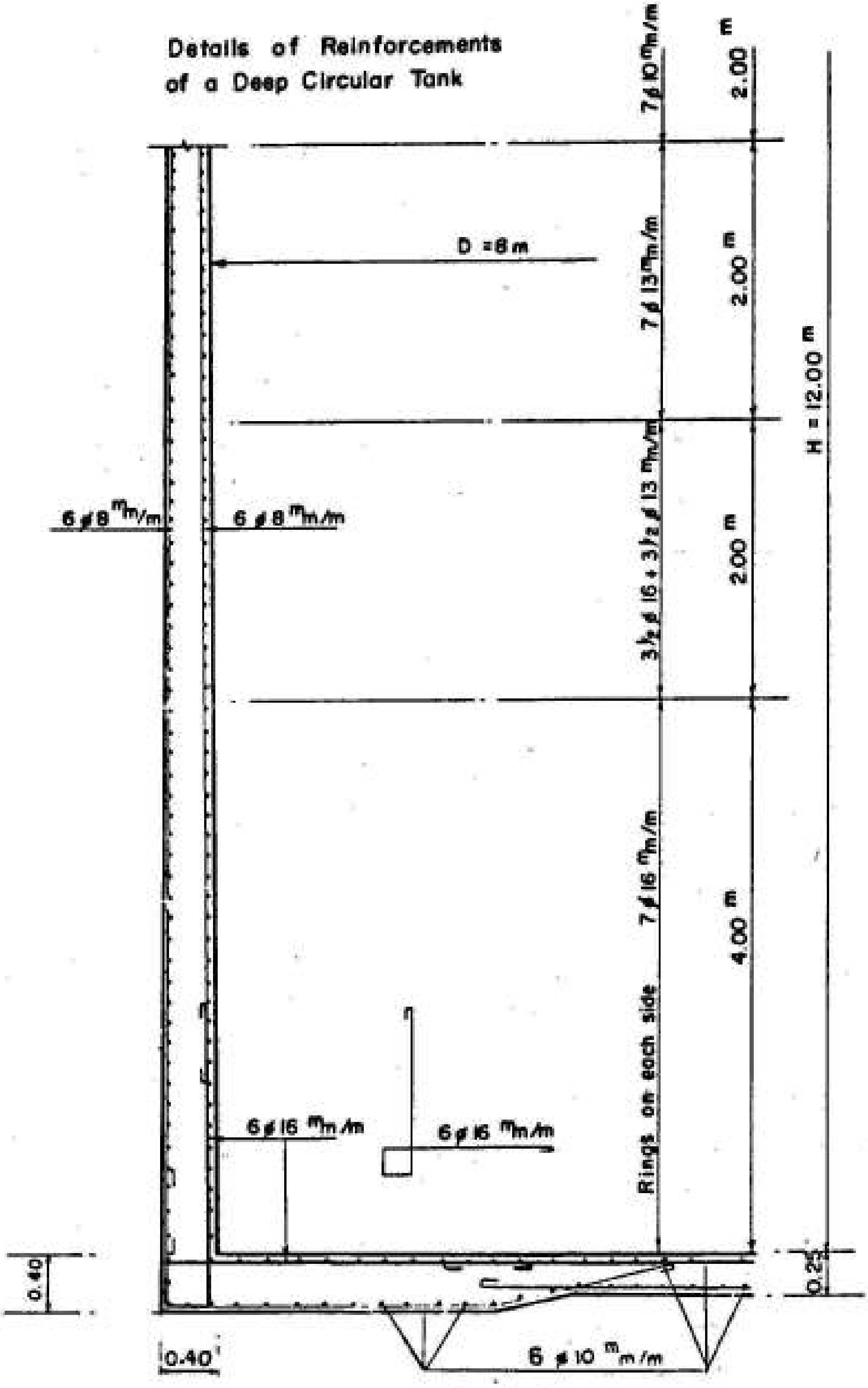


Fig. III-6

Other vertical reinforcements necessary to bind the rings and to resist shrinkage stresses and eventual field bending moments may be chosen $\geqslant$ 20 % of max. ring reinforcements and not less than $5 \phi 8$ mm/m.

In our case : $M^{+}_{max} \simeq M_f/5 = 5100/5 = 1020$ kg m

wall thickness at position of max. field moment $\simeq 33$ cms

$$A_s = \frac{1020}{1300 \times .30} = 2.6 \text{ cm}^2$$

20 % of max. rings ($7 \phi 16$ mm/m) $= 14 \times 0.2 = 2.8$ cm^2

Choose 6ϕ 8 mm/m on outside and inside surfaces of wall. The details of reinforcements in the lower part of the wall are shown in figure III.6 .

III.3. THEORY OF THICK CYLINDERS

If the thickness of the wall t of a circular tank is not small in proportion to the radius R, the stress σ_t due to T will not be uniformly distributed over the cross-section. Lamé has solved this problem in 1852 as follows : Fig. III.7

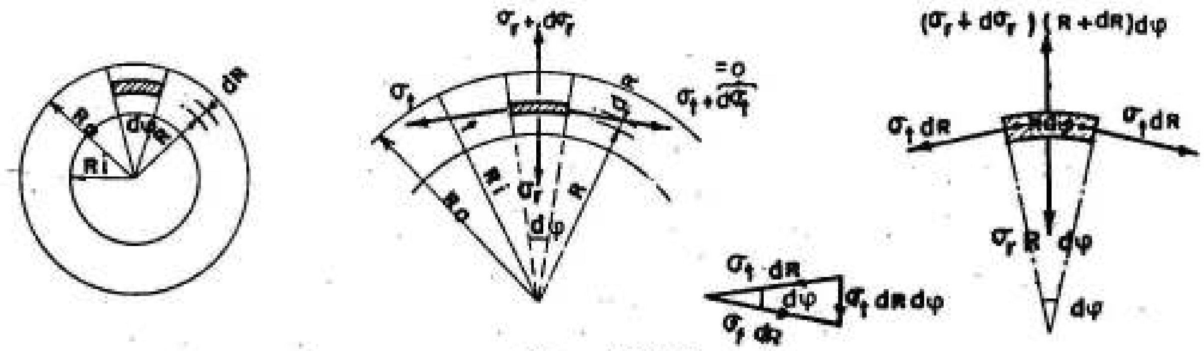


Fig. III-7

The stress distribution σ_t on the cross-section of a thick wall of a circular tank subject to hydrostatic pressure p can be determined if we consider the equilibrium of an element of a tank enclosing an angle $d\varphi$ as shown. Assume :

R = radius of any fiber inside the wall.

R_i = radius of inner face of wall.

R_o = radius of outer face of wall.

σ_r & ε_r = radial stress and strain

σ_t & ε_t = tangential stress and strain

$\vartheta = \frac{1}{m}$ = Poisson's ratio = $\frac{1}{5} \div \frac{1}{6}$ for concrete

Due to symmetry in shape and loading we have :

in tangential direction : $d\sigma_t = 0$ and

in radial direction : the shearing force $Q = 0$

Due to equilibrium in radial direction, we get the following relation :

$$(\sigma_r + d\sigma_r) (R + dR) \; d\varphi - \sigma_r R \, d\varphi - \sigma_t \, dR \, d\varphi = 0$$

$$\sigma_r R \, d\varphi + d\sigma_r R \, d\varphi + \sigma_r \, dR \, d\varphi + d\sigma_r \, dR \, d\varphi - \sigma_r R \, d\varphi -$$
$$- \sigma_t \, dR \, d\varphi = 0$$

Reducing by $d\varphi$ and neglecting $d\sigma_r \, dR$ being a small value of the second degree, we get :

$$d\sigma_r R + \sigma_r \, dR = \sigma_t \, dR \qquad \text{i.e.}$$

$$\sigma_t = \frac{d(\sigma_r R)}{dR} \tag{15}$$

According to theory of elasticity, one can express the relation between stresses and strains in the following manner :

$$\left.\begin{aligned} \varepsilon_r &= \frac{1}{E} (\sigma_r - \vartheta \sigma_t) \\ \varepsilon_t &= \frac{1}{E} (\sigma_t - \vartheta \sigma_r) \end{aligned}\right\} \tag{16}$$

These two equations give :

$$\left.\begin{aligned} \sigma_r &= \frac{E}{1 - \vartheta^2} (\varepsilon_r + \vartheta \varepsilon_t) \\ \sigma_t &= \frac{E}{1 - \vartheta^2} (\varepsilon_t + \vartheta \varepsilon_r) \end{aligned}\right\} \tag{17}$$

Due to the deformation of the wall of the tank caused by the hydrostatic pressure, the radius R will be increased by the radial displacemen y i.e. it will be (R + y). Therefore the strain in the radial direction ε_r is given by :

$$\varepsilon_r = \frac{dy}{dR} \tag{18}$$

We have further :

Circumference of tank before deformation	$= 2\pi R$
Circumference of tank after deformation	$= 2\pi (R + y)$
Increase in length of circumference	$= 2\pi y$

Strain in tangential direction

$$\varepsilon_t = 2\pi y / 2\pi R \qquad \text{or}$$

$$\varepsilon_t = y / R \tag{19}$$

Substituting the values of ε_r and ε_t as given by equations 18 & 19 in equations 17, we get :

$$\left.\begin{aligned} \sigma_r &= \frac{E}{1-\nu^2}\left(\frac{dy}{dR} + \nu\frac{y}{R}\right) \\ \sigma_t &= \frac{E}{1-\nu^2}\left(\frac{y}{R} + \nu\frac{dy}{dR}\right) \end{aligned}\right\} \tag{20}$$

Introducing these values in equation 15, we get :

$$\frac{y}{R} + \nu\frac{dy}{dR} = \frac{d}{dR}\left(R\frac{dy}{dR} + \nu y \right) \qquad \text{or}$$

$$\frac{y}{R} + \nu\frac{dy}{dR} = \frac{dy}{dR} + R\frac{d^2y}{dR^2} + \nu\frac{dy}{dR} \qquad \text{i.e.}$$

$$R^2\frac{d^2y}{dR^2} + R\frac{dy}{dR} - y = 0 \tag{21}$$

The solution of this differential equation is :

$$y = C_1 R + \frac{C_2}{R} \tag{22}$$

in which

C_1 and C_2 are the integration constants. Differentiating equation 22 with respect to R, we get :

y i.e. it will be (R + y). Therefore the strain in the radial direction ε_r is given by :

$$\varepsilon_r = \frac{dy}{dR} \qquad (18)$$

We have further :

Circumference of tank before deformation	=	$2\pi R$
Circumference of tank after deformation	=	$\underline{2\pi (R + y)}$
Increase in length of circumference	=	$2\pi y$

Strain in tangential direction

$$\varepsilon_t = 2\pi y / 2\pi R \qquad \text{or}$$

$$\varepsilon_t = y / R \qquad (19)$$

Substituting the values of ε_r and ε_t as given by equations 18 & 19 in equations 17, we get :

$$\left.\begin{aligned} \sigma_r &= \frac{E}{1-\nu^2}\left(\frac{dy}{dR} + \nu \frac{y}{R}\right) \\ \sigma_t &= \frac{E}{1-\nu^2}\left(\frac{y}{R} + \nu \frac{dy}{dR}\right) \end{aligned}\right\} \qquad (20)$$

Introducing these values in equation 15, we get :

$$\frac{y}{R} + \nu \frac{dy}{dR} = \frac{d}{dR}\left(R \frac{dy}{dR} + \nu y \right) \qquad \text{or}$$

$$\frac{y}{R} + \nu \frac{dy}{dR} = \frac{dy}{dR} + R \frac{d^2y}{dR^2} + \nu \frac{dy}{dR} \qquad \text{i.e.}$$

$$R^2 \frac{d^2y}{dR^2} + R \frac{dy}{dR} - y = 0 \qquad (21)$$

The solution of this differential equation is :

$$y = C_1 R + \frac{C_2}{R} \qquad (22)$$

in which

C_1 and C_2 are the integration constants. Differentiating equation 22 with respect to R, we get :

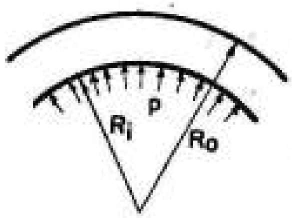
$$\frac{dy}{dR} = C_1 - \frac{C_2}{R^2} \quad \text{and} \quad \frac{d^2y}{dR^2} = \frac{2\,C_2}{R^3}$$

Substituting these values in the elements of equation 21, we get:

$$R^2 \frac{2\,C_2}{R^3} + R\,(C_1 - \frac{C_2}{R^2}) - (C_1 R + \frac{C_2}{R}) = \frac{2\,C_2}{R} + R\,C_1 - \frac{C_2}{R} - C_1 R - \frac{C_2}{R} = 0$$

which means that the solution is correct.

Determination of C_1 & C_2 Fig. III.8



i) For $R = R_i$ $\sigma_r = -p$

ii) For $R = R_o$ $\sigma_r = 0$ (no outside pressure)

Fig. III-8

therefore

$$\sigma_r = \frac{E}{1-\nu^2}\left(\frac{dy}{dR} + \nu\frac{y}{R}\right) = \frac{E}{1-\nu^2}\left(C_1 - \frac{C_2}{R^2} + \nu\,C_1 + \nu\frac{C_2}{R^2}\right)$$

$$= \frac{E}{1-\nu^2}\left[C_1\,(1+\nu) - \frac{C_2}{R^2}\,(1-\nu)\right]$$

Because of condition ii, we have

$$0 = C_1\,(1+\nu) - \frac{C_2}{R_o^2}\,(1-\nu) \qquad \text{and}$$

Because of condition i we have

$$-p = \frac{E}{1-\nu^2}\left[C_1\,(1+\nu) - \frac{C_2}{R_i^2}\,(1-\nu)\right]$$

These two equations give

$$C_1\,(1+\nu)\left(1 - \frac{R_i^2}{R_o^2}\right) = p\,\frac{1-\nu^2}{E}\,\frac{R_i^2}{R_o^2} \qquad \text{or}$$

$$C_1 = p\,\frac{1-\nu}{E}\,\frac{R_i^2}{R_o^2 - R_i^2} \tag{23}$$

and

$$C_2 = p\,\frac{1+\nu}{E}\,\frac{R_o^2\,R_i^2}{R_o^2 - R_i^2} \tag{24}$$

Therefore

$$\sigma_t = \frac{E}{1-\vartheta^2}\left(\frac{y}{R} + \vartheta\frac{dy}{dR}\right) = \frac{E}{1-\vartheta^2}\left[C_1 + \frac{C_2}{R^2} + \vartheta\left(C_1 - \frac{C_2}{R^2}\right)\right]$$

$$= \frac{E}{1-\vartheta^2}\left[C_1(1+\vartheta) + \frac{C_2}{R^2}(1-\vartheta)\right]$$

$$= \frac{E}{1-\vartheta^2}\left[p\,\frac{1-\vartheta}{E}\;\frac{R_i^2}{R_o^2-R_i^2}(1+\vartheta) + \frac{p}{R^2}\;\frac{1+\vartheta}{E}\;\frac{R_o^2\;R_i^2}{R_o^2-R_i^2}(1-\vartheta)\right]$$

or

$$\underline{\sigma_t = p\,\frac{R_i^2}{R_o^2 - R_i^2}\left(1 + \frac{R_o^2}{R^2}\right)} \qquad (25) \text{ Hyperbola}$$

Which gives the formula of Lamé for stress distribution on the cross-section of thick cylinders subject to internal pressure p. Therefore the stress at the inner surface of the tank :

$R = R_i$ is :

$$\underline{\sigma_{ti} = p\,\frac{R_o^2 + R_i^2}{R_o^2 - R_i^2}} \qquad (25a)$$

and at the outer surface : $R = R_o$

$$\underline{\sigma_{to} = p\,\frac{2\,R_i^2}{R_o^2 - R_i^2}} \qquad (25b)$$

Assuming $R_o = \lambda R_i$ we get :

$$\sigma_{ti} = p\,\frac{\lambda^2+1}{\lambda^2-1}, \qquad \sigma_{to} = p\,\frac{2}{\lambda^2-1} \quad \text{and} \quad \frac{\sigma_{ti}}{\sigma_{to}} = \frac{\lambda^2+1}{2}$$

For $\lambda = 1$ $\frac{\sigma_{ti}}{\sigma_{to}} = 1$

$\lambda = 1.2$ " " $= 1.22$ difference not big

$\lambda = 2$ " " $= 2.5$

The stresses are graphically represented in figure III.9

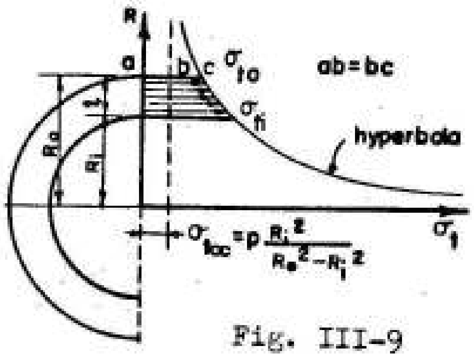


Fig. III-9

For $R = \infty$ $\quad \sigma_{t\infty} = p \dfrac{R_i^2}{R_o^2 - R_i^2} = 1/2\ \sigma_{to}$

This means that the stresses are bigger at the inside surface of the tank. In reinforced concrete circular walls with one mesh , the ring reinforcement is to be placed nearer to the inner surface of the wall and in walls with double meshes the inner rings may be bigger than the outer ones.

In reinforced concrete circular tanks $R_o < 1.2\ R_i$ (i.e. $\lambda < 1.2$) which means that the error by use of cylinder formula ($\sigma_t = T/t$) is less than 10 % .

For very thin cylinders $R_i \simeq R_o \simeq R$, the formula of Lamé gives the same result as the cylinder formula.

$$\sigma_t = p \frac{R_i^2}{\underbrace{(R_o + R_i)}_{2R_i}\ \underbrace{(R_o - R_i)}_{t}} \left(1 + \frac{R_o^2}{R^2}\right) = p\ \frac{R_i^2}{2\ R_i\ t} \times 2$$

$$= p\ \frac{R_i}{t} = \frac{T}{t}$$

Whereas in relatively thick plain concrete circular walls, the average tensile stresses calculated according to the cylinder formula may give serious not allowed errors.

III.4. TANK WALLS FIXED TO FLOOR

III.4.1 Theory of Reissner and Lewe $\quad$ Fig. III.10

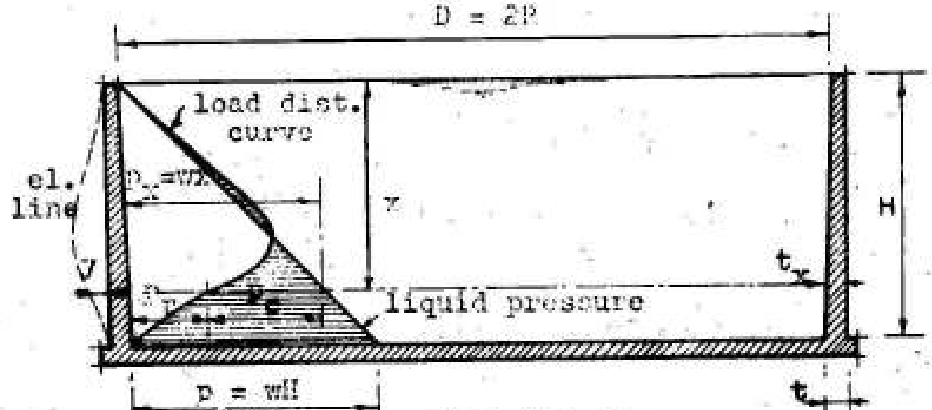


Fig. III-10

The liquid pressure p_x at any depth x will be counteracted by the combined resistance of rings and cantilevers, thus :

$$p_x = w\,x = p_r + p_c \qquad (26)$$ in which

p_r = the part of the liquid pressure resisted in the horizontal direction by ring action.

p_c = the part of the liquid pressure resisted in the vertical direction by cantilever action.

The value of p_r can be determined as follows :

Strain in tangential direction $\varepsilon_t = \frac{y}{R}$ (refer to equation 19)

Stress in tangential direction $\sigma_t = \varepsilon_t E = \frac{y}{R} E$ (27)

Ring tension $T = \sigma_t t_x = \frac{y}{R} t_x E$ (28)

Pressure resisted by ring action $p_r = \frac{T}{R} = \frac{y}{R^2} t_x E$ (29)

This equation means that if we know the equation of the elastic line of the wall, we can find p_r and respectively p_c . The equation of the elastic line can be determined as follows :

It is known from the general theory of elasticity that the relation between the external force q, the shearing force Q , the bending moment M and the deflection y are :

$$\frac{d^2y}{dx^2} = -M / EI, \quad \frac{dM}{dx} = Q, \quad \frac{dQ}{dx} = -q \quad \& \quad \frac{d^4y}{dx^4} = q / EI \qquad (30)$$

in which q = force acting on wall in radial direction (Fig. III.11).

$q = w\,x\,R\,d\varphi - \sigma_t t_x\,d\varphi$ but $\sigma_t = \frac{y}{R} E$ then

$q = w\,x\,R\,d\varphi - \frac{y}{R} E\,t_x\,d\varphi = (w\,x\,R - \frac{y}{R} t_x E)\,d\varphi$ but

$$I = \frac{R\,d\varphi\,t_x^3}{12}$$

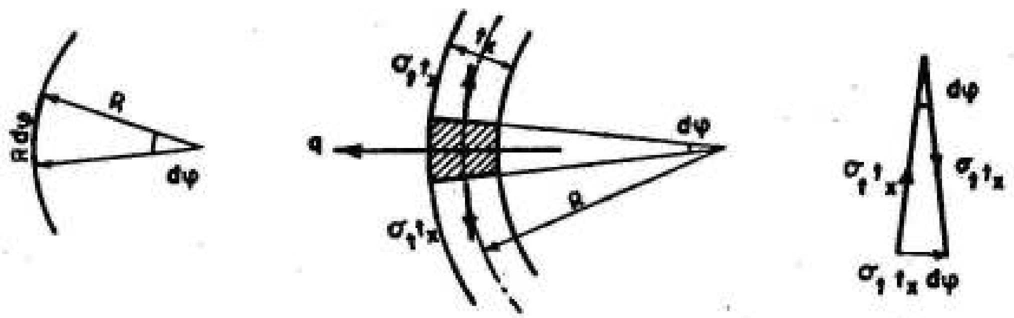


Fig. III-11

Substituting these values in the differential equation of the elastic line - 30 - we get :

$$E \frac{R\, d\varphi\, t_x^3}{12} \cdot \frac{d^4 y}{dx^4} = \left(w\, x\, R - \frac{y}{R}\, t_x\, E \right) d\varphi \qquad \text{or}$$

$$\frac{R^2\, t_x^2}{12} \cdot \frac{d^4 y}{dx^4} = \frac{w\, R^2}{E\, t_x}\, x - y$$

Assuming a tank with constant wall thickness t and putting

$$K = \frac{12\, H^4}{R^2\, t^2}, \quad n = \sqrt[4]{\frac{K}{4}} \quad \text{and} \quad \xi = \frac{x}{H} \qquad (31) \qquad \text{we get:}$$

$$\frac{H^4}{4\, n^4} \cdot \frac{d^4 y}{dx^4} + y - \frac{w\, R^2}{E\, t} \cdot x = 0 \qquad (32)$$

This equation gives the final form of the differential equation of the elastic line. Its solution gives the value of y and respectively p_r and p_c or the ring tension T and the cantilever moment M.

The solution of the differential equation 32 is given by :

$$y = \frac{w\, R^2}{E\, t} \cdot x + \left(A_1\, e^{n\xi} + A_2\, e^{-n\xi} \right) \cos n\xi + \left(B_1\, e^{n\xi} + B_2\, e^{-n\xi} \right) \sin n\xi \qquad (33)$$

in which A_1 , A_2 , B_1 & B_2 are 4 integration constants to be determined from the conditions at the supports. The derivatives of y are given by :

$$\frac{H}{n} \cdot \frac{dy}{dx} = (A_1\, e^{n\xi} - A_2\, e^{-n\xi}) \cos n\xi - (A_1\, e^{n\xi} + A_2\, e^{-n\xi}) \sin n\xi$$

$$+ (B_1\, e^{n\xi} - B_2\, e^{-n\xi}) \sin n\xi + (B_1\, e^{n\xi} + B_2\, e^{-n\xi}) \cos n\xi$$

$$+ \frac{w\, R^2\, H}{E\, t\, n}$$

$$\frac{H^2}{2\, n^2} \cdot \frac{d^2y}{dx^2} = - (A_1\, e^{n\xi} - A_2\, e^{-n\xi}) \sin n\xi + (B_1\, e^{n\xi} - B_2\, e^{-n\xi}) \cos n\xi$$

$$\frac{H^3}{2\, n^3} \cdot \frac{d^3y}{dx^3} = - (A_1\, e^{n\xi} + A_2\, e^{-n\xi}) \sin n\xi - (A_1\, e^{n\xi} - A_2\, e^{-n\xi}) \cos n\xi$$

$$+ (B_1\, e^{n\xi} + B_2\, e^{-n\xi}) \cos n\xi - (B_1\, e^{n\xi} - B_2\, e^{-n\xi}) \sin n\xi$$

$$\frac{H^4}{4\, n^4} \cdot \frac{d^4y}{dx^4} = - (A_1\, e^{n\xi} + A_2\, e^{-n\xi}) \cos n\xi - (B_1\, e^{n\xi} + B_2\, e^{-n\xi}) \sin n\xi$$

$$= - (y - w\, \frac{R^2}{E\, t}\, x)$$

which means that the solution is correct.

The integration constants for an open tank fixed at the base can be determined from the following conditions :

a) at base $x = H$ $\qquad y = 0\, ,\ \dfrac{dy}{dx} = 0$

b) at top $x = 0$ $\qquad M = Q = 0\, ,\ \dfrac{d^2y}{dx^2} = \dfrac{d^3y}{dx^3} = 0$

From condition b : $M \propto \dfrac{d^2y}{dx^2} = 0$ or $\qquad B_1 - B_2 = 0$
and

$$Q \propto \frac{d^3y}{dx^3} = 0 \quad \text{or} \quad (A_1 - A_2) + (B_1 + B_2) = 0$$

From these relations we get $B_1 = B_2$ and $A_1 = A_2 - 2\, B_1$, thus

$$y = \left[A_1 e^{n\xi} + (A_1 - 2B_1) e^{-n\xi}\right] \cos n\xi + B_1 (e^{n\xi} + e^{-n\xi})$$

$$\sin n + w \frac{R^2}{E\,t} x \,,$$ or

$$y = A_1 (e^{n\xi} + e^{-n\xi}) \cos n\xi + B_1 \left[(e^{n\xi} + e^{-n\xi}) \sin n\xi - 2 e^{-n\xi} \cos n\xi \right] + \frac{w R^2}{E\,t} x$$

and

$$\frac{H}{n} \cdot \frac{dy}{dx} = A_1 \left[(e^{n\xi} - e^{-n\xi}) \cos n\xi - (e^{n\xi} + e^{-n\xi}) \sin n\xi \right]$$
$$+ B_1 \left[(e^{n\xi} - e^{-n\xi}) \sin n\xi + (e^{n\xi} + e^{-n\xi}) \cos n\xi + 2 e^{-n\xi} \cos n\xi + e^{-n\xi} \sin n\xi \right] + \frac{w R^2 H}{E\,t\,n}$$

at the base $x = H$, $\xi = 1$, $y = 0$, $\frac{dy}{dx} = 0$ and $e^{-n} = e^{-n\xi} =$ small value

Assuming $\frac{H}{R} \approx 1$ $\frac{H}{t} > 10$ $\therefore$ $n = \sqrt[4]{3 \times 1 \times 100} = 4.2$

$e^{-4.2} = 0.018$, neglected !

$$y = 0 = A_1 e^{n} \cos n + B_1 e^{n} \sin n + \frac{w R^2 H}{E\,t}$$ and

$$\frac{H}{n} \cdot \frac{dy}{dx} = 0 = A_1 e^{n} (\cos n - \sin n) + B_1 e^{n} (\sin n + \cos n) + \frac{w R^2 H}{E\,t\,n}$$

The solution of these two equations gives :

$$\left.\begin{aligned} A_1 &= - \frac{w R^2 H}{E\,t\,e^{n}} \left(\cos n + \sin n - \frac{\sin n}{n}\right) \\ B_1 &= + \frac{w R^2 H}{E\,t\,e^{n}} \left(\cos n - \sin n - \frac{\cos n}{n}\right) \end{aligned}\right\} \qquad (34)$$

Substituting these constants in the equation of y, we can find the ring tension and respectively the load distribution.

The second derivative of y gives the bending moment because

$\frac{d^2y}{dx^2} = - \frac{M}{E\,I}$. The moment at the foot can be determined from the relation :

$$\frac{H^2}{2\,n^2} \cdot \frac{d^2y}{dx^2} = - A_1\, e^{n\xi} \sin n\xi + B_1\, e^{n\xi} \cos n\xi$$

For x = H , we get :

$$M_f = \frac{2\,n^2}{H^2}\, E\, I\, e^{n\xi}\, (A_1 \sin n - B_1 \cos n)$$

$$= \frac{2\,n^2}{H^2}\, E\, I \left[- \frac{w\,R^2\,H}{E\,t} (\cos n + \sin n - \frac{\sin n}{n}) \sin n \right]$$

$$- \left[\frac{w\,R^2\,H}{E\,t} (\cos n - \sin n - \frac{\cos n}{n}) \cos n \right] \quad \text{or}$$

$$M_f = - 2\,n^2\, I. \quad \frac{w\,R^2}{t\,H} (\cos n \sin n + \sin^2 n - \frac{\sin^2 n}{n}$$

$$+ \cos^2 n - \sin n \cos n - \frac{\cos^2 n}{n}) \quad \text{or}$$

$$M_f = - 2\,n^2\, I\, \frac{w\,R^2}{t\,H} (1 - \frac{1}{n}) \quad \text{but} \quad I = \frac{t^3}{12} \quad \text{then}$$

$$M_f = - 2\,n^2 \cdot \frac{t^3}{12} \cdot \frac{w\,R^2}{t\,H} \cdot \frac{n - 1}{n} \quad \text{or}$$

$$\underline{M_f = - \frac{w\,R^2\,t^2}{12\,H}\, 2\,n\,(n - 1)} \qquad (35)$$

Calculating the max. shearing force at the base from the relation

$Q = - E\, I\, \frac{d^3y}{dx^3}$, we get :

$$\underline{Q_{max} = \frac{w\,R^2\,t^2}{12\,H^2} \cdot 2\,n^2\,(2\,n - 1)} \qquad (36)$$

In the same way other cases can be calculated according to their edge conditions e.g.

in which $\lambda = \frac{12\ H^5}{E\ t^3}\ w$ and $\xi = \frac{x}{H}$

In order to use these curves proceed as follows :

1) Determine the value of n from equation 37, namely

$$n = 0.5 + \sqrt{\frac{\sigma_{tb}\ H}{R^2\ w} + 0.25}$$ in which

σ_{tb} = 18 kg/cm^2 for normal cases and w = 0.001 kg/cm^3 for water .

2) Calculate the thickness at the foot of the wall from equation 38 which gives

$$t = 1.73\ \frac{H^2}{R\ n^2}$$

then choose a convenient profile for the wall.

3) To determine the ring tension in the wall draw for the calculated value of (n) a vertical line to meet the projection line for interpolation in a point - a -; through - a - draw a horizontal line to meet the maxima curve in - b -. This last point - b - gives the apex of the distribution curve if the wall were rectangular. For trapezoidal walls, linear interpolation is sufficient. Having drawn the load distribution curve, the ring tension in every section can be determined from the relation :

$$T = C_o\ p\ R$$

4) To draw the bending moment diagram, it is recommended to determine the maximum fixing moment M_f from equation 35, namely :

$$M_f = - \frac{w\ R^2\ t^2}{12\ H} \cdot 2\ n\ (\ n - 1\ )$$

In the same way as given under (3) determine the apex of the positive part of the bending moment diagram for a rectangular wall. If the wall were trapezoidal multiply the maximum positive value by $(\ t_x\ /\ t\ )^3$ where t_x is the thickness of the wall at position of maximum positive moment. The curves show that the cantilever action increases for diminishing values of K (or n) i.e. the smaller the height, the bigger the radius and the stiffer the wall. By $R = \infty$, $K = 0$ (the limiting case of a straight cantilever) the water pressure is resisted totally by cantilever action.

Example :

It is required to design the wall of a cylinderical reinforced concrete open water tank, 5.0 ms deep and 20 ms diameter. Assume the wall to be fixed to the floor.

Solution :

$H = 5.00$ ms $\qquad R = 10.00$ ms $\qquad$ Assume $\sigma_{tb} = 18 \ kg/cm^2$

& $w = 0.001 \ kg/cm^2$

$$n = 0.5 + \sqrt{\frac{\sigma_{tb}}{R^2}\frac{H}{w} + 0.25} = 0.5 + \sqrt{\frac{18 \times 500}{1000^2 \times .001} + 0.25} = 3.53$$

$$t = 1.73 \frac{H^2}{R \ n^2} = 1.73 \frac{500^2}{1000 \times 3.53^2} \quad \text{or} \quad t = 35 \text{ cms}$$

Choose thickness at top 20 cms and at bottom 35 cms.

From the curves of Reissner (Fig. III.13), we find :

for $n = 3.53$, $\xi = 0.5$ and $C_o = 0.425$ at T_{max}, thus

$T_{max} = C_o \, w \, H \, R = 0.425 \times 1 \times 5 \times 10 = 21.25$ tons/m

acting at 2.5 ms from top surface.

The max. ring reinforcement is given by :

$$\max. A_s = \frac{T_{max}}{\sigma_s} = \frac{21.25}{1.4} = 15.2 \ cm^2/m$$

chosen 12 ϕ 13 mm/m ($A_s = 15.9 \ cm^2$) on both sides i.e. 6 ϕ 13 mm/m on each side. Thickness of wall at position of max. ring tension is given by :

$$t_x = \frac{20 + 35}{2} = 27.5 \text{ cms}$$

$$M+_{max} = 7.1 \times \frac{1 \times 10^2 \times 0.25^2}{12 \times 5} = 0.74 \text{ m t}$$

$$A_s = \frac{740}{1300 \times 0.22} = 2.6 \text{ cm}^2 \quad \text{chosen } 6\,\phi\,8 \text{ mm/m}$$

The details of reinforcement are shown in Fig. III.14

III.4.2 <u>Simplified Methods for Determining the Fixing Moment, the Shearing Force and the Thickness of the Wall at the base.</u>

It has been proved in equation 38 that $t = 1.73 \frac{H^2}{R\,n^2}$ from which one can determine the value of n, thus :

$$n = \frac{1.315\,H}{\sqrt{R\,t}} \qquad (39)$$

The max. fixing moment is given according to equation 35 by the relation :

$$M_f = -\frac{w\,R^2\,t^2}{12\,H} \cdot 2\,n\,(\,n - 1\,)$$

Substituting for t the value given in equation 38, we get :

$$M_f = -\frac{w\,R^2\,t^2}{12\,H}\left(1.73\,\frac{H^2}{R\,n^2}\right) 2\,n\,(n-1)$$

$$= -\frac{w\,H\,(2\,R)\,t}{6.95} \cdot \frac{n-1}{n}$$

Assuming $C_1 = \frac{6.95}{1 - \frac{1}{n}}$,

we get $$M_f = -\frac{w\,H\,D\,t}{C_1} \qquad (40)$$

The shearing force at the base of the wall is given accor - ding to equation 36 by the relation :

$$Q_{max} = \frac{w\,R^2\,t^2}{12\,H^2} \cdot 2\,n^2\,(\,2\,n - 1\,)$$

But according to equation 38 $t^2 = \frac{3\,H^4}{R^2\,n^4}$

Substituting this value of t^2 in the equation of Q_{max}, we get :

$$Q_{max} = \frac{w\,R^2}{6\,H^2} \cdot \frac{3\,H^4}{R^2\,n^4} \cdot n^2\,(2\,n - 1) = \frac{w\,H^2}{2} \cdot \frac{2\,n - 1}{n^2}$$

Assuming $\frac{2\,n - 1}{n^2} = C_2$ and substituting, we get :

$$\underline{Q_{max} = C_2\,\frac{w\,H^2}{2}} \qquad (41)$$

It is further known that the bending tensile stress at the foot of the wall is given by : $\sigma_{tb} = \frac{6\,M_f}{t^2}$

Assuming $\sigma_{tb} = 18$ kg/cm^2 = 180 t/m^2 and substituting for M_f the value given in equation 40, we get :

$$180 = \frac{6\,w\,H\,D\,t}{C_1\;t^2} = \frac{12\,w\,H\,R}{C_1\,t} \quad \text{or} \quad t = \frac{w\,H\,R}{15\,C_1}$$

Assuming further $\frac{1}{15\,C_1} = C_3'$ then $t = C_3'\,w\,H\,R$

In order to get t in cms for w in t/m^3 (equals 1 for water), H and R in meters, we should have :

$$\underline{t = C_3\,w\,H\,R} \qquad (42)$$

in which $C_3 = 100\,C_3' = \frac{100}{15\,C_1} = \frac{6.66}{C_1}$

The values of M_f, Q_{max} and t at the base of a wall fixed to the floor as given by equations 40, 41 and 42 depend on the value of n which in turn depends on the magnitude of the thickness t. In this manner a method of trial and error is to be applied i.e. we have first to assume a convenient value for t, then determine the corresponding value of n according to equation 39. Applying equation 42, it is possible to check the assumed value of t.

It is however possible to avoid these trials in special cases e.g. water tanks in the following manner. Equation 37 gives :

$$Q_{max} = C_2 \frac{w\,H^2}{2} = 0.211 \times \frac{1 \times 10^2}{2} = 10.5 \text{ t}$$

According to Gray : Fig. III.16

Average thickness of wall $t = 31$ cms

$$\frac{H}{t} = \frac{10}{0.31} = 32 \,, \quad \frac{H}{D} = \frac{10}{10} = 1, \quad C_o = 0.8 \,, \quad C = .225$$

$$T_{max} = C_o\, w\, H\, R = 0.8 \times 1 \times 10 \times 5 = 40 \text{ t}$$

$$x' = C\,H = 0.225 \times 10 = 2.25 \text{ ms from base}$$

2) Depth $H = 5$ m , diameter $D = 20$ m $w = 1$ t/m^3

For section at base $\frac{H}{R^2} = \frac{5}{10^2} = 0.05$ medium tank

According to table $C_1 = 9.7$ $C_3 = 0.685$

$$t_{max} = C_3\, w\, H\, R = 0.685 \times 1 \times 5 \times 10 = 34.25 \text{ cms chosen } 35 \text{ cms}$$

$$M_f = - \frac{w\,H\,D\,t_{max}}{C_1} = - \frac{1 \times 5 \times 20 \times .35}{9.7} = - 3.6 \text{ m.t.}$$

$$M_{max}^{+} = \frac{M_f}{5} = \frac{3.6}{5} = 0.72 \text{ m.t.}$$

Tha wall will be chosen of constant thickness 25 cms and provided with a haunch at the base of 10 x 40 cms. The max. ring tension and its position can be determined according to curves of Gray, thus :

$$\frac{H}{t} = \frac{5.00}{0.25} = 20 \,, \quad \frac{H}{D} = \frac{5}{20} = 0.25 \quad \therefore \quad C_o = 0.51, \; C = 0.47$$

$$T_{max} = C_o\, w\, H\, R = 0.51 \times 1 \times 5 \times 10 = 25.5 \text{ tons at } 2.35 \text{ ms from base}$$

The shear at the base for a wall 25 cms thick can be calculated as follows :

$$n = \frac{1.315\,H}{\sqrt{R\ t}} = \frac{1.315 \times 5}{\sqrt{10 \times 25}} = 4.16$$

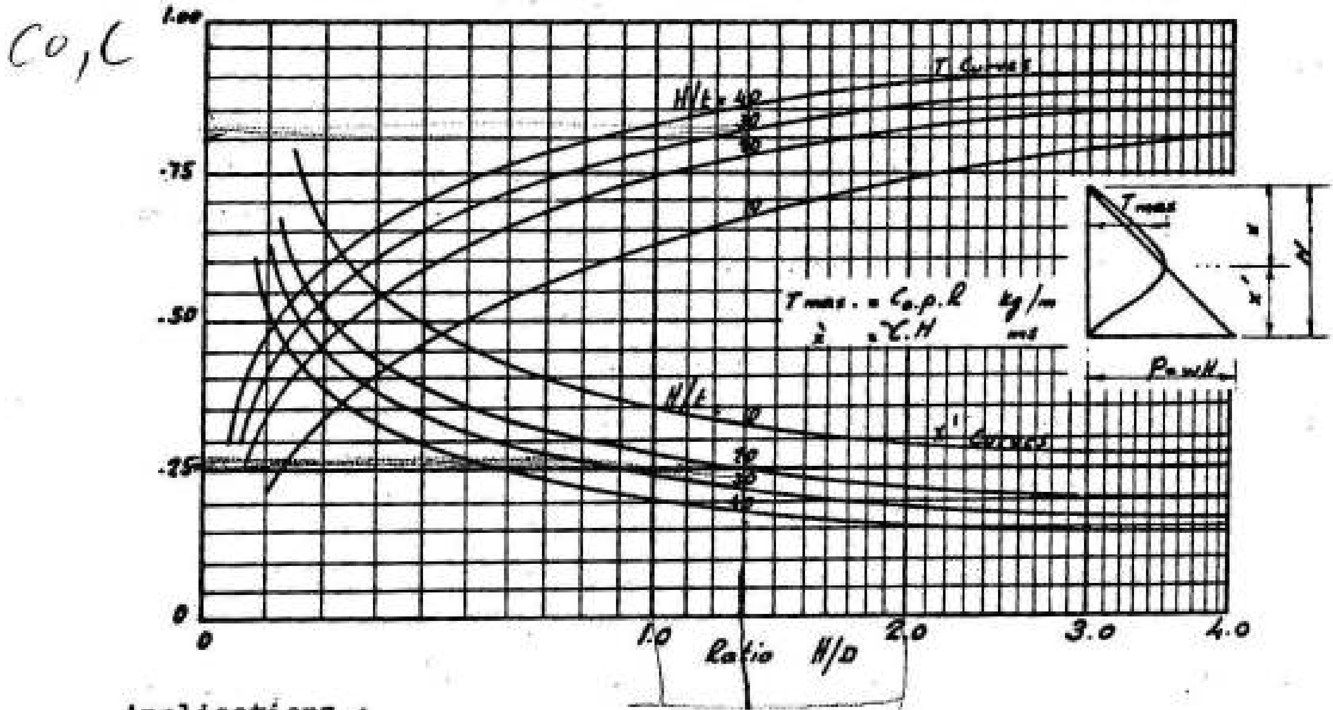
The corresponding value of C_2 is given by 0.425 and

$$Q_{max} = C_2 \frac{w\,H^2}{2} = 0.425 \times \frac{5^2}{2} = 5.3 \text{ tons}$$

been given by W.S. Gray in his book " Reinforced Concrete Water Towers, Bunkers, Silos and Gantries " in the following curves (Fig. III.16):

VALUES OF MAX RING-TENSION & ITS POSITION

Fig. III-16



Applications :

We show in the following examples, the direct application of the given simplified method for determining the internal forces in open circular water tanks with walls fixed at base.

1) Depth H = 10^m , Diameter D = 10 m , $w = 1\ t/m^3$

$$H / R^2 = 10 / 25 = 0.4$$

the corresponding n = 9 i.e. case of deep tank.

According to table $C_1 = 7.8$ $C_2 = 0.211$ $C_3 = 0.855$

Wall is chosen trapezoidal : 20 cms thick at its top end, the thickness at the base is given by :

$t_{max} = C_3\ w\ H\ R = 0.855 \times 1 \times 10 \times 5 = 42.75$ cms chosen 42 cm.

$$M_f = - \frac{w\ H\ D\ t_{max}}{C_1} = - \frac{1 \times 10 \times 10 \times 42}{7.8} = 5.40 \text{ m.t,}$$

$$M_{max} = M_f / 5 = 1.08 \text{ m.t.}$$

The use of this method for determining the internal forces in walls of circular tanks is very convenient, because when one is used to it, he can easily estimate the coefficients C_1 required for determining the fixing moment, C_3 required for determining the max. thickness of the wall, C_o required for determining the max. ring tension without appreciable errors which may affect the design as can be seen from the following example :

3) It is required to design the wall of an open circular tank 7 ms deep and 15 ms diameter. Make a quick estimate and check the results:

a) <u>Estimate</u> :

This tank is a medium one, its diameter $\simeq$ twice its depth then the max. ring tension may be estimated by :

$T_{max} = 0.65\ w\ H\ R = 0.65 \times 1 \times 7 \times 7.5 = 34$ ton

Wall thickness (equation 7)

$t = 0.8\ T_{max} = 0.8 \times 34 = 27$ cms

Max. ring reinforcements

$A_{s_{max}} = \dfrac{T_{max}}{\sigma_s} = \dfrac{34}{1.4} = 24.5\ cms'$ 6 ϕ 16 mm/m on each side

Max. thickness at base :

$t_{max} = 0.8\ w\ H\ R = 0.8 \times 1 \times 7 \times 7.5 = 42$ cms

The wall is chosen 27 cms and provided with a haunch 15 x 50 cms at the base.

The fixing moment :

$$M_f = - \frac{w\ H\ D\ t_{max}}{8} = - \frac{1 \times 7 \times 15 \times .42}{8} = - 5.50 \text{ m.t.}$$

Thickness required to resist this moment safely

$t_{max} = \sqrt{M_f / 3} = \sqrt{5500 / 3} = 42.7$ cms

i.e. chosen thickness of 42 cms is convenient

Vertical tension steel required at base :

$$A_s = \frac{M}{k_2\, d} = \frac{5500}{1300 \times .385} = 11\ cm^2$$ 6 ϕ 16/m

Max. field moment

$$M+_{max} = M_f\ /\ 5 = 5500\ /\ 5 = 1100\ kgm$$

Vertical reinforcement required

$$A_s = \frac{M+_{max}}{k_2\, d} = \frac{1100}{1300 \times .24} = 3.50\ cm^2$$ 7 ϕ 8 mm/m

<u>Check</u> :

Section at base $\frac{H}{R^2} = \frac{7}{7.5^2} = 0.125$

According to table $C_3 = 0.77$

$$t_{max} = C_3\ w\ H\ R = 0.77 \times 1 \times 7 \times 7.5 = 40.5\ cm$$

Chosen thickness of 42 cms is convenient.

The corresponding value of n is given by :

$$n = \frac{1.315\ H}{\sqrt{R\ t_{max}}} = \frac{1.315 \times 7}{\sqrt{7.5 \times 42}} = 5.20 \quad \text{giving} \quad C_1 = 8.65$$

So that :

$$M_f = \frac{-\ w\ H\ D\ t}{8.65} = -\frac{7 \times 15 \times .42}{8.65} = 5.10\ \ mt$$

The thickness required to resist this moment is given by :

$$t_{max} = \sqrt{\frac{5100}{3}} = 41\ cms < 42\ cms$$

According to Gray : (t = 27 cms)

$$\frac{H}{t} = \frac{7}{.27} = 26,\ \frac{H}{D} = \frac{7}{15} = 0.465\ ,\ C_o = 0.65\ ,\ C = .34$$

$$T_{max} = C_o\ w\ H\ R = 0.65 \times 1 \times 7 \times 7.5 = 34\ t$$ at

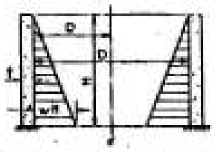
$$x' = C\ H = 0.34 \times 7 = 2.4\ ms$$ from base

The internal forces are shown in figure III.17

The details of reinforcements are similar to those shown in figure III.14

Table I

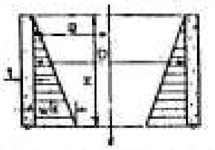
Tension in circular rings
Triangular load
Fixed base, free top
T = coef. × wHR
Positive sign indicates tension



$\frac{H^2}{Dt}$	Coefficients at point									
	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	+0.149	+0.134	+0.120	+0.101	+0.082	+0.066	+0.049	+0.029	+0.014	+0.004
0.8	+0.263	+0.239	+0.215	+0.190	+0.160	+0.130	+0.096	+0.063	+0.034	+0.010
1.2	+0.283	+0.271	+0.254	+0.234	+0.209	+0.180	+0.142	+0.099	+0.054	+0.016
1.6	+0.265	+0.268	+0.268	+0.266	+0.250	+0.226	+0.185	+0.134	+0.075	+0.023
2.0	+0.234	+0.251	+0.273	+0.285	+0.285	+0.274	+0.232	+0.172	+0.104	+0.031
3.0	+0.134	+0.203	+0.267	+0.322	+0.357	+0.362	+0.330	+0.262	+0.157	+0.052
4.0	+0.067	+0.164	+0.256	+0.339	+0.403	+0.429	+0.409	+0.334	+0.210	+0.073
5.0	+0.025	+0.137	+0.245	+0.346	+0.428	+0.477	+0.469	+0.398	+0.259	+0.092
6.0	+0.018	+0.119	+0.234	+0.344	+0.441	+0.504	+0.514	+0.447	+0.301	+0.112
8.0	-0.011	+0.104	+0.218	+0.335	+0.443	+0.534	+0.575	+0.530	+0.381	+0.151
10.0	-0.011	+0.098	+0.208	+0.323	+0.437	+0.542	+0.608	+0.589	+0.440	+0.179
12.0	-0.005	+0.097	+0.202	+0.312	+0.429	+0.543	+0.628	+0.633	+0.494	+0.211
14.0	-0.002	+0.098	+0.200	+0.306	+0.420	+0.539	+0.639	+0.666	+0.541	+0.241
16.0	0.000	+0.099	+0.199	+0.304	+0.412	+0.531	+0.641	+0.687	+0.582	+0.265

Table II

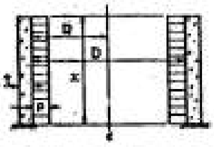
Tension in circular rings
Triangular load
Hinged base, free top
T = coef. × wHR
Positive sign indicates tension



$\frac{H^2}{Dt}$	Coefficients at point									
	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	+0.474	+0.440	+0.395	+0.352	+0.308	+0.264	+0.215	+0.165	+0.111	+0.057
0.8	+0.423	+0.402	+0.381	+0.358	+0.330	+0.297	+0.249	+0.202	+0.145	+0.076
1.2	+0.350	+0.355	+0.361	+0.362	+0.358	+0.343	+0.309	+0.256	+0.186	+0.098
1.6	+0.271	+0.303	+0.341	+0.369	+0.385	+0.385	+0.362	+0.314	+0.233	+0.124
2.0	+0.205	+0.260	+0.321	+0.373	+0.411	+0.434	+0.419	+0.369	+0.280	+0.151
3.0	+0.074	+0.179	+0.281	+0.375	+0.449	+0.506	+0.519	+0.479	+0.375	+0.210
4.0	+0.017	+0.137	+0.253	+0.367	+0.469	+0.545	+0.579	+0.553	+0.447	+0.256
5.0	-0.008	+0.114	+0.235	+0.356	+0.469	+0.562	+0.617	+0.606	+0.503	+0.294
6.0	-0.011	+0.103	+0.223	+0.343	+0.463	+0.566	+0.639	+0.643	+0.547	+0.327
8.0	-0.015	+0.096	+0.208	+0.324	+0.443	+0.564	+0.661	+0.697	+0.621	+0.386
10.0	-0.008	+0.095	+0.200	+0.311	+0.428	+0.552	+0.666	+0.730	+0.678	+0.433
12.0	-0.002	+0.097	+0.197	+0.302	+0.417	+0.541	+0.664	+0.750	+0.720	+0.477
14.0	0.000	+0.098	+0.197	+0.299	+0.408	+0.531	+0.659	+0.761	+0.752	+0.513
16.0	+0.002	+0.100	+0.198	+0.299	+0.403	+0.521	+0.650	+0.764	+0.776	+0.536

Table III

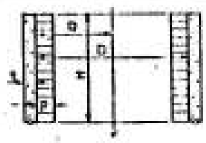
Tension in circular rings
Rectangular load
Fixed base, free top
T = coef. × pR
Positive sign indicates tension



$\frac{H^2}{Dt}$	Coefficients at point									
	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	+0.582	+0.505	+0.431	+0.353	+0.277	+0.206	+0.145	+0.092	+0.046	+0.013
0.8	+1.052	+0.921	+0.796	+0.669	+0.542	+0.415	+0.289	+0.179	+0.089	+0.024
1.2	+1.218	+1.078	+0.946	+0.808	+0.665	+0.519	+0.378	+0.246	+0.127	+0.034
1.6	+1.257	+1.141	+1.009	+0.881	+0.742	+0.600	+0.449	+0.294	+0.153	+0.045
2.0	+1.253	+1.144	+1.041	+0.929	+0.806	+0.667	+0.514	+0.345	+0.186	+0.055
3.0	+1.160	+1.112	+1.061	+0.998	+0.912	+0.796	+0.646	+0.459	+0.258	+0.081
4.0	+1.085	+1.073	+1.057	+1.029	+0.977	+0.887	+0.746	+0.553	+0.322	+0.105
5.0	+1.037	+1.044	+1.047	+1.042	+1.015	+0.949	+0.825	+0.629	+0.379	+0.128
6.0	+1.010	+1.024	+1.038	+1.045	+1.034	+0.986	+0.879	+0.694	+0.430	+0.149
8.0	+0.989	+1.005	+1.022	+1.036	+1.044	+1.026	+0.953	+0.788	+0.519	+0.189
10.0	+0.989	+0.998	+1.010	+1.023	+1.039	+1.040	+0.996	+0.859	+0.591	+0.226
12.0	+0.994	+0.997	+1.003	+1.014	+1.031	+1.043	+1.022	+0.911	+0.652	+0.262
14.0	+0.997	+0.998	+1.000	+1.007	+1.022	+1.040	+1.035	+0.949	+0.705	+0.294
16.0	+1.000	+0.999	+0.999	+1.003	+1.015	+1.032	+1.040	+0.975	+0.750	+0.321

Table IV

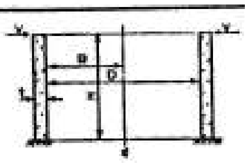
Tension in circular rings
Rectangular load
Hinged base, free top
T = coef. × pR
Positive sign indicates tension



$\frac{H^2}{Dt}$	Coefficients at point									
	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	+1.474	+1.340	+1.195	+1.052	+0.908	+0.764	+0.615	+0.465	+0.311	+0.154
0.8	+1.423	+1.302	+1.181	+1.058	+0.930	+0.797	+0.649	+0.502	+0.345	+0.166
1.2	+1.350	+1.255	+1.161	+1.062	+0.958	+0.843	+0.709	+0.556	+0.386	+0.198
1.6	+1.271	+1.203	+1.141	+1.069	+0.985	+0.885	+0.756	+0.614	+0.433	+0.224
2.0	+1.205	+1.160	+1.121	+1.073	+1.011	+0.934	+0.819	+0.669	+0.480	+0.251
3.0	+1.074	+1.079	+1.081	+1.075	+1.049	+1.006	+0.919	+0.779	+0.575	+0.310
4.0	+1.017	+1.037	+1.053	+1.067	+1.069	+1.045	+0.979	+0.853	+0.647	+0.356
5.0	+0.992	+1.014	+1.035	+1.056	+1.069	+1.062	+1.017	+0.906	+0.703	+0.394
6.0	+0.989	+1.003	+1.023	+1.043	+1.063	+1.066	+1.039	+0.943	+0.747	+0.427
8.0	+0.985	+0.996	+1.008	+1.024	+1.043	+1.064	+1.061	+0.997	+0.821	+0.486
10.0	+0.992	+0.995	+1.000	+1.011	+1.028	+1.052	+1.066	+1.030	+0.878	+0.533
12.0	+0.998	+0.997	+0.997	+1.002	+1.017	+1.041	+1.064	+1.050	+0.920	+0.577
14.0	+1.000	+0.998	+0.997	+0.999	+1.008	+1.031	+1.059	+1.061	+0.952	+0.613
16.0	+1.002	+1.000	+0.998	+0.999	+1.003	+1.021	+1.050	+1.064	+0.976	+0.636

Table V

Tension in circular rings
Shear / m ., V, applied at top
Fixed base, free top
T = coef. × VR/H
Positive sign indicates tension

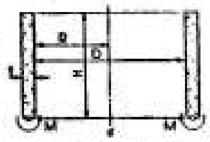


$\frac{H^2}{Dt}$	Coefficients at point*									
	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	- 1.57	- 1.32	- 1.08	- 0.86	- 0.65	- 0.47	- 0.31	- 0.18	- 0.08	- 0.02
0.8	- 3.09	- 2.55	- 2.04	- 1.57	- 1.15	- 0.80	- 0.51	- 0.28	- 0.13	- 0.03
1.2	- 3.95	- 3.17	- 2.44	- 1.79	- 1.25	- 0.81	- 0.48	- 0.25	- 0.10	- 0.02
1.6	- 4.57	- 3.54	- 2.60	- 1.80	- 1.17	- 0.69	- 0.36	- 0.16	- 0.05	- 0.01
2.0	- 5.12	- 3.83	- 2.68	- 1.74	- 1.02	- 0.52	- 0.21	- 0.05	+ 0.01	+ 0.01
3.0	- 6.32	- 4.37	- 2.70	- 1.43	- 0.58	- 0.02	+ 0.15	+ 0.19	+ 0.13	+ 0.04
4.0	- 7.34	- 4.73	- 2.60	- 1.10	- 0.19	+ 0.26	+ 0.38	+ 0.33	+ 0.19	+ 0.06
5.0	- 8.22	- 4.99	- 2.45	- 0.79	+ 0.11	+ 0.47	+ 0.50	+ 0.37	+ 0.20	+ 0.06
6.0	- 9.02	- 5.17	- 2.27	- 0.50	+ 0.34	+ 0.59	+ 0.53	+ 0.35	+ 0.17	+ 0.01
8.0	-10.42	- 5.36	- 1.85	- 0.02	+ 0.63	+ 0.66	+ 0.46	+ 0.24	+ 0.09	+ 0.01
10.0	-11.67	- 5.43	- 1.43	+ 0.36	+ 0.78	+ 0.62	+ 0.33	+ 0.12	+ 0.02	0.00
12.0	-12.76	- 5.41	- 1.03	+ 0.63	+ 0.83	+ 0.52	+ 0.21	+ 0.04	- 0.02	0.00
14.0	-13.77	- 5.34	- 0.68	+ 0.80	+ 0.81	+ 0.42	+ 0.13	0.00	- 0.03	- 0.01
16.0	-14.74	- 5.22	- 0.33	+ 0.96	+ 0.76	+ 0.32	+ 0.05	- 0.04	- 0.05	- 0.02

*When this table is used for shear applied at the base, while the top is fixed, 0.0H is the bottom of the wall and 1.0H is the top. Shear acting inward is positive, outward is negative.

Table VI

Tension in circular rings
Moment / m ., M, applied at base
Hinged base, free top
T = coef. × MR/H^2
Positive sign indicates tension

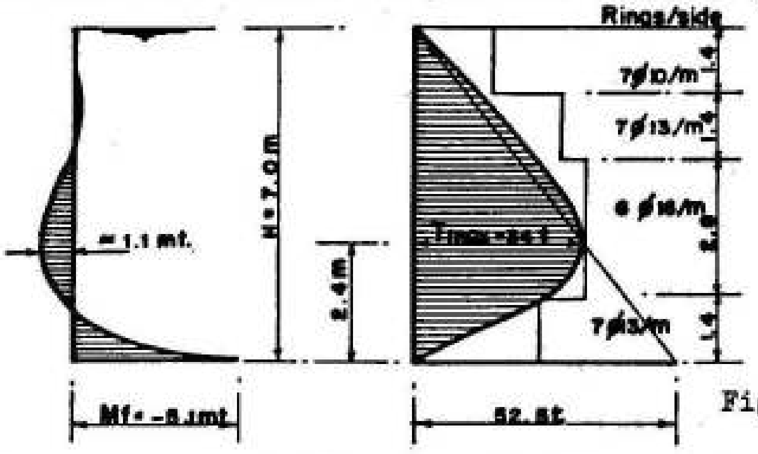


$\frac{H^2}{Dt}$	Coefficients at point*									
	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
0.4	+ 2.70	+ 2.50	+ 2.30	+ 2.12	+ 1.91	+ 1.69	+ 1.41	+ 1.13	+ 0.80	+ 0.44
0.8	+ 2.02	+ 2.06	+ 2.10	+ 2.14	+ 2.10	+ 2.02	+ 1.95	+ 1.75	+ 1.39	+ 0.80
1.2	+ 1.06	+ 1.42	+ 1.79	+ 2.03	+ 2.46	+ 2.65	+ 2.80	+ 2.60	+ 2.22	+ 1.37
1.6	+ 0.12	+ 0.79	+ 1.43	+ 2.04	+ 2.72	+ 3.25	+ 3.56	+ 3.59	+ 3.13	+ 2.01
2.0	- 0.68	+ 0.22	+ 1.10	+ 2.02	+ 2.90	+ 3.69	+ 4.30	+ 4.54	+ 4.08	+ 2.75
3.0	- 1.78	- 0.71	+ 0.43	+ 1.60	+ 2.95	+ 4.29	+ 5.66	+ 6.58	+ 6.55	+ 4.71
4.0	- 1.87	- 1.00	- 0.08	+ 1.04	+ 2.47	+ 4.31	+ 6.34	+ 8.17	+ 8.82	+ 6.81
5.0	- 1.54	- 1.03	- 0.42	+ 0.45	+ 1.86	+ 3.92	+ 6.60	+ 9.41	+11.03	+ 9.02
6.0	- 1.04	- 0.86	- 0.59	- 0.05	+ 1.21	+ 3.34	+ 6.54	+10.26	+13.08	+11.41
8.0	- 0.24	- 0.53	- 0.73	- 0.67	- 0.02	+ 2.05	+ 5.87	+11.32	+16.52	+16.06
10.0	+ 0.21	- 0.23	- 0.64	- 0.94	- 0.73	+ 0.82	+ 4.79	+11.63	+19.48	+20.87
12.0	+ 0.32	- 0.05	- 0.46	- 0.98	- 1.15	- 0.18	+ 3.52	+11.27	+21.80	+25.73
14.0	+ 0.26	+ 0.04	- 0.28	- 0.76	- 1.29	- 0.87	+ 2.29	+10.55	+23.50	+30.34
16.0	+ 0.22	+ 0.07	- 0.08	- 0.64	- 1.28	- 1.30	+ 1.12	+ 9.67	+24.53	+34.65

*When this table is used for moment applied at the top, while the top is hinged, 0.0H is the bottom of the wall and 1.0H is the top. Moment applied at an edge is positive when it causes outward rotation at that edge.

If the wall of the tank were free at top and hinged at bottom, the maximum ring tension will be increased by ca 10% in deep tanks and 20 to 25 % in medium tanks and its position moves a small distance towards the base of the wall. In this case the fixing moment M_f

Bending Mom. Diag.



Ring Tension Diag.

Fig. III-17

will be equal to zero and the field moment will be increased to ca M_f / 3 . The distribution of the ring tension and cantilever moments will have the form shown in fig. III.19.

III.4.3 - Tables of the Portland Cement Association

The American Portland Cement Association has published the following series of tables* giving the ring tension and cantilever moments in rectangular walls of cylinderical tanks free at top and fixed or hinged at bottom for different cases of loading. The tables include further a lot of data very useful in the design of circular tanks.

Illustrative Example :

The use of the " Portland Cement Association " tables will be explained in the following example :

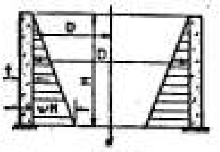
Figure III.18 shows an open cylinderical reinforced concrete water tank 5 ms deep and 20 ms diameter. It is required to determine the internal forces for the following cases :

---

* "Circular Concrete Tanks without Prestressing ". Concrete Information of the Portland Cement Association. ST - 57 - 1.

Table VII

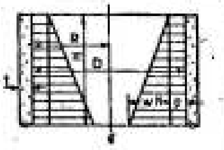
Moments in cylindrical wall
Triangular load
Fixed base, free ton
Mom. = coef. × wH^3
Positive sign indicates tension in the outside



$\frac{H^2}{Dt}$	Coefficients at point									
	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
0.4	+.0005	+.0014	+.0021	+.0007	-.0042	-.0150	-.0302	-.0529	-.0816	-.1205
0.8	+.0011	+.0037	+.0063	+.0080	+.0070	+.0023	-.0068	-.0224	-.0465	-.0795
1.2	+.0012	+.0042	+.0077	+.0103	+.0112	+.0090	+.0022	-.0108	-.0311	-.0602
1.6	+.0011	+.0041	+.0075	+.0107	+.0121	+.0111	+.0058	-.0051	-.0232	-.0505
2.0	+.0010	+.0035	+.0068	+.0099	+.0120	+.0115	+.0075	-.0021	-.0185	-.0436
3.0	+.0006	+.0024	+.0047	+.0071	+.0090	+.0097	+.0077	+.0012	-.0119	-.0333
4.0	+.0003	+.0015	+.0028	+.0047	+.0066	+.0077	+.0069	+.0023	-.0080	-.0268
5.0	+.0002	+.0008	+.0016	+.0029	+.0046	+.0059	+.0059	+.0028	-.0058	-.0222
6.0	+.0001	+.0003	+.0008	+.0019	+.0032	+.0046	+.0051	+.0029	-.0041	-.0187
8.0	.0000	+.0001	+.0002	+.0008	+.0016	+.0028	+.0038	+.0029	-.0022	-.0140
10.0	.0000	.0000	+.0001	+.0004	+.0007	+.0019	+.0029	+.0028	-.0012	-.0122
12.0	.0000	-.0001	+.0001	+.0002	+.0003	+.0013	+.0023	+.0026	-.0005	-.0104
14.0	.0000	.0000	.0000	.0000	+.0001	+.0008	+.0019	+.0023	-.0001	-.0090
16.0	.0000	.0000	-.0001	-.0002	-.0001	+.0004	+.0013	+.0019	+.0001	-.0079

Table VIII

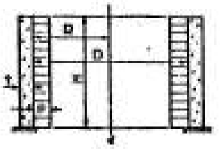
Moments in cylindrical wall
Trapezoidal load
Hinged base, free top
Mom. = coef. × $(wH^3 + pH^2)$
Positive sign indicates tension in the outside



$\frac{H^2}{Dt}$	Coefficients at point									
	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
0.4	+.0020	+.0072	+.0151	+.0230	+.0301	+.0348	+.0357	+.0312	+.0197	0
0.8	+.0019	+.0064	+.0133	+.0207	+.0271	+.0319	+.0329	+.0292	+.0187	0
1.2	+.0016	+.0058	+.0111	+.0177	+.0237	+.0280	+.0296	+.0263	+.0171	0
1.6	+.0012	+.0044	+.0091	+.0145	+.0195	+.0236	+.0255	+.0232	+.0155	0
2.0	+.0009	+.0033	+.0073	+.0114	+.0158	+.0199	+.0219	+.0205	+.0145	0
3.0	+.0004	+.0018	+.0040	+.0063	+.0092	+.0127	+.0152	+.0153	+.0111	0
4.0	+.0001	+.0007	+.0016	+.0033	+.0057	+.0083	+.0109	+.0118	+.0092	0
5.0	.0000	+.0001	+.0006	+.0016	+.0034	+.0057	+.0080	+.0094	+.0078	0
6.0	.0000	.0000	+.0002	+.0008	+.0019	+.0039	+.0062	+.0078	+.0068	0
8.0	.0000	.0000	-.0002	.0000	+.0007	+.0020	+.0038	+.0057	+.0054	0
10.0	.0000	.0000	-.0002	-.0001	+.0002	+.0011	+.0025	+.0043	+.0045	0
12.0	.0000	.0000	-.0001	-.0002	.0000	+.0005	+.0017	+.0032	+.0039	0
14.0	.0000	.0000	-.0001	-.0001	-.0001	.0000	+.0012	+.0026	+.0033	0
16.0	.0000	.0000	.0000	-.0001	-.0002	-.0004	+.0008	+.0022	+.0029	0

Table IX

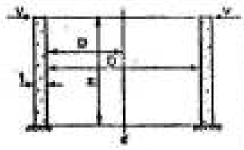
Moments in cylindrical wall
Rectangular load
Fixed base, free top
Mom. = coef. × pH^2
Positive sign indicates tension in the outside



$\frac{H^2}{Dt}$	Coefficients at point									
	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
0.4	-.0023	-.0093	-.0227	-.0439	-.0710	-.1018	-.1455	-.2000	-.2593	-.3310
0.8	.0000	-.0006	-.0025	-.0083	-.0185	-.0362	-.0594	-.0917	-.1325	-.1835
1.2	+.0008	+.0026	+.0037	+.0029	-.0009	-.0089	-.0227	-.0468	-.0815	-.1178
1.6	+.0011	+.0036	+.0062	+.0077	+.0068	+.0011	-.0093	-.0267	-.0529	-.0876
2.0	+.0010	+.0036	+.0066	+.0088	+.0089	+.0059	-.0019	-.0167	-.0389	-.0719
3.0	+.0007	+.0026	+.0051	+.0074	+.0091	+.0083	+.0042	-.0053	-.0223	-.0483
4.0	+.0004	+.0015	+.0033	+.0052	+.0068	+.0075	+.0053	-.0013	-.0145	-.0365
5.0	+.0002	+.0008	+.0019	+.0035	+.0051	+.0061	+.0052	+.0007	-.0101	-.0293
6.0	+.0001	+.0004	+.0011	+.0022	+.0036	+.0049	+.0048	+.0017	-.0073	-.0242
8.0	.0000	+.0001	+.0003	+.0008	+.0018	+.0031	+.0038	+.0024	-.0040	-.0184
10.0	.0000	-.0001	.0000	+.0002	+.0009	+.0021	+.0030	+.0026	-.0022	-.0147
12.0	.0000	.0000	-.0001	.0000	+.0004	+.0014	+.0024	+.0022	-.0012	-.0123
14.0	.0000	.0000	.0000	.0000	+.0002	+.0010	+.0018	+.0021	-.0007	-.0105
16.0	.0000	.0000	.0000	-.0001	+.0001	+.0006	+.0012	+.0020	-.0005	-.0091

Table X

Moments in cylindrical wall
Shear / m V, applied at top
Fixed base, free top
Mom. = coef. × VH
Positive sign indicates tension in outside

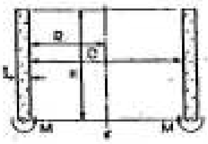


$\frac{H^2}{Dt}$	Coefficients at point*									
	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.6H	1.0H
0.4	+0.093	+0.172	+0.240	+0.300	+0.354	+0.402	+0.448	+0.492	+0.535	+0.578
0.8	+0.085	+0.145	+0.185	+0.208	+0.220	+0.224	+0.223	+0.219	+0.214	+0.208
1.2	+0.082	+0.132	+0.157	+0.164	+0.159	+0.145	+0.127	+0.106	+0.084	+0.062
1.6	+0.079	+0.122	+0.139	+0.138	+0.125	+0.105	+0.081	+0.056	+0.030	+0.004
2.0	+0.077	+0.115	+0.126	+0.119	+0.103	+0.080	+0.056	+0.031	+0.006	-0.019
3.0	+0.072	+0.100	+0.100	+0.086	+0.066	+0.044	+0.025	+0.006	-0.010	-0.024
4.0	+0.068	+0.088	+0.081	+0.063	+0.043	+0.025	+0.010	-0.001	-0.010	-0.019
5.0	+0.064	+0.078	+0.067	+0.047	+0.028	+0.013	+0.003	-0.003	-0.007	-0.011
6.0	+0.062	+0.070	+0.056	+0.036	+0.018	+0.006	0.000	-0.003	-0.005	-0.006
8.0	+0.057	+0.058	+0.041	+0.021	+0.007	0.000	-0.002	-0.003	-0.002	0.001
10.0	+0.053	+0.049	+0.029	+0.012	+0.002	-0.002	-0.002	-0.002	-0.001	0.000
12.0	+0.049	+0.042	+0.022	+0.007	0.000	-0.002	-0.002	-0.001	0.000	0.000
14.0	+0.046	+0.036	+0.017	+0.004	-0.001	-0.002	-0.001	-0.001	0.000	0.000
16.0	+0.044	+0.031	+0.012	+0.001	-0.002	-0.002	-0.001	0.000	0.000	0.000

*When this table is used for shear applied at the base, while the top is fixed, 0.0H is the bottom of the wall and 1.0H is the top. Shear acting inward is positive, outward is negative.

Table XI

Moments in cylindrical wall
Moment/m , M, applied at base
Hinged base, free top
Mom. = coef. × M
Positive sign indicates tension in outside

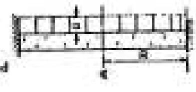


$\frac{H^2}{Dt}$	Coefficients at point*									
	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
0.4	+0.013	+0.051	+0.109	+0.196	+0.296	+0.414	+0.547	+0.692	+0.843	+1.000
0.8	+0.009	+0.040	+0.090	+0.164	+0.253	+0.375	+0.503	+0.659	+0.824	+1.000
1.2	+0.006	+0.027	+0.063	+0.125	+0.206	+0.316	+0.454	+0.616	+0.802	+1.000
1.6	+0.003	+0.011	+0.035	+0.078	+0.157	+0.253	+0.393	+0.570	+0.775	+1.000
2.0	-0.002	-0.002	+0.012	+0.034	+0.096	+0.193	+0.340	+0.519	+0.748	+1.000
3.0	0.007	-0.022	-0.030	-0.029	+0.010	+0.087	+0.227	+0.426	+0.692	+1.000
4.0	-0.008	-0.026	-0.044	-0.051	-0.034	+0.073	+0.150	+0.354	+0.645	+1.000
5.0	-0.007	-0.024	-0.045	-0.061	-0.057	-0.015	+0.095	+0.296	+0.606	+1.000
6.0	-0.005	-0.018	-0.040	[illegible]	-0.065	-0.037	+0.057	+0.252	+0.572	+1.000
8.0	0.001	-0.009	0.022	-0.044	-0.068	-0.062	+0.002	+0.178	+0.515	+1.000
10.0	0.000	-0.007	-0.009	-0.028	-0.053	-0.067	-0.031	+0.123	+0.467	+1.000
12.0	0.000	0.000	-0.003	-0.016	-0.040	-0.064	-0.049	+0.081	+0.424	+1.000
14.0	0.000	0.000	0.000	-0.008	-0.029	-0.059	-0.060	+0.048	+0.387	+1.000
16.0	0.000	0.000	+0.002	-0.003	-0.021	-0.051	-0.066	+0.025	+0.354	+1.000

*When this table is used for moment applied at the top, while the top is hinged, 0.0H is the bottom of the wall and 1.0H is the top. Moment applied at an edge is positive when it causes outward rotation at that edge.

Table XII

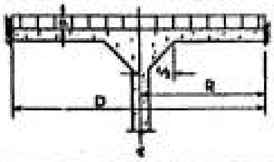
Moments in circular slab without center support
Uniform load
Fixed edge
Mom. = coef. × pR^2
Positive sign indicates compression in surface loaded



Coefficients at point										
0.00R	0.10R	0.20R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.00R
Radial moments, M_r										
+.075	+.073	+.067	+.057	+.043	+.025	+.003	-.023	-.053	-.087	-.125
Tangential moments, M_t										
+.075	+.074	+.071	+.066	+.059	+.050	+.039	+.026	+.011	.000	-.025

Table XIII

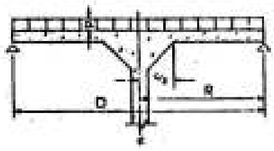
Moments in circular slab with center support
Uniform load
Fixed edge
Mom. = coef. × pR^2
Positive sign indicates compression in surface loaded



c/D	0.05R	0.10R	0.15R	0.20R	0.25R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.00R
	Radial moments, M_r												
0.05	-0.2100	-0.0729	-0.0275	-0.0026	+0.0133	+0.0238	+0.0342	+0.0347	+0.0277	+0.0142	-0.0049	-0.0294	-0.0569
0.10		-0.1433	-0.0624	-0.0239	-0.0011	+0.0136	+0.0290	+0.0326	+0.0278	+0.0158	-0.0021	-0.0255	-0.0541
0.15			-0.1089	-0.0521	-0.0200	+0.0002	+0.0220	+0.0293	+0.0269	+0.0169	+0.0008	-0.0216	-0.0490
0.20				-0.0862	-0.0429	-0.0161	+0.0133	+0.0249	+0.0254	+0.0178	+0.0029	-0.0178	-0.0441
0.25					-0.0698	-0.0351	+0.0029	+0.0194	+0.0231	+0.0177	+0.0049	-0.0143	-0.0393
	Tangential moments, M_t												
0.05	-0.0417	-0.0700	-0.0541	-0.0381	-0.0251	-0.0145	+0.0002	+0.0085	+0.0118	+0.0109	+0.0065	-0.0003	-0.0118
0.10		-0.0287	-0.0421	-0.0354	-0.0258	-0.0168	-0.0027	+0.0059	+0.0099	+0.0098	+0.0061	-0.0009	-0.0108
0.15			-0.0318	-0.0284	-0.0243	-0.0177	-0.0051	+0.0031	+0.0080	+0.0086	+0.0057	-0.0000	-0.0098
0.20				-0.0172	-0.0203	-0.0171	-0.0070	+0.0013	+0.0063	+0.0075	+0.0052	-0.0003	-0.0088
0.25					-0.0140	-0.0150	-0.0083	-0.0005	+0.0046	+0.0064	+0.0048	0.0000	-0.0078

Table XIV

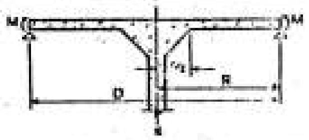
Moments in circular slab with center support
Uniform load
Hinged edge
Mom. = coef. × pR^2
Positive sign indicates compression in surface loaded



c/D	0.05R	0.10R	0.15R	0.20R	0.25R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.00R
	Radial moments, M_r												
0.05	-0.3658	-0.1388	-0.0640	-0.0221	+0.0058	+0.0255	+0.0501	+0.0614	+0.0629	+0.0560	+0.0437	+0.0247	0
0.10		-0.2487	-0.1180	-0.0557	-0.0178	+0.0081	+0.0391	+0.0539	+0.0578	+0.0532	+0.0418	+0.0237	0
0.15			-0.1869	-0.0977	-0.0467	-0.0135	+0.0258	+0.0451	+0.0518	+0.0494	+0.0393	+0.0226	0
0.20				-0.1465	-0.0800	-0.0381	+0.0109	+0.0352	+0.0452	+0.0451	+0.0368	+0.0215	0
0.25					-0.1172	-0.0645	-0.0055	+0.0245	+0.0381	+0.0404	+0.0340	+0.0200	0
	Tangential moments, M_t												
0.05	-0.0731	-0.1277	-0.1040	-0.0780	-0.0569	-0.0391	-0.0121	+0.0051	+0.0175	+0.0234	+0.0251	+0.0228	+0.0158
0.10		-0.0498	-0.0708	-0.0684	-0.0539	-0.0394	-0.0153	+0.0020	+0.0134	+0.0197	+0.0218	+0.0199	+0.0145
0.15			-0.0374	-0.0516	-0.0470	-0.0375	-0.0175	-0.0014	+0.0097	+0.0163	+0.0186	+0.0172	+0.0121
0.20				-0.0293	-0.0367	-0.0333	-0.0184	-0.0042	+0.0065	+0.0132	+0.0158	+0.0148	+0.0103
0.25					-0.0234	-0.0263	-0.0184	-0.0052	+0.0038	+0.0103	+0.0112	+0.0122	+0.0085

Table XV

Moments in circular slab with center support
Moment / m , M, applied at edge
Hinged edge
Mom. = coef. × M
Positive sign indicates compression in top surface



c/D	0.05R	0.10R	0.15R	0.20R	0.25R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.00R
	Radial moments, M_r												
0.05	-2.650	-1.121	-0.622	-0.333	-0.129	+0.029	+0.268	+0.450	+0.596	+0.718	+0.824	+0.917	+1.000
0.10		-1.950	-1.026	-0.584	-0.305	-0.103	+0.187	+0.394	+0.558	+0.692	+0.808	+0.901	+1.000
0.15			-1.594	-0.930	-0.545	-0.280	+0.078	+0.323	+0.510	+0.663	+0.790	+0.900	+1.000
0.20				-1.366	-0.847	-0.499	-0.057	+0.236	+0.451	+0.624	+0.768	+0.891	+1.000
0.25					-1.204	-0.765	-0.216	+0.130	+0.392	+0.577	+0.740	+0.880	+1.000
	Tangential moments, M_t												
0.05	-0.530	-0.980	-0.847	-0.688	-0.544	-0.418	-0.211	-0.042	+0.095	+0.212	+0.314	+0.405	+0.488
0.10		-0.388	-0.641	-0.605	-0.518	-0.419	-0.233	-0.072	+0.065	+0.185	+0.290	+0.384	+0.469
0.15			-0.319	-0.472	-0.453	-0.404	-0.251	-0.100	+0.035	+0.157	+0.263	+0.363	+0.451
0.20				-0.272	-0.372	-0.368	-0.261	-0.123	+0.007	+0.129	+0.240	+0.340	+0.433
0.25					-0.239	-0.305	-0.253	-0.145	-0.020	+0.099	+0.214	+0.320	+0.414

Table XVI

Shear at base of cylindrical wall

$V = \text{coef.} \times \begin{cases} wH^2 & \text{(triangular)} \\ pH & \text{(rectangular)} \\ M/H & \text{(mom. at base)} \end{cases}$

Positive sign indicates shear acting inward

$\frac{H^2}{Dt}$	Triangular load, fixed base	Rectangular load, fixed base	Triangular or rectangular load, hinged base	Moment at edge
0.4	+0.436	+0.755	+0.245	-1.58
0.8	+0.374	+0.552	+0.234	-1.75
1.2	+0.339	+0.460	+0.220	-2.00
1.6	+0.317	+0.407	+0.204	-2.28
2.0	+0.299	+0.370	+0.189	-2.57
3.0	+0.262	+0.310	+0.158	-3.18
4.0	+0.236	+0.271	+0.137	-3.68
5.0	+0.213	+0.243	+0.121	-4.10
6.0	+0.197	+0.222	+0.110	-4.49
8.0	+0.174	+0.193	+0.096	-5.18
10.0	+0.158	+0.172	+0.087	-5.81
12.0	+0.145	+0.158	+0.079	-6.38
14.0	+0.135	+0.147	+0.073	-6.88
16.0	+0.127	+0.137	+0.068	-7.36

Table XVIII

Stiffness of cylindrical wall
Near edge hinged, far edge free
$k = \text{coef.} \times Et^3/H$

$\frac{H^2}{Dt}$	Coefficient	$\frac{H^2}{Dt}$	Coefficient
0.4	0.139	5	0.713
0.8	0.270	6	0.783
1.2	0.345	8	0.903
1.6	0.399	10	1.010
2.0	0.445	12	1.108
3.0	0.548	14	1.198
4.0	0.635	16	1.281

Table XVII

Load on center support for circular slab

$\text{Load} = \text{coef.} \times \begin{cases} pR^2 & \text{(hinged and fixed)} \\ M & \text{(moment at edge)} \end{cases}$

c/D	0.05	0.10	0.15	0.20	0.25
Hinged	1.320	1.387	1.463	1.542	1.625
Fixed	0.839	0.919	1.007	1.101	1.200
M at edge	8.16	8.66	9.29	9.99	10.81

Table XIX

Stiffness of circular plates
With center support
$k = \text{coef.} \times Et^3/R$

c/D	0.05	0.10	0.15	0.20	0.25
Coef.	0.290	0.309	0.332	0.358	0.387

Without center support
Coef. = 0.104

Table XX. Supplementary Coefficients for Values of H^2/Dt Greater than 16 (*Extension of Tables I to XI, XVI and XVIII*)*

$\frac{H^2}{Dt}$	TABLE I					TABLE II					TABLE III					TABLE IV				
	Coefficients at point					Coefficients at point					Coefficients at point					Coefficients at point				
	.75H	.80H	.85H	.90H	.95H	.75H	.80H	.85H	.90H	.95H	.75H	.80H	.85H	.90H	.95H	.75H	.80H	.85H	.90H	.95H
20	+0.716	+0.654	+0.520	+0.325	+0.115	+0.812	+0.817	+0.756	+0.603	+0.344	+0.949	+0.825	+0.629	+0.379	+0.128	+1.062	+1.017	+0.906	+0.703	+0.394
24	+0.746	+0.702	+0.577	+0.372	+0.137	+0.816	+0.839	+0.793	+0.647	+0.377	+0.986	+0.879	+0.694	+0.430	+0.149	+1.056	+1.039	+0.943	+0.747	+0.427
32	+0.782	+0.768	+0.663	+0.459	+0.182	+0.814	+0.861	+0.847	+0.721	+0.436	+1.026	+0.953	+0.788	+0.519	+0.189	+1.064	+1.061	+0.997	+0.821	+0.486
40	+0.800	+0.805	+0.731	+0.530	+0.217	+0.802	+0.866	+0.880	+0.778	+0.483	+1.040	+0.996	+0.859	+0.591	+0.226	+1.052	+1.066	+1.030	+0.878	+0.533
48	+0.791	+0.828	+0.785	+0.593	+0.254	+0.791	+0.864	+0.900	+0.820	+0.527	+1.043	+1.022	+0.911	+0.652	+0.262	+1.041	+1.064	+1.050	+0.920	+0.577
56	+0.763	+0.838	+0.824	+0.636	+0.285	+0.781	+0.859	+0.911	+0.852	+0.563	+1.040	+1.035	+0.949	+0.705	+0.294	+1.021	+1.059	+1.061	+0.952	+0.613

$\frac{H^2}{Dt}$	TABLE V					TABLE VI					TABLE VII					TABLE VIII				
	Coefficients at point					Coefficients at point					Coefficients at point					Coefficients at point				
	.00H	.05H	.10H	.15H	.20H	.75H	.80H	.85H	.90H	.95H	.80H	.85H	.90H	.95H	1.00H	.75H	.80H	.85H	.90H	.95H
20	-18.44	- 9.98	- 4.90	- 1.50	+ 0.22	+15.30	+ 25.9	+ 36.9	+ 43.3	+ 35.3	+.0015	+.0014	+.0005	-.0018	-.0063	+.0008	+.0014	+.0020	+.0024	+.0020
24	-18.04	-10.34	- 4.54	- 1.00	+ 0.68	+13.20	+ 25.9	+ 40.7	+ 51.8	+ 45.3	+.0012	+.0012	+.0007	-.0013	-.0053	+.0005	+.0010	+.0015	+.0020	+.0017
32	-20.84	-10.72	- 3.70	- 0.04	+ 1.26	+ 8.10	+ 23.2	+ 45.9	+ 65.4	+ 63.6	+.0007	+.0009	+.0007	-.0008	-.0040	.0000	+.0005	+.0009	+.0014	+.0013
40	-23.34	-10.86	- 2.86	+ 0.72	+ 1.56	+ 3.28	+ 19.2	+ 46.5	+ 77.9	+ 83.5	+.0002	+.0005	+.0006	-.0005	-.0032	.0000	+.0003	+.0006	+.0011	+.0011
48	-25.52	-10.82	- 2.00	+ 1.26	+ 1.68	- 0.70	+ 14.1	+ 45.1	+ 87.2	+103.0	.0000	+.0001	+.0006	-.0003	-.0026	.0000	+.0001	+.0004	+.0008	+.0010
56	-27.54	-10.68	- 1.38	+ 1.60	+ 1.82	- 3.40	+ 9.2	+ 42.2	+ 94.0	+121.0	.0000	.0000	+.0004	-.0001	-.0023	.0000	.0000	+.0003	+.0007	+.0008

$\frac{H^2}{Dt}$	TABLE IX					TABLE X					TABLE XI					TABLE XVI				TABLE XVIII
	Coefficients at point					Coefficients at point					Coefficients at point									
	.80H	.85H	.90H	.95H	1.00H	.05H	.10H	.15H	.20H	.25H	.80H	.85H	.90H	.95H	1.00H	Tri. Fixed	Rect. Fixed	T. or R. Hinged	Mom. at Edge	Stiffness
20	+.0015	+.0013	+.0002	-.0024	-.0073	+0.032	+0.033	+0.033	+0.023	+0.014	-0.015	+0.095	+0.296	+0.606	+1.000	+0.114	+0.122	+0.062	- 8.20	1.430
24	+.0012	+.0012	+.0004	-.0018	-.0061	+0.031	+0.035	+0.028	+0.018	+0.009	-0.037	+0.057	+0.250	+0.572	+1.000	+0.102	+0.111	+0.055	- 8.94	1.566
32	+.0008	+.0009	+.0006	-.0010	-.0046	+0.028	+0.029	+0.020	+0.011	+0.004	-0.062	+0.002	+0.178	+0.515	+1.000	+0.089	+0.096	+0.048	-10.36	1.810
40	+.0005	+.0007	+.0007	-.0005	-.0037	+0.026	+0.025	+0.015	+0.006	+0.001	-0.067	-0.031	+0.123	+0.457	+1.000	+0.080	+0.086	+0.043	-11.62	2.025
48	+.0004	+.0006	+.0006	-.0003	-.0031	+0.024	+0.021	+0.011	+0.003	0.000	-0.064	-0.049	+0.081	+0.424	+1.000	+0.072	+0.079	+0.039	-12.76	2.220
56	+.0002	+.0004	+.0005	-.0001	-.0026	+0.023	+0.018	+0.008	+0.002	0.000	-0.059	-0.060	+0.048	+0.387	+1.000	+0.067	+0.074	+0.036	-13.78	2.400

*For points not shown in the supplementary tables, ring tension and moment may be determined approximately by sketching curves similar to those in the text.

1) Bottom edge sliding

2) " " hinged

3) " " fixed

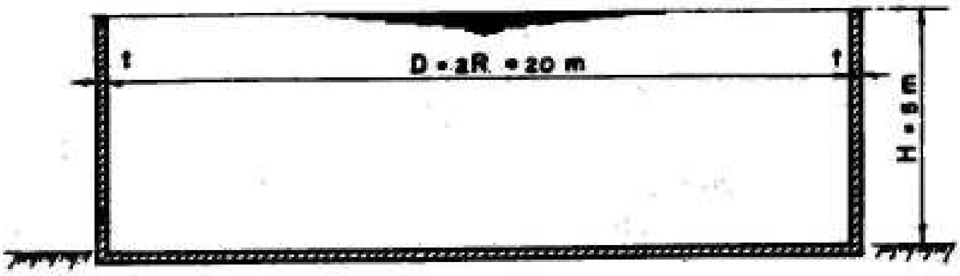


Fig. III-18

1) Top Edge Free and Bottom Edge Sliding :

$$H = 5.00 \text{ ms} \quad \text{and} \quad R = 10 \text{ ms}$$

The maximum wall thickness can be determined according to equation 7 from the relation :

$$t_{max} = 0.8 \text{ w H R}$$

thus :

$$t_{max} = 0.8 \times 1 \times 5 \times 10 = 40 \text{ cms}$$

As the water pressure increases with the depth, and is fully resisted by ring action, it is convenient to choose a trapezoidal wall with a minimum thickness t = 20 cms at top and a maximum thickness of 40 cms at bottom.

Table 1 Ring Tension in Cylinderical Tank Wall with Free Top & Sliding Base $T = x R$

Distance from top edge x	0	0.2H	0.4H	0.6H	0.8H	1.0H
Ring Tension T t/m	0	10.000	20.000	30.000	40.000	50.000

2) Top Edge Free Bottom Edge Hinged

As the max. ring tension and bending moment take place at the middle part of the wall, it is recommended to choose a wall of constant thickness with :

$$t \cong 0.5\ H\ R \tag{26}$$

In our case $t = 0.5 \times 5 \times 10 = 25$ cms

Therefore $\frac{H^2}{D\ t} = \frac{5^2}{20 \times 0.25} = 5$

Ring tension per meter height will be computed by multiplying $p_{max}R$ by the coefficients for H^2 / Dt = 5.0 taken from table II , $p_{max}R$ = 5 x 10 = 50 t/m

Bending moments in the vertical wall strips 1^m wide are computed by multiplying the coefficient given in table VIII by

$p_{max}H^2 = 5 \times 5^2 = 125$ mt/m

It has to be noted that :

* The coefficients are given such that point 0.0H denotes the top and point 1.0H the base of the wall.
* The positive ring forces are tensile while the negative are compressive.
* Positive bending moments cause tensile stresses in the outer surface and compressive stresses in the inner surface of the wall.

Table 2 : Ring Tension & Bending Moments in Cylinderical Tank Wall with Free Top and Hinged Base.

Point		0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
Ring Tension	Coef. II	- .008	+ .114	+ .235	+ .356	+ .469	+ .562	+ .617	+ .606	+ .503	+ .294
	T t/m	- 0.4	+ 5.70	+ 11.75	+ 17.8	+ 23.45	+ 28.1	+ 30.85	+ 30.5	+ 25.1	+ 14.7
B.M.	Coef. VIII	.00	+ .00	+ .0001	+ .0006	+ .0016	+ .0034	+ .0057	+ .008	+ .0094	+ .0078
	M mt/m	.00	+ .00	+ .013	+ .075	+ .20	+ .425	+ .712	+ 1.00	+ 1.172	+ .975

Reaction at base (coefficient table XVI)

Q_{max} = 0.121 w H^2 = 0.121 x 1 x 25 = 3.0 t/m'

The max. ring tension in the wall T_{max} = 30.85 tons and acts at 0.6H = 3.00 ms from the top edge. The required ring reinforcement at x = 0.6H is given by :

$$A_s = \frac{T}{\sigma_s} = \frac{30850}{1400} = 22 \text{ cms} \quad 16 \phi 13 \text{ mm/m (} 8 \phi 13 \text{ mm/m on each side) } 21.2 \text{ cm}^2$$

The tensile stress in concrete, including shrinkage, is given by :

$$\sigma_t = \frac{T + \varepsilon_{sh} E_s A_s}{A_c + n A_s} = \frac{30850 + 0.00025 \times 2100000 \times 21.2}{100 \times 25 \times 10 \times 21.2}$$

$$= \frac{30850 + 11100}{2500 + 212} = \frac{41950}{2712} = 15.5 \text{ kg/cm}^2 \quad \text{safe.}$$

The max. moment M_{max} + = 1172 kgm and acts at x = 0.8H = 4 ms. from top :

$d = t - 3 = 25 - 3 = 22$ cms $\qquad d = k_1\sqrt{M}$ or

$22 = k_1 \sqrt{1172}$ i.e $k_1 = 0.643$

For σ_s = 1400 kg/cm^2 , $\alpha = 0$, σ_c = 25 kg/cm^2 & k_2 = 1300 and

$$A_s = \frac{M}{k_2\, d} = \frac{1172}{1300 \times .22} = 4.1 \text{ cm}^2 \quad \text{choose } 6 \phi 10 \text{ mm/m}$$

3) <u>Top Edge Free, Bottom Edge Fixed</u> :

In this case, the water pressure will be resisted by ring tension in the horizontal direction and cantilever action in the vertical direction. The max. ring tension and max. positive moment will be smaller than in case 2, wall hinged at base, while relatively big fixing moments will be induced at the fixed bottom edge of the wall.

For this reason, it is recommended to choose for this case a wall of constant thickness t = 25 cms stiffened with a haunch (ca 10 x 40 cms) at its foot.

Due to the existance of the haunch, it is recommended to determine the fixing moment and respectively the thickness of tha wall at its foot either by method of Reissner or by the simplified method given before while the ring tension and positive moments can be determined from the P.C.A. tables neglecting the effect of the haunch. Thus,

For section at base : Refer to example 2 in the simplified methods :

$$A_s = \frac{3600}{1284 \times .32} = 8.8 \text{ cm}^2 \qquad 7\ \phi\ 13 \text{ mm.}$$

The ring tension per meter height will be computed by multiplying $P_{max} R = 5 \times 10 = 50$ t/m by the coefficients for $H^2 / D\ t$ taken from table 1. The positive bending moments in the vertical wall strips 1 m wide are computed by multiplying the coefficient given in table VII by $P_{max} H^2 = 5 \times 5^2 = 125$ mt/m.

The ring tension and the cantilever moments at the different depths of the wall are given in table 3 in which all the values are determined from the P.C.A. tables for $H^2 / D\ t = 5$ with the exception of the fixing moment at the foot of the wall which equals − 3.6 mt as given in example 2 page 45 .

Table 3 . Ring Tension & Bending Moments in Cylinderical Tank Wall with Free Top & Fixed Base.

	Point	0.0H	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H
Ring Ten.	Coef. I	+ .025	+ .137	+ .245	+ .346	+ .428	+ .477	+ .469	+ .398	+ .259	+ .092
	T t/m	+ 1.25	+ 6.85	+ 12.25	+ 17.3	+ 21.4	+ 23.85	+ 23.45	+ 19.9	+ 12.9	+ 4.6
B.M.	Coef. II	+ .00	+ .0002	+ .0008	+ .0016	+ .0029	+ .0046	+ .0059	+ .006	+ .003	+ .006
	M mt/m	.00	+ .025	+ .10	+ .20	+ .36	+ .58	+ .74	+ .74	+ .35	− .72

The shear at base of wall equals p . H x coef. of 0.213 for H/D t = 5 taken from table XVI. i.e. Q_{max} = 0.213 x 5 x 5 = 5.33 t/m.

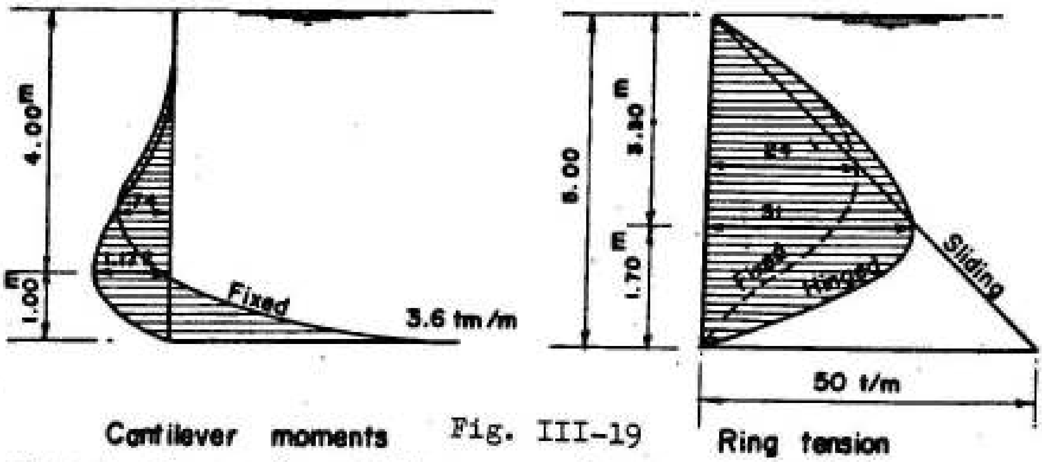


Fig. III-19

The ring tension and cantilever moments for the 3 cases are shown in figure III.19.

In cases of tank walls monolithically cast with the floor supported on clayey soils liable to unknown rotations, it is recommended to design the section at the foot of the wall for the max. possible negative bending moment M = - 3600 kgm - case of total fixation, the max. positive bending moment M_t = 1172 kgm. and the max. ring tension T_{max} = 31 t are to be taken from the case of hinged base i.e. we make the design for the hatched values shown in figure. The tension in the floor due to cantilever action Q_{max} = 5.33 t/m'

III.5. Tank Walls Continuous with Roof or Floor :

When the bottom of the wall and the edge of the floor slab are made continuous as shown in figure III.20, the elastic deformation of the floor slab tends to rotate the lower point of the wall and introduces a moment M in the corner. The procedure for obtaining M, is so much like moment distribution applied to continuous frames, only there is no carry over moments. The data in tables numbers XVIII and XIX of the P.C.A. , are stiffnesses which denote moments required to cause a unit rotation at the edge of the wall and the floor slab. Only relative values of stiffness are required in this application. The example shown in fig. III.20 gives a good explanation for the problem. The main steps for the solution are :

1) Determine the fixing moment at the bottom of the wall and at the edge of the floor, assuming joint between wall and floor totaly fixed. The two fixing moments are generally not equal.

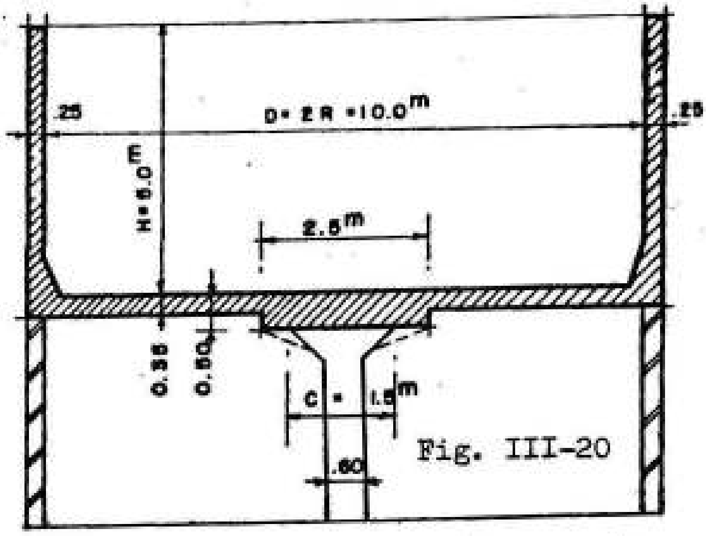


Fig. III-20

2) The difference between the two fixing moments will be dis - tributed between wall and floor by moment distribution method according to the relative stiffness and distribution factors of both wall and floor.

3) Find out the effects of the distributed moments on both and add it to the solution of fixed joint.

4) Determine the concrete dimensions and steel reinforcements such that there is adequate strength and sufficient safety against crac - king.

Example :

A reinforced concrete water tank 10 ms diameter and 5.0 ms deep is supported on a cylinderical wall at its outside edge & on a central column at the centre as shown in fig. III.20. The wall is free at its top edge and continuous with the floor slab, at its bottom edge. The column capital is 1.5 m diameter, and the drop panel, 50 cms thick, is 2.5 ms diameter .

It is required to clarify the effect of continuity between wall and floor on both of them.

Step 1 : Fixed End Moments of Wall & Floor Slab

The fixed end moment of the wall is computed by multiplying the coeff. given in table VII for the corresponding $H^2/D\,t$ at 1.0 H by $p_{max}\,H^2 = 5 \times 5^2 = 125$

$$\frac{H^2}{D\,t} = \frac{5^2}{10 \times .25} = 10 \qquad \text{coeff. (table VII) at 1.0 H} = -\,0.0122$$

$$M = -\,0.0122 \times 125 = -\,1.53 \text{ mt.}$$

The radial moments of the floor slab are effected by the drop panel, but since the ratio of the panel area to the total slab area is about 1/20, its effect on the loads and moments may be neglected and the coefficients given in table XIII, XIV & XV can be used.

load/m^2 floor slab = weight of water + own weight

$$p = 5.0 + 0.35 \times 2.5 = 5.875 \text{ t/m}^2$$

The radial fixed end moment equals the coefficient of 0.049 from table XIII for c/D = 1.5/10 = 0.15 at point 1.0 R multiplied by $p\,R^2$

i.e. $M = -\,0.049 \times 5.875 \times 5^2 = -\,7.2$ mt.

Step 2 : The Connecting Moment :

The fixed end moments of the wall (- 1.53 mt) and floor (- 7.2 mt) being not equal, the difference will be distributed between wall and floor slab according to their relative stiffness, thus:
The relative stiffness of the wall equals $t^3 / H = 25^3/500 = 31.25$ multiplied by coefficient taken from table XVIII for $H^2 / D\,t = 10$.
i.e. 31.25 x 1.01 = 31.56
The relative stiffness of the floor slab equals $t^3 / R = 35^3 / 500 =$ 85.75 multiplied by coefficient taken from table XIX for c/D = 1.5/10 = 0.15

i.e. 85.75 x 0.332 = 28.47

The distribution factors are therefore :

for wall : $\dfrac{31.56}{31.56 + 28.47} = \dfrac{31.56}{60.03} = 0.523$

for floor : $\dfrac{28.47}{31.56 + 28.47} = \dfrac{28.47}{60.03} = 0.477$

The final connecting moment will be determined by moment distribution as follows : (fig. III.21).

	Wall	Floor
Distribution Factor	0.523	0.477
Fixed end Moment	+1.53	-7.20
Distributed Moment	+2.97	+2.70
	+4.50	-4.50

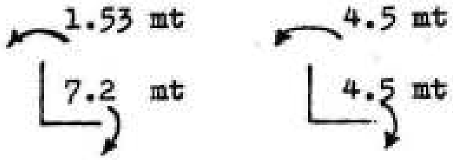


Fig.III.21

The quick damping of the bending moments makes the simple moment distribution shown sufficient for determining the connecting moment in the corner.

Step 3 : <u>Ring Tension & Cantilever Moment of Wall</u>

Ring tension per meter height & cantilever moment for 1m strip will be calculated for a wall with fixed base, subject to triangular water pressure plus the effect of an induced moment equal to - 2.97 mt

a) Ring tension due to triangular water pressure will be computed by multiplying coefficients from table 1 for (H^2 / Dt) = 10 by $p_{max.}$ R = 5 x 5 = 25.

b) Ring tension due to an induced moment of - 2.97 mt will be computed by multiplying coefficient from table VI for $H^2 / Dt = 10$ by $M R / H^2 = - \dfrac{2.97 \times 5}{5^2} = 0.594$

c) Cantilever moment due to triangular water pressure will be computed by multiplying coefficient from table VII for $H^2 / Dt = 10$ by $p_{max} H^2 = 5 \times 5^2 = 125$

d) Cantilever moment due to an induced moment of - 2.97 mt will be computed by multiplying coefficient from table XI for H^2 / Dt = 10 by M = - 2.97 mt.

The final value will be as given in the following table and figure III.22 :

Point	Ring Tension					Cantilever Moment				
	Coeff. Table I Δ load	Coeff. Table VI moment	T t/m due to load	T t/m due to moment	T total in t/m	Coeff. Table VII load	Coeff. Table XI moment	M mt/m due to Δ load	M mt/m due to moment	M total in mt/t
0 H	− 0.011	+ 0.21	− 0.265	− 0.124	− 0.389	.000	0.000	0.000	0.000	0.000
.1H	+ 0.098	− 0.23	+ 2.46	+ 0.136	+ 2.596	.000	0.000	0.000	0.000	0.000
.2H	+ 0.208	− 0.64	+ 5.20	+ 0.38	+ 5.58	.000	− 0.002	+ 0.000	+ 0.006	+ 0.006
.3H	+ 0.323	− 0.94	+ 8.10	+ 0.56	+ 8.66	+ .0001	− 0.009	+ 0.013	+ 0.027	+ 0.040
.4H	+ 0.437	− 0.73	+ 11.00	+ 0.43	+ 11.43	+ .0004	− 0.028	+ 0.050	+ 0.084	+ 0.134
.5H	+ 0.542	+ 0.82	+ 13.55	− 0.49	+ 13.06	+ .0007	− 0.053	+ 0.088	+ 0.159	+ 0.247
.6H	+ 0.608	+ 4.79	+ 15.20	− 2.84	+ 12.36	+ .0019	− 0.067	+ 0.238	+ 0.200	+ 0.438
.7H	+ 0.589	+ 11.63	+ 14.73	− 6.82	+ 7.91	+ .0029	− 0.031	+ 0.363	+ 0.093	+ 0.456
.8H	+ 0.440	+ 19.48	+ 11.00	− 11.55	− 0.55	+ .0028	+ 0.123	+ 0.350	− 0.369	− 0.019
.9H	+ 0.179	+ 20.87	+ 4.47	− 12.40	− 7.93	− .0012	+ 0.467	− 0.150	− 1.400	− 1.550
H	0	0	0	0	0	− .0122	+ 1.000	− 1.530	− 2.970	− 4.500

Shear at base : $Q = Q_1 + Q_2$ or

$Q = 0.158\ P\ H + 5.81\ M / H$

$= 0.158 \times 5 \times 5 + 5.81 \times 2.97 / 5 = 3.95 + 3.45 = 7.4$ t/m

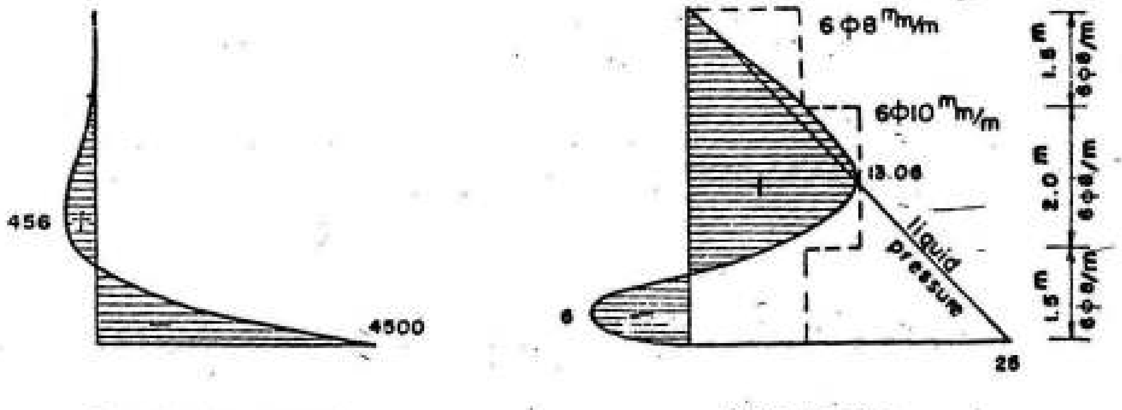


Fig. III-22

Step 4 : <u>Radial and Tangential Moments in Floor Slab</u> :

The radial moments are computed by selecting coefficients for c/D = 0.15 from tables XIII & XV , and multiplying them by $p\ R^2$ = $5.875 \times 25 = 147$ mt/m (for fixed edge) and by M = 2.7 mt/m for moment at edge :

Radial moments in the last line are for a segment having an arc 1 m long at the edge (point 1.OR). They are obtained by multiplying the original moment per meter by the fraction indicating its distance from the centre, for illustration $5.33 \times 0.6 = 3.2$ mt The moments in the 2 last lines of the table are plotted in Fig.

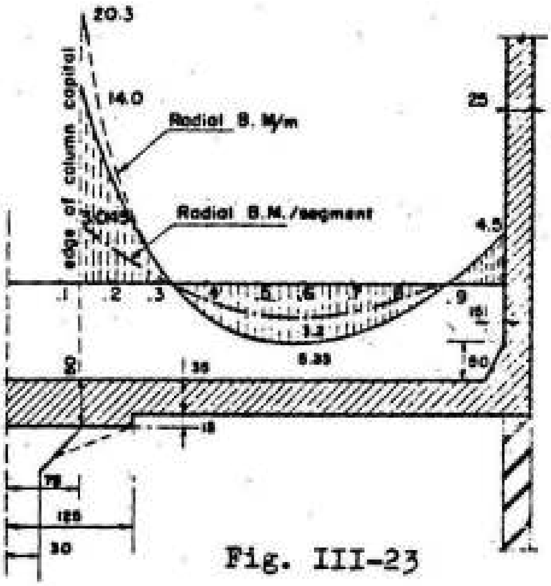


Fig. III-23

III.22 . The radial moment around the central column is relatively high, and the actual value is only = 0.7 the theoretical 0.7(16 + 4.3) = 14 mt. It is however recommended, in this case to enlarge the column capital as shown by the dotted line fig. III.23 so that :

at	0.1 H	0.15 H	0.2 H	0.25 H
thickness	75 cm	66.66 cm	58.33 cm	50 cm

Point	.15R	.20R	.25R	.30R	.40R	.50R	.60R	.70R	.80R	.90R	1.0R
Coeff. T.XIII Fixed, 10^{-2}x	- 10.9	- 5.21	- 2.00	+ 0.02	+ 2.20	+ 2.93	+ 2.69	+ 1.69	+ 0.06	- 2.16	- 4.9
Coeff. Table XV, M at edge	- 1.59	- .930	- .545	- .280	+ .078	+ .323	+ .510	+ .653	+ .790	+ .900	+ 1.0
Radial Moment Fixed Edge	- 16.0	- 7.67	- 2.94	+ 0.03	+ 3.23	+ 4.31	+ 3.95	+ 2.46	+ 0.09	- 3.18	- 7.2
Radial Moment M at edge	- 4.30	- 2.51	- 1.47	- 0.76	+ 0.21	+ 0.87	+ 1.38	+ 1.79	+ 2.14	+ 2.43	+ 2.7
Total Radial Moment mt/m	- 14.0	- 8.50	- 3.50	- 0.73	+ 3.44	+ 5.18	+ 5.33	+ 4.27	+ 2.23	+ 0.75	- 4.5
Total Radial Mon/Segment	- 3.05	- 2.04	- 1.10	- 0.22	+ 1.38	+ 2.59	+ 3.20	+ 3.00	+ 1.78	+ 0.68	- 4.5

The tangential moments are computed by selecting coefficients for c/D = 0.15 from tables XIII & XV and multiplying them by $w R^2$ = 147 kgm/m (for fixed edge), and by M = 2.7 mt/m (for moment at edge).

The total tangential bending moments are plotted in figure III.24. The effective depth within the drop panel is 46 centimeters instead of 32 cms. in the rest of the floor slab, and if the moments

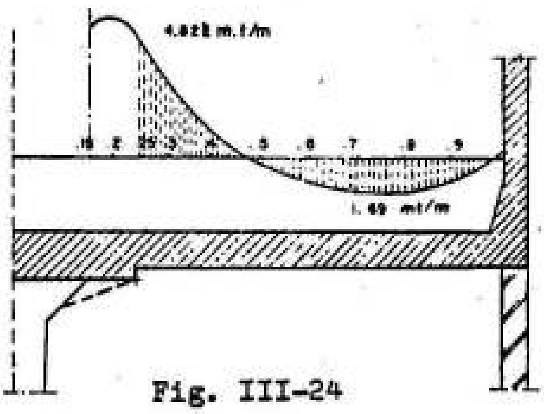


Fig. III-24

in that region are reduced in the ratio 32/46 it is seen that the critical moment, 4.822 occurs at the edge of the drop panel.

Point	.15R	.20R	.25R	.30R	.40R	.50R	.60R	.70R	.80R	.90R	1.0R
Coeff. T. XIII Fixed, 10^{-2}x	− 2.18	− 2.84	− 2.43	− 1.77	− 0.51	+ 0.31	+ 0.80	+ 0.86	+ 0.57	− 0.06	− 0.98
Coeff. Table XV, M at edge	− .319	− .472	− .463	− .404	− .251	− .100	+ .035	+ .157	+ .263	+ .363	+ .451
Tang. Moment Fixed edge	− 3.21	− 4.18	− 3.57	− 2.60	− 0.75	+ 0.47	+ 1.18	+ 1.27	+ 0.84	− 0.09	− 1.44
Tang. Moment M at edge	− 0.86	− 1.27	− 1.25	− 1.09	− 0.68	− 0.27	+ 0.10	+ 0.42	+ 0.71	+ 0.98	+ 1.22
Total Tang. Moment mt/m	− 4.07	− 5.45	− 4.82	− 3.69	− 1.43	+ 0.20	+ 1.28	+ 1.69	+ 1.55	+ 0.89	− 0.22

Step 5 : <u>Design of the Different Elements</u> :

a) Wall : Max. ring tension : T_{max} = 13.06 t at 0.5 H t =25 cm

max. ring reinforcements : (σ_s = 1400 kg/cm^2)

$$A_s = \frac{T}{\sigma_s} = \frac{13060}{1400} = 9.3 \text{ cm}^2 \quad 6 \, \Phi \, 10 \ \text{/m on each side } (9.4 \text{ cm}^2)$$

max. tensile stress in concrete (including shrinkage) :

Assuming ϵ_{sh} = 0.00025 , E_s = 2100 000 kg/cm^2

$$\sigma_t = \frac{T + \epsilon_{sh} E_s A_s}{A_c + n \ A_s} = \frac{13060 + 0.00025 \times 2100000 \times 9.4}{100 \times 25 + 10 \times 9.4}$$

$$= \frac{13060 + 5740}{2594} = 7.27 \text{ kg/cm}^2$$

The max. positive moment in wall = 456 kgm at 0.7 H from top

Vertical reinforcement on outside surface :

$$A_s = \frac{456}{1300 \times .22} = 1.6 \text{ cm}^2 \text{ chosen } 5 \, \Phi \, 8 \text{ mm (min).}$$

Section at foot of wall : max. negative moment M_{max} − = 4500 kgm.

$t = \sqrt{\frac{4500}{3}}$ = 38.7 cms chosen 40 cms

$37 = k_1 \sqrt{4500}$ $k_1 = 0.553$ for σ_s = 1400 kg/cm^2 k_2 = 1280

$A_s = \frac{4500}{1280 \times .37}$ = 9.5 cm^2 chosen 7 Ø 13 mm/m

at top end of haunch M = 1550 kgm

$A_s = \frac{1550}{1300 \times .22}$ = 5.44 cm^2 chosen 7 Ø 13 mm/m O.K.

b) Floor Slab :

The column load is determined by multiplying coefficients taken from table XVII for c/D = 0.15 by p R^2, for edge fixed, and by M = 2.7 mt/m, for induced moment :

Col. load when edge is fixed : 1.007 x 5.875 x 5^2 = 147 t

Col. load due to induce moment at edge : 9.29 x 2.7 = 34 t

Total 181 t

Diam. of critical section for shear around theoretical capital is 1.50 + 0.50 = 2.00 ms

Length of this section is given by π x 2.00 = 6.28 ms

Load on area within the section 5.875 x π x 1^2 = 18.5 t

Shear stress $\tau = \frac{Q}{.87\ b\ d} = \frac{181000 - 18500}{.87 \times 628 \times 47} = \frac{162500}{25600}$ = 6.35 kg/cm^2

Diam. of critical section for shear around drop panel = 250 + 35=285 cm

Length of this section = π x 2.85 = 9. ms

Load on area within the section 5.875 x π x 1.425^2 = 37.5 t

Shear stress $\tau = \frac{Q}{.87\ b\ d} = \frac{181000 - 37500}{.87 \times 900 \times 32} = \frac{143500}{25000}$ = 5.85 kg/cm^2

Shear at edge of wall = p π R^2 - column load

= 5.875 x 3.14 x 5^2 - 181 = 460 - 181=269 t

Length of this section = 3.14 x 10 = 31.4 ms

Shear stresses $\tau = \frac{Q}{.87\ b\ d} = \frac{269000}{.87 \times 3140 \times 32} = 3.10\ kg/cm^2$

The horiz, reaction of the wall $Q = 7.4\ t/m$ will be resisted by ring reinforcement at the foot of the wall.

$$A_s = \frac{7.4 \times 5}{1.4} = 24.5\ cm^2 \quad 10\ \emptyset\ 19\ mm.$$

The max. positive radial moment $M^{+}_{max} = 5.33\ mt \quad t = 35\ cm$

$32 = k_1 \sqrt{5330} \quad k_1 = 0.44 \quad \sigma_c = 40\ kg/cm^2 \quad k_2 = 1240$

$A_s = \frac{5330}{1240 \times .32} = 13.4\ cm^2 \quad 10\ \emptyset\ 13\ mm/m$

Max. - ve B.M. at wall surface $M- = 3500\ kgm$

$\sigma_t = \frac{6 \times 350000}{100 \times 35^2} = 17\ kg/cm^2$

$32 = k_1 \sqrt{3500} \quad k_1 = 0.54 \quad \sigma_c = 30 \quad k_2 = 1280$

$A_s = \frac{3500}{1280 \times .32} = 8.6\ cm^2$ chosen $7\ \emptyset\ 13\ mm/m$

Top radial reinforcements at middle

$M_{max} - = 14\ mt \quad t = 66.66\ cms \quad$ at $r = 75\ cms$

$\sigma_t = \frac{6 \times 14 \times 10^5}{100 \times 66.66^2} = 19\ kg/cm^2$

$A_s = \frac{14000}{1250 \times 63} = 18\ cm^2/m \quad$ total $A_s = 18 \times \pi \times 1.5 = 85 cm^2$

$M -= 8.5\ mt \quad t = 58.33\ cms \quad$ at $r = 1.00\ m$

$\sigma_t = \frac{6 \times 2.5 \times 10^5}{100 \times 58.33^2} = 15\ kg/cm^2$

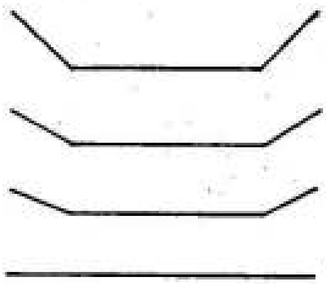
$A_s = \frac{8500}{1250 \times .55} = 12.3\ cm^2/m \quad$ total $A_s = 12.3 \times \pi \times 2 = 78 cm^2$

$M -= 3.5\ mt \quad t = 35\ cms \quad$ at $r = 1.25\ ms$

$\sigma_t = 17\ cms^2 \quad A_s/m = 8.6\ cm^2 \quad$ total $A_s = 8.6 \times \pi \times 25 = 67.5 cm^2$

The above calculations show that the total amount of the top radial reinforcement does not vary much in the central part of the slab around the column head so that the same number of bars may be used, at smaller distances in the middle and at bigger distances as we move outwards. Therefore :

Use 32 Ø 19 mm arranged as shown in fig. III.25 in which we use 4 types of bars of the following form :



4 Ø 19
4 Ø 19
4 Ø 19
4 Ø 19
32 Ø 19

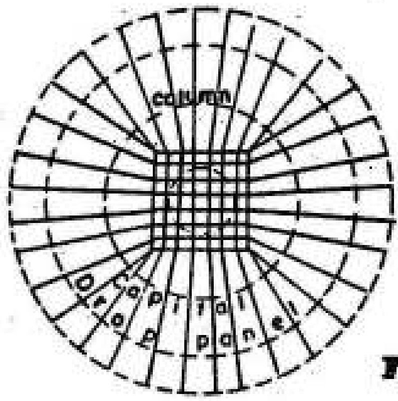


Fig. III.25

Max. ring reinforcements in lower fiber at $x = 0.75\ r$:

$$A_s = \frac{M_{tang}}{k_2\ d} = \frac{1690}{1300 \times 0.3} = 4.4\ cm^2 \qquad Ø\ 10\ mm\ @\ 15\ cm$$

Ring reinforcements in upper fiber at $x = 0.25\ r$

$$A_s = \frac{4822}{1250 \times 0.3} = 13\ cm^2 \qquad Ø\ 16\ mm\ @\ 15\ cms$$

Ring reinforcements in upper fiber at $x = 0.35\ r$

$$A_s = \frac{2750}{1250 \times 0.3} = 7.35\ cm^2 \qquad Ø\ 13\ mm\ @\ 15\ cms$$

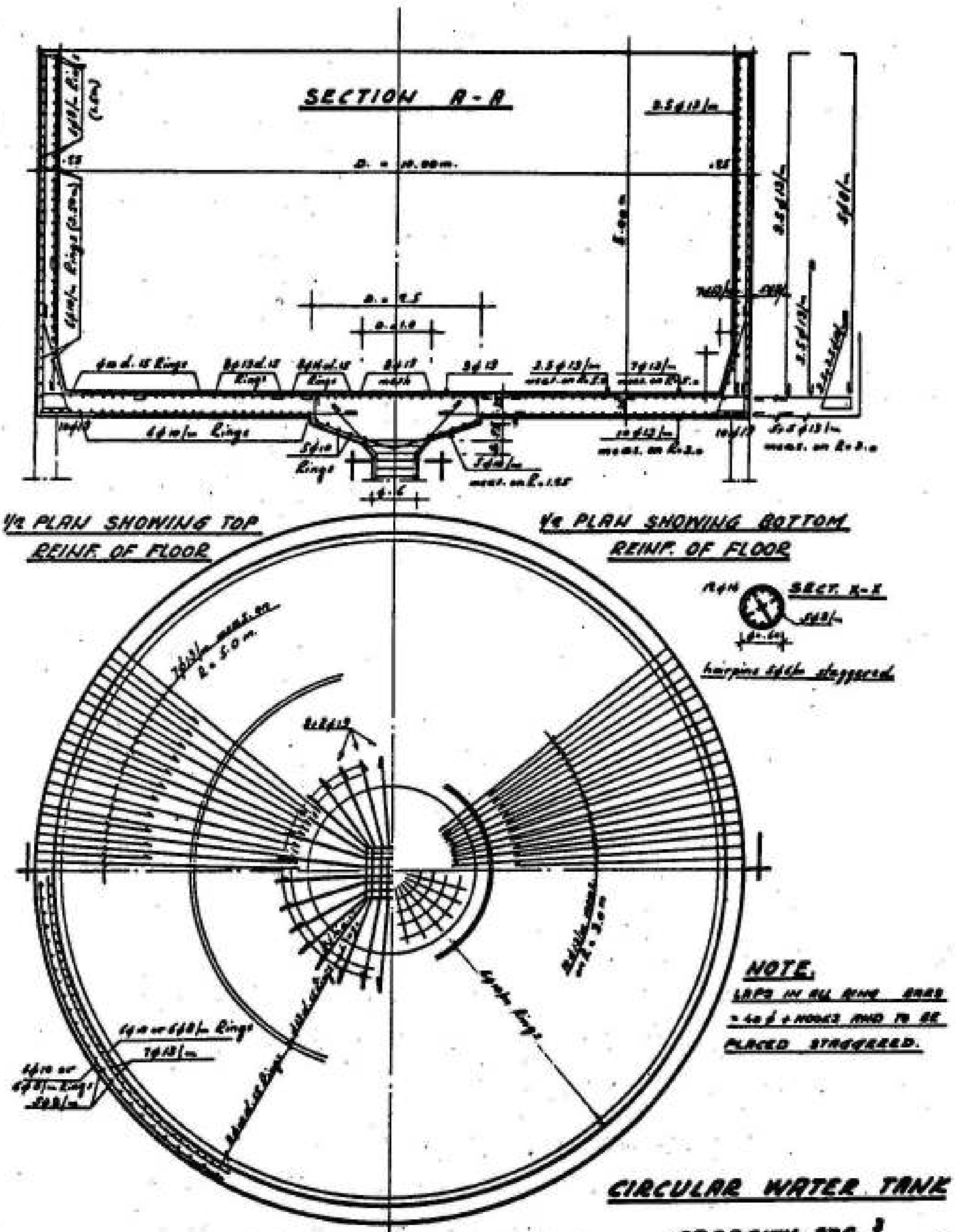
c) Central Column : $P = 181$ t, $\phi = 60$ cms, $A_s = 12\ Ø\ 16$ mm

$P = \sigma_c\ (A_c + n\ A_s)$ or $181000 = \sigma_c\ (2850 + 15 \times 24) = 24$ cms

or $\sigma_c = 56\ kg/cm^2$ O.K.

The details of reinforcements are shown in fig. III.26

It has to be noted that in the given details, bent up bars have been avoided because the shear stresses are low (max. value = 6.35 kg/cm^2), moreover, such bars in walls are liable to move from their position during concreting operations and the laying of straight bars in circular elements is easier.



CIRCULAR WATER TANK

CAPACITY 375 m^3

Fig. III - 26

IV . ROOFS AND FLOORS OF CIRCULAR TANKS

IV.1. INTRODUCTION

Roofs and floors of circular tanks may be of the beam and slab type as shown in figures IV - 1 a and b

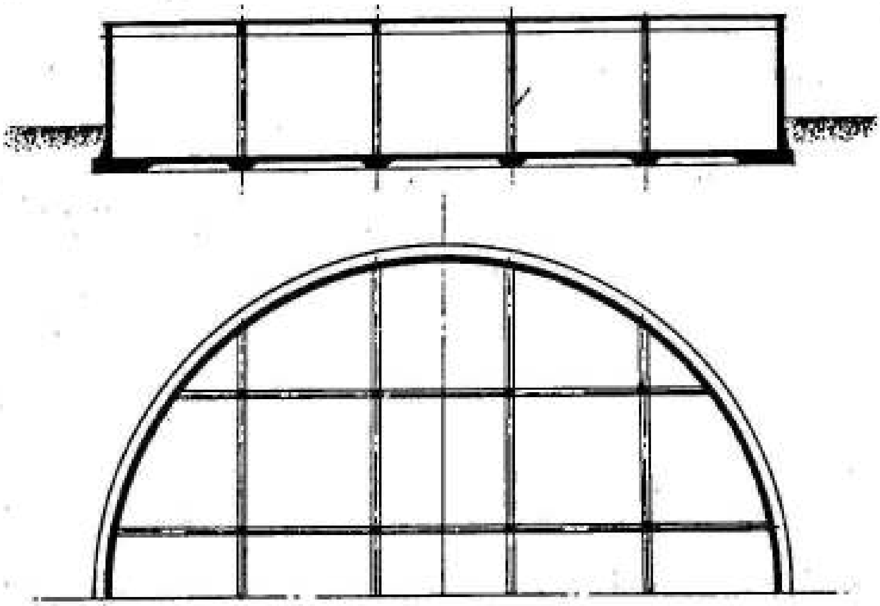


Fig. IV-1a

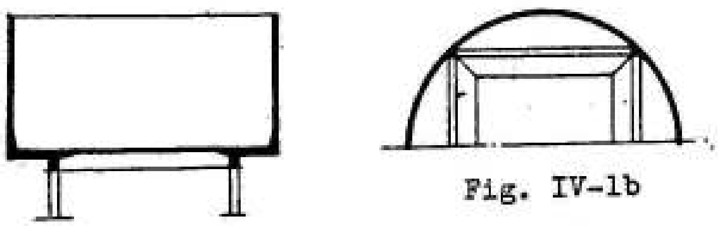


Fig. IV-1b

or of the flat slab type as shown in figure IV- 2

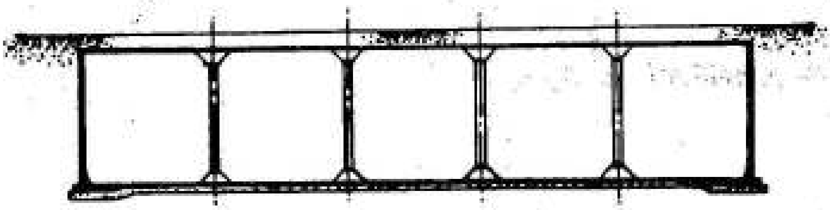
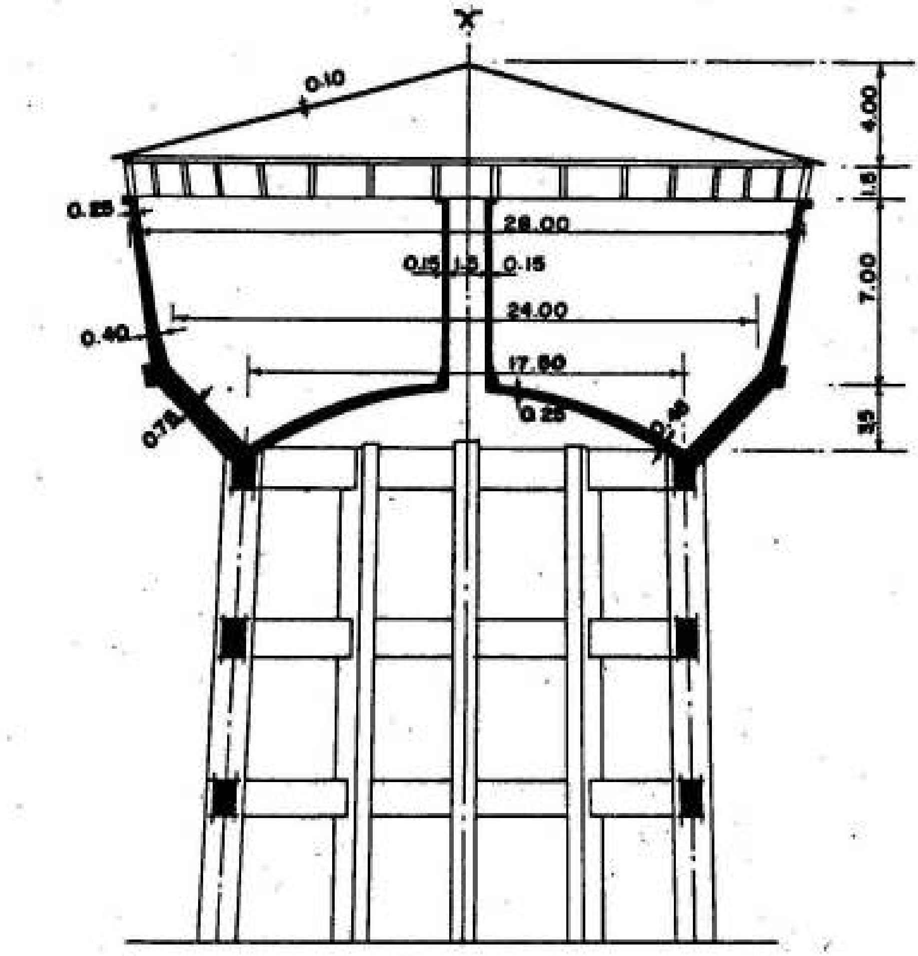


Fig. IV-2

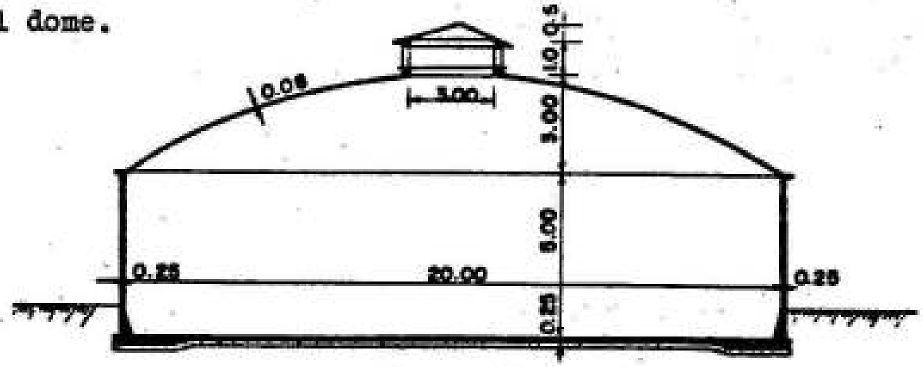
shuttering are taken in consideration. Figure IV-4 shows an Inze tank 5000 cubic meters capacity designed by the author and executed at El Nasr City, Cairo. Figure IV-5 gives another example



Reinforced concrete Inze water tower with conical roof

Fig. IV-4

of a circular reinforced concrete reservoir covered by an 8 cms thick spherical dome.



Reinforced concrete dome covering a circular tank

Fig. IV-5

IV-2. INTERNAL FORCES IN CIRCULAR FLAT PLATES

It will be assumed that :

p = Uniform load per unit area.

P = Total load, concentrated load or live load per unit length.

t = Plate thickness.

E = Modulus of elasticity (generally = 210 000 kg/cm^2 for concrete.)

v = $1/m$ = Poisson's ratio. For concrete, it varies between 1/10 and 1/5 . It is generally assumed in the following equal to 1/6. Some neglected its effect and assume it equal to zero.

$$D = \frac{E\,t^3}{12\,(1 - v^2)}$$ = Flexural rigidity of a plate.

δ = Deflection of the middle surface of the plate.

φ = Slope angle " " " " " " "

A = Reaction at the support.

Q_r = Radial shearing force.

M_r = Radial bending moment.

M_t = Tangential bending moment.

a = Radius of plate from its middle axis to the support.
  = Span of a circular strip of a continuous plate.

$b = \beta a$ = radius of a specified circle on the plate.

$r = \rho a$ = radius of any circle on the plate.

Notes :

All logarithms appearing in the following equations* are natural (i.e. to the base e) .

Reference : Beton-Kalender 1952 , Volume II. Page 271. Published by Wilhelm Ernst & Sohn. Berlin. München. Düsseldorf.

1) **Solid Circular Plate Subjected to a Uniformly Distributed Load.**

(Fig.IV.6)

Load p = constant Total load $P = p\,\pi\,a^2$

$$Q_r = -\frac{P}{2\pi a}\cdot\rho \qquad A = \frac{P}{2\pi a}$$

a) Fixed Edge

$$\delta = \frac{P a^2}{64\pi D}\cdot(1-\rho^2)^2 \qquad \varphi = \frac{P a}{16\pi D}\cdot\rho\,(1-\rho^2)$$

$$M_r = \frac{P}{16\pi}\left[1+\nu-(3+\nu)\rho^2\right] \qquad M_t = \frac{P}{16\pi}\left[1+\nu-(1+3\nu)\rho^2\right]$$

b) Simply Supported Edge

$$\delta = \frac{P a^2}{64\pi D}\cdot(1-\rho^2)\left(\frac{5+\nu}{1+\nu}-\rho^2\right) \qquad \varphi = \frac{P a}{16\pi D}\cdot\rho\left(\frac{3+\nu}{1+\nu}-\rho^2\right)$$

$$M_r = \frac{P}{16\pi}(3+\nu)(1-\rho^2) \qquad M_t = \frac{P}{16\pi}\left[3+\nu-(1+3\nu)\rho^2\right]$$

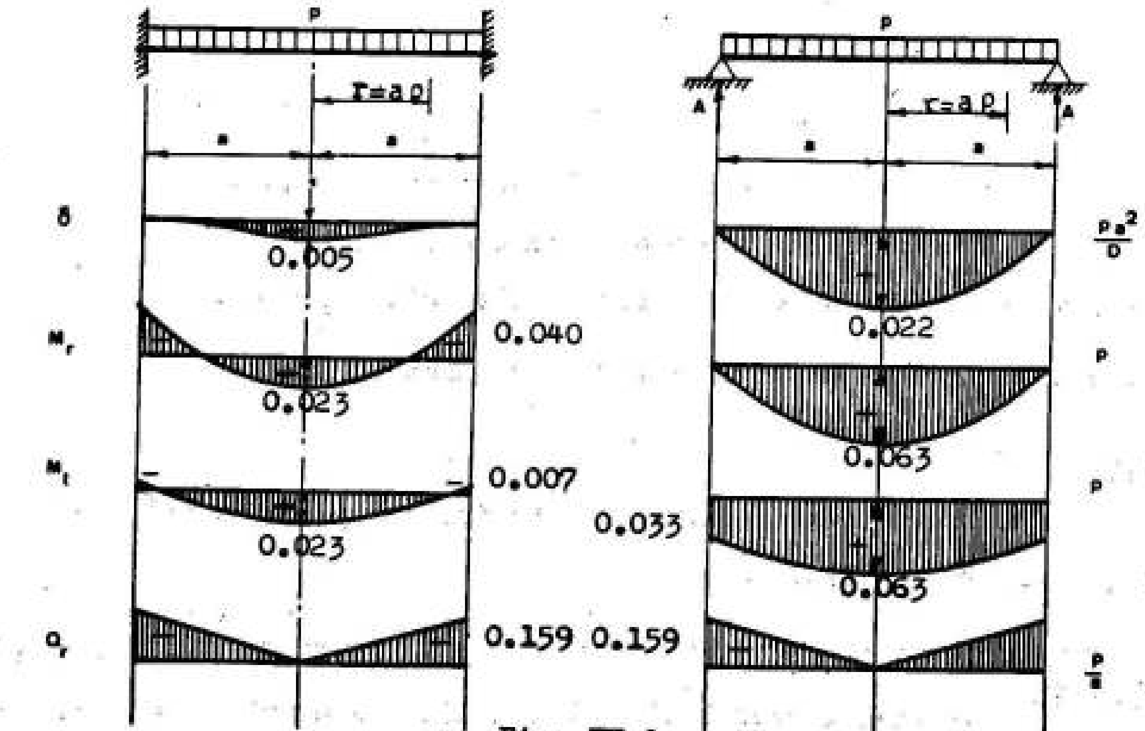


Fig. IV-6

2) Solid Circular Plate Subjected to a Concentrated Load P at Center (Fig. IV.7)

$Q_r = -\frac{P}{2\pi a}\cdot\frac{1}{\rho}$ for $\rho > \beta$, $Q_r = 0$ for $\rho = 0$, $A = \frac{P}{2\pi a}$

a) Fixed Edge

$\delta = \frac{P a^2}{16\pi D}(1 - \rho^2 + 2\rho^2 \ln\rho)$ $\qquad \varphi = -\frac{Pa}{4\pi D}\cdot\rho\cdot\ln\rho$

$M_r = -\frac{P}{4\pi}\left[1 + (1+\nu)\ln\rho\right]$
$M_t = -\frac{P}{4\pi}\left[\nu + (1+\nu)\ln\rho\right]$ $\Big\}$ for $\rho > \beta$

$M_r = M_t = -\frac{P}{4\pi}(1+\nu)\ln\beta$ for $\rho = 0$

b) Simply Supported Edge

$\delta = \frac{P a^2}{16\pi D}\left[\frac{3+\nu}{1+\nu}(1-\rho^2) + 2\rho^2\ln\rho\right]$ $\qquad \varphi = \frac{Pa}{4\pi D}\cdot\rho\left(\frac{1}{1+\nu} - \ln\rho\right)$

$M_r = -\frac{P}{4\pi}(1+\nu)\ln\rho$
$M_t = \frac{P}{4\pi}\left[1 - \nu - (1+\nu)\ln\rho\right]$ $\Big\}$ for $\rho > \beta$

$M_r = M_t = \frac{P}{4\pi}\left[1 - (1+\nu)\ln\beta\right]$ for $\rho = 0$

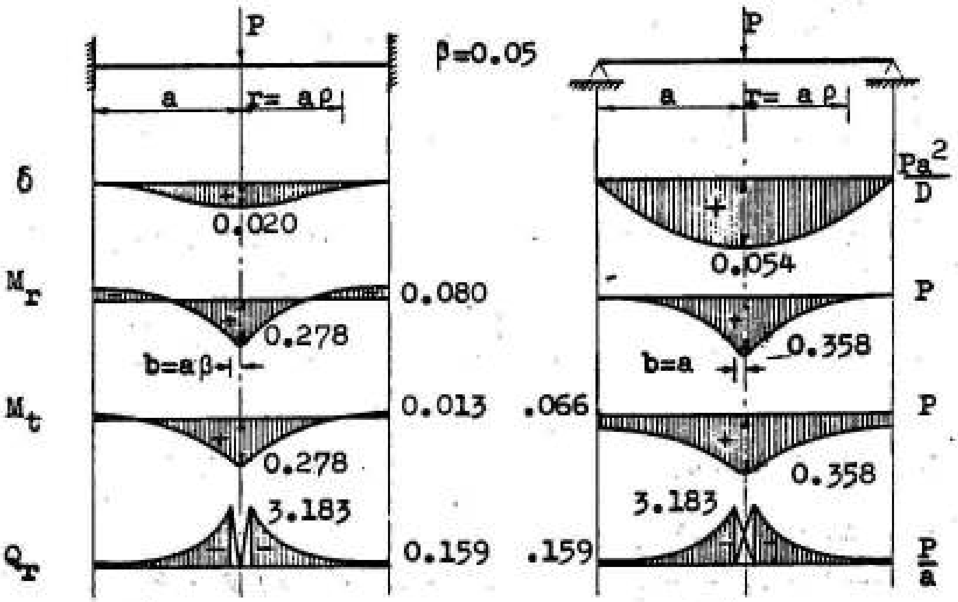


Fig. IV-7

3) Solid Circular Plate Subjected to Partial Uniform Load at the Middle

(Fig. IV.8)

Total load $P = p\,\pi\,b^2 = p\,\pi\,a^2\,\beta^2$ $\qquad A = P / 2\pi a$

$$Q_r = -\frac{P}{2\pi a \beta^2}\,\rho \qquad \text{for} \qquad 0 < \rho < \beta$$

$$Q_r = -\frac{P}{2\pi a}\;\frac{1}{\rho} \qquad \text{for} \qquad \beta \leq \rho \leq 1$$

a) Fixed Edge

i) Inner loaded part $\qquad 0 \leq \rho \leq \beta$

$$\delta = \frac{P a^2}{64\pi D}\left[4 - 3\beta^2 + 4\beta^2 \ln\beta - 2(\beta^2 - 4\ln\beta)\rho^2 + \frac{\rho^4}{\beta^2}\right]$$

$$\varphi = \frac{Pa}{16\pi D}\cdot\rho\left(\beta^2 - 4\ln\beta - \frac{\rho^2}{\beta^2}\right)$$

$$M_r = \frac{P}{16\pi}\left[(1+\nu)(\beta^2 - 4\ln\beta) - \frac{3+\nu}{\beta^2}\rho^2\right]$$

$$M_t = \frac{P}{16\pi}\left[(1+\nu)(\beta^2 - 4\ln\beta) - \frac{1+3\nu}{\beta^2}\rho^2\right]$$

ii) Outer unloaded part $\qquad \beta \leq \rho \leq 1$

$$\delta = \frac{P a^2}{32\pi D}\left[(2-\beta^2)(1-\rho^2) + 2(\beta^2 + 2\rho^2)\ln\rho\right]$$

$$\varphi = \frac{Pa}{16\pi D}\cdot\rho\left[\beta^2\left(1 - \frac{1}{\rho^2}\right) - 4\ln\rho\right]$$

$$M_r = \frac{P}{16\pi}\left[-4 + (1+\nu)\frac{\beta^2}{\rho^2} + (1+\nu)(\beta^2 - 4\ln\rho)\right]$$

$$M_t = \frac{P}{16\pi}\left[-4\nu - (1-\nu)\frac{\beta^2}{\rho^2} + (1+\nu)(\beta^2 - 4\ln\rho)\right]$$

b) Simply Supported Edge $\qquad 0 < \rho < \beta$

i) Inner loaded part

$$\delta = \frac{P a^2}{64\pi D}\cdot\frac{1}{1+\nu}\left\{4(3+\nu) - (7+3\nu)\beta^2 + 4(1+\nu)\beta^2\ln\beta - 2\left[4 - (1-\nu)\beta^2 - 4(1+\nu)\ln\beta\right]\rho^2 + \frac{1+\nu}{\beta^2}\rho^4\right\}$$

$$\varphi = \frac{Pa}{16\,\pi\,D} \cdot \frac{\rho}{1+\nu}\left[4 - (1-\nu)\,\beta^2 - (1+\nu)\ln\beta - \frac{1+\nu}{\beta^2}\,\rho^2\right]$$

$$M_r = \frac{P}{16\,\pi}\left[4 - (1-\nu)\beta^2 - 4\,(1+\nu)\ln\beta - \frac{3+\nu}{\beta^2}\,\rho^2\right]$$

$$M_t = \frac{P}{16\,\pi}\left[4 - (1-\nu)\beta^2 - 4\,(1+\nu)\ln\beta - \frac{1+3\nu}{\beta^2}\,\rho^2\right]$$

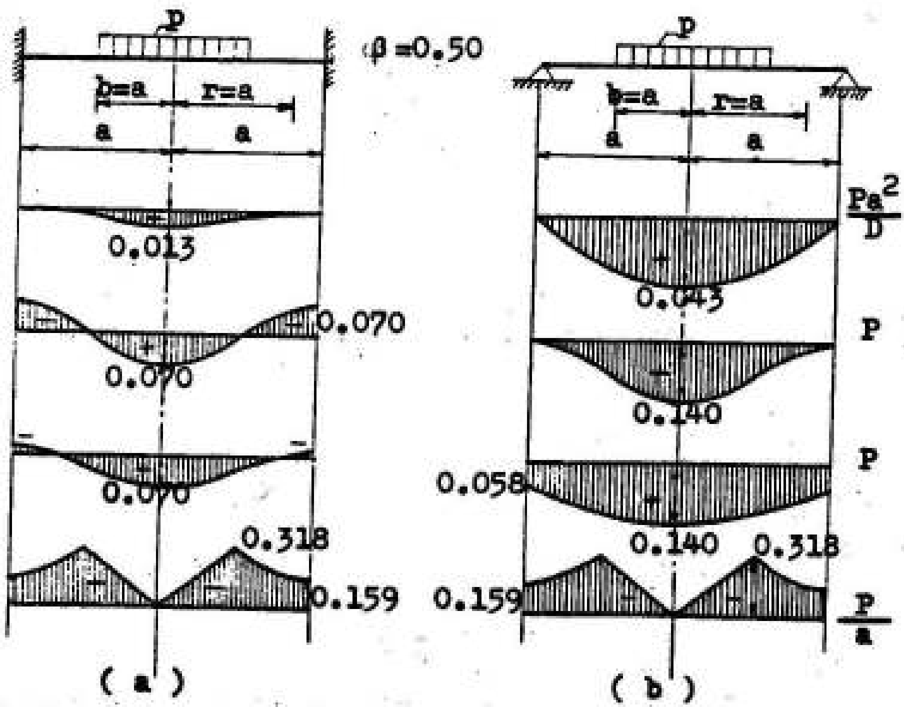
ii) Outer unloaded part $\quad \beta \leqslant \rho < 1$

$$\delta = \frac{P\,a^2}{32\,\pi\,D} \cdot \frac{1}{1+\nu}\left\{\left[2\,(3+\nu) - (1-\nu)\beta^2\right](1-\rho^2) + 2\,(1+\nu)\beta^2\ln\rho + 4\,(1+\nu)\rho^2\ln\rho\right\}$$

$$\varphi = \frac{Pa}{16\,\pi\,D} \cdot \frac{1}{1+\nu}\left\{\left[4-(1-\nu)\,\beta^2\right]\rho - (1+\nu)\,\frac{\beta^2}{\rho} - 4\,(1+\nu)\,\rho\,\ln\rho\right\}$$

$$M_r = \frac{P}{16\,\pi}\left[(1-\nu)\,\beta^2\left(\frac{1}{\rho^2} - 1\right) - 4\,(1+\nu)\,\ln\rho\right]$$

$$M_t = \frac{P}{16\,\pi}\left\{(1-\nu)\left[4 - \beta^2\left(\frac{1}{\rho^2} + 1\right)\right] - 4(1+\nu)\ln\rho\right\}$$



Solid Circular Plate Subjected to Partial Uniform Load at the Middle

Fig. IV-8

4) Solid Circular Plate Loaded by a Ring Load

(Fig.IV-9)

Ring load P/m ; $A = P \beta$

$Q_r = 0$ for $0 \leqslant \rho < \beta$ $\qquad Q_r = -\frac{P\beta}{\rho}$ for $\beta < \rho \leqslant 1$

a) Fixed Edge

i) Inner part $\qquad 0 < \rho \leqslant \beta$

$$\delta = \frac{P a^3}{8 D} \beta \left[1 - \beta^2 + 2 \beta^2 \ln \beta + (1 + \beta^2 + 2 \ln \beta) \rho^2 \right]$$

$$\varphi = -\frac{P a^2}{8 D} \beta \rho (1 - \beta^2 + 2 \ln \beta)$$

$$M_r = M_t = -\frac{Pa}{4} \beta (1 + \nu) (1 - \beta^2 + 2 \ln \beta)$$

ii) Outer part $\qquad \beta \leqslant \rho \leqslant 1$

$$\delta = \frac{P a^3}{8 D} \beta \left[(1 + \beta^2) (1 - \rho^2) + 2 (\beta^2 + \rho^2) \ln \rho \right]$$

$$\varphi = \frac{P a^2}{4 D} \beta \rho \left[\beta^2 (1 - \frac{1}{\rho^2}) - 2 \ln \rho \right]$$

$$M_r = -\frac{P a}{4} \beta \left[2 - (1 - \nu) \frac{\beta^2}{\rho^2} - (1 + \nu) (\beta^2 - 2 \ln \rho) \right]$$

$$M_t = -\frac{P a}{4} \beta \left[2\nu + (1 + \nu) \frac{\beta^2}{\rho^2} - (1 + \nu) (\beta^2 - 2 \ln \rho) \right]$$

b) Simply Supported Edge

i) Inner part $\qquad 0 \leqslant \rho < \beta$

$$\delta = \frac{P a^3}{8 D} \cdot \frac{\beta}{1 + \nu} \left\{ (3 + \nu) (1 - \beta^2) + 2 (1 + \nu) \beta^2 \ln \beta - \left[(1 - \nu)(1 - \beta^2) - 2 (1 + \nu) \ln \beta \right] \rho^2 \right\}$$

$$\varphi = \frac{P a^2}{4 D} \cdot \frac{\beta \rho}{1 + \nu} \left[(1 - \nu)(1 - \beta^2) - 2 (1 + \nu) \ln \beta \right]$$

$$M_r = M_t = \frac{P a}{4} \beta \left[(1 - \nu)(1 - \beta^2) - 2 (1 + \nu) \ln \beta \right]$$

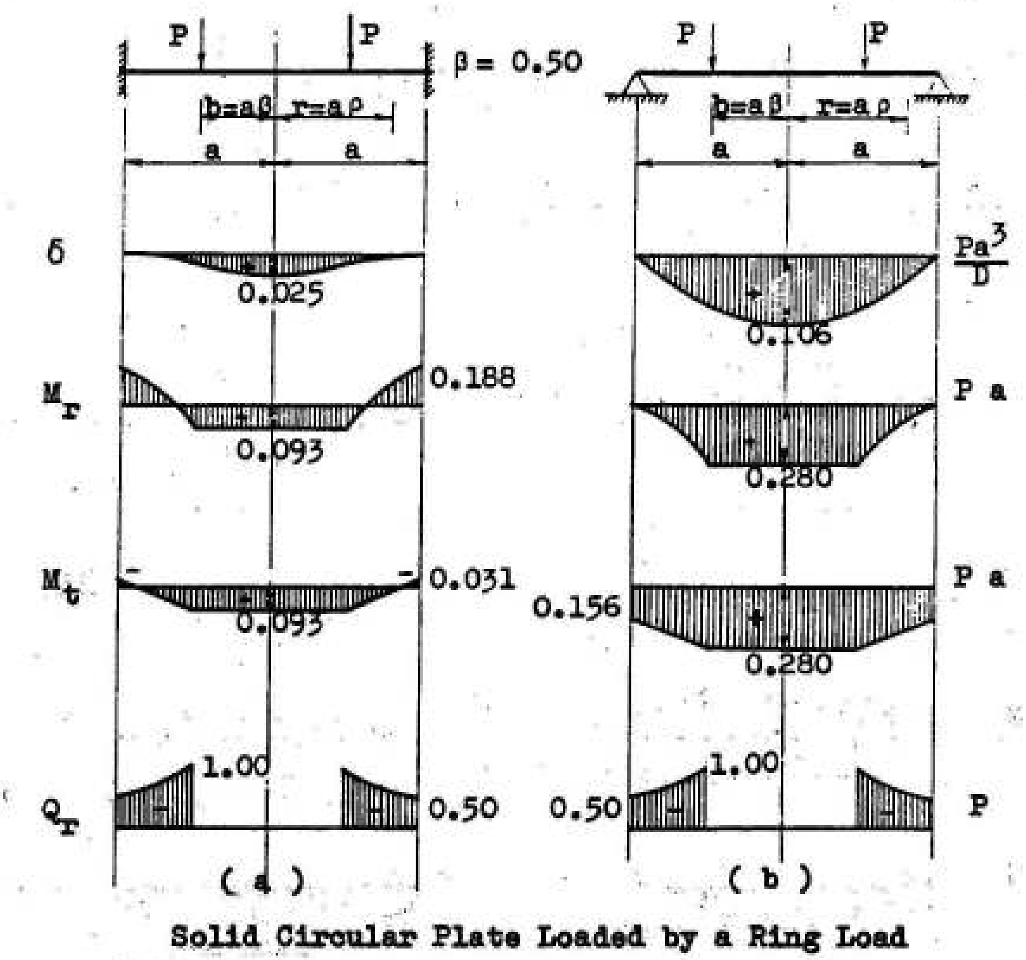
Outer part $\quad \beta < \rho < 1$

$$= \frac{P a^3}{8 D} \cdot \frac{\beta}{1 + \nu} \left\{ \left[3 + \nu - (1 - \nu) \beta^2 \right] (1 - \rho^2) + 2 (1 + \nu)(\beta^2 + \rho^2) \ln \rho \right\}$$

$$= \frac{P a^2}{4 D} \quad \frac{\beta \rho}{1 + \nu} \left[2 - (1 - \nu) \beta^2 - (1 + \nu) \frac{\beta^2}{\rho^2} - 2 (1 + \nu) \ln \rho \right]$$

$$r = \frac{P a}{4} \beta \left[(1 - \nu) \beta^2 (\frac{1}{\rho^2} - 1) - 2 (1 + \nu) \ln \rho \right]$$

$$_t = \frac{P a}{4} \beta \left\{ (1 - \nu) \left[2 - \beta^2 (\frac{1}{\rho^2} + 1) \right] - 2(1 + \nu) \ln \rho \right\}$$



Solid Circular Plate Loaded by a Ring Load

Fig. IV-9

Table XVI

Shear at base of cylindrical wall

$V = \text{coef.} \times \begin{cases} wH^2 & \text{(triangular)} \\ pH & \text{(rectangular)} \\ M/H & \text{(mom. at base)} \end{cases}$

Positive sign indicates shear acting inward

$\frac{H^2}{Dt}$	Triangular load, fixed base	Rectangular load, fixed base	Triangular or rectangular load, hinged base	Moment at edge
0.4	+0.436	+0.755	+0.245	-1.58
0.8	+0.374	+0.552	+0.234	-1.75
1.2	+0.339	+0.460	+0.220	-2.00
1.6	+0.317	+0.407	+0.204	-2.28
2.0	+0.299	+0.370	+0.189	-2.57
3.0	+0.262	+0.310	+0.158	-3.18
4.0	+0.236	+0.271	+0.137	-3.68
5.0	+0.213	+0.243	+0.121	-4.10
6.0	+0.197	+0.222	+0.110	-4.49
8.0	+0.174	+0.193	+0.096	-5.18
10.0	+0.158	+0.172	+0.087	-5.81
12.0	+0.145	+0.158	+0.079	-6.38
14.0	+0.135	+0.147	+0.073	-6.88
16.0	+0.127	+0.137	+0.068	-7.36

Table XVIII

Stiffness of cylindrical wall
Near edge hinged, far edge free
$k = \text{coef.} \times Et^3/H$

$\frac{H^2}{Dt}$	Coefficient	$\frac{H^2}{Dt}$	Coefficient
0.4	0.139	5	0.713
0.8	0.270	6	0.783
1.2	0.345	8	0.903
1.6	0.399	10	1.010
2.0	0.445	12	1.108
3.0	0.548	14	1.198
4.0	0.635	16	1.281

Table XVII

Load on center support for circular slab

$\text{Load} = \text{coef.} \times \begin{cases} pR^2 & \text{(hinged and fixed)} \\ M & \text{(moment at edge)} \end{cases}$

c/D	0.05	0.10	0.15	0.20	0.25
Hinged	1.320	1.387	1.463	1.542	1.625
Fixed	0.839	0.919	1.007	1.101	1.200
M at edge	8.16	8.66	9.29	9.99	10.81

Table XIX

Stiffness of circular plates
With center support
$k = \text{coef.} \times Et^3/R$

c/D	0.05	0.10	0.15	0.20	0.25
Coef.	0.290	0.309	0.332	0.358	0.387

Without center support
Coef. = 0.104

Table XX. Supplementary Coefficients for Values of H^2/Dt Greater than 16 (*Extension of Tables I to XI, XVI and XVIII*)*

$\frac{H^2}{Dt}$	TABLE I					TABLE II				
	Coefficients at point					Coefficients at point				
	.75H	.80H	.85H	.90H	.95H	.75H	.80H	.85H	.90H	.95H
20	+0.716	+0.654	+0.520	+0.325	+0.115	+0.812	+0.817	+0.756	+0.603	+0.344
24	+0.746	+0.702	+0.577	+0.372	+0.137	+0.816	+0.839	+0.793	+0.647	+0.377
32	+0.782	+0.764	+0.663	+0.459	+0.182	+0.814	+0.861	+0.847	+0.721	+0.436
40	+0.800	+0.805	+0.731	+0.530	+0.217	+0.802	+0.866	+0.880	+0.778	+0.483
48	+0.791	+0.828	+0.785	+0.593	+0.254	+0.791	+0.864	+0.900	+0.820	+0.527
56	+0.763	+0.838	+0.824	+0.636	+0.285	+0.781	+0.859	+0.911	+0.852	+0.563

$\frac{H^2}{Dt}$	TABLE III					TABLE IV				
	Coefficients at point					Coefficients at point				
	.75H	.80H	.85H	.90H	.95H	.75H	.80H	.85H	.90H	.95H
20	+0.949	+0.825	+0.629	+0.379	+0.128	+1.062	+1.017	+0.906	+0.703	+0.394
24	+0.986	+0.879	+0.694	+0.430	+0.149	+1.056	+1.039	+0.943	+0.747	+0.427
32	+1.026	+0.953	+0.788	+0.519	+0.189	+1.064	+1.061	+0.997	+0.821	+0.486
40	+1.040	+0.996	+0.859	+0.591	+0.226	+1.052	+1.068	+1.030	+0.878	+0.533
48	+1.043	+1.022	+0.911	+0.652	+0.262	+1.041	+1.064	+1.050	+0.920	+0.577
56	+1.040	+1.035	+0.949	+0.705	+0.294	+1.021	+1.059	+1.061	+0.952	+0.613

$\frac{H^2}{Dt}$	TABLE V					TABLE VI				
	Coefficients at point					Coefficients at point				
	.00H	.05H	.10H	.15H	.20H	.75H	.80H	.85H	.90H	.95H
20	-18.44	- 9.98	- 4.90	- 1.59	+ 0.22	+15.30	+ 25.1	+ 36.9	+ 43.3	+ 35.3
24	-18.04	-10.34	- 4.54	- 1.00	+ 0.68	+13.20	+ 25.9	+ 40.7	+ 51.8	+ 45.3
32	-20.84	-10.72	- 3.70	- 0.04	+ 1.26	+ 8.10	+ 23.2	+ 45.9	+ 65.4	+ 63.6
40	-23.34	-10.86	- 2.86	+ 0.72	+ 1.56	+ 3.28	+ 19.2	+ 46.5	+ 77.9	+ 83.5
48	-25.52	-10.82	- 2.06	+ 1.26	+ 1.66	- 0.70	+ 14.1	+ 45.1	+ 87.2	+103.0
56	-27.54	-10.68	- 1.36	+ 1.60	+ 1.82	- 3.40	+ 9.2	+ 42.2	+ 94.0	+121.0

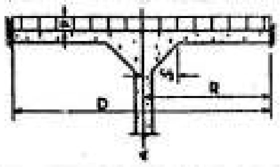
$\frac{H^2}{Dt}$	TABLE VII					TABLE VIII				
	Coefficients at point					Coefficients at point				
	.80H	.85H	.90H	.95H	1.00H	.75H	.80H	.85H	.90H	.95H
20	+.0015	+.0014	+.0005	-.0018	-.0063	+.0008	+.0014	+.0020	+.0024	+.0020
24	+.0012	+.0012	+.0007	-.0013	-.0053	+.0005	+.0010	+.0015	+.0020	+.0017
32	+.0007	+.0009	+.0007	-.0008	-.0040	.0000	+.0005	+.0009	+.0014	+.0013
40	+.0002	+.0005	+.0006	-.0005	-.0032	.0000	+.0003	+.0006	+.0011	+.0011
48	.0000	+.0001	+.0006	-.0003	-.0026	.0000	+.0001	+.0004	+.0008	+.0010
56	.0000	.0000	+.0004	-.0001	-.0023	.0000	.0000	+.0003	+.0007	+.0008

$\frac{H^2}{Dt}$	TABLE IX					TABLE X				
	Coefficients at point					Coefficients at point				
	.80H	.85H	.90H	.95H	1.00H	.05H	.10H	.15H	.20H	.25H
20	+.0015	+.0013	+.0002	-.0024	-.0073	+0.032	+0.039	+0.033	+0.023	+0.014
24	+.0012	+.0012	+.0004	-.0018	-.0061	+0.031	+0.035	+0.028	+0.018	+0.009
32	+.0008	+.0009	+.0006	-.0010	-.0046	+0.028	+0.029	+0.020	+0.011	+0.004
40	+.0005	+.0007	+.0007	-.0005	-.0037	+0.026	+0.025	+0.015	+0.006	+0.001
48	+.0004	+.0006	+.0006	-.0003	-.0031	+0.024	+0.021	+0.011	+0.003	0.000
56	+.0002	+.0004	+.0005	-.0001	-.0026	+0.023	+0.018	+0.008	+0.002	0.000

$\frac{H^2}{Dt}$	TABLE XI					TABLE XVI				TABLE XVIII
	Coefficients at point									
	.80H	.85H	.90H	.95H	1.00H	Tri. Fixed	Rect. Fixed	T. or R. Hinged	Mom. at Edge	Stiffness
20	-0.015	+0.095	+0.296	+0.606	+1.000	+0.114	+0.122	+0.062	- 8.20	1.430
24	-0.037	+0.057	+0.250	+0.572	+1.000	+0.102	+0.111	+0.055	- 8.94	1.566
32	-0.062	+0.002	+0.178	+0.515	+1.000	+0.089	+0.096	+0.048	-10.36	1.810
40	-0.067	-0.031	+0.123	+0.467	+1.000	+0.080	+0.086	+0.043	-11.62	2.025
48	-0.064	-0.049	+0.081	+0.424	+1.000	+0.072	+0.079	+0.039	-12.76	2.220
56	-0.059	-0.060	+0.048	+0.387	+1.000	+0.067	+0.074	+0.036	-13.78	2.400

*For points not shown in the supplementary tables, ring tension and moment may be determined approximately by sketching curves similar to those in the text.

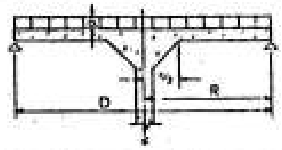
Table XIII



Moments in circular slab with center support
Uniform load
Fixed edge
Mom. = coef. × pR^2
Positive sign indicates compression in surface loaded

c/D	0.05R	0.10R	0.15R	0.20R	0.25R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.00R
	Radial moments, M_r												
0.05	−0.2100	−0.0729	−0.0275	−0.0026	+0.0133	+0.0238	+0.0342	+0.0347	+0.0277	+0.0142	−0.0049	−0.0294	−0.0569
0.10		−0.1433	−0.0624	−0.0239	−0.0011	+0.0136	+0.0290	+0.0326	+0.0278	+0.0168	−0.0021	−0.0255	−0.0541
0.15			−0.1089	−0.0521	−0.0200	+0.0002	+0.0220	+0.0293	+0.0269	+0.0169	+0.0006	−0.0216	−0.0490
0.20				−0.0862	−0.0429	−0.0161	+0.0133	+0.0249	+0.0254	+0.0176	+0.0029	−0.0178	−0.0441
0.25					−0.0598	−0.0351	+0.0029	+0.0194	+0.0231	+0.0177	+0.0049	−0.0143	−0.0393
	Tangential moments, M_t												
0.05	−0.0417	−0.0700	−0.0541	−0.0381	−0.0251	−0.0145	+0.0002	+0.0085	+0.0118	+0.0109	+0.0065	−0.0003	−0.0118
0.10		−0.0287	−0.0421	−0.0354	−0.0258	−0.0168	−0.0027	+0.0059	+0.0099	+0.0098	+0.0061	−0.0003	−0.0108
0.15			−0.0218	−0.0294	−0.0243	−0.0177	−0.0051	+0.0031	+0.0080	+0.0086	+0.0057	−0.0000	−0.0098
0.20				−0.0172	−0.0203	−0.0171	−0.0070	+0.0013	+0.0063	+0.0075	+0.0052	−0.0003	−0.0088
0.25					−0.0140	−0.0150	−0.0083	−0.0005	+0.0048	+0.0064	+0.0048	0.0000	−0.0078

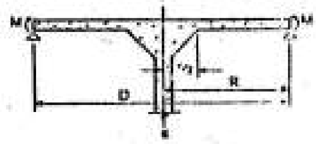
Table XIV



Moments in circular slab with center support
Uniform load
Hinged edge
Mom. = coef. × pR^2
Positive sign indicates compression in surface loaded

c/D	0.05R	0.10R	0.15R	0.20R	0.25R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.00R
	Radial moments, M_r												
0.05	−0.3658	−0.1388	−0.0640	−0.0221	+0.0058	+0.0255	+0.0501	+0.0614	+0.0629	+0.0566	+0.0437	+0.0247	0
0.10		−0.2487	−0.1180	−0.0557	−0.0170	+0.0081	+0.0391	+0.0539	+0.0578	+0.0532	+0.0416	+0.0237	0
0.15			−0.1869	−0.0977	−0.0487	−0.0135	+0.0258	+0.0451	+0.0518	+0.0494	+0.0393	+0.0226	0
0.20				−0.1465	−0.0800	−0.0381	+0.0109	+0.0352	+0.0452	+0.0451	+0.0368	+0.0215	0
0.25					−0.1172	−0.0645	−0.0055	+0.0245	+0.0381	+0.0404	+0.0340	+0.0200	0
	Tangential moments, M_t												
0.05	−0.0731	−0.1277	−0.1040	−0.0789	−0.0569	−0.0391	−0.0121	+0.0061	+0.0175	+0.0234	+0.0251	+0.0228	+0.0168
0.10		−0.0498	−0.0758	−0.0684	−0.0539	−0.0394	−0.0153	+0.0020	+0.0134	+0.0197	+0.0218	+0.0199	+0.0145
0.15			−0.0374	−0.0518	−0.0470	−0.0375	−0.0175	−0.0014	+0.0097	+0.0163	+0.0186	+0.0172	+0.0121
0.20				−0.0293	−0.0367	−0.0333	−0.0184	−0.0042	+0.0065	+0.0132	+0.0158	+0.0148	+0.0103
0.25					−0.0234	−0.0263	−0.0184	−0.0062	+0.0038	+0.0103	+0.0132	+0.0122	+0.0085

Table XV

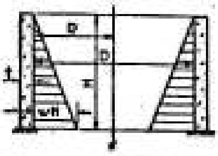


Moments in circular slab with center support
Moment / m , M, applied at edge
Hinged edge
Mom. = coef. × M
Positive sign indicates compression in top surface

c/D	0.05R	0.10R	0.15R	0.20R	0.25R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.00R
	Radial moments, M_r												
0.05	−2.650	−1.121	−0.622	−0.333	−0.129	+0.029	+0.268	+0.450	+0.598	+0.718	+0.824	+0.917	+1.000
0.10		−1.950	−1.026	−0.584	−0.305	−0.103	+0.187	+0.394	+0.558	+0.692	+0.808	+0.907	+1.000
0.15			−1.594	−0.930	−0.545	−0.280	+0.076	+0.323	+0.510	+0.663	+0.790	+0.900	+1.000
0.20				−1.366	−0.847	−0.499	−0.057	+0.236	+0.451	+0.624	+0.768	+0.891	+1.000
0.25					−1.204	−0.765	−0.216	+0.130	+0.392	+0.577	+0.740	+0.880	+1.000
	Tangential moments, M_t												
0.05	−0.530	−0.980	−0.847	−0.688	−0.544	−0.418	−0.211	−0.042	+0.095	+0.212	+0.314	+0.405	+0.486
0.10		−0.388	−0.641	−0.608	−0.518	−0.419	−0.233	−0.072	+0.066	+0.185	+0.290	+0.384	+0.469
0.15			−0.319	−0.472	−0.463	−0.404	−0.251	−0.100	+0.035	+0.157	+0.263	+0.363	+0.451
0.20				−0.272	−0.372	−0.368	−0.261	−0.123	+0.007	+0.129	+0.240	+0.340	+0.433
0.25					−0.239	−0.305	−0.759	−0.145	−0.020	+0.099	+0.714	+0.370	+0.414

Table VII

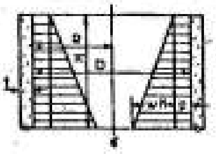
Moments in cylindrical wall
Triangular load
Fixed base, free top
Mom. = coef. × wH^3
Positive sign indicates tension in the outside



$\frac{H^2}{Dt}$	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
	Coefficients at point									
0.4	+.0005	+.0014	+.0021	+.0007	-.0042	-.0150	-.0302	-.0529	-.0816	-.1205
0.8	+.0011	+.0037	+.0063	+.0080	+.0070	+.0023	-.0068	-.0224	-.0465	-.0795
1.2	+.0012	+.0042	+.0077	+.0103	+.0112	+.0090	+.0022	-.0108	-.0311	-.0602
1.6	+.0011	+.0041	+.0075	+.0107	+.0121	+.0111	+.0058	-.0051	-.0232	-.0505
2.0	+.0010	+.0035	+.0068	+.0099	+.0120	+.0115	+.0075	-.0021	-.0185	-.0436
3.0	+.0006	+.0024	+.0047	+.0071	+.0090	+.0097	+.0077	+.0012	-.0119	-.0333
4.0	+.0003	+.0015	+.0028	+.0047	+.0066	+.0077	+.0069	+.0023	-.0080	-.0268
5.0	+.0002	+.0008	+.0016	+.0029	+.0046	+.0059	+.0059	+.0028	-.0058	-.0222
6.0	+.0001	+.0003	+.0008	+.0019	+.0032	+.0046	+.0051	+.0029	-.0041	-.0187
8.0	.0000	+.0001	+.0002	+.0008	+.0016	+.0028	+.0038	+.0029	-.0022	-.0146
10.0	.0000	.0000	+.0001	+.0004	+.0007	+.0019	+.0029	+.0028	-.0012	-.0122
12.0	.0000	-.0001	+.0001	+.0002	+.0003	+.0013	+.0023	+.0026	-.0005	-.0104
14.0	.0000	.0000	.0000	.0000	+.0001	+.0008	+.0019	+.0023	-.0001	-.0090
16.0	.0000	.0000	-.0001	-.0002	-.0001	+.0004	+.0013	+.0019	+.0001	-.0079

Table VIII

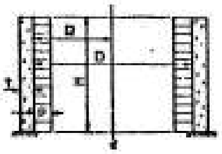
Moments in cylindrical wall
Trapezoidal load
Hinged base, free top
Mom. = coef. × $(wH^3 + pH^2)$
Positive sign indicates tension in the outside



$\frac{H^2}{Dt}$	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
	Coefficients at point									
0.4	+.0020	+.0072	+.0151	+.0230	+.0301	+.0348	+.0357	+.0312	+.0197	0
0.8	+.0019	+.0064	+.0133	+.0207	+.0271	+.0319	+.0329	+.0292	+.0187	0
1.2	+.0016	+.0058	+.0111	+.0177	+.0237	+.0280	+.0296	+.0263	+.0171	0
1.6	+.0012	+.0044	+.0091	+.0145	+.0195	+.0236	+.0255	+.0232	+.0155	0
2.0	+.0009	+.0033	+.0073	+.0114	+.0158	+.0199	+.0219	+.0205	+.0145	0
3.0	+.0004	+.0018	+.0040	+.0063	+.0092	+.0127	+.0152	+.0153	+.0111	0
4.0	+.0001	+.0007	+.0016	+.0033	+.0057	+.0083	+.0109	+.0118	+.0092	0
5.0	.0000	+.0001	+.0006	+.0016	+.0034	+.0057	+.0080	+.0094	+.0078	0
6.0	.0000	.0000	+.0002	+.0008	+.0019	+.0039	+.0062	+.0078	+.0068	0
8.0	.0000	.0000	-.0002	.0000	+.0007	+.0020	+.0038	+.0057	+.0054	0
10.0	.0000	.0000	-.0002	-.0001	+.0002	+.0011	+.0025	+.0043	+.0045	0
12.0	.0000	.0000	-.0001	-.0002	.0000	+.0005	+.0017	+.0032	+.0039	0
14.0	.0000	.0000	-.0001	-.0001	-.0001	.0000	+.0012	+.0026	+.0033	0
16.0	.0000	.0000	.0000	-.0001	-.0002	-.0004	+.0008	+.0022	+.0029	0

Table IX

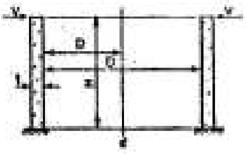
Moments in cylindrical wall
Rectangular load
Fixed base, free top
Mom. = coef. × pH^2
Positive sign indicates tension in the outside



$\frac{H^2}{Dt}$	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
	Coefficients at point									
0.4	-.0023	-.0093	-.0227	-.0439	-.0710	-.1018	-.1455	-.2000	-.2593	-.3310
0.8	.0000	-.0006	-.0025	-.0083	-.0185	-.0362	-.0594	-.0917	-.1325	-.1835
1.2	+.0008	+.0026	+.0037	+.0029	-.0009	-.0089	-.0227	-.0468	-.0815	-.1178
1.6	+.0011	+.0036	+.0062	+.0077	+.0068	+.0011	-.0093	-.0267	-.0529	-.0876
2.0	+.0010	+.0036	+.0066	+.0088	+.0089	+.0059	-.0019	-.0167	-.0389	-.0719
3.0	+.0007	+.0026	+.0051	+.0074	+.0091	+.0083	+.0042	-.0053	-.0223	-.0483
4.0	+.0004	+.0015	+.0033	+.0052	+.0068	+.0075	+.0053	-.0013	-.0145	-.0365
5.0	+.0002	+.0008	+.0019	+.0035	+.0051	+.0061	+.0052	+.0007	-.0101	-.0293
6.0	+.0001	+.0004	+.0011	+.0022	+.0036	+.0049	+.0048	+.0017	-.0073	-.0242
8.0	.0000	+.0001	+.0003	+.0008	+.0018	+.0031	+.0038	+.0024	-.0040	-.0184
10.0	.0000	-.0001	.0000	+.0002	+.0009	+.0021	+.0030	+.0026	-.0022	-.0147
12.0	.0000	.0000	-.0001	.0000	+.0004	+.0014	+.0024	+.0022	-.0012	-.0123
14.0	.0000	.0000	.0000	.0000	+.0002	+.0010	+.0018	+.0021	-.0007	-.0105
16.0	.0000	.0000	.0000	-.0001	+.0001	+.0006	+.0012	+.0020	-.0005	-.0091

Table X

Moments in cylindrical wall
Shear / m V, applied at top
Fixed base, free top
Mom. = coef. × VH
Positive sign indicates tension in outside

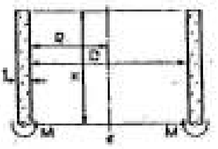


$\frac{H^2}{Dt}$	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
	Coefficients at point*									
0.4	+0.093	+0.172	+0.240	+0.300	+0.354	+0.402	+0.448	+0.492	+0.535	+0.578
0.8	+0.085	+0.145	+0.185	+0.208	+0.220	+0.224	+0.223	+0.219	+0.214	+0.208
1.2	+0.082	+0.132	+0.157	+0.164	+0.159	+0.145	+0.127	+0.106	+0.084	+0.062
1.6	+0.079	+0.122	+0.139	+0.138	+0.125	+0.105	+0.081	+0.056	+0.030	+0.004
2.0	+0.077	+0.115	+0.126	+0.119	+0.103	+0.080	+0.056	+0.031	+0.006	-0.019
3.0	+0.072	+0.100	+0.100	+0.086	+0.066	+0.044	+0.025	+0.006	-0.010	-0.024
4.0	+0.068	+0.088	+0.081	+0.063	+0.043	+0.025	+0.010	-0.001	-0.010	-0.015
5.0	+0.064	+0.078	+0.067	+0.047	+0.028	+0.013	+0.003	-0.003	-0.007	-0.011
6.0	+0.062	+0.070	+0.056	+0.036	+0.018	+0.006	0.000	-0.003	-0.005	-0.006
8.0	+0.057	+0.058	+0.041	+0.021	+0.007	0.000	-0.002	-0.003	-0.002	0.001
10.0	+0.053	+0.049	+0.029	+0.012	+0.002	-0.002	-0.002	-0.002	-0.001	0.000
12.0	+0.049	+0.042	+0.022	+0.007	0.000	-0.002	-0.002	-0.001	0.000	0.000
14.0	+0.046	+0.036	+0.017	+0.004	-0.001	-0.002	-0.001	-0.001	0.000	0.000
16.0	+0.044	+0.031	+0.012	+0.001	-0.002	-0.002	-0.001	0.000	0.000	0.000

*When this table is used for shear applied at the base, while the top is fixed, 0.0H is the bottom of the wall and 1.0H is the top. Shear acting inward is positive, outward is negative.

Table XI

Moments in cylindrical wall
Moment/m , M, applied at base
Hinged base, free top
Mom. = coef. × M
Positive sign indicates tension in outside

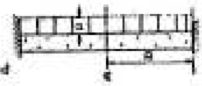


$\frac{H^2}{Dt}$	0.1H	0.2H	0.3H	0.4H	0.5H	0.6H	0.7H	0.8H	0.9H	1.0H
	Coefficients at point*									
0.4	+0.013	+0.051	+0.109	+0.196	+0.296	+0.414	+0.547	+0.692	+0.843	+1.000
0.8	+0.009	+0.040	+0.090	+0.164	+0.253	+0.375	+0.503	+0.659	+0.824	+1.000
1.2	+0.006	+0.027	+0.063	+0.125	+0.206	+0.316	+0.454	+0.616	+0.802	+1.000
1.6	+0.003	+0.011	+0.035	+0.078	+0.152	+0.253	+0.393	+0.570	+0.775	+1.000
2.0	-0.002	-0.002	+0.012	+0.034	+0.096	+0.193	+0.340	+0.519	+0.748	+1.000
3.0	-0.007	-0.022	-0.030	-0.029	+0.010	+0.087	+0.227	+0.426	+0.692	+1.000
4.0	-0.008	-0.026	-0.044	-0.051	-0.034	+0.073	+0.150	+0.354	+0.645	+1.000
5.0	-0.007	-0.024	-0.045	-0.061	-0.057	-0.015	+0.095	+0.296	+0.606	+1.000
6.0	-0.005	-0.018	-0.040	[illegible]	-0.065	-0.037	+0.057	+0.252	+0.572	+1.000
8.0	0.001	-0.009	0.022	-0.044	-0.068	-0.062	+0.002	+0.178	+0.515	+1.000
10.0	0.000	-0.002	-0.009	-0.028	-0.053	-0.067	-0.031	+0.123	+0.467	+1.000
12.0	0.000	0.000	-0.003	-0.016	-0.040	-0.064	-0.049	+0.081	+0.424	+1.000
14.0	0.000	0.000	0.000	-0.008	-0.029	-0.059	-0.060	+0.048	+0.387	+1.000
16.0	0.000	0.000	+0.002	-0.003	-0.021	-0.051	-0.066	+0.025	+0.354	+1.000

*When this table is used for moment applied at the top, while the top is hinged, 0.0H is the bottom of the wall and 1.0H is the top. Moment applied at an edge is positive when it causes outward rotation at that edge.

Table XII

Moments in circular slab without center support
Uniform load
Fixed edge
Mom. = coef. × pR^2
Positive sign indicates compression in surface loaded



0.00R	0.10R	0.20R	0.30R	0.40R	0.50R	0.60R	0.70R	0.80R	0.90R	1.00R
Coefficients at point										
Radial moments, M_r										
+.075	+.073	+.067	+.057	+.043	+.025	+.003	-.023	-.053	-.087	-.125
Tangential moments, M_t										
+.075	+.074	+.071	+.066	+.059	+.050	+.039	+.026	+.011	-.006	-.025

5) Solid Circular Plate Subjected to an Intermediate Radial Ring Moment

(Fig. IV-10)

$M \frac{kgm}{m}$ $\qquad Q_r = 0 \qquad A = 0$

a) Fixed Edge

i) Inner part $\qquad 0 \leqslant \rho < \beta$

$$\delta = -\frac{M a^2}{4 D}\left[2\beta^2 . \ln \beta + (1-\beta^2)\,\rho^2\right] \qquad \varphi = \frac{M a}{2 D}(1-\beta^2)\,\rho$$

$$M_r = M_t = \frac{M}{2}(1+\nu)(1-\beta^2)$$

ii) Outer part $\qquad \beta < \rho < 1$

$$\delta = -\frac{M a^2}{4 D} . \beta^2 (1-\rho^2 + 2 \ln \rho) \qquad \varphi = \frac{M a}{2 D} . \beta^2 \left(\frac{1}{\rho} - \rho\right)$$

$$M_r = -\frac{M \beta^2}{2}\left[1 + \nu + (1-\nu)\frac{1}{\rho^2}\right] \qquad M_t = -\frac{M \beta^2}{2}\left[1 + \nu - (1-\nu)\frac{1}{\rho^2}\right]$$

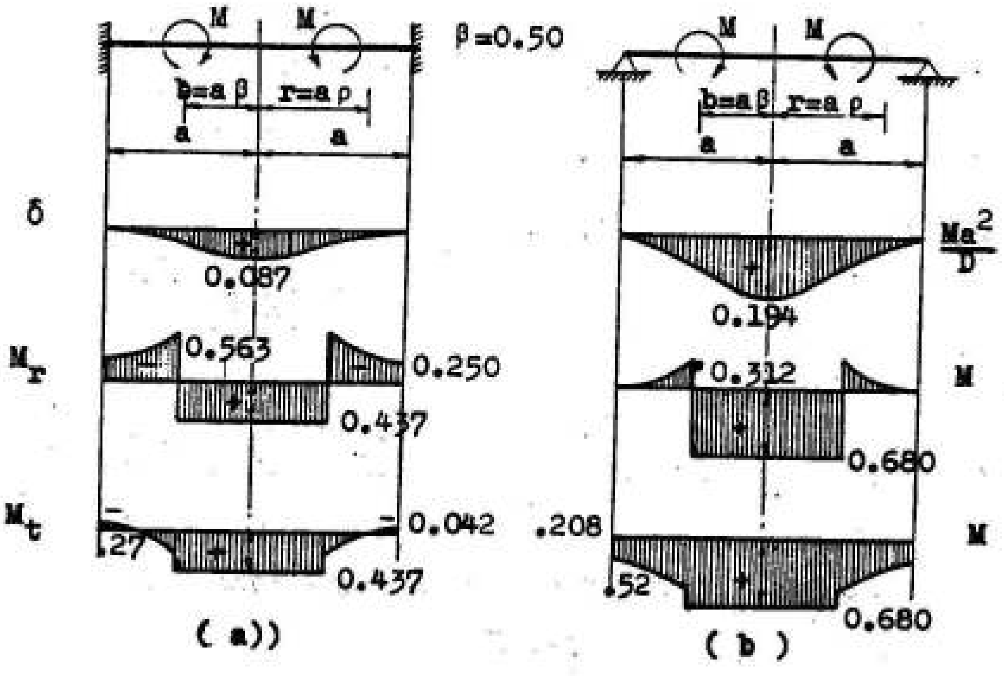


Fig. IV-10

b) Simply Supported Edge

i) Inner part $0 < \rho < \beta$

$$\delta = \frac{M a^2}{4 D} \cdot \frac{1}{1+\nu}\left\{2 \beta^2\left[1-(1-\nu) \ln \beta\right]-\left[1+\nu+(1-\nu) \beta^2\right] \rho^2\right\}$$

$$\varphi = \frac{M a}{2 D} \cdot \frac{1}{1+\nu} \rho\left[1+\nu+(1-\nu) \beta^2\right]$$

$$M_r = M_t = \frac{M}{2}\left[1+\nu+(1-\nu) \beta^2\right]$$

ii) Outer part

$$\delta = \frac{M a^2}{4 D} \cdot \frac{\beta^2}{1+\nu}\left[(1-\nu)(1-\rho^2)-2(1+\nu) \ln \rho\right]$$

$$\varphi = \frac{M a}{2 D} \cdot \frac{\beta^2}{1+\nu}\left[(1-\nu) \rho+(1+\nu) \frac{1}{\rho}\right]$$

$$M_r = \frac{M}{2}(1-\nu)\beta^2\left(1-\frac{1}{\rho^2}\right) \qquad M_t = \frac{M}{2}(1-\nu) \beta^2\left(1-\frac{1}{\rho^2}\right)$$

6) Simply Supported Solid Circular Plate Subjected to Radial Ring Moments at its Edges

Fig. IV-11

Radial moment $M \frac{kgm}{m}$

$$\delta = \frac{M a^2}{2 D (1+\nu)} \cdot (1-\rho^2)$$

$$\varphi = \frac{M a}{D (1+\nu)} \cdot \rho$$

$$M_r = M_t = M$$

$$Q_r = 0 \qquad A = 0$$

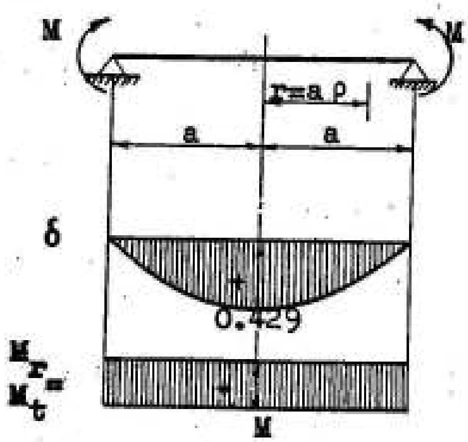


Fig. IV-11

7) Circular Plate with a Circular Hole at the Center and Subjected to a Uniformly Distributed Load p/m² (Fig. IV-12)

$$A = \frac{p\,a}{2}(1-\beta^2) \quad \text{for} \quad \beta < 1 \qquad A = \frac{p\,a}{2}(\beta^2 - 1) \quad \text{for} \quad \beta > 1$$

$$Q_r = -\frac{p\,a}{2}\left(\rho - \frac{\beta^2}{\rho}\right)$$

a) Fixed Edge

assuming $k_1 = \beta^2 \dfrac{(1-\nu)\beta^2 + (1+\nu)(1 + 4\beta^2 \ln\beta}{1 - \nu + (1+\nu)\beta^2}$ then

$$\delta = \frac{p\,a^4}{64\,D}\left[-1 + 2(1 - k_1 - 2\beta^2)(1-\rho^2) + \rho^4 - 4k_1 \ln\rho - 8\beta^2\rho^2 \ln\rho\right]$$

$$\varphi = \frac{p\,a^3}{16\,D}\left[(1-k_1)\rho - \rho^3 + k_1\frac{1}{\rho} + 4\beta^2\rho\ln\rho\right]$$

$$M_r = \frac{p\,a^2}{16}\left[(1+\nu)(1-k_1) + 4\beta^2 - (3+\nu)\rho^2 - (1-\nu)k_1 . \frac{1}{\rho^2} + 4(1+\nu)\beta^2 \ln\rho\right]$$

$$M_t = \frac{p\,a^2}{16}\left[(1+\nu)(1-k_1) + 4\nu\beta^2 - (1+3\nu)\rho^2 + (1-\nu)k_1 . \frac{1}{\rho^2} + 4(1+\nu)\beta^2 \ln\rho\right]$$

b) Simply Supported Edge

assuming $k_2 = \beta^2\left[3 + \nu + 4(1+\nu)\dfrac{\beta^2}{1-\beta^2}\ln\beta\right]$ then

$$\delta = \frac{p\,a^4}{64\,D}\left\{\frac{2}{1+\nu}\left[(3+\nu)(1-2\beta^2) + k_2\right](1-\rho^2) - (1-\rho^4) - \frac{4}{1-\nu} . k_2 . \ln\rho - 8\beta^2\rho^2\ln\rho\right\}$$

$$\varphi = \frac{p\,a^3}{16\,D}\left[\frac{1}{1+\nu}(3 + \nu - 4\beta^2 + k_2)\rho - \rho^3 + \frac{k_2}{1-\nu} . \frac{1}{\rho} + 4\beta^2\rho\ln\rho\right]$$

$$M_r = \frac{p\,a^2}{16}\left[(3+\nu)(1-\rho^2) + k_2\left(1 - \frac{1}{\rho^2}\right) + 4(1+\nu)\beta^2\ln\rho\right]$$

$$M_t = \frac{p\,a^2}{16}\left[2(1-\nu)(1-2\beta^2) + (1+3\nu)(1-\rho^2) + k_2\left(1 + \frac{1}{\rho^2}\right) + 4(1+\nu)\beta^2\ln\rho\right]$$

Circular Plate with a Circular Hole at the Center Subjected to a Uniformly Distributed Load p / m²

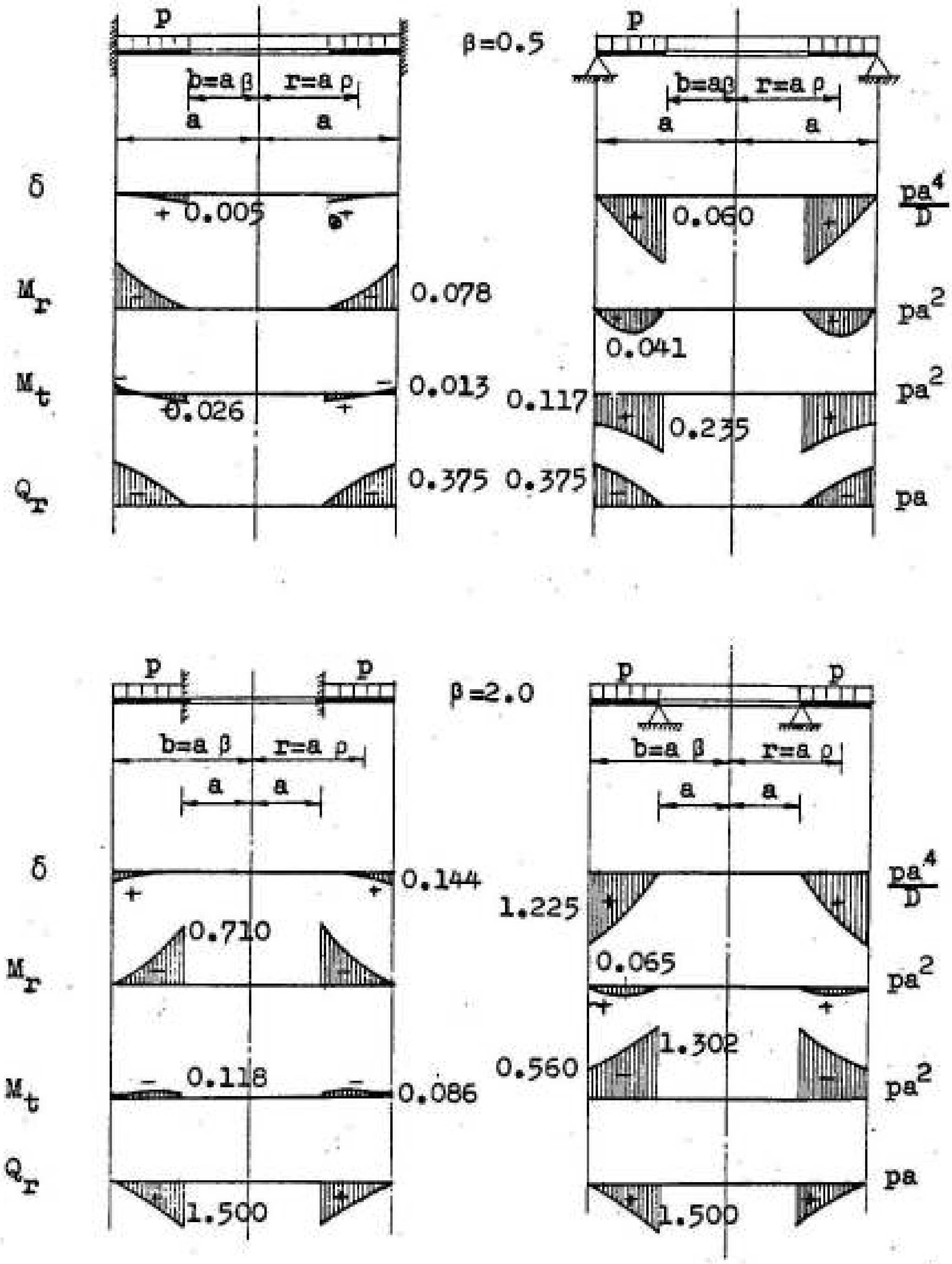


Fig. IV-12

8) <u>Circular Plate with a Circular Hole at the Center and Subjected to a Ring Load along the Inner Edge $\rho = \beta$</u>

(Fig. IV-13)

Load P / m

$$Q_r = - P \beta \frac{1}{\rho}$$

A = P β for β > 1 A = − P β for β > 1

a) <u>Fixed Edge</u>

Assuming $k_3 = \beta^2 \dfrac{1 + (1 + \nu) \ln \beta}{1 - \nu + (1 + \nu)\beta^2}$ then

$$\delta = \frac{P a^3}{8 D} \beta \left[(1 + 2 k_3)(1 - \rho^2) + 4 k_3 \ln \rho + 2 \rho^2 \ln \rho \right]$$

$$\varphi = \frac{P a^2}{2 D} \beta \left[k_3 (\rho - \frac{1}{\rho}) - \rho \ln \rho \right]$$

$$M_r = \frac{P a}{2} \beta \left[- 1 + (1 + \nu) k_3 + (1 - \nu) k_3 \frac{1}{\rho^2} - (1 + \nu) \ln \rho \right]$$

$$M_t = \frac{P a}{2} \beta \left[- \nu + (1 + \nu) k_3 - (1 - \nu) k_3 \frac{1}{\rho^2} - (1 + \nu) \ln \rho \right]$$

b) <u>Simply Supported Edge</u>

Assuming $k_4 = (1 + \nu) \dfrac{\beta^2}{1 - \beta^2} \ln \beta$ then

$$\delta = \frac{P a^3}{8 D} \beta \left[\frac{3 + \nu - 2 k_4}{1 + \nu} (1 - \rho^2) + \frac{4 k_4}{1 - \nu} \ln \rho + 2 \rho^2 \ln \rho \right]$$

$$\varphi = \frac{P a^2}{2 D} \beta \left[\frac{1 - k_4}{1 + \nu} \rho - \frac{k_4}{1 - \nu} \cdot \frac{1}{\rho} - \rho \ln \rho \right]$$

$$M_r = \frac{P a}{2} \beta \left[k_4 (\frac{1}{\rho^2} - 1) - (1 + \nu) \ln \rho \right]$$

$$M_t = \frac{P a}{2} \beta \left[1 - \nu - k_4 (\frac{1}{\rho^2} + 1) - (1 + \nu) \ln \rho \right]$$

Circular Plate with a Circular Hole at the Center Subjected to a Ring Load along the Inner Ring

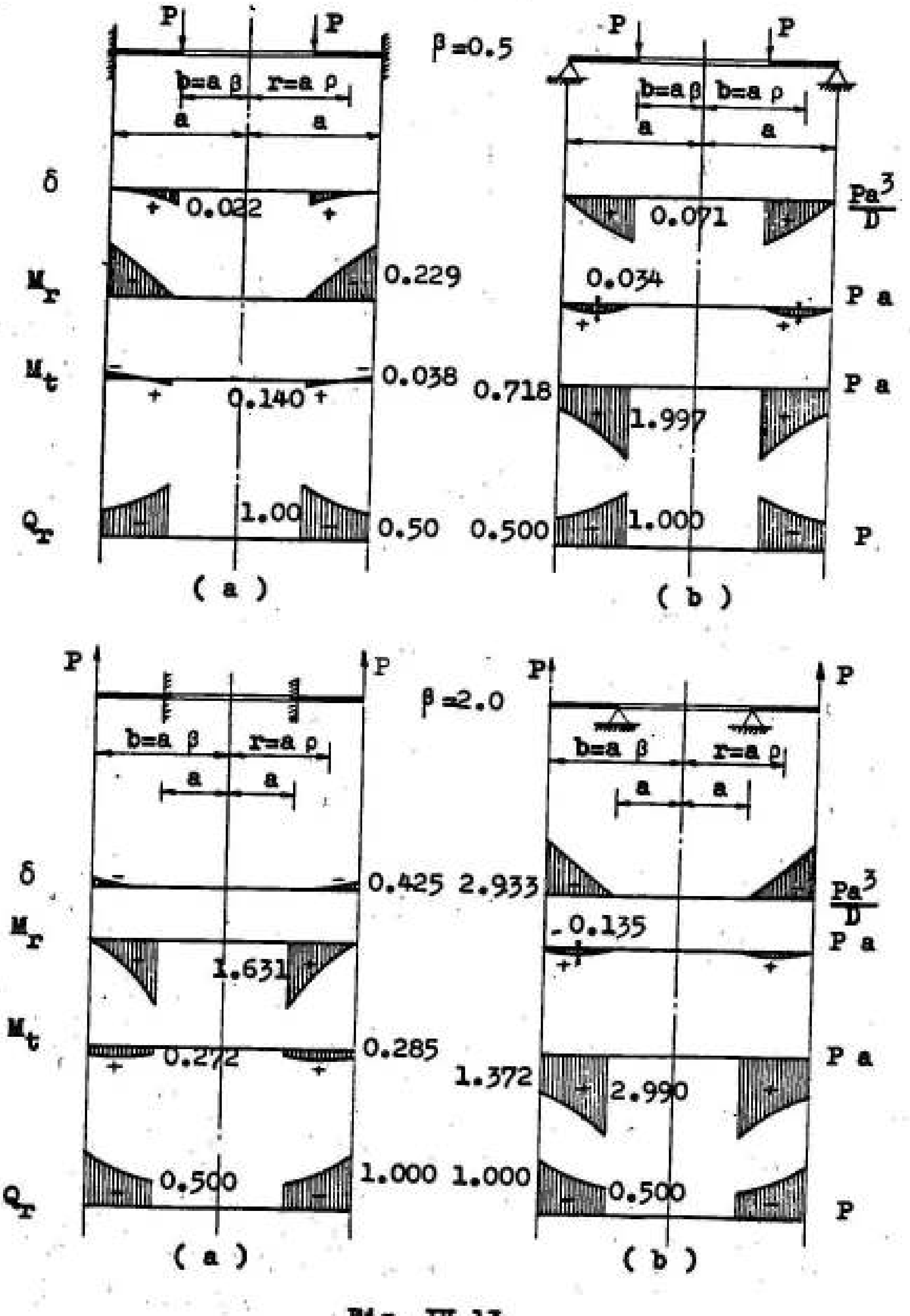
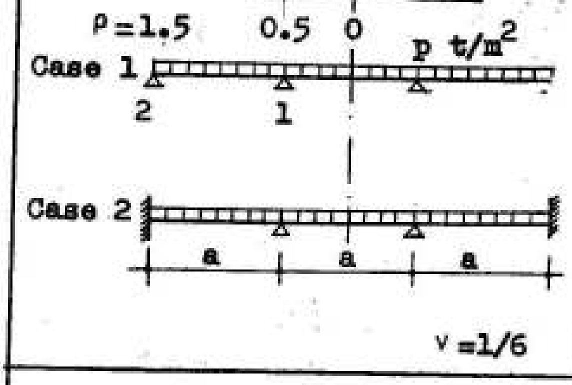


Fig. IV-13

II. RADIAL MOMENTS (M_r), TANGENTIAL MOMENTS (M_t) AND REACTIONS OF CIRCULAR FLAT PLATES CONTINUOUS OVER RING SUPPORTS AT EQUAL DISTANCES (a)

a) **Three equal spans**

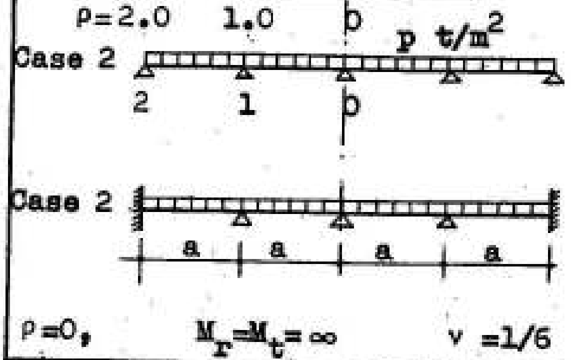


ρ	Case 1		Case 2	
	M_r	M_t	M_r	M_t
1.5	0	+.0188	-.0835	-.0139
1.3	+.0578	+.0271	-.0132	+.0049
1.1	+.0889	+.0225	+.0330	+.0119
1.0	+.0925	+.0151	+.0455	+.0109
0.9	+.0875	+.0042	+.0506	+.0068
0.7	+.0411	-.0267	+.0300	-.0097
0.5	-.0851	-.0591	-.0552	-.0291
0.3	-.0535	-.04[illegible]	-.0235	-.0141
0.1	-.0376	-.0366	-.0077	-.0066
0.0	-.0357	-.0357	-.0057	-.0057
Mult	p a²	p a²	p a²	p a²

REACTIONS

Sup.	Case 1	Case 2
2	+ 0.3618	+ 0.4535
1	+ 1.1642	+ 0.8898
Mult	p a	p a

b) **Four equal spans**



ρ	Case 1		Case 2	
	M_r	M_t	M_r	M_t
2.0	0	+.0129	-.0811	-.0135
1.6	+.0847	+.0179	+.0346	+.0093
1.5	+.0848	+.0127	+.0444	+.0086
1.4	+.0748	+.0053	+.0449	+.0057
1.0	-.1069	-.0314	-.0769	-.0172
0.6	-.0008	+.0080	+.0179	+.0109
0.5	+.0137	+.0110	+.0284	+.0099
0.4	*.0225	+.0104	+.0322	+.0044
0.1	-.0035	-.0310	-.0244	-.0676
Mult	p a²	p a²	p a²	p a²

REACTIONS

Sup.	Case 1	Case 2
2	+0.3655	+0.5612
1	+1.2001	+0.9131
0	+0.4330	+0.6709
Mult	p a p a²	p a p a²

RADIAL MOMENTS (M_r), TANGENTIAL MOMENTS (M_t) AND REACTIONS OF CIRCULAR FLAT PLATES CONTINUOUS OVER RING SUPPORTS AT EQUAL DISTANCES (a)

c) **Five equal spans**

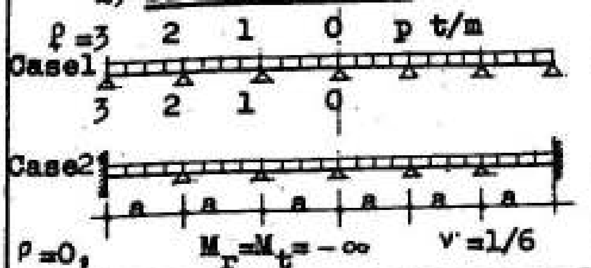
BENDING MOMENTS

ρ	Case 1		Case 2	
	M_r	M_t	M_r	M_t
2.5	0	+.0098	-.0787	-.0131
2.1	+.0818	+.0159	+.0348	+.0076
2.0	+.0799	+.0118	+.0425	+.0075
1.9	+.0676	+.0058	+.0403	+.0051
1.5	-.1130	-.0254	-.0875	-.0155
1.1	+.0132	+.0082	+.0302	+.0115
1.0	+.0289	+.0094	+.0432	+.0107
0.9	+.0375	+.0077	+.0488	+.0069
0.5	-.0446	-.0186	-.0537	-.0277
0.1	+.0029	+.0039	-.0062	-.0052
0.0	+.0048	+.0048	-.0043	-.0043
Mult	$p a^2$	$p a^2$	$p a^2$	$p a^2$

REACTIONS

Sup.	Case 1	Case 2
3	+ 0.3675	+ 0.4594
2	+ 1.2065	+ 1.0255
1	+ 0.7931	+ 0.8765
Mult	p a	p a

d) **Six equal spans**



BENDING MOMENTS

ρ	Case 1		Case 2	
	M_r	M_t	M_r	M_t
3.0	0	+.0081	-.0799	-.0133
2.6	+.0814	+.0153	+.0354	+.0076
2.5	+.0789	+.0118	+.0426	+.0073
2.4	+.0659	+.0066	+.0401	+.0052
2.0	-.1106	-.0232	-.0852	-.0149
1.6	+.0164	+.0062	+.0321	+.0089
1.5	+.0296	+.0069	+.0422	+.0084
1.4	+.0340	+.0058	+.0433	+.0057
1.0	-.0660	-.0121	-.0753	-.0164
0.6	+.0247	+.0120	+.0189	+.0110
0.5	+.0338	+.0095	+.0292	+.0098
0.4	+.0357	+.0022	+.0327	+.0040
0.1	-.0320	-.0809	-.0255	-.0695
Mult	$p a^2$	$p a^2$	$p a^2$	$p a^2$

REACTIONS

Sup.	Case 1	Case 2
3	+0.3725	+0.4675
2	+1.1500	+0.9733
1	+0.9622	+1.0420
0	+0.7573	+0.6834
	p a $p a^2$	p a $p a^2$

IV-3. MEMBRANE FORCES IN SURFACES OF REVOLUTION *

It is assumed here that the thickness of the shell is so small that it can be considered as a membrane which can resist meridian and ring forces, in the plane of the surface, only i.e. the bending moments due to fixation at supports, unsymmetrical loading and similar effects are neglected .

IV-3.1. Notations.

It will be assumed that : (Fig. IV-16)

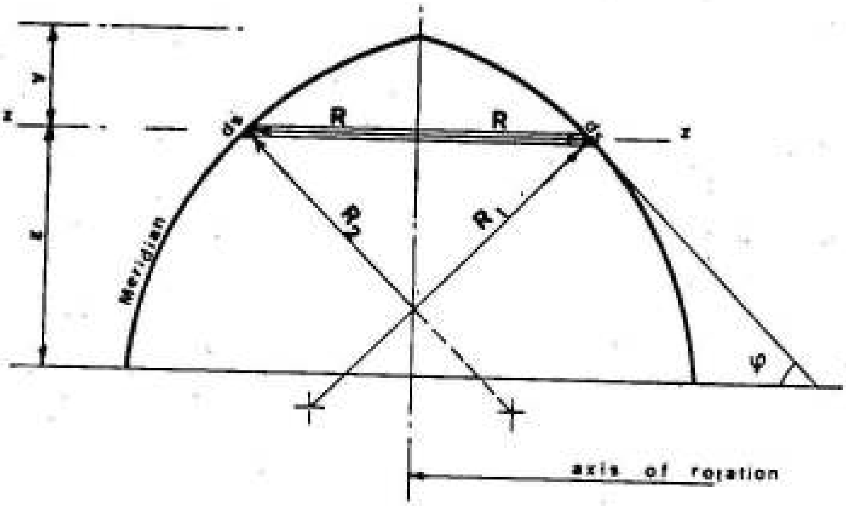


Fig. IV-16

R = radius normal to axis of revolution of any circular ring at any plane z - z

R_1 = radius of curvature of meridian

R_2 = cross radius curvature along the normal - to axis of rotation.

N_φ (evt. T_1) = resultant meridian force per unit length of circumference.

N_θ (evt. T_2) = resultant ring force per unit length of meridian

H = horizontal thrust of shell per unit length of circumference

W_φ = Sum of vertical forces above z - z (expressed through the angle φ)

---

* Refer to M. Hilal " Design of Reinforced Concrete Halls " First Edition. Published by J. Marco & Co. Cairo

IV-3.2 The Meridian Force

In order to have equilibrium at any horizontal section z z , the vertical component of the meridian forces N_φ must be equal to the vertical load above z - z per meter run circumference. Hence we get : (Fig. IV-17)

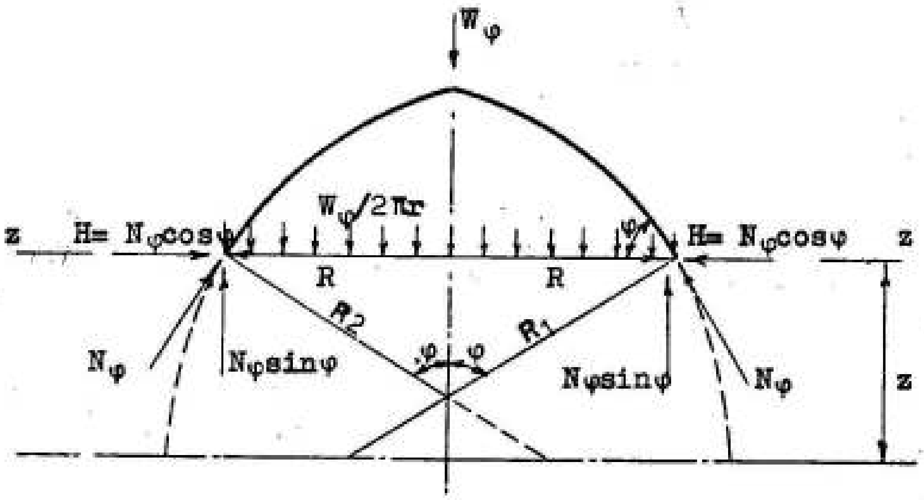


Fig. IV-17

$$W_\varphi / 2 \pi R = N_\varphi \sin \varphi$$ or

$$N_\varphi = W_\varphi / 2 \pi R \sin \varphi \qquad (43)$$

But $R = R_2 \sin \varphi$, so that N_φ can also be given in the form :

$$N_\varphi = W_\varphi / 2 \pi R_2 \sin^2 \varphi \qquad (44)$$

The horizontal thrust H per unit length of circumference is :

$$H = W_\varphi / 2 \pi R \tan \varphi = N \cos \varphi \qquad (45)$$

IV-3.3 The Ring Force

Assuming that the radial component of the external loads per square meter surface is p_r and considering the equilibrium of the external and internal forces normal to the surface, one can prove that:

$$\frac{N_\varphi}{R_1} + \frac{N_\theta}{R_2} = p_r \qquad (46)$$

For a spherical surface $R_1 = R_2 = a$ and

$$N_\varphi + N_\theta = p_r a \qquad (47)$$

For a conical surface $R_1 = \infty$ and

$$\underline{N_\theta = p_r R_2} \qquad (48)$$

IV-3.4 Application to Popular Reinforced Concrete Surfaces of Revolution.

a) Spherical Shells

The relation between a, R and y is given by (Fig. IV-18)

$$a = \frac{R^2 + y^2}{2y} \qquad (49)$$

The surface area of a spherical shell is

$$A = 2 \pi a y \qquad (50)$$

i.e. it is equal to the surface area of a cylinder having the same radius a and height y.

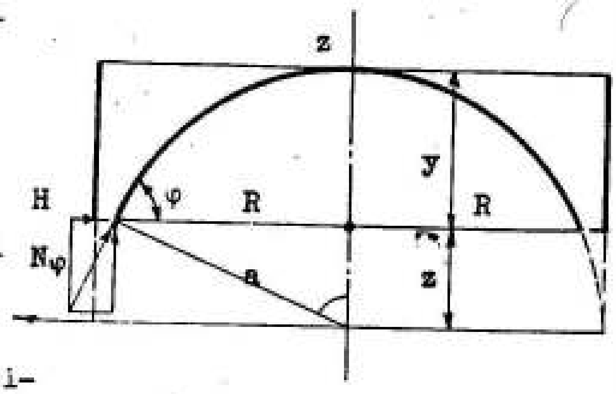


Fig. IV-18

The internal forces due to a vertical dead load g/m² surface is shown in figure IV- 19

$$W_\varphi = g. \; 2 \pi a y \qquad (51)$$

$$H = g a \frac{\cos \varphi}{1+\cos \varphi} = g a \frac{z}{a + z} \qquad (52)$$

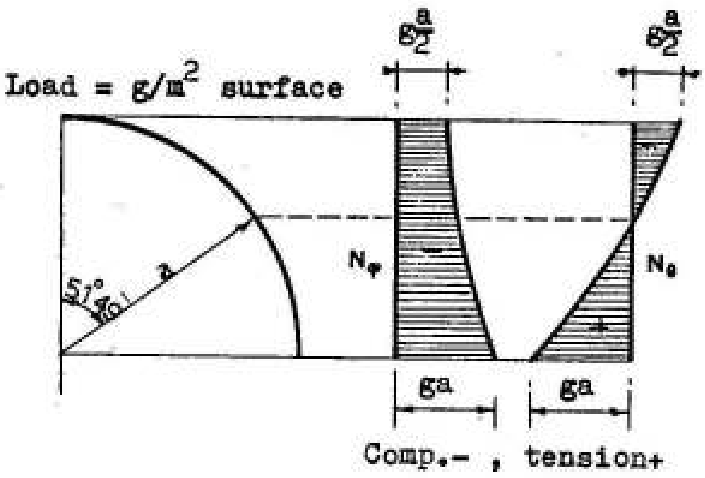


Fig. IV-19

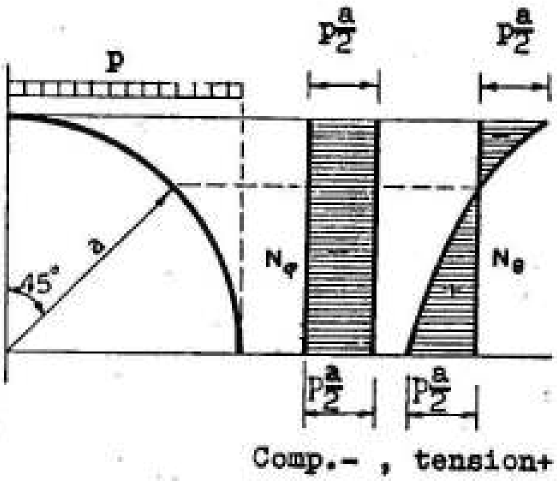


Fig. IV-20

$$N_\varphi = g \frac{a}{1 + \cos\varphi} = g \frac{a^2}{a + z} \qquad (53)$$

$$N_\theta = g\, a \left(\cos\varphi - \frac{1}{1 + \cos\varphi} \right) = g \left(z - \frac{a^2}{a + z} \right) \qquad (54)$$

The internal forces due to a vertical load <u>p/m^2 horizontal</u> is shown in figure IV- 20

$$W_\varphi = p\, \pi\, a^2 \sin^2\varphi \qquad (55)$$

$$H = p\, a \frac{\cos\varphi}{2} = p\, z/2 \qquad (56)$$

$$N_\varphi = p\, a/2 \quad = \text{constant}$$

$$N_\theta = p \frac{a \cos 2\varphi}{2} = \frac{p}{2a} (2 z^2 - a^2) \qquad (57)$$

The internal forces due to a liquid pressure as that shown in Fig. IV-21 is given by :

Assuming w = weight / m^3 liquid, then

$$p_r = w\, (h_1 + a - a \cos\varphi) \qquad (58)$$

$$N_\varphi = -w \frac{a}{6} \left[3 h_1 + a \frac{1 - \cos\varphi}{1 + \cos\varphi} (1 + 2\cos\varphi) \right] \qquad (59)$$

$$N_\theta = +w \frac{a}{6} \left[3 h_1 + a \frac{1 - \cos\varphi}{1 + \cos\varphi} (5 + 4\cos\varphi) \right] \qquad (60)$$

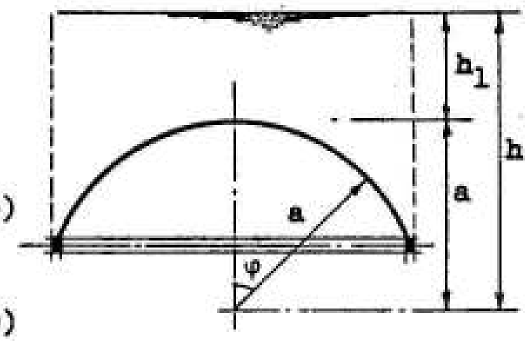


Fig. IV-21

b) <u>Conical Shells</u>

The surface area of a conical shell (fig. IV-22) is given by :

$$A = 2\, \pi\, R\, s / 2 \qquad (61)$$

The meridian force is

$$N_s = W_\varphi / 2\pi\, y \cos\varphi \qquad (62)$$

The ring force is

$$N_\theta = p_r\, y \cos\varphi / \sin^2\varphi \qquad (63)$$

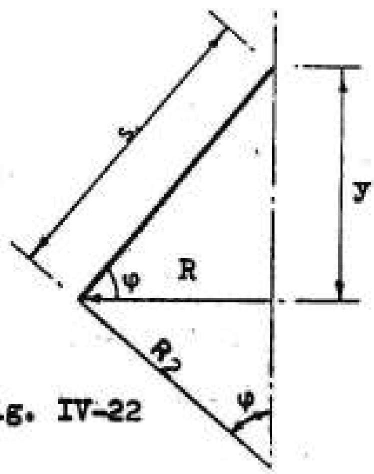


Fig. IV-22

Internal forces due to a vertical dead load g/m^2 surface are :

the forces at the footring, it is recommended to increase the thickness of the shell in the region of the transition curve.

IV-3-6. Fixed and Continuous Surfaces of Revolution

The bending moments shown in figure IV-25 give only a rough estimate, they may however be used in cases where the expected values of the bending moments are small.

Surfaces of revolution in tanks are generally continuous with each other, e.g. the Inze tank shown in figure IV-4. The connecting moments at the joints can be determined by the moment distribution method if the fixed end moments and the relative stiffness of the elements meeting at a joint are known.

We give in the following*, the fixing moments and the stiffness factor of some simple cases generally met with in tank problems.

1) Cylindrical wall of variable thickness (Fig. IV-27)

Assuming $k_1 = \frac{\sqrt[4]{3\,(1-\nu^2)}}{\sqrt{R\,t_2}}$ and $\nu = 1/6$

then $k_1 = \frac{1.3068}{\sqrt{R\,t_2}}$ and

the fixing moment $\bar{M}$ is :

$$\bar{M} = \frac{E\,t_2}{2\,R^2\,k_1^2}\left(\Delta r_o + \frac{\theta_o}{k_1}\right)$$

and the stiffness S is

$$S = 2\,t_2^3\,k_1$$

For water pressure

$$\Delta r_o = \frac{w\,R^2}{E\,t_2}\,H \quad \text{and} \quad \theta_o = -\frac{w\,R^2\,(y_2 - H)}{E\,t_2\,y_2}$$

Hence

$$\bar{M} = \frac{p}{2\,k_1^2}\left(1 + \frac{1}{y_2\,k_1} - \frac{1}{H\,k_1}\right)$$

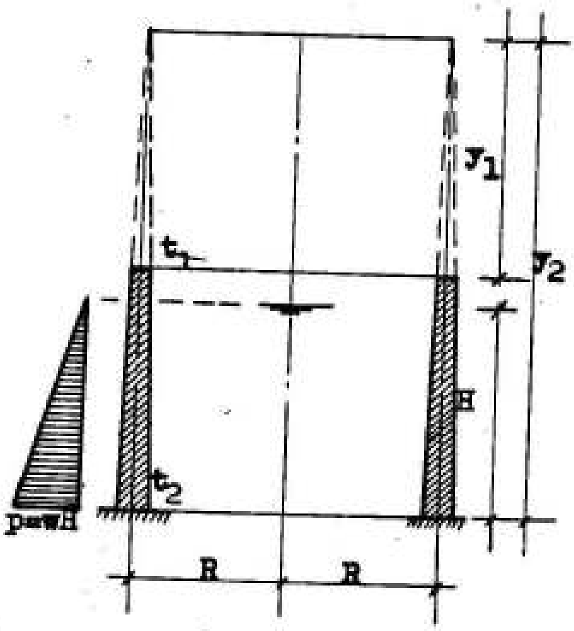


Fig. IV-27

* Refer to Markus " Theorie und Berechung Rotationssymmetrischer Bauwerke" Published by Werner - Verlag. Dusseldorf.

For a wall of constant thickness ($t_1 = t_2 = t$) subject to hydrostatic pressure

$$\bar{M} = \frac{p}{2 k_1^2} \left(1 - \frac{1}{k_1 H}\right)$$

2) Spherical dome of variable thickness (Fig. IV-28)

Assume $k_2 = \sqrt{\frac{a}{t}} \sqrt{3 (1 - \nu^2)}$

and $\nu = 1/6$ then

$$k_2 = 1.3068 \sqrt{\frac{a}{t}}$$

$$\bar{M} = - \frac{Et}{2 k_2^2} \left(\frac{\Delta r_o}{\sin\varphi} - \frac{a}{k_2} \theta_o \right)$$

$$S = 2 t^3 k_2 / a$$

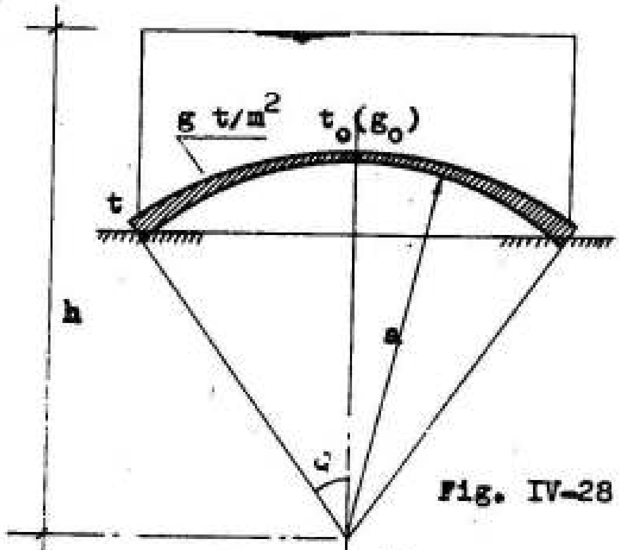


Fig. IV-28

For dead load g t/m²

$$\Delta r_o = \frac{g_o a^2}{Et\, t_o} \left[\frac{1 + \nu}{\sin^2\varphi} \left(t_o - t \cos\varphi + \frac{t - t_o}{\varphi} \sin\varphi \right) - t \cos\varphi \right]$$

$$\theta_o = \frac{g_o a}{Et\, t_o} \left\{ \frac{t - t_o}{\varphi} \cos\varphi - (2 + \nu) t \sin\varphi + \frac{(1 + \nu)(t - t_o)}{t\, \varphi \sin^2\varphi} \times \left[t_o - t \cos\varphi + \frac{t - t_o}{\varphi} \sin\varphi - \frac{t}{1 + \nu} \cos\varphi \sin^2\varphi \right] \right\}$$

For water pressure ($w = 1.0$ t/m³)

$$\Delta r_o = - \frac{w a^3}{Et} \left[\frac{(1 - \nu) h}{2 a} - \cos\varphi + \frac{1 + \nu}{3} \left(\cos\varphi + \frac{1}{1 + \cos\varphi} \right) \right] \sin\varphi$$

$$\theta_o = \frac{w a^2}{Et} \left\{ \sin\varphi + \frac{t - t_o}{t\, \varphi} (1 + \nu) \left[\frac{h}{2 a} - \frac{1}{3} \left(\cos\varphi + \frac{1}{1 + \cos\varphi} \right) \right] - \frac{t - t_o}{t\, \varphi} \left(\frac{h}{a} - \cos\varphi \right) \right\}$$

For a dome of constant thickness $t_o = t$

$$\bar{M} = \frac{Et}{2 k_2^2} \left(\frac{\Delta r_o}{\sin\varphi} \right) + \frac{a}{k_2} \theta_o)$$

Case of dead load g t/m^2 surface

$$\Delta r_o = \frac{g a^2}{Et} \left(\frac{1 + \nu}{1 + \cos\varphi} - \cos\varphi \right) \sin\varphi$$

$$\theta_o = - \frac{g a}{Et} (2 + \nu) \sin\varphi$$

Case of live load p t/m^2 horizontal

$$\Delta r_o = \frac{p a^2}{Et} \left(\frac{1 + \nu}{2} - \cos^2\varphi \right) \sin\varphi$$

$$\theta_o = - \frac{p a}{Et} (3 + \nu) \sin\varphi \cos\varphi$$

Case of water pressure

$$\Delta r_o = - \frac{w a^3}{Et} \left[\frac{(1 - \nu)h}{2 a} - \cos\varphi + \frac{1 + \nu}{3} \left(\cos\varphi + \frac{1}{1 + \cos\varphi} \right) \right] \sin\varphi$$

$$\theta_o = \frac{w a^2}{Et} \sin\varphi$$

3) Conical surface of constant thickness (Fig. IV-29)

Assume $k_3 = \sqrt{\frac{\tan\varphi}{t} \sqrt{3 (1 - \nu^2)}}$ and $\nu = 1/6$ then

$$k_3 = 1.3068 \sqrt{\tan\varphi / t}$$

$$\bar{M} = \frac{t \sin\varphi}{2 s^2 k_3^2 \cos^2\varphi} \left(\Delta r_o - \frac{\sin\varphi}{k_3} \theta_o \right)$$

$$S = 2 t^3 k_3$$

Case of dead load g t/m^2 surface

$$\Delta r_o = \frac{g s^2}{Et} \left[1 - \frac{\nu}{2 \cos^2\varphi} \left(1 - \frac{l^2}{s^2} \right) \right] \cos^2\varphi \cot\varphi$$

$$\theta_o = \frac{g s}{Et} \left[\frac{1}{2} \left(1 - \frac{l^2}{s^2} \right) + \nu - (2+\nu)\cos^2\varphi \right] \frac{\cot\varphi}{\sin\varphi}$$

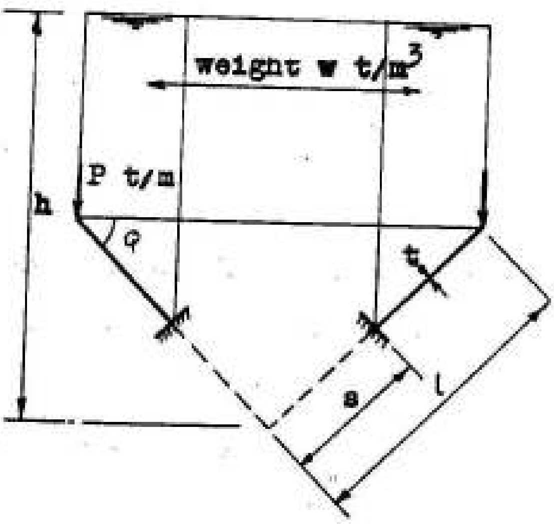


Fig. IV-29

Case of a concentrated load P/m

$$\Delta r_o = \frac{P l}{Et} \cot\varphi \quad \text{and} \quad \theta_o = - \frac{P l}{Et s} \cdot \frac{\cos\varphi}{\sin^2\varphi}$$

Case of water pressure

$$\Delta r_o = \frac{w\,s}{E\,t}\left\{ s\left(\frac{h}{\sin\varphi} - s\right) + \frac{\nu}{2\,s}\left[\frac{h}{\sin\varphi}\,(l^2 - s^2) - \frac{2}{3}\,(l^3 - s^3)\right]\right\}\cos^2\varphi$$

$$\theta_o = \frac{w}{E\,t}\left[3\,s^2\sin\varphi - 2\,h\,s - \frac{h}{2\,s}\,(l^2 - s^2) + \frac{\sin\varphi}{3\,s}\,(l^3 - s^3)\right]\cot^2\varphi$$

4) Ring Beams

At joints of different continuous surfaces of revolution (e.g. a cone and a dome) there exists a ring beam which resists a part of the distributed moment according to its relative stiffness.

The stiffness of a circular ring beam of radius R, breadth b and total depth t is given by

$$S = b\,(1 - \nu^2)\,\frac{t^3}{R^2} \qquad \text{for} \quad \nu = 1/6 \qquad \text{then}$$

$$\underline{S = 0.972\ b\ t^3 / R^2}$$

IV-3-7 Circular Beams

Circular beams loaded and supported normal to their plane (Fig. IV-30) are dealt with in detail in text books on theory of elasticity. We give in the following , the internal forces in a circular beam subjected to uniform load p/m and supported symmetrically on n columns. Thus :

$$2\,\varphi_o = 2\,\pi/n \qquad \text{i.e.} \qquad \varphi_o = \pi/n \qquad (72)$$

Reaction at any column

$$V = 2\,\pi\,R\,p/n \qquad (73)$$

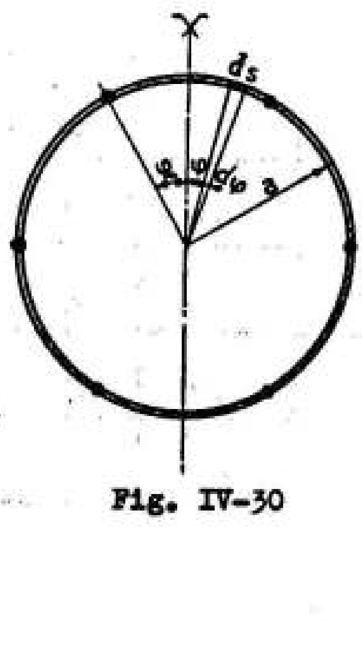


Fig. IV-30

Max. shearing force to the right or left of any support

$$Q_{max} = \pm \pi R\, p/n \qquad (74)$$

The bending moment M , the torsional moment M_t and the shearing force Q in any section at an angle φ from the center line between two successive supports are given by :

$$M = R^2 p \left(\frac{\cos\varphi}{n \sin\varphi_o} - 1 \right) \qquad (75)$$

$$M_t = R^2 p \left(\frac{\pi \sin\varphi}{n \sin\varphi_o} - \varphi \right) \qquad (76)$$

$$Q = - R\, p\varphi \qquad (77)$$

We give in the following table, the reactions, the maximum shearing forces, bending moments and torsional moments in a circular beam of radius R supported symmetrically on n columns and subjected to a total uniformly distributed load P. where

$$P = 2\pi R\, p$$

Number of Cols.	Load on each Column	Max. Shearing Force	Max. Bending Moment		Max. Torsional Moment	Angle bet. axis of Col. & Sec. of max. M_t
			at C.L. of Span	Over C.L. of Columns		
n	V	Q_{max}	M (+)	M (-)	M_t	Deg.
4	P/4	P/8	.0176 P R	.0053 P R	.0053 P R	19° 21'
6	P/6	P/12	.0075 P R	.0148 P R	.0015 P R	12° 44'
8	P/8	P/16	.0042 P R	.0083 P R	.0006 P R	9° 33'
12	P/12	P/24	.0019 P R	.0037 P R	.0002 P R	6° 21'

V. PRESTRESSED CIRCULAR TANKS

V.1 INTRODUCTION.

The design of prestressed structures is based on a knowledge of the fundamental principles of prestressed concrete. This is as true for the design of tanks as for beams and slabs. Before analysing the stresses in a prestressed tank it may be of advantage to give the most important basic principles of prestressed concrete as may be required for design of circular tanks.

Prestressed concrete denotes concrete in which effective internal stresses are induced generally by the use of high tensile steel. This operation is done in such a way as to completely eliminate or at least to effectively reduce tensile stresses under the action of working loads together with the provision of an ample factor of safety against cracking or collapse.

The most effective use of prestressing can only be obtained if the concrete and steel are of very high quality. The higher the crushing strength of concrete and the tensile strength of steel , the greater the effectiveness with which the prestress can be utilised.

High quality concrete can be achieved through careful selection of aggregates, suitable granular composition, use of low water-cement ratio, sufficient cement content and thorough mixing, compaction and curing.

The minimum cement content in the concrete mix is 350 kg/m^3 of finished concrete, the minimum crushing strength is 300 kg/cm^2 after 28 days. The allowable compressive stress may be assumed one third of the crushing strength. The modulus of elasticity E_c may be assumed equal to 300 t/cm^2 .

The steel to be used for prestressing is generally hard drawn wires of an ultimate strength not less than 15 t/cm^2 and having diameters varying between 3 and 7 mms.

The permissible stress in the prestressing steel shall be based on the ultimate strength and the 0.2 % proof stress as follows :

The maximum tensile stress at transfer σ_{so} shall not exeed 80 % of the proof stress or 65 % of the ultimate tensile strength which over is the lesser. Accordingly, for hard drawn wires 16.5/14 having an ultimate tensile strength of 16.5 t/cm^2 and a proof stress of 14.0 t/cm^2 the maximum allowable tensile stress σ_{so} = 10 t/cm^2 .

The modulus of elasticity E_s is generally equal to 2000 t/cm^2. The modular ratio $n = E_s/E_c$ varies between 6 and 7 .

Due to shrinkage, creep .... etc. the prestress is reduced by 15 to 20 % of its initial value, so that the final prestress σ_{soo} will be equal to 0.80 to 0.85 σ_{so} .

Stresses in prestressed elements under working loads may be computed by the elastic theory both at transfer with full prestress ($\sigma_s = \sigma_{so}$) and no live loads and, under final conditions after losses have taken place ($\sigma_s = \sigma_{soo}$) and full live loads.

Prestressing of circular tanks is made in order to eliminate the tensile stresses created by the hydrostatic pressure. In most prestressed circular structures , prestress is applied both circumferentially and longitudinally, the circumferential prestress being circular and the longitudinal prestress actually linear. It is however easier to use circumferential prestressing only. This is possible in walls with freely sliding edges.

V-2 CIRCUMFERENTIAL PRESTRESSING.

Prestressed concrete circular tanks* are generally constructed by winding prestressing wires around the walls using a special winding machine. The normal method of prestressing consists of the following process : First, the walls of the tank are built of either concrete or pneumatic mortar, mortar being generally used if the walls are less than 12 cms thick. Often, the walls are poured in alternative vertical slices keyed together. After the concrete walls have attained sufficient strength they are prestressed circumferentially by a self propelled machine, which winds the wire around the walls in a continuous operation, stressing it and spacing it at the same time.

After the circumferential prestressing is completed for each layer, a coat of pneumatic mortar is placed around the tank for protection. Two or more layers of prestressing are used for large tanks(Fig. V.1).

Vertical prestressing for the tanks can be applied using any system of linear prestressing, whichever may be the most economic.

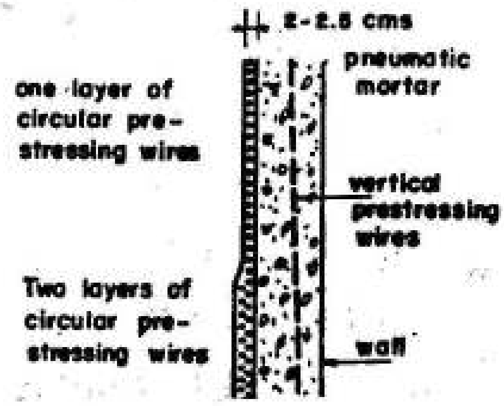


Fig. V-1

Circumferential prestress in tanks is designed to resist ring tension produced by liquid pressure.

Consider one half of a thin horizontal slice of a tank as a free body, fig. V.2. Under the action of the initial prestress F_o in the steel, the total compression C in the concrete is equal to F_o causing an initial compressive stress in the concrete equal to $\sigma_o = F_o / A_c$. After the losses due to shrinkage and creep are developped the pres-

* Lin " Prestressed Concrete Structures "

$$A_c = F_o / \sigma_c = \sigma_{so} A_s / \sigma_c$$

and for $\sigma_{so} = 10\ t/cm^2$, we get :

$$A_c = 10\ A_s / 0.1 = 100\ A_s$$

but $A_c = 100\ t$ then $t\ cms = A_s\ cm^2$ i.e.

$$\underline{A_s = t = 0.145\ p\ R} \qquad (82)$$

If it is required to limit σ_c to 80 kg/cm^2 only, the amount of the prestressing steel will not be affected while the thickness will be increased to :

$$\underline{t = 0.18\ p\ R} \qquad (83)$$

<u>Example</u> :

Determine max. t and A_s required for a prestressed open circular slurry tank 10 ms diameter and 12 ms deep with sliding base. Assume $w = 25\ t/m^3$, $\sigma_{so} = 10\ t/cm^2$, $\sigma_c = 100\ kg/cm^2$, $P_{max} = 2.5 \times 12 = 30\ t/m^2$, $t = A_s = 0.145\ P\ R$ or

$t\ cms = A_s\ cm^2 = 0.145 \times 30 \times 5 = \underline{22}$

V-3. <u>VERTICAL PRESTRESSING</u>

Due to hydrostatic pressure, the walls of circular tanks are subject to ring tension in the horizontal direction and cantilever moments in the vertical direction in case they are hinged or fixed to the floor.

It is evident that circumferential prestressing will also cause ring forces and cantilever moments in a sense opposite to that of hydrostatic pressure and that they exist by themselves when the tank is empty and act jointly with the ring forces and moments produced by liquid pressure when the tank is full.

In order to reinforce the walls against these moments, vertical prestressing may be applied. If vertical prestress is axially applied to the concrete, only direct compressive stress is produced and the solution is simple. If the vertical tendons are bent or curved, the vertical prestress produces radial components which, in turn, influence the circumferential prestress and the analysis can become quite complicated.

Let us investigate the effect of circumferential prestressing on the vertical moments. If the circumferential prestress varies triangularly from zero at the top to a maximum at the bottom, its effect is equal but opposite to the application of an equivalent liquid pressure. If the circumferential prestress is constant throughout the entire height of the wall, it is the same as the application of an equivalent gaseous pressure. For both cases, the P.C.A. tables* or any similar ones are available for the computation of vertical moments.

Vertical prestressing should be designed to stand the stresses produced by various possible combinations of the following forces :

1) The vertical weight of the roof and the walls themselves.
2) The vertical moments produced by the applied circumferential prestress.
3) The vertical moments produced by internal liquid pressure.

It must be noted that the maximum stresses in the concrete usually exist when the tank is empty, because then the circumferential prestress would have its full effect. When the tank is filled, the liquid pressure tends to counterbalance the effect of circumferential prestress and the vertical moments are smaller. Since it is convenient to use the same amount of vertical prestress throughout the entire height of the wall, the amount will be controlled by the point of maximum moment. By properly locating the vertical tendons to resist such moment, a most economical design can be obtained. However, efforts are

* Refer to P . C. A. tables of circular tanks

seldom made to do so, and the amount of prestress as well as the location of the tendons is generally determined empirically rather than by any logical method of design.

In order to avoid complications in the position of the vertical prestressing cables due to the varying sign of the bending moments - as in case of circular tanks with fixed base - the walls may be made hinged to the floor at their foot. The vertical prestressing cables may then be located nearer to the inside surface of the wall at its bottom end where the maximum moments occur. At the upper end, the cables may be anchored at the center of gravity of the concrete section as shown in figure V.3.

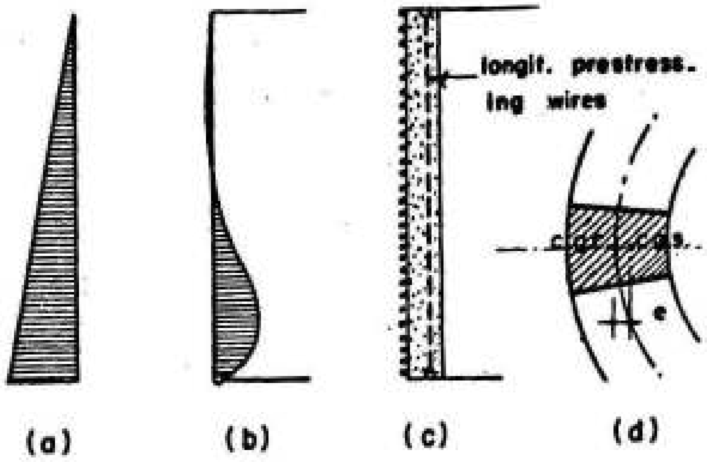


Fig. V-3

a) Pressure on wall due to circumferential prestress.

b) Corresponding B.M.D. for a wall hinged at base.

c) Longitudinal and circumferential prestressing wires.

d) Location of long prestressing wires.

In order to determine the magnitude of the prestressing force , one can proceed as follows :

1) The magnitude of the maximum bending moment due to circumferential prestressing can be determined from table VIII of the P.C.A. tables.

2) The corresponding stress in the section is then computed.

3) Assuming the eccentricity of the longitudinal prestressing steel at position of maximum bending moment is e, compute the stresses in

the section due to a vertical prestressing force F = 1 acting in the center of gravity of the steel (c.g.s.)

4) In order to get zero stress in the inner surface of the tank due to the combined action of the longitudinal prestressing force and bending moments due to the circumferantial prestressing (case of full prestressing), divide the maximum tensile stress computed under 2 by the maximum compressive stress computed under 3.

The stresses in the section under the different combinations of the forces acting will be shown in the example given in V.5.

V-4. DOME PRESTRESSING

In the following, only the general principles and practice of dome prestressing especially as applied to tank roofs will be mentioned. Generally speaking, for domes with diameter greater than 30 ms, the economy of prestressing should be seriously considered. Domes for tanks up to 75 ms in diameter have been constructed.

The dome roof itself is made of concrete or pneumatic morter with thickness varying from 6 to 15 cms. For domes of large diameter, variable thicknesses may be employed and thicknesses greater than 15 cms are used for the lower portion. Before concreting the dome, some erection bars are compressed around the base of the dome. After the hardening of the shell concrete, wires are prestressed around it. During this operation, the dome shell rises from its forms as it is compressed, thus simplifying the careful proceedure of decentering required for non-prestressed domes.

The conventional method of dome prestressing consists of prestressing the edge ring to induce sufficient compressive stresses to counteract the tensile stresses set up in the ring under the maximum live and dead loads. With this process, it is usually possible to raise the dome from its false work, since only the dead load is actually

Loads :

Own weight (assumed 10 cms thick)	g	= 250	kg/m^2
Superimposed dead and live loads	p	= 150	"
	total	= 400	kg/m^2

Total roof own weight $W_g = 0.25 \times 1335$ = 334 ton

Total roof superimposed loads $W_p = 0.15 \times 1335$ = 200 "

total W = 534 ton

Total load / m' wall $V = \frac{W}{2 \pi R} = \frac{534}{2 \times 3.14 \times 20}$ = 4.25 t/m

The horizontal thrust/m $H = V \cot \varphi = 4.25 \times \frac{37.5}{20}$ = 8.0 "

The ring tension $T = H R = 8 \times 20$ = 160 t

The ring reinforcement $A_s = T / \sigma_s = 160 / 1.4$ = 114 cm^2

chosen 23 Φ 25 mm

As a preliminary estimate for the wall assume :

The max. ring tension $T = 0.5\ p_{max}\ R = 0.5 \times 8 \times 20$ = 80 t/m

Required wall thickness $t = 0.75\ T = 0.75 \times 80$ = 60 cms

According to P.C.A. table II, we get :

for $H^2 / D\,t = 8^2 / 40 \times 0.6 = 2.67$,

the maximum ring tension :

$T_{max}\ 0.5\ H\ R = 0.5 \times 8 \times 20$ = 80 ton at 0.6 H from top i.e. the assumed thickness of 60 cms is acceptable.

The required ring reinforcement at 0.6 H is given by :

$A_s \max = T_{max} / \sigma_s = 80 / 1.4 = 57\ cm^2$

chosen 8 Φ 22 mm/m on each surface giving $A_s = 60\ cm^2$

The max. tensile stress in concrete is given by :

$$\sigma_{t\,max} = \frac{T_{max} + \varepsilon_{sh}\ E_s\ A_s}{A_c + n\ A_s}$$

$$= \frac{80000 + 0.00025 \times 2000000 \times 60}{100 \times 60 + 10 \times 60} = 16.6 \text{ kg/cm}^2 < 20$$

b) Design of dome and wall in prestressed concrete

For the roof dome, only the foot ring will be prestressed thus , the final prestressing force required to counterbalance the max. ring tension $T_{max.}$ of 160 t is according to equation 84 given by :

$$F_{\infty} = W \cot \varphi / 2\pi = T_{max.} = 160 \text{ t}$$

The required prestressing steel

$$A_s = F_{\infty} / \sigma_s = 160/8 = \underline{20 \text{ cm}^2}$$

which is less than 20% of the amount required in case of ordinary rein forced concrete.

The max. compressive stress in the foot ring will take place at transfer with a prestressing force F_o and a ring force T_o due to won weight of dome. Thus,

$$F_o = F_{\infty} / 0.8 = 160/0.8 = 200 \text{ t.}$$

and

$$T_o = T_{max.} W_g / W = 160 \times 334 / 534 = 100 \text{ t.}$$

The net compressive force acting on the foot ring

$$N = F_o - T_o = 200 - 100 = 100 \text{ t.}$$

Assuming the max. allowable compressive stress in concrete = 50 kg/cm^2 then the area of concrete section required for the foot ring is given by :

$$A_c = N / \sigma_c = 100000/50 = \underline{2000 \text{ cm}^2}$$

A foot ring 70 x 30 cms will be chosen .

Prestressed circular tanks require a wall thickness much smaller than that required for reinforced concrete tanks and therefore the pressure resisted by ring action P_r is much bigger. As a first estimate, assume :

$$\max. P_r = 0.7 \text{ w H} = 0.7 \times 1 \times 8 = 5.6 \text{ t/m}^2$$

The thickness required for the wall can be estimated according to equation 101 from the relation

$$t = 0.145\ p_r\ R = 0.145 \times 5.6 \times 20 = 16.5 \text{ cms}$$

A wall 17 cms thick will be chosen .

According to P.C.A. table II, we get :

$$\text{for } H^2 / D\,t = 8^2 : 40 \times 0.17 = 9.4$$

The maximum ring tension :

$$T_{max} = p\,R = 0.713\ w\,H\,R = 0.713 \times 1 \times 8 \times 20 = 114 \text{ t}$$

at 0.7 H from top.

The maximum area of circumferential prestressing steel at 0.7 H from top is, according to equation 100 b, given by :

$$A_s = \frac{s\,p\,R}{\sigma_s + 6\,\sigma_c} = \frac{1.25 \times 114}{8 + 6 \times 0.1} = 16.6 \text{ cm}^2$$

This value of the area of the steel, as well as the wall thickness can be directly determined from equation 101 ; thus,

$$A_s = t = 0.145\,p\,R = 0.145\,T_{max.} = 0.145 \times 114 \qquad \text{i.e.}$$

$$A_s = 16.5 \text{ cm}^2 \;\&\; t = 16.5 \text{ cm}$$

which means that the chosen thickness of 17 cms is acceptable.

The residual stress in the concrete under the effective prestress F_∞ and the internal pressure at position of maximum ring tension can be calculated according to equation 78 as follows :

$$\sigma_c = -\frac{F_\infty}{A_c} + \frac{p\,R}{A_v} = -\frac{16.5 \times 8000}{100 \times 17} + \frac{114000}{100 \times 17 + 7 \times 16.5} = -77.5 + 63$$

or $\underline{\sigma_c = -\ 14.5}$ kg/cm^2

which provides a margin of safety of 25 % up to zero compression in concrete.

The cantilever moment due to the hydrostatic pressure can be calculated from the P.C.A. table VIII, thus ,

For $H^2 / D t = 9.4$, $M_{max} = 0.005 \text{ w } H^3 = 0.005 \times 1 \times 8^3 = 2.56$ mt. acting at 0.8 H from top and causes tensile stresses on the outer face of the wall.

The initial and final maximum cantilever moments due to the prestressing steel (with tension on the inside face of the wall) can be determined as follows :

16.5 cm^2 prestressing circumferential wires tensioned to an initial stress of 10 t/cm^2 will cause a ring compression of :

$C = A_s \sigma_s = 16.5 \times 10 = 165$ t.

The corresponding intensity of radial compression will be :

$P_o = C / R = 165 / 20.17 = 8.2 \; t/m^2$

This pressure causes a cantilever moment equal to :

$M_o = 2.56 \times 8.2 / 0.713 \text{ w } H = 2.56 \times 8.2/0.713 \times 1 \times 8 = 3.68$ mt.

For a final prestress of 8 t/cm^2 this value reduces to :

$M_{oo} = M_o \times 8 / 10 = 3.68 \times .8 = 2.94$ mt.

The circumferential prestressing will counterbalance the internal pressure in case of full tank, but in case of empty tank, the tensile stresses due to the cantilever moments induced by the prestressing force can be counterbalanced by vertical prestressing wires placed in the tension zone - near the inside surface - at a distance of say 5 cms from it.

The magnitude of the vertical prestressing force can be determined by dividing the magnitude of the maximum tensile stress due to the moment caused prestressing - 3.68 mt - by the magnitude of the maximum compressive stress caused by a prestressing vertical force of 1 ton acting at 5 cms from the inside face of the wall, thus, max. tensile stress

Initial and Final Stresses in a Prestressed Circular Tank under Different Cases of Loading

Conditions	Initial Stre in kg/cm^2		Final Stresses in kg/cm^2	
Fiber	Inside	Outside	Inside	Outside
A) Weight of Roof 4250/1700	- 2.50	- 2.50	- 2.50	-- 2.50
B) Weight of Wall 0.17 x 2400 = 410 kg/m^2 410 x 0.8 x 8 / 1700	- 1.54	- 1.54	- 1.54	- 1.54
C) Axial Compression of Vertical Prestress Initial 58000 / 1700 Final 46400 / 1700	- 34.10	- 34.10	- 27.30	- 27.30
D) Eccentricity of Vertical Prestress Initial 6 x 58000 x 3.5/100 x 17^2 Final 6 x 46400 x 3.5/100 x 17^2	- 42.20	+ 42.20	- 33.80	+ 33.80
E) Vertical Moment due to Circumferential Prestress Initial 6 x 3.68 x 10^5/100 x 17^2 Final 6 x 2.94 x 10^5/100 x 17^2	+ 76.5	- 76.5	+ 61.20	- 61.20
Total for Tank Empty	- 3.84	- 72.44	- 3.94	- 58.74
F) Vertical Moment due to Hydrostatic Pressure 6 x 2.56 x 10^5 / 100 x 17^2	- 53.00	+ 53.00	- 53.00	+ 53.00
Total for Tank Full	- 56.84	- 19.44	- 56.94	- 5.74

due to M_o = 3.68 mt is given by :

$$\sigma = 6\ M / b\ t^2 = 6 \cdot x\ 3.68 \times 10^5 / 100 \times 17^2 = 76.5\ kg/cm^2$$

Maximum compressive stress due to a vertical prestressing force of 1.0 ton acting at 5 cms from the inside face (e = 3.5 cm) is given by :

$$\sigma = - F/A - 6\ F\ e / b\ t^2 = - 1000/1700 - 6 \times 1000 \times 3.5/100 \times 17^2$$
$$= - 0.59 - 0.73 = - 1.32\ kg/cm^2$$

Vertical initial prestressing force is therefore given by :

$$F_o = 76.5 / 1.32 = 58\ t.$$

The corresponding final vertical prestressing force is

$$F = 0.8\ F_o = 0.8 \times 58 = 46.4\ t.$$

The stresses for both the inside and outside fibers under both initial and final conditions at position of maximum cantilever moment are computed and listed as in the table.

It is seen that the wall is subject to compressive stresses under all load stages.

The details of reinforcements are shown in figure V.6.

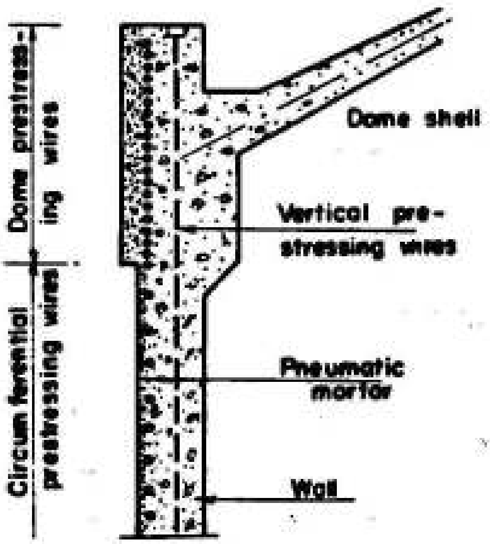


Fig. V-6

V.6. ECONOMIC PROPORTIONS OF PRESTRESSED CIRCULR TANKS :

The ratio of diameter to height is of some importance with regard to the cost of circular cylindrical prestressed concrete tanks. Favourable dimensional proportions are given in the following table Fig. V.7 which has been published by Preload Engineers, New York , on the basis of many year's experience.

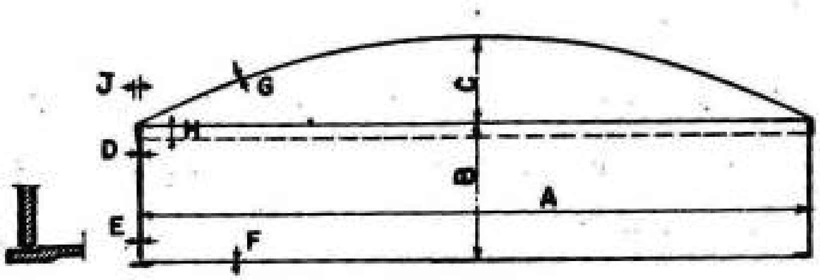


Fig. V-7

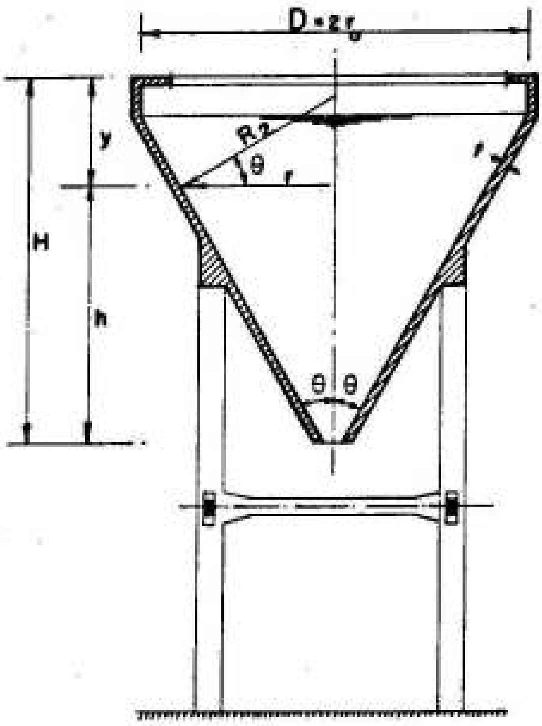
Capacity in m³	Dimensions in m								
	A	B	C	D	E	F	G	H	J
378	12.50	3.50	1.56	.12	.12	.05	.05	.20	.15
945	16.90	4.30	2.11	.12	.12	.05	.05	.22	.15
1890	21.35	5.35	2.67	.12	.12	.05	.05	.30	.17
2835	24.40	6.10	3.05	.12	.15	.05	.05	.36	.19
3780	26.95	6.70	3.36	.12	.18	.05	.05	.38	.22
5670	30.80	7.80	3.86	.12	.23	.05	.05	.43	.25
7550	33.85	8.55	4.23	.12	.24	.05	.06	.48	.27
9450	36.40	9.15	4.55	.22	.26	.05	.06	.51	.30
18900	46.00	11.45	5.75	.22	.44	.05	.10	.69	.38
37800	57.90	14.50	7.24	.22	.74	.05	.11	.89	.49
	Economic Proportion in U.S.A. B:A = 1 : 4								

The wall thickness is probably rather too small for normal conditions ; the increase in thickness from top to bottom should be noted. The thickness of the base slab has in all cases been given as 5 cm . because in the U.S.A. the base is usually constructed as a thin but closely reinforced gunite layer.

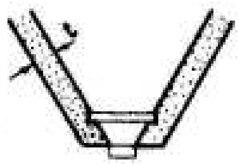
VI. EXAMPLES OF CIRCULAR TANKS

As typical examples showing the use of surfaces of revolution in tanks, we give the following three structures :

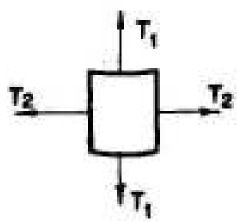
VI.1 - PASTE CONTAINER* : Fig. VI.1.



Paste container



Discharge funnel



Meridian and ring forces

Data :

Fig. VI-1

w = weight of paste per cubic meter.

r = h tan θ = horizontal radius of any section.

$$R_2 = \frac{r}{\cos\theta} = \frac{h\tan\theta}{\cos\theta} = \text{cross radius of curvature.}$$

---

* The notations used in this example differ from those used in the membrane theory. All equations being derived from the first principles, it has been found that changing the notations is not essential

The internal forces in the wall of the container due to weight of paste can be determined as follows :

a) Ring Forces Due to Paste :

For determining the ring tension T_2 in any section, we apply the fundamental equation 46 using the given notations, thus :

$$\frac{T_1}{R_1} + \frac{T_2}{R_2} = p_r$$ where

R_1 = radius of curvature of meridian = ∞

p_r = component of load of paste normal to surface of container assuming angle of friction between paste and reinforcement concrete = 0 . thus :

$$p_r = w\,y$$

Therefore :

$$T_2 = R_2\,p_r = \frac{w\,y\,h\,\tan\theta}{\cos\theta} \quad \text{tension} \qquad (86)$$

Where :

$$h = (H - y)$$

The depth y at which the ring force T_2 is maximum can be determined from the condition :

$$\frac{dT_2}{dy} = 0$$ but

$$\frac{dT_2}{dy} = w\,\frac{\tan\theta}{\cos\theta} \cdot \frac{dy\,(H - y)}{dy} = w\,\frac{\tan\theta}{\cos\theta}\,(H - 2\,y)$$

Therefore, for maximum ring tension, we must have :

$$w\,\frac{\tan\theta}{\cos\theta}(H - 2\,y) = 0$$

or

$$\underline{y = H/2}$$

and

$$\max.\ T_2 = w\,\frac{\tan\theta}{\cos\theta} = w\frac{H^2}{4} \qquad (87)$$

b) Meridian Forces Due to Paste :

When determining the meridian force T_1 , two separate cases have to be considered, namely above and below the ring beam as follows:

i) Above the Ring Beam : Fig. VI.2 :

If we equate the weight of the paste above the horizontal section at depth y (hatched volume) to the vertical component of the compressive meridian force T_1 , we get : Fig. 2 .

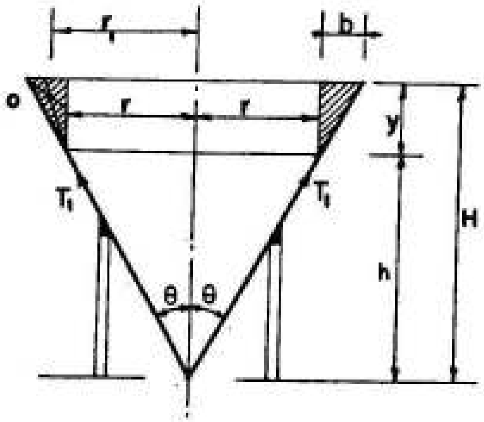


Fig. VI-2

$$T_1\ 2\ \pi\ r\cos\theta = w\,\frac{b\,y}{2} .\ 2\ \pi\ r_1$$

in which

$r\ = (H - y)\ \tan\theta$

$b\ = y\tan\theta$ and

r_1 = radius of center of gravity of hatched volume

$= (H - \frac{2}{3} y)\tan\theta$

Therefore :

$$T_1\ 2\ \pi (H - y)\tan\theta .\cos\theta\ = w\ \pi\ y^2\ (H - \frac{2}{3} y)\tan^2\theta$$

or

$$T_1 = \frac{w}{6} .\ \frac{3H - 2y}{H - y} .\ y^2\ \frac{\tan\theta}{\cos\theta}\ \text{compression} \qquad (88)$$

ii) Below the Ring Beam : Fig.VI.3 :

In this case, the weight of the paste acting on the lower conical part (hatched volume) will be hung to the wall of the container causing meridian tensile stresses which can be treated in a similar way as follows. (Fig.VI.3)

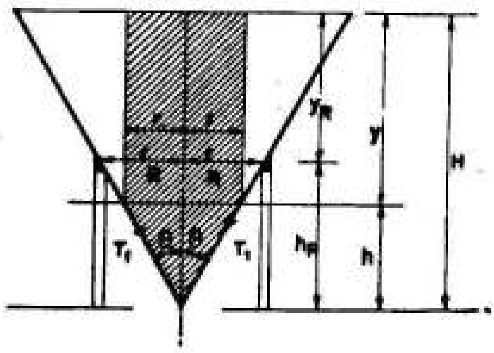


Fig. VI- 3

$$T_1\ 2\ \pi\ r .\cos\theta = w\left[\pi\ r^2 .\ y + \frac{1}{3}\ \pi\ r^2\ h\right]$$

Substituting $r = (H - y) \tan\theta$ and $h = H - y$, we get :

$$T_1 . 2\pi (H - y) \tan\theta \cos\theta = w \left[\pi (H - y)^2 \tan^2\theta \; y + \frac{1}{3} \pi (H - y)^2 \tan^2 (H - y) \right]$$

or

$$T_1 = \frac{w}{6} (H - y)(H + 2y) \frac{\tan\theta}{\cos\theta} \quad \text{tension} \qquad (89)$$

The depth y at which the tensile meridian force below the ring beam is max. can be determined from the condition' $\frac{dT_1}{dy} = 0$ but

$$\frac{dT_1}{dy} = \frac{w}{6} \; \frac{\tan\theta}{\cos\theta} (H - 4y)$$

For max. meridian tension T_1 $\quad y = \frac{H}{4}$ and

$$\max. T_1 = \frac{3}{16} . w H^2 . \frac{\tan\theta}{\cos\theta} \qquad (90)$$

provided the ring beam is at a distance from the apex greater than 3/4 H . If not, the max. T_{1R} occurs at the ring beam and can be determined according to equation 90 from the relation :

$$T_{1R} = \frac{w}{6} (H - y_R)(H + 2 y_R) \frac{\tan\theta}{\cos\theta} \qquad (90b)$$ in which

$y_R > \frac{H}{4}$ is the distance of the ring beam from the surface of the container.

c) Internal Forces in Ring Beam Due to Paste.

The ring beam is subject to a compressive force C_R determined by : (Fig. VI.4)

$C_R = H_R \, r_R = T_1 \sin\theta \, r_R$ and $T_1 = T_1' + T_1''$ in which

T_1' is the compressive meridian force just above the middle of the ring beam and

T_1'' is the tensile meridian force just below the middle of the ring beam.

d) Internal Forces Due to Own Weight of Container.

The ring and meridian forces due to the own weight of the container are usually small compared with those caused by the weight of the past, they can however be calculated as follows : (Fig. VI.5)

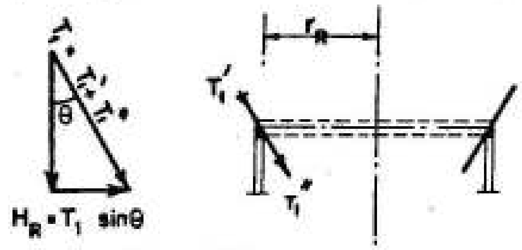


Fig. VI-4

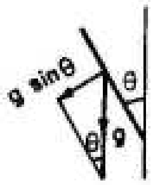


Fig. VI-5

Assuming g = own weight of tank / m^2 , the ring force can be determined from the relation :

$$T_2 = R_2 \cdot Z \qquad \text{in which}$$

$$R_2 = \frac{h \tan\theta}{\cos\theta} \text{ and } Z = g \sin\theta, \qquad \text{therefore,}$$

$$\underline{T_2 = g\,h\,\tan^2\theta} \qquad (91)$$

which means that T_2 is proportional to h, so that :

T_2 is maximum at the top where h is maximum and equals H , thus,

$$\text{max. } T_2 = g\,H\,\tan^2\theta \qquad (91a)$$

The meridian compressive force T_1 above the ring beam can be calculated as follows : (Fig. VI.6)

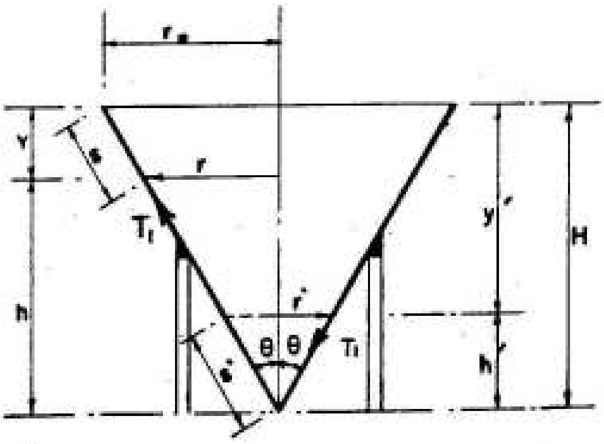


Fig. VI-6

$$T_1 \; 2\pi r. \cos\theta = g\left(2\pi r_o \frac{H}{2\cos\theta} - 2\pi r \frac{h}{2\cos}\right)$$

but $\qquad r = h\tan\theta \quad$ and $\quad r_o = H\tan\theta$

so that

$$T_1\, h\tan\theta\cos\theta = \frac{g}{2\cos\theta}\left(H^2\tan\theta - h^2\tan\theta\right) \qquad \text{or}$$

$$\left.\begin{aligned} T_1 &= \frac{g}{2\cos^2\theta}\cdot\frac{H^2-h^2}{h} \qquad \text{or} \\ T_1 &= \frac{g}{2\cos^2\theta}\cdot y\,\frac{2H-y}{h} \end{aligned}\right\} \qquad (92)$$

The meridian tensile force T_1 below the ring beam can be calculated from the relation :

$$2\pi r T_1\cos\theta = g\, 2\pi r \frac{h}{2\cos\theta} \qquad \text{or}$$

$$\underline{T_1 = \frac{g\,h}{2\cos^2\theta}} \qquad (93)$$

For the special case : $\theta = 30°$ we have :

$\sin\theta = 1/2 \qquad \cos\theta = \sqrt{3}/2 \qquad \tan\theta = 1/\sqrt{3}$.

and $\qquad \cos^2\theta = 3/4 \qquad \tan^2\theta = 1/3$

If we assume further that the ring beam is arranged at mid. height i.e. $y = H/2$, the equations giving the internal forces can be much simplified as shown in table given on page **121.**

e) <u>Numerical Example</u> :

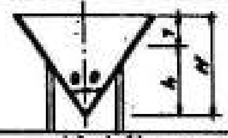
The design of a paste container of the form shown in fig. VI.1 having D = 10 ms, $\theta = 30°$, supported symmetrically at mid-height on six columns and used for mixing paste weighing 2 tons per cubic meter can accordingly be done as follows :

$D = 10$ ms $\qquad r_o = 5$ ms $\qquad \theta = 30° \qquad w = 2\ t/m^3$

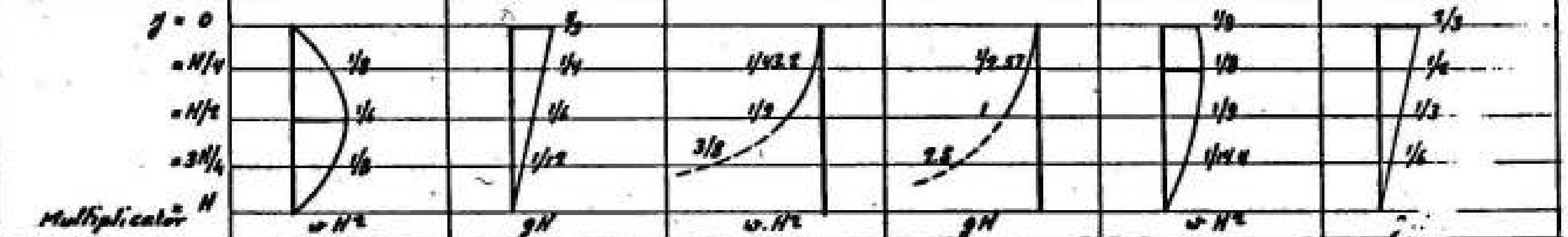
Therefore $\qquad H = r_o/\tan\theta = 5\sqrt{3} = 8.66$ ms

For the ring beam $\qquad r_R = 25$ ms $\qquad y_R = h_R = \frac{H}{2} = 4.33$ ms

INTERNAL FORCES IN PASTE CONTAINERS.



		Ring tensile force T_2		Meridian comp. force above ring beam		Meridian tensile force below ring beam	
		Due to paste. w/m^3	Due to own weight g/m^2	Due to paste w/m^3	Due to own weight g/m^2	Due to paste w/m^3	Due to own weight g/m^2
GENERAL		(86) $\frac{w\,y\,h\tan\theta}{\cos\theta}$	(91) $g\,h\tan^2\theta$ (91a) max: $g\,H\tan^2\theta$ at top	(88) $\frac{w}{6}\cdot\frac{3H-2y}{h}\cdot y^2\,\frac{\tan\theta}{\cos\theta}$	(92) $\frac{g}{2\cos^2\theta}\cdot y\cdot\frac{2H-y}{h}$	(89) $\frac{wh}{6}(H+2y)\frac{\tan\theta}{\cos\theta}$ (90) max: $\frac{3}{16}wH^2\frac{\tan\theta}{\cos\theta}$ at $y=H/4$	(93) $\frac{gh}{2\cos^2\theta}$
$\theta = 30°$	at any depth	(86a) $\frac{2}{3}\,w\,y\,h$	(91b) $y\,\frac{h}{3}$	(88a) $\frac{wy^2}{9}\cdot\frac{3H-2y}{h}$	(92a) $\frac{2gy}{3}\cdot\frac{2H-y}{h}$	(89a) $\frac{wh}{9}(H+2y)$	(93a) $\frac{2gh}{3}$
	at mid. height $y = h = \frac{H}{2}$	(87) $\frac{wH^2}{6}$	(91c) $g\,\frac{H}{6}$	(88b) $\frac{wH^2}{9}$	(92b) $g\,H$	(89b) $\frac{wH^2}{9}$	(93b) $\frac{gH}{3}$



Note: Figures between brackets give the number of the equations.

The container wall will be assumed 20 cms thick, so that :

$$g = 0.2 \times 2.5 = 0.5 \quad t/m^2$$

Max. ring force T_2 max. at $y = H/2$

Due to pressure of paste (equation 87)

$$T_2 = \frac{w\,H^2}{6} = 2 \times \frac{8.66^2}{6} = 25.00 \quad t/m$$

Due to own weight (equation 91c)

$$T_2 = g\,\frac{H}{6} = 0.5 \times \frac{8.66}{6} = 0.72 \quad t/m$$

Total T_2 max. = 25.72 t/m

Meridian force T_1 max. above ring beam

Due to pressure of paste (equation 88b)

$$T_1 = \frac{w\,H^2}{9} = \frac{2 \times 8.66^2}{9} = 16.7 \quad t/m$$

Due to own weight (equation 92b)

$$T_1 = g\,H = 0.5 \times 8.66 = 4.33 \quad t/m$$

Total compressive meridian force above ring beam

Total T_1' max. = 16.7 + 4.33 = 21.03 t/m

Meridian force T_1'' max. below ring beam .

Due to pressure of paste (equation 89b)

$$T_1 = \frac{w\,H^2}{9} = \frac{2 \times 8.66^2}{9} = 16.7 \quad t/m$$

Due to own weight (equation 93b)

$$T_1 = g\,\frac{H}{3} = \frac{0.5 \times 8.66}{3} = 1.44 \quad t/m$$

Total tensile meridian force below ring beam

Total T_1'' max. = 16.7 + 1.44 = 18.14 t/m

<u>Total meridian force above and below ring beam.</u>

$T_{1R} = T_1' + T_1'' = 21.03 + 18.14 \qquad = 39.17$ t/m

Vertical load on ring beam $P_R = T_{1R}$ cos

$= 39.17 \times \frac{\sqrt{3}}{2} \qquad = 34.00$ t/m

Horizontal load on ring beam $H_R = T_{1R} \sin\theta$

$= 39.17 \times 1/2 \qquad = 19.59$ t/m

Compressive force in ring beam $C_R = H_R\, r_R$

$= 19.59 \times 2.5 \qquad = 49.00$ t

Having determined the maximum values of the internal forces in the container, one can design its different elements as follows :

<u>Design of container wall slab</u> $t = 20$ cms

Max. ring tensile force T_2 max = 25.72 t/m

Max. meridian tensile force T_1'' max = 18.14 t/m

Max. meridian compressive force T_1' max = 21.03 t/m

Max. ring reinforcement at ring beam $A_s = \frac{T_{2max.}}{\sigma_s} = \frac{25.72}{1.4} = 17.6\ cm^2$

Choose 7 ϕ 13 mm/m on each side $A_s = 18.5\ cm^2$

Assuming shrinkage strain $\epsilon_{sh} = 0.25$ mm/m & $E_s = 2100\ t/cm^2$, the max. ring tensile stress in concrete σ_t is given by :

$$\sigma_t = \frac{T_{2\,max} + \epsilon_{sh} E_s A_s}{A_c + n A_s} = \frac{25720 + 0.00025 \times 2100000 \times 18.5}{100 \times 20 + 10 \times 18.5}$$

$$= \frac{25720 + 9730}{2185} = 16.4\ kg/cm^2$$

Max. longitudinal reinforcement below ring beam

$A_s = \frac{T''_{max}}{\sigma_s} = \frac{18.14}{1.4} = 13\ cm^2$ choose 9 ϕ 10 mm/m on each side

$A_s = 14.2\ cm^2$

Max. longitudinal tensile stress in concrete σ_t is given by ;

$\frac{e_s}{d} = \frac{79.3}{82} = 0.97$ **a case of medium eccentricity.**

Reinforcement assumed (about 2/3 % of section) 5 Φ 19, top and 5 Φ 19 bottom.

Percentage of tension or compression steel :

$$\mu = \mu' = \frac{A_s}{b\ d} = \frac{14.2}{50 \times 82} \times 100 = 0.346\ \%$$

$$\sigma_c = C_1 \frac{M_s}{b\ d^2} = 4 \times \frac{39.65 \times 10^5}{50 \times 82^2} = 47 \quad kg/cm^2$$

$$\sigma_s = C_2 \frac{M_s}{b\ d^2} = 55 \times \frac{39.65 \times 10^5}{50 \times 82^2} = 645 \quad kg/cm^2$$

Note : C_1 & C_2 are coefficients depending on μ , μ' and $\frac{e_s}{d}$ and are extracted from the design curves of reinforced concrete.

Shear stresses :

$$Q_{max} = 45.85 \text{ t.}, \quad N = 50 \text{ t.}$$

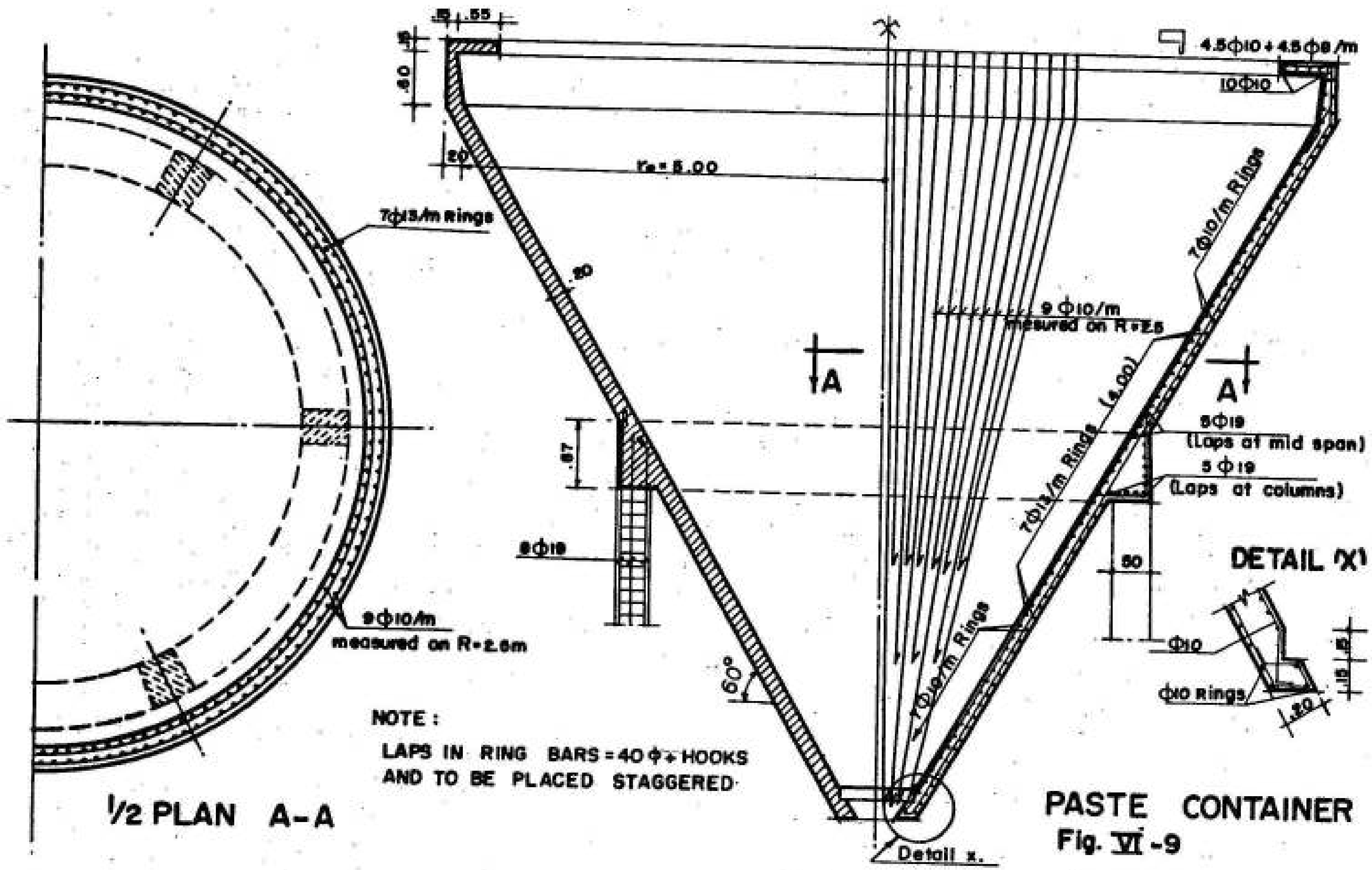
$$\tau_{max} = \frac{Q_{max}}{0.87\ b\ d} = \frac{45850}{0.87 \times 50 \times 82} = 12.8\ kg/cm^2$$

Assuming the allowable shear stress in reinforced concrete = 8 kg/cm^2 then the part of the beam subject to shear stresses bigger than 8 kg/cm^2 is about 36 cms from the face of the column (fig. VI.8), this length being smaller than 3/4 the theoretical depth of the beam, there is no need for bent bars. Vertical 2 branch stirrups Φ 10 mm spaced every 10 cms will be used. In addition, 5 Φ 13 horizontal bars will be arranged on each side of the beam (fig. VI.9).

Torsional Shear Stresses

M_t = 2.06 mt   b = 50 cms   t = 87 cms

$$\psi = 3 + \frac{2.6}{t/b + 0.45} = 3 + \frac{2.6}{87/50 + 0.45} = 4.19$$



NOTE :

LAPS IN RING BARS = 40 ϕ + HOOKS AND TO BE PLACED STAGGERED.

PASTE CONTAINER
Fig. VI -9

The torsional shear stress is therefore given by :

$$\tau_t = \psi \frac{M_t}{b^2\ t} \qquad = 4.19 \times \frac{2.06 \times 10^5}{50^2 \times 87} = 4 \ \ kg/cm^2$$

The stress being low, no additional provisions are necessary . The details of reinforcements are shown in fig. VI.9 .

VI.2. <u>AN INZE WATER TANK OF CAPACITY 850 m^3</u> (Fig. VI.10)

Capacity :

	Cylinder	$\pi \times 6^2 \times 6.5$	= 735 m^3	
+	Cone	$\pi \times 2/3\ (6^2 + 4^2 + 6 \times 4)$	= 159 m^3	894 m^3
- Internal cylinder		$\pi \times 0.75^2 \times 7.4$	= 13 m^3	
- Bottom Dome		$\frac{\pi \times 1.1}{6}\ (3 \times 4^2 + 1.1^2)$	= 28 m^3	41 m^3
				capacity = 853 m^3

1) <u>Design of roof dome</u>

For the roof dome which is a part of a hemi-sphere, we have

$$\text{Radius } a = \frac{R^2 + y^2}{2\ y} = \frac{6^2 + 1.4^2}{2 \times 1.4} = 13.5 \text{ m}, \quad \sin \varphi = \frac{6}{13.5} = 0.445$$

$$\cos \varphi = \frac{12}{13.5} = 0.89$$

Area $A = 2\ \pi\ a\ .\ y = 2\ \pi \times 13.5 \times 1.4$

$= 118.5\ m^2$ $\qquad$ $\tan \varphi = \frac{6}{12} = 0.50$

Loads on roof :

Own weight (~ 8 cms) = 200 kg/m^2

Superimposed dead and live loads = 150 "

total g = 350 " (lantern neglected !)

Total roof load W = 350 x 118.5 = 41500 kgs

Vertical load/meter run of cylinderical wall

$$P_1 = \frac{W}{2\ \pi\ R} = \frac{14500}{2\ \pi \times 6} = 1100 \ \ kg/m$$

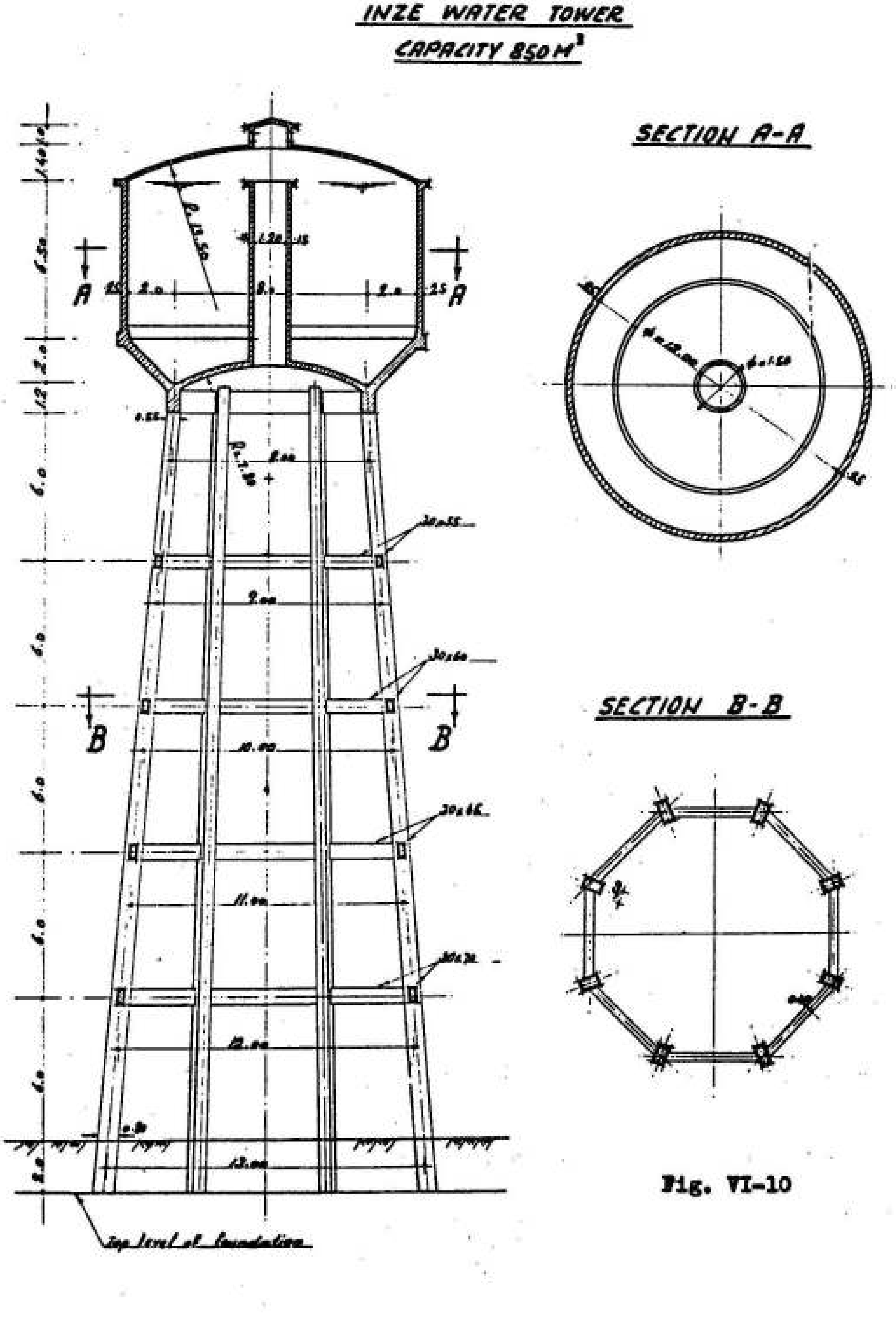

INZE WATER TOWER
CAPACITY 850 M³
SECTION A-A
SECTION B-B
Top level of foundation

Fig. VI-10

The outward horizontal thrust

$$H_1 = \frac{P_1}{\tan\varphi} = \frac{1100}{0.5} = 2200 \text{ kg/m}$$

The force H_1 must be resisted by a tension ring. The tension in the ring :

$$T_1 = H_1 R = 2200 \times 6 = 13200 \text{ kg}$$

The steel required in the ring :

$$A_s = \frac{T_1}{\sigma_s} = \frac{13200}{1400} = 9.4 \text{ cm}^2 \text{ chosen } 8\ \phi\ 13\ (10.3 \text{ cm}^2)$$

Assuming the average cross section of the ring beam 15 x 50 cms the tensile stress in concrete is :

$$\sigma_t = \frac{T_1}{A_c + n A_s} = \frac{13200}{15 \times 50 + 10 \times 10.3} = \frac{13200}{853} = 15.5 \text{ kg/cm}^2$$

Stress in dome shell

Meridian force : $N_\varphi = \frac{g\ a}{1 + \cos}$

Crown $\varphi = 0$, $\cos\varphi = 1$, $N_\varphi = \frac{g\ a}{2} = \frac{350 \times 13.5}{2} = 2350$ kg/m compression

Foot ring $\cos\varphi = 0.89$, $N_\varphi = \frac{g\ a}{1.89} = \frac{350 \times 13.5}{1.89} = 2480$ kg/m compression

or $N_\varphi = \frac{P_1}{\sin\varphi} = 1100 \times \frac{13.5}{6} = 2480$ kg/m compression

Ring force $N_\theta = g\ a \left(\cos\varphi - \frac{1}{1 + \cos\varphi} \right)$

Crown $\varphi = 0$, $\cos\varphi = 1$, $N_\theta = \frac{g\ a}{2} = 350 \times \frac{13.5}{2} = 2350$ kg/m compression

Foot ring $\cos\varphi = 0.89$, $N_\theta = 0.36\ g\ a = 36 \times 350 \times 13.5$

$= 1730$ kg/m compression

The max. compressive stress in concrete :

Crown : $\sigma_c = \frac{2350}{100 \times 8} = 2.93$ kg/cm^2

Foot-ring: σ_c = 2480/100 x 15 = 1.58 kg/cm^2 meridian

and = 1730/100 x 15 = 1.15 " ring

The compressive stress in concrete of dome shell is very low and a very thin shell might be used. An 8 cm shell is easier to execute and gives better isolation and insulation.

The foot ring being subjected to high tensile stresses (15.5 kg/cm^2) and strains while the dome shell at the same ring is subjected to low compressive stresses (1.15 kg/cm^2) and strains, bending moments will take place at the junction between the shell and the foot ring along the meridian. It is thus advisable to increase the thickness of the shell at the footring for a length $x = 0.6 \sqrt{a\,t}$ as shown in fig. VI - 11

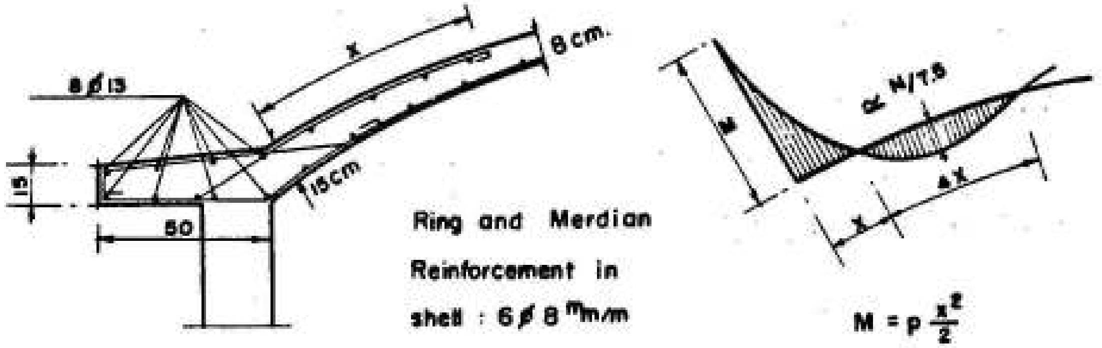


Fig. VI-11

Assuming the thickness of the dome shell at footring is equal to 15 cms, we get :

$$x = 0.6 \sqrt{a\,t} = 0.6 \sqrt{13.5 \times 0.15} = 0.85 \text{ ms.}$$

The fixing moment is therefore approximately given by :

$$M = p\,x^2 / 2 = 350 \times 0.85^2 / 2 = 125 \text{ kgm}$$

In spite of the small value of the fixing moment, the recommended gradual increase of the shell at the footring and reinforcing it on both surfaces is a good practice.

Design of Tank :

The cylinderical wall, the conical floor and the spherical bottom dome of the tank are rigidly connected together and may be considered as continuous in which case, the statically indeterminate connecting moments can be determined by the moment distribution method as follows (fig. VI.12)

Assume the cylinderical wall to be fixed at a, the conical floor to be fixed at a and b and the bottom spherical dome to be fixed at b and determine the corresponding fixing moments due to the acting loads and forces namely water pressure, own weight, temperature changes ..etc.

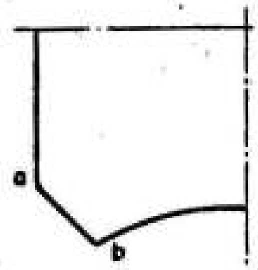


Fig. VI-12

The difference between the fixing moments at a and at b will be distributed between any two adjacent elements according to their relative stiffness and the corresponding distribution factors.

In order to simplify the calculations and at the same time to have ample safety, one can proceed as follows :

Design of Cylindrical Wall :

The ring tension, field moments and shear at base can be determined for the case of wall with hinged base, while the bending moment at the bottom of the wall may be determined for the case of wall with fixed base. According to these assumption, the thickness of the wall will be governed by the maximum ring tension which can be estimated by using the simplified method shown in III.4.2. Having chosen the thickness, the internal forces in the wall and the corresponding design can be determined with the help of the P.C.A. tables. Thus :

$$T_{max} = 0.75 \text{ w H R} = 0.75 \times 1 \times 6.5 \times 6 = 29 \text{ t} \quad \text{for } \frac{D}{H} = \frac{12}{6.5} = 1.85$$

Therefore $\quad t = 0.8 \times 29 \quad = 23.2$ cms

The wall will be chosen of constant thickness equal to 25 cms According to the P.C.A. tables we have :

$$\frac{H^2}{D\ t} = \frac{6.5^2}{12 \times 0.25} = \underline{14}$$ and

for a wall with <u>hinged base</u> we get :

According to table II : $T_{max} = 0.761\ w\ H\ R$ at 0.7 H or

$T_{max} = 0.761 \times 1 \times 6.5 \times 6 = \underline{30\ t/m}$ acting at ~ <u>4.5 ms</u> from top

According to table VIII $M_{max} = +\ 0.0033\ w\ H^3$ at 0.9 H or

$M_{max} = +\ 0.0033 \times 1 \times 6.5^3 = \underline{0.91\ m\ t}$

According to table XVI the shear at base is given by :

$V_{max} = 0.073\ w\ H^2 = .073 \times 1 \times 6.5^2 = \underline{3.1\ t}$ and

for a wall with <u>fixed base</u> we get :

The maximum fixing moment, according to table VII is given by :

$M_{max} = -\ 0.009\ w\ H^3 = -\ 0.009 \times 1 \times 6.5^3 = -\ 2.5$ m t

Further, the wall is subject to compressive stresses due to the loads from the roof and its own weight .

Thrust at top edge

$$N_1 = P_1 = 1.1\ t/m$$

Thrust at bottom edge

$$N_2 = N_1 + \text{own weight of wall}$$

$$\text{or}\quad N_2 = 1.1 + 0.25 \times 6.5 \times 2.5$$
$$= 5.15\ t/m$$

Accordingly, the internal forces for which the wall is designed are as follows : (fig. VI. 13)

<u>Design of section of max. ring tension</u> :

$T_{max} = 30\ t/m$

Max. ring reinforcement, $A_s = \frac{T}{\sigma_s} = \frac{30}{1.4} = 21.5\ cm^2$

chosen 6 Φ 16 mm/m on each face ($A_s = 24\ cm^2$)

Thickness of wall $t = 0.8\ T = 0.8 \times 30 = 24$ cms chosen 25 cms

The ring reinforcements will be reduced in the upper sections of the wall as shown in fig. VI. 13

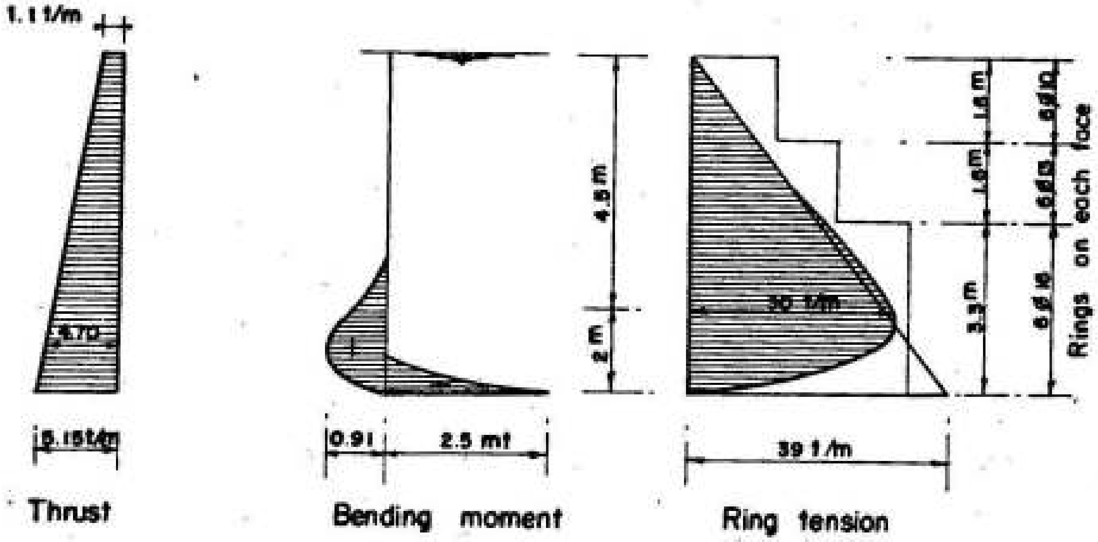


Fig. VI-13

<u>For upper 1.6 m :</u> $T_{max} = 1.6 \times 6 = 9.6$ t/m

$A_s = \frac{9.6}{1.4} = 6.8$ cm^2 Use 6 ϕ 10/m on each face

<u>For second 1.6 m:</u> $T_{max} = 3.2 \times 6 = 19.2$ t/m

$A_s = \frac{19.2}{1.4} = 13.6$ Use 6 ϕ 13/m on each face

<u>Section of max. positive moment :</u>

$M_{max} + = 910$ kgm $N = 4700$ kgs compression $t = 25$ cms

$e = \frac{M}{N} = \frac{910}{4700} = 0.193$ ms $e_s = e + \frac{t}{2} - \text{cover} = 19.3 + 12.5 - 3$
$= 28.8$ cms.

$M_s = N.e_s = 4700 \times 0.288 = 1350$ kgm

$d = k_1 \sqrt{M_s}$ or $22 = k_1 \sqrt{1350}$ from which $k_1 = 0.6$

For $\sigma_s = 1400$ kg/cm^2 $\sigma_c = 26$ kg/cm^2 and $k_2 = 1300$, therefore

$$A_s = \frac{M_s}{k_2 d} - \frac{N}{\sigma_s} = \frac{1350}{1300 \times 0.22} - \frac{4700}{1400} = 4.75 - 3.35 = 1.4 \text{ cm}^2$$

The minimum vertical reinforcement is 20 % of the rings i.e.

$$0.2 \times 12 = 2.4 \ \text{cm}^2$$

The vertical reinforcements will be chosen 5 ϕ 8 mm/m on each face

Section at base of wall :

$M = 2500$ kgm , $N = 5150$ kg (compression)

The tension being on the water side then

$$t = \sqrt{M/3} - 2 \text{ cms} = \sqrt{2500/3} - 2 = 27 \text{ cms}$$

The wall at its bottom edge will be provided by a haunch 10 x 40 cms so that $t_{max} = 35$ cms and $d = 32$ cms , therefore

$$e = \frac{M}{N} = \frac{2500}{5150} = 0.485 \text{ ms} \qquad e_s = e - \frac{t}{2} + \text{cover} = 48.5 + \frac{35}{2} - 3$$

$$= 63 \text{ cms.}$$

$M_s = N.e_s = 5150 \times .63 = 3250$ kgm and

$32 = k_1 \sqrt{3250}$ or $k_1 = 0.56$

for $\sigma_s = 1400$ kg/cm^2, $\sigma_c = 30$ kg/cm^2 and $k_2 = 1280$

$$A_s = \frac{M_s}{k_2 d} - \frac{N}{\sigma_s} = \frac{3250}{1280 \times 0.32} - \frac{5150}{1400} = 7.95 - 3.65 = 4.30 \text{ cm}^2$$

chosen 6 ϕ 10 mm/m

3. **Design of Ring Beam at Foot of Wall :**

The load of the roof and cylinderical wall (N_2) will cause meridian compressive forces N_{s1} in the conical floor. But N_{s1} is not colinear with N_2, thus, in order to have equilibrium the horizontal thrust H is created at joint a, its magnitude can be determined from the triangle of forces shown in figure VI. 14 . In order to be able

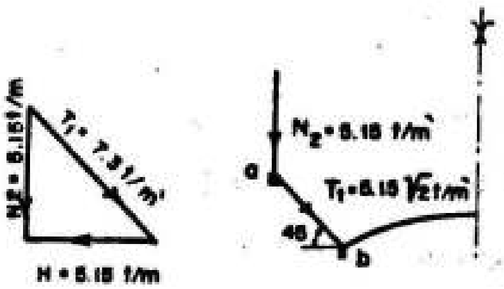


Fig. VI-14

to resist this horizontal force H, a ring beam can be arranged at joint a. This beam has, in addition, to resist the horizontal shear V_{max} at the base of the wall. So that the total horizontal force H_2 acting on the ring beam is given by :

$$H_2 = 5.15 + 3.1 = 8.25 \ t/m$$

The corresponding ring tension T is given by :

$$T = H_2 \times R = 8.25 \times 6 = 49.5 \ t$$

Required ring reinforcement

$$A_s = T / \sigma_s = 49.5/1.4 = 35.3 \ cm^2$$

Chosen 10 Φ 22 ($A_s = 38 \ cm^2$)

The details of reinforcements are shown in figure VI. 15 .

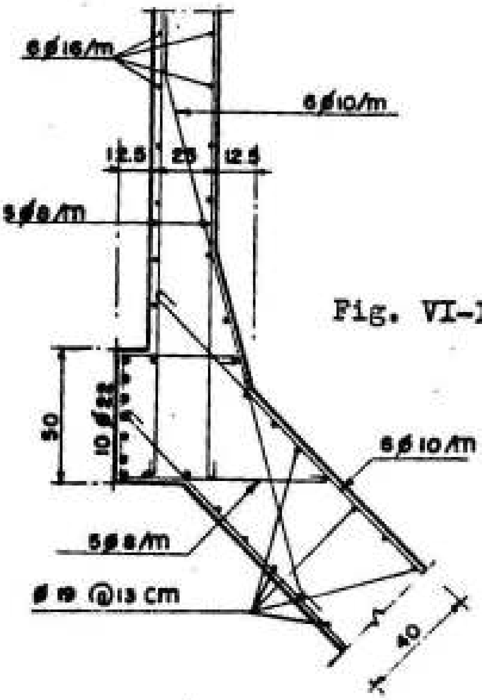


Fig. VI-15

4. <u>Design of Conical Part of the Tank Floor</u> :

The conical floor is subject to meridian compressive forces N_s and ring tensile forces N_θ . Their values can be determined according to the fundamental principles given in chapter IV.3.4.6 as follows :

The vertical component of the meridian forces at any section is equal to the vertical forces per meter acting at that action, thus,

The meridian force at the upper edge of the cone is given by :

$$N_s = \frac{W / m'}{\sin \varphi} = 5.15 \times \sqrt{2} = 7.3 \ t/m$$

at mid-height of cone : (fig. VI. 16)

Depth of water = 7.5 m   horizontal radius R = 5 m

$$\text{Weight of roof and wall} = \frac{5.15 \times \pi \times 12}{\pi \times 10} = 6.18 \ t/m$$

Assuming thickness of cone 40 cms ($1 \ t/m^2$)

Weight of upper half of cone :

$$\pi \times \frac{12 + 10}{2} \times 1\sqrt{2} \times \frac{1}{\pi \times 10} = \frac{22\sqrt{2}}{20}$$

$$= 1.70 \text{ t/m}$$

Weight of water on upper half of cone

$$\pi \times (6^2 - 5^2) \times 7 \times \frac{1}{\pi \times 10} = \frac{11 \times 7}{10}$$

$$= 7.70 \text{ t/m}$$

Total load $W = 6.18 + 1.70 + 7.70$

$$= 15.58 \text{ t/m}$$

Meridian force $N_s = \frac{W}{\sin\varphi} = 15.58\sqrt{2}$

$$= 22 \text{ t/m}$$

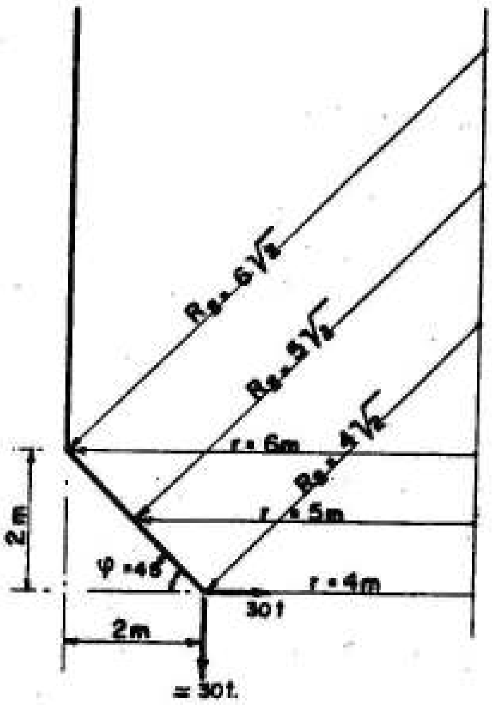


Fig. VI-16

At foot of cone :

Weight of roof and wall $= \frac{5.15 \times \pi \times 12}{\pi \times 8}$

$$= 7.72 \text{ t/m}$$

Weight of cone $\pi \times \frac{12 + 8}{2} \times 2\sqrt{2} \times \frac{1}{\pi \times 8} = 3.55$ t/m

Weight of water $\pi\ (6^2 - 4^2) \times 7.5 \times \frac{1}{\pi \times 8} = \underline{18.75}$ t/m

Total load $W = 7.72 + 3.55 + 18.75 \qquad = 30.02$ t/m

Meridian force $N_s = 30.02\sqrt{2} \qquad = 42.6$ t/m

Ring force N_θ can be determined from the relation :

$N_\theta \doteq R_2\, p_r$ in which p_r = water pressure + g cos φ

at upper edge of cone : $R_2 = 6\sqrt{2} = 8.5$ ms, $p_r = 6.5 + 1/\sqrt{2}$

$= 7.207 \text{ t/m}^2$ i.e.

$N_\theta = 8.5 \times 7.207 \qquad = 61.3$ t/m

at mid-height of cone : $R_2 \qquad = 5\sqrt{2} = 7.07$ ms, $p_r = 7.5 + 1/\sqrt{2}$

$= 8.207 \text{ t/m}^2$ i.e.

$N_\theta = 7.07 \times 8.207 \qquad = 58.1$ t/m

P_r = water pressure + g cos = $8.5 + 0.75 \times \frac{6.7}{7.8} = 9.142$ t/m^2

So that $N_\theta = 7.8 \times 9.142 - 33 = 38$ t/m compression

The bending moments at the lower ring beam can be determined according to Markus - refer to pages (92-94) - as follows :

i) <u>Fixing end moment and stiffness of conical floor</u>

Referring to figure IV - 29, we have :

$h = 12.5$ m $s = 5.66$ m $l = 8.49$ m $t = 0.40$ $\varphi = 45^\circ$

$g = 0.4 \times 2.5 = 1.00$ t/m^2 $P = 5.15$ t/m

Therefore

$k_3 = 1.3068 \cdot \sqrt{1/5.66 \times 0.40} = 0.87$ and

$$E\,\Delta r_o = \frac{1.00 \times 5.66^2}{0.40}\left[1 - \frac{0.167}{2 \times 0.5}\left(1 - \frac{8.49^2}{5.66^2}\right)\right] \times 0.5 \times 1.0$$

$$+ 0.167 \times \frac{5.15 \times 8.49}{0.4} \times 1.0$$

$$+ \frac{1.00 \times 5.66}{0.40} 5.66 \left(\frac{12.50}{0.707} - 5.66\right) + \frac{0.167}{2 \times 5.66}\left[\frac{12.50}{0.707}(8.49^2 - 5.66^2) - \frac{2}{3}(8.49^3 - 5.66^3)\right]\Bigg\} \times 0.5$$

$$= 48.6 + 18.5 + 524.9 = 592$$

$$E\,\theta_o = \frac{1.00 \times 5.66}{0.40}\left[\frac{1}{2}\left(1 - \frac{8.49^2}{5.66^2}\right) + 0.167 - 2.167 \times 0.5\right]\frac{1}{0.707}$$

$$- \frac{5.15 \times 8.49}{0.4 \times 5.66} \times \frac{0.707}{0.50}$$

$$+ \frac{1.00}{0.40}\left[3 \times 5.66^2 \times 0.707 - 2 \times 12.5 \times 5.66 - \frac{12.50}{2 \times 5.66}(8.49^2 - 5.66^2) + \frac{0.707}{3 \times 5.66}(8.49^3 - 5.66^3)\right] \times 1.0$$

$$= -30.7 - 27.3 - 250.0 = -308$$

hence,

The fixed end moment $\bar{M}$ is

$$\bar{M} = \frac{0.40 \times 0.707}{2 \times 5.66^2 \times 0.87^2 \times 0.5} \left(592 + \frac{.707}{.87} \times 308 \right) = \underline{9.8 \text{ m t}}$$

The stiffness S is

$$S = 2 \times 0.40^3 \times 0.87 = \underline{0.1115}$$

ii) <u>Fixed end moment and stiffness of floor-spherical dome</u>

Referring to figure IV - 28, we have

$a = 7.8$ m $t_o = 15$ cm $t = 30$ cms $h = 15.2$ ms

$\sin\varphi = 4.0 / 7.80 = 0.5128$ $\sin^2\varphi = 0.263$

$\varphi = 30° 51' = 0.538$ radians $\cos\varphi = 0.8585$

$h = 8.5 + \frac{4.0}{\tan\varphi} = 15.20$ ms $g_o = 0.15 \times 2.5 = 0.375$ t/m^2

$$k_2 = 1.3068 \sqrt{\frac{7.80}{0.3}} = 6.66$$

$$E\Delta r_o = \frac{0.375 \times 7.8^2}{0.30 \times 0.15} \left[\frac{1.167}{0.263} \left(0.15 - 0.3 \times 0.8585 + \frac{0.30-0.15}{0.538}\right) - 0.3 \times 0.8585 \right]$$

$$- \frac{1.00 \times 78^3}{0.30} \left[\frac{0.833 \times 15.20}{2 \times 7.80} - 0.8585 + \frac{1.167}{3} \left(0.8585 + \frac{1}{1.8585}\right) \right] \times 0.5128$$

$$= - 51 - 403 = - 454$$

$$\theta_o = \frac{0.375 \times 7.8}{0.30 \times 0.15} \left\{ \frac{0.30-0.15}{0.538} \times 0.8585 - 2.167 \times 0.30 \times 0.5128 + \frac{1.167(0.30-0.15)}{0.30 \times 0.538 \times 0.263} \right.$$

$$\left. \times \left[0.15 - 0.3 \times 0.8585 + \frac{0.30-0.15}{0.538} \times 0.5128 - \frac{0.30}{1.167} \times 0.8585 \times 0.263 \right] \right\}$$

$$+ \frac{1.00 \times 7.8^2}{0.30} \left\{ 0.5128 + \frac{0.30-0.15}{0.30 \times 0.538} \times 1.167 \left[\frac{15.20}{2 \times 7.8} - \frac{1}{3} \left(0.8585 + \frac{1}{1.8585}\right) \right] \right.$$

$$\left. - \frac{0.30 - 0.15}{0.30 \times 0.538} \left(\frac{15.2}{7.8} - 0.8585 \right) \right\}$$

$$= 18.9 + 10.1 = + 29$$

Vertical reaction $= 429/\pi \times 8 = 17$ t/m

Horizontal reaction $= 17 \times \frac{6.7}{4} = 28.6$ t/m

6. <u>Design of Lower Circular Beam</u> :

This beam is supported symmetrically on eight columns.

The resultant vertical force on the beam $= 30 + 17 = 47$ t/m and

The resultant inward horizontal force $= 30 - 28.6 = 1.4$ t/m i.e.

The axial compression in the beam $= 1.4 \times 4 = 5.6$ t

It is however recommended to choose the different elements of the tank such that the axial force in this beam is minimum.

The total vertical load on the beam $P = (47 + \text{own weight})\,\pi D$

Assuming own weight = 1.2 t/m , then $P = 48.2 \times \pi \times 8 = 1212$ ton

According to table we get :

Max. shearing force $Q_{max} = \frac{P}{16} = \frac{1212}{16} = 76$ ton

Max. bending moment at center of each span $M+ = .00416\ P\ R$ or

$M_{max}+ = .00416 \times 1212 \times 4 = 20$ mt

Max. bending moment over center line of support $M_{max}- = .00827\ P\ R$ or

$M_{max}- = .00827 \times 1212 \times 4 = 40$ mt

Max. torsional moment due to vertical loads: $M_t = .00063\ P\ R$

$M_t = .00063 \times 1212 \times 4 = 3.1$ mt

Torsional moment due to unbalanced moment $= 1.68$ mt

Total torsional moment $= 4.78$ mt

Assuming the ring beam 50 x 120 cms, we get :

For middle section of any span : $M = 20$ mt $N = 5.6$ t

$e = \frac{M}{N} = \frac{20}{5.6} = 3.57$ ms $\qquad e_s = 3.57 + 0.6 - 0.05 = 4.12$ms

$M_s = N\ e_s = 5.6 \times 4.12 = 23.1$ mt

$115 = k_1 \sqrt{\frac{23100}{0.5}} \qquad k_1 \equiv 0.54$, for $\sigma_s = 1400$ kg/cm^2 , $\alpha = 0$

$\sigma_c = 31$ kg/cm^2 $\qquad k_2 = 1285$ $\qquad$ and

$$A_s = \frac{M_s}{k_2\ d} - \frac{N}{\sigma_s} = \frac{23100}{1285 \times 1.15} - \frac{5600}{1400} = 15.6 - 4 = 11.6 \text{ cm}^2$$

which must be bigger than the minimum steel (0.25 % of cross-section)

minimum $A_s = \frac{0.25}{100} \times 50 \times 120 = 15$ cm^2 chosen 4 ϕ 22.

For section over center line of supports : $M = 40$ mt, $N = 5.6$ t

$e = \frac{40}{5.6} = 7.14$ ms $\qquad e_s = 7.14 + 0.6 - 0.05 = 7.69$ ms

$M_s = 5.6 \times 7.69 = 43$ mt

$115 = k_1 \sqrt{\frac{43000}{0.5}} \qquad k_1 = 0.392$, for $\sigma_s = 1400$ kg/cm^2, $\alpha = 0$

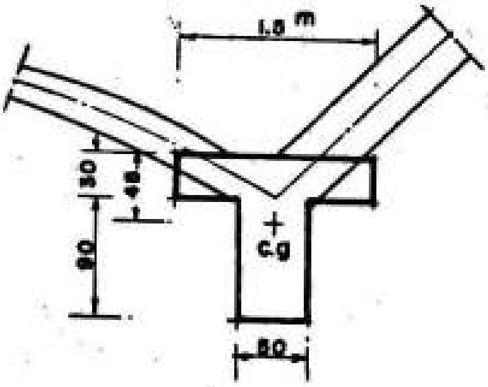
$\sigma_c = 45$ kg/cm^2 $\qquad k_2 = 1250$ $\qquad$ and

$$A_s = \frac{43000}{1250 \times 1.15} - \frac{5600}{1400} = 30 - 4 = 26 \text{ cm}^2$$ chosen 7 ϕ 22

check of tensile stresses in concrete :

At supports, the bending moments are negative and the concrete on the water side is subject to tensile stresses which must be smaller than the tensile strength of concrete if tension cracks are to be avoided.

Fig. VI-19



In order to determine the tensile stresses, a part of the dome and cone slabs may be assumed acting with the web, their effect can be replaced by a horizontal flange 150 cms wide and 30 cms deep as shown in figure VI.19

Area of concrete section A_c $= 50 \times 90 + 150 \times 30 = 9000 \text{ cm}^2$

Distance of c.g. from upper fiber $y = \dfrac{50 \times 120 \times 60 + 100 \times 30 \times 15}{9000}$

$= 45$ cms.

Moment of inertia of section :

$$I = \frac{50 \times 120^3}{12} + 50 \times 120 \times 15^2 + \frac{100 \times 30^3}{12} + 100 \times 30 \times 30^2 = 90.45 \times 10^5 \text{ cm}^4$$

The tensile stress $\sigma_t = -\dfrac{N}{A_c} + \dfrac{M\,Y}{I} = -\dfrac{5600}{9000} + \dfrac{40 \times 10^5 \times 45}{90.45 \times 10^5}$

$= 19 \text{ kg/cm}^2$

which can be safely allowed.

Shear Stresses : The axial compressive force of 5.6 t being small relative to the shearing force of 76 t, its effect on the principal diagonal tensile stresses may be neglected and therefore :

$$\tau = \frac{Q}{.87\ bd} = \frac{76000}{0.87 \times 50 \times 115} = 15.2 \text{ kg/cm}^2$$

We will show in the following whether this high value of τ_{max} needs special web reinforcements or not .

Assuming that the allowed shear stress in reinforced concrete is 8 kg/cm^2., then the part of the beam subject to shear stresses higher than this value is only 50 cms from the face of the column (figure VI. 20 this length being smaller than 3/4 the depth of the beam , there is no need for bent bars.

Torsional shear stresses :

The torsional moment of 1.68 mt due to the unbalanced moment is constant along the perimeter of the beam and does not cause shear stresses. Hence, the torsional moment causing shear is $M_t = 3.1$ mt.

Knowing that b = 50 cm and t = 120 cm, then

$$\psi = 3 + \frac{2.6}{t/b + 0.45} = 3 + \frac{2.6}{120/50 + 0.45} = 3.91$$

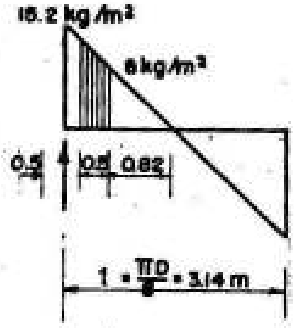


Fig. VI-20

The torsional shear stress is therefore given by

$$\tau_t = \psi \frac{M_t}{b^2\, t} = 3.91 \times \frac{3.1 \times 10^5}{50^2 \times 100} = 4.85 \text{ kg/cm}^2$$

The stresses are low and no special provisions are needed.

The details of reinforcements of the tank are shown in figure VI. (a & b).

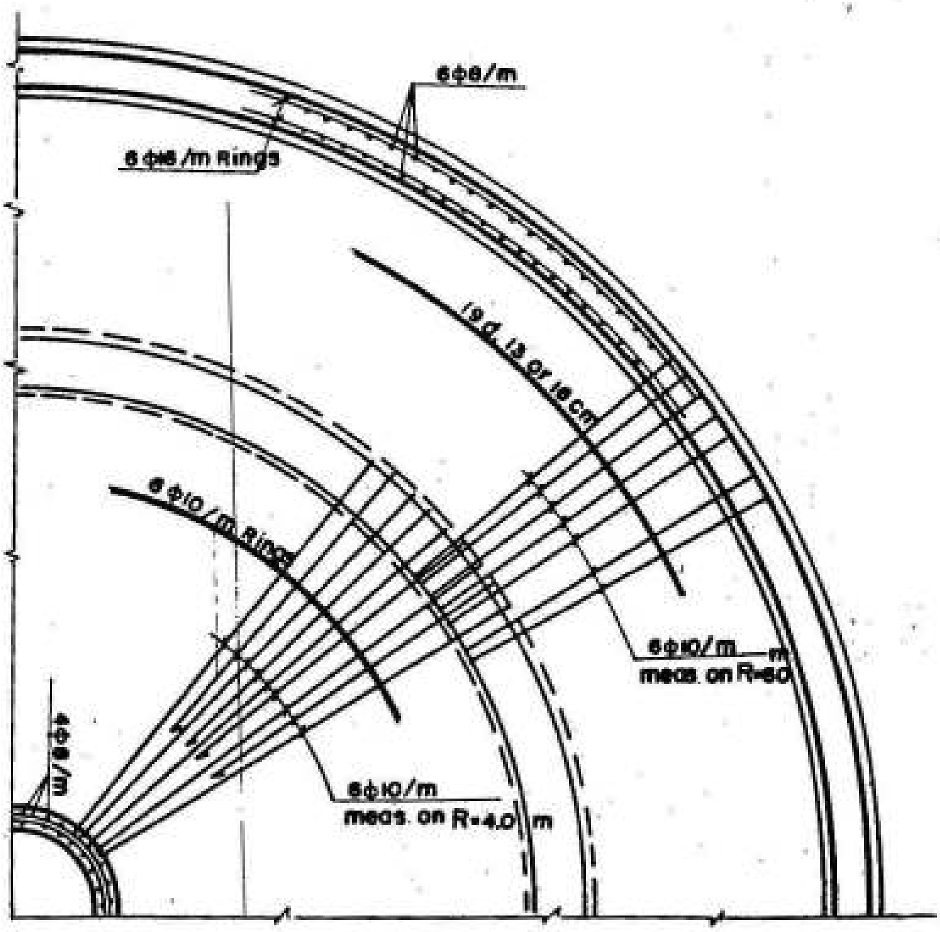
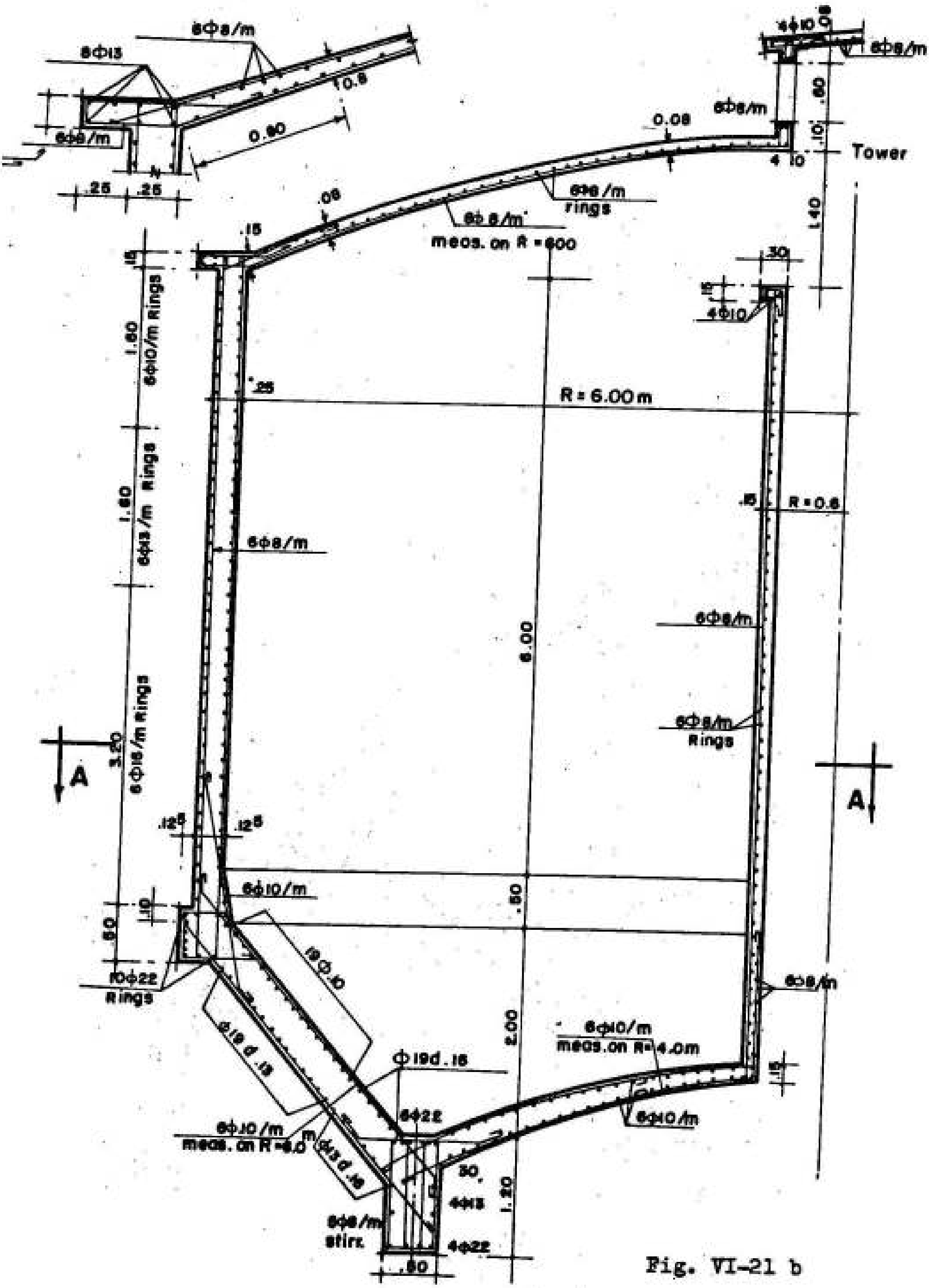


Fig. VI-21a



6Φ8/m
8Φ13
0.8
0.90
6Φ8/m
.25
.25
4Φ10
6Φ8/m
.60
6Φ8/m
0.08
.10
Tower
6Φ8/m
rings
6Φ 8/m
meas. on R = 600
.08
.15
1.40
.30
.15
4Φ10
1.60
6Φ10/m Rings
.25
R = 6.00 m
1.60
6Φ13 /m Rings
.15
R = 0.6
6Φ8/m
6.00
6Φ8/m
6Φ8/m
Rings
3.20
6Φ16/m Rings
A
A
.125
.125
6Φ10/m
.50
.10
.60
19Φ10
10Φ22
Rings
6Φ8/m
Φ19 d.13
2.00
6Φ10/m
meas. on R=4.0m
Φ19d.16
.15
6Φ22
6Φ10/m
6Φ10/m
meas. on R=6.0m
Φ13 d.16
.30
4Φ13
1.20
6Φ8/m
stirr.
4Φ22
.60

Fig. VI-21 b

7. Design of the Supporting Tower-Vertical Loads :

The supporting tower is to be treated as a space frame. We give in the following an approximate method for determining the internal forces in the columns and struts.

Vertical load of tank plus water to level 0 - 0	= 1212 t
" " " " " " per column = 1218/8	= 152 t
Weight of water = 853 t and " " = 853/8	= 106.5 t
Own weight of tank to level 0 - 0 per"	= 45.5 t
Assuming own weight of column or strut = 0.75 t/m	
Weight of column between sections 0 - 0 & I - I = 0.75 x 6	= 4.5 t
Total dead weights per column to level I-I	= 50 t

As the length of any side of the octagonal strut = 0.383 D, where :

D = the diameter of the octagon , we get :

Weight of column + strut between sections I - I & II - II = (6 + 0.383 x 9) 0.7	= 6.6 t
Total dead weights per column to level II-II	= 56.6 t
Weight of column + strut between section II-II & III - III = (6 + 0.383 x 10) 0.7	= 6.9 t
Total dead weights per column to level III-III	= 63.5 t
Weight of column + strut between sections III-III & IV - IV = (6 + 0.383 x 11) 0.7	= 7.2 t
Total dead weights per column to level IV - IV	= 70.7 t
Weight of column + strut between sections IV-IV & V - V = (8 + 0.383 x 12) 0.7	= 8.8 t
Total dead weights per column to level V - V	= 79.5 t
Therefore total dead loads + water per column = 79.5 + 106.5	= 186 t

Wind pressure on tank and supporting tower :

Intensity of wind pressure on tank = 150 kg/m^2 horizontal. As it acts on surfaces of revolution, this pressure may be reduced to 100 kg/m^2

The wind pressure on the supporting tower is assumed equal to 100 kg/m^2 and considered as concentrated at the joints where the columns and the struts intersect.

Magnitude and position of wind horizontal forces acting on tank (fig. VI. 22).

Element	Area A	m^2	y from 0 - 0 m	A y m^3
Lantern	1.0 x 1.5	= 1.5	11.5	17.3
Upper dome	12.5 x 1.5	= 18.7	10.3	129.0
Cylinder	12.5 x 6.5	= 81.0	6.25	505.0
Cone	$\frac{12.7 + 8.6}{2} \times 2$	= 21.3	2.00	42.6
Ring	8.6 x 1	= 8.6	0.50	4.3
		131.1		698.2

Total wind force = 131.1 x 0.1 = 13 t

Arm above 0 - 0 = 698.2/131.1 = 5.3 m

Wind per meter of column or strut

= 0.1 x 0.6 = 0.06 t/m

Section	Wind load W	
0 - 0	8 x 3 x 0.06	= 1.4 t
I - I	(8 x 6 + 9 x 2) x 0.06	= 4.0 t
II - II	(8 x 6 + 10 x 2) x 0.06	= 4.1 t
III-III	(8 x 6 + 11 x 2) x 0.06	= 4.2 t
IV - IV	(8 x 6 + 12 x 2) x 0.06	= 4.3 t

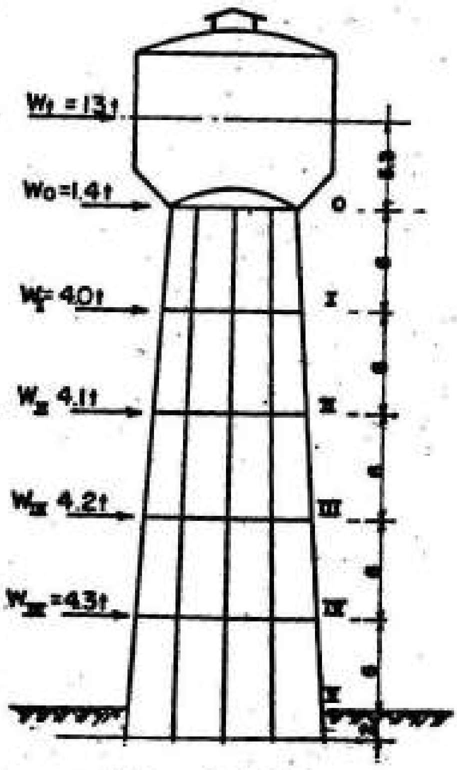


Fig. VI-22

Vertical forces due to wind :

It will be assumed that the points of zero bending moments will take place at the middle of any unsupported length of column or strut.

The vertical forces due to wind can be determined from the con - ditions :

The moment of the horizontal wind forces about any horizontal plane at mid-height between two successive struts (plane of assumed zero bending moments in columns) M_w must be equal to the couple caused by vertical wind forces in columns at the same plane. Assuming these vertical forces to be V_1 , V_2 and V_3 , we get : (fig. VI.23).

$M_w = V_1 \times 2\,r + 2\,V_2 \times 2\,a$ in which

$V_2 = V_1 \times \frac{a}{r}$ and $a = \frac{r}{\sqrt{2}}$ or $a^2 = \frac{r^2}{2}$

thus :

$M_w = V_1 \times 2\,r + 4\,\frac{V_1\,a^2}{r} = 4\,V_1\,r$ or

$$V_1 = \frac{M_w}{4\,r} = \text{max. vertical force}$$

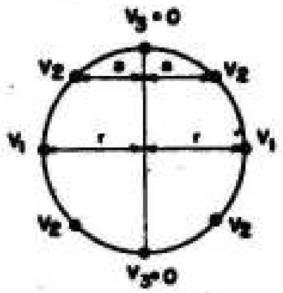


Fig. VI-23

Accordingly, the max. vertical forces in columns due to wind can be calculated as follows :

Section	H	Tons	M_w	M_w	r	V_{max}
		13.0				
0 - 0	1.4	14.4	13 x 8.3 + 1.4 x 3 = 112.2	112.2	4.25	6.6
I - I	4.0	18.4	14.4 x 6 + 4 x 3 = 98.4	210.6	4.75	11.2
II - II	4.1	22.5	18.4 x 6 + 4.1 x 3 = 122.7	333.3	5.25	15.8
III-III	4.2	26.7	22.5 x 6 + 4.2 x 3 = 147.6	480.9	5.75	20.8
IV - IV	4.3	31.0	26.7 x 7 + 4.3 x 4 = 204.1	685.0	6.33	27.2
V - V						

The horizontal forces in the columns at points of zero bending moments may be assumed equal, accordingly the bending moments and normal forces on the columns are given by :

Column	H/Column tons	Vertical Force tons	wind force tons	max. Vert. force tons	Max. B.M. m t.
0 - I	14.4/8 = 1.80	152 + 4.5 = 156.5	± 6.6	163	1.8 x3= 5.4
I - II	18.4/8 = 2.30	156.5 + 6.6 = 163.1	± 11.2	174	2.3 x3= 6.9
II -III	22.5/8 = 2.82	163.1 + 6.9 = 170.0	± 15.8	186	2.82x3= 8.5
III-IV	26.7/8 = 3.34	170 + 7.2 = 177.2	± 20.8	198	3.34x3=10.0
IV - V	31.0/8 = 3.88	177.2 + 8.8 = 186	± 27.2	213	3.88x4=14.6

Approximate Determination of Internal Forces in Struts :

The internal forces in the struts vary according to the direction of wind. In some cases, the struts will be subject to torsional moments which, if neglected, the internal forces can approximately be determined for the maximum horizontal forces acting on any of the sides assumed as a plane frame. The wind forces will be assumed as concentrated at the joints and points of zero bending moments lie at middle of struts or columns.

The max. horizontal force acting in the plane of a side due to any of the wind loads W shown in Fig. VI. 22 is for octagonal towers equal to 0.146 W and can be determined as follows :

a) Distribute the wind load W on the different sides according to figure VI.24. Note that

$l' = D \cos 67° \ 30' = 0.383 D$

$$l = \frac{l'}{\sqrt{2}} = 0.27 D$$

$D' = 0.923 D$

$$w = \frac{W}{D'} = \frac{W}{0.923 D}$$

$$= 1.08 \frac{W}{D}$$

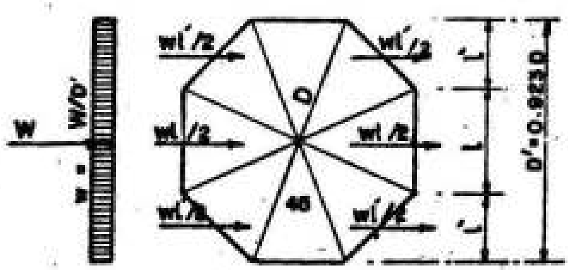


Fig. VI-24

b) Determine the normal component of the wind load on the different sides (fig. VI-25). Thus the normal component on the inclined side is given by

$$\frac{w\,l}{2} \times \cos 45° = \frac{w}{2\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{w\,l}{4}$$

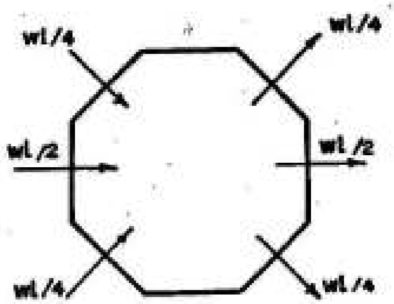


Fig. VI-25

c) Distribute the wind loads given in fig. VI 25 on the different joints as shown in fig. VI-26.

d) Determine the corresponding wind forces in the different sides by drawing the triangles of forces for joints a and b (figs. VI.27- A, B and C).

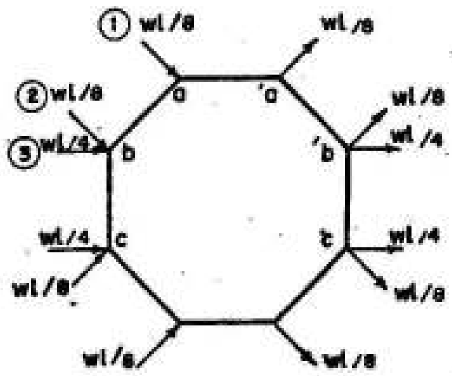


Fig. VI-26

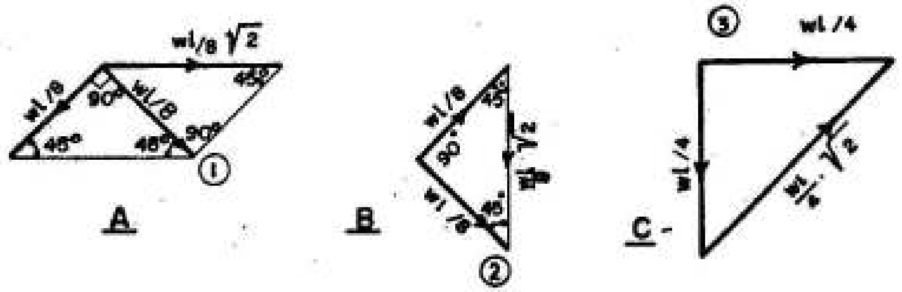


Fig. VI-27

Accordingly, the wind force in side a - a' is given by $\frac{w\,l}{8}\sqrt{2}$ at a and an equal value at a' so that the total force is :

$$2 \times \frac{w\,l}{8}\sqrt{2} = \frac{1.08\ W}{D} \times \frac{0.383\ D}{4}\sqrt{2} = 0.146\ W$$

The wind forcein side a b is given by the components of (1) and (2) which cancel each other plus the component of (3) so that the total force is :

$$w \frac{1}{4} \sqrt{2} = 0.146\ W$$

The wind force along b c due to W is equal to zero. The final forces along the different sides are shown in figure (VI. 28). Accordingly, the wind forces on the assumed plane frame will be as shown in figure VI.29 , each of the forces shown is 0.146 W. The shear at points of zero bending moments at the middle of the free length of any column assumed equally resisted by the columns are also given. The max. bending moment in any of the struts will be equal to the sum of the moments of the two columns just above and below it, thus

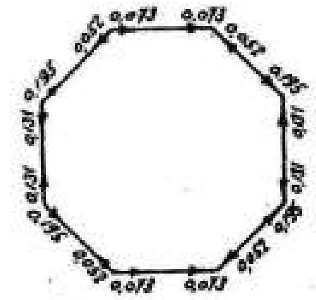


Fig. VI-28

Wind forces as factor of w

M_I = (1.053 + 1.344) x 3 = 2.397
x 3 = 7.2 mt

M_{II} = (1.344 + 1.644) x 3 = 2.988
x 3 = 9.0 mt

M_{III} = (1.644 + 1.95) x 3 = 3.639
x 3 = 10.9 mt

M_{IV} = 1 1.95 x 3 + 2.264 x 4 = 5.85 + 9.05 = 14.9 m t

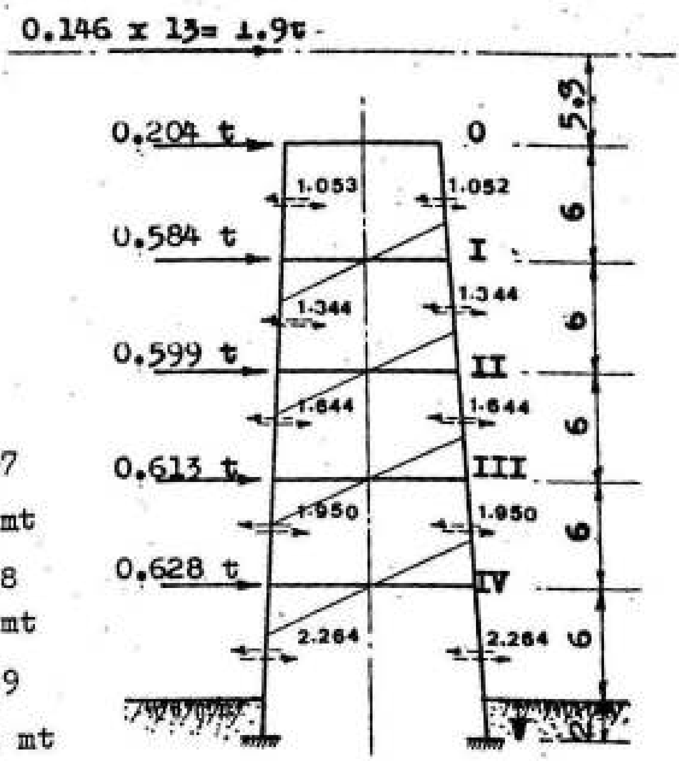


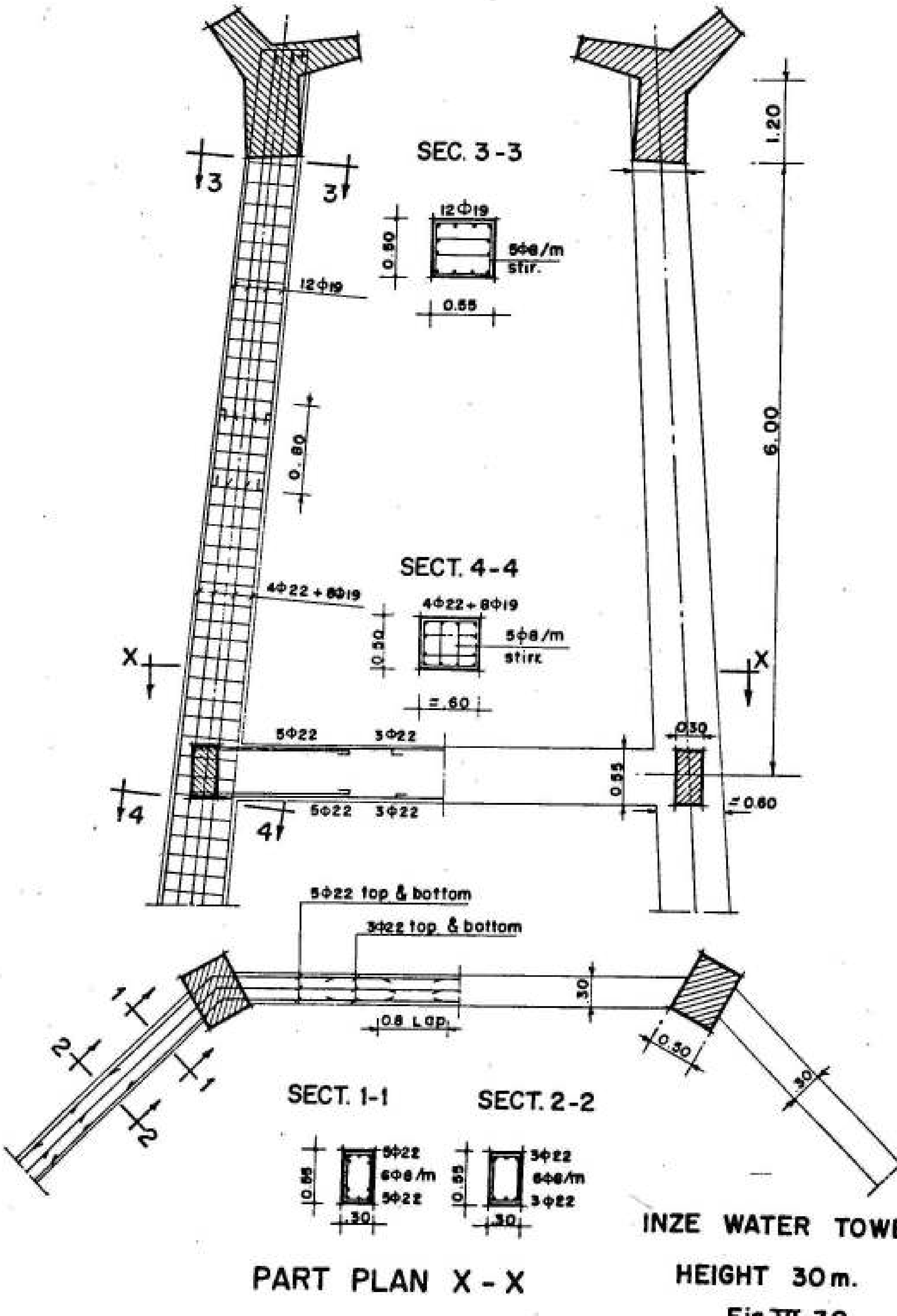
Fig. VI-29

Comparing these values with the bending moments in the columns due to wind, we find that, in our case, the max. bending moments in the struts at the face of the columns may be assumed equal to the bending moments at the upper ends of the columns just below them. The thrust in the struts are small and can be neglected in the design.

Following the general principles of reinforced concrete design the dimensions and reinforcements of the columns and struts (designed with relatively low stresses) may be assumed as follows :

	Element	Normal Force H in t.	Bending Moment M in m t	Concrete Dimensions cms	Reinforcement
Columns	0 - I	163	± 5.4	50 x 55	12 Φ 19
	I - II	174	± 6.9	50 x 60	4 Φ 22 + 8 Φ 19
	II - III	186	± 8.5	50 x 65	6 Φ 22 + 4 Φ 19
	III - IV	198	± 10.0	50 x 70	8 Φ 22 + 4 Φ 19
	IV - V	213	± 14.6	50 x 75	12 Φ 22
Struts	I - I	neglected	± 7.2	30 x 55	5 Φ 22 top. & 5 Φ 22 bot.
	II - II	"	± 9.0	30 x 60	6 Φ 22 top & 6 Φ 22 bot.
	III - III	"	± 10.0	30 x 65	6 Φ 22 top & 6 Φ 22 bot.
	IV - IV	"	± 14.9	30 x 70	7 Φ 22 top & 7 Φ 22 bot.

The details of reinforcements of the supporting tower are shown in figure VI. 30.



INZE WATER TOWER

HEIGHT 30 m.

Fig. VI-30

VI-3 : Water Tower of Figure IV-3a

In this example, the detailed design of the circular plates of the floor of the elevated tank and the foundation of the tower will be shown.

1) The floor of the tank

The floor of the tank under consideration is a hollow circular plate with an overhanging cantilever ring subject to uniform and concentrated loads as well as to edge moments as shown in fig. VI-31

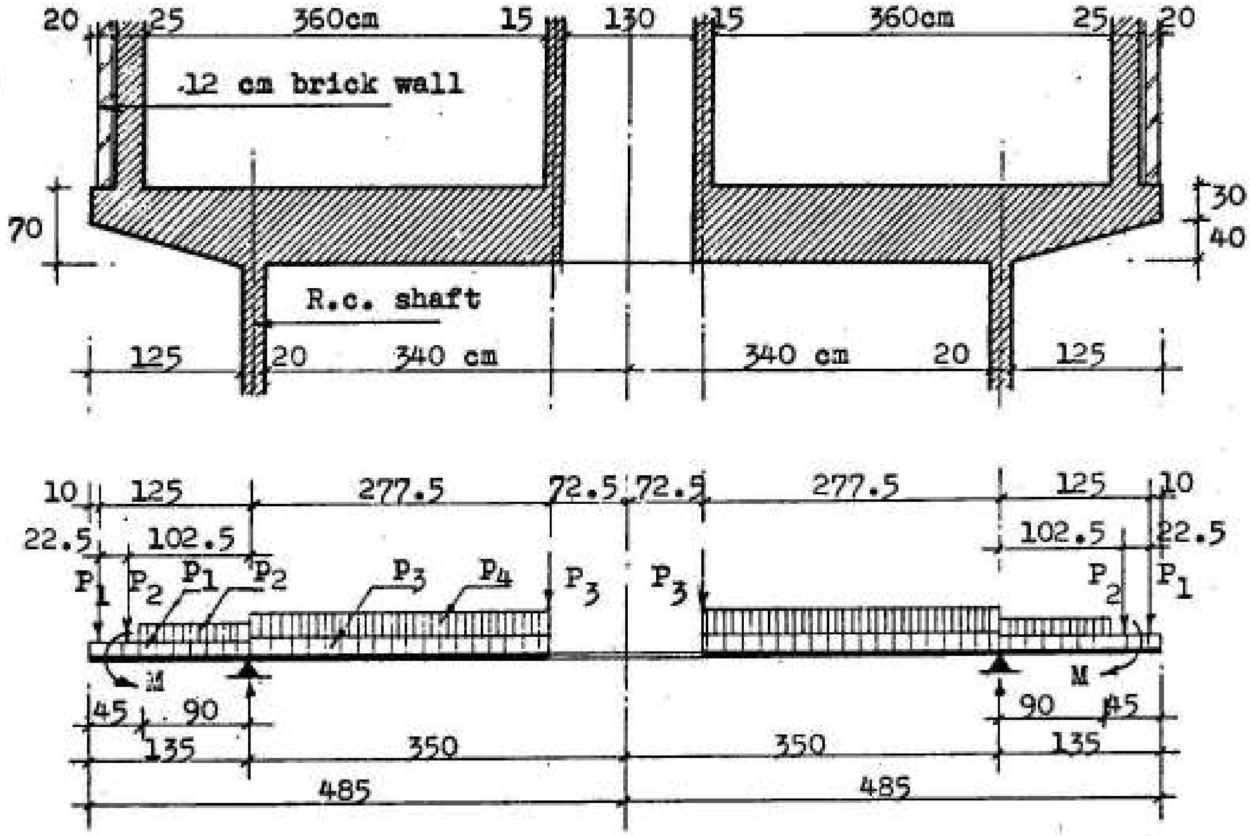


Fig. VI-31

It will be considered as simply supported on the 20 cms thick reinforced concrete shaft.

Data :

Depth of tank	= 5.35 ms
Depth of water	= 5.1 m
Weight of R.C.	= 2.5 t/m^3
Weight of brick walls	= 1.8 t/m^3
Superimposed load on roof slab	= 400 kg/m^2

a) Loads

Brick wall	$P_1 = 1.15$ t/m
Roof + R.C. outside wall	$P_2 \simeq 3.20$ t/m
Roof + internal shaft	$P_3 \simeq 5.65$ t/m
Own weight of cantilever ring	$p_1 = \frac{0.3 + 0.7}{2} \times 2.5 = 1.25$ t/m^2
Weight of water	$p_2 = p_4 = 5.10$ t/m^2
Own weight of interm. slab	$p_3 = 0.7 \times 2.5 = 1.75$ t/m^2

Due to the big rigidity of the tank floor and in order to simplify the calculations, the wall shall be considered as fixed to the floor.

The bending moment transmitted from wall $M \simeq 1.40$ m.t/m

Radial reaction of wall at base $\simeq 3.45$ t/m

The bending moment transmitted from the intermediate shaft to the floor is small and can be neglected.

b) Bending Moments

A simply supported circular slab with overhanging cantilevers is not a statically determinate problem. In order to determine the internal forces in the cantilever ring and the hollow ring slab each shall be solved for complete fixation at the support, then, the unbalanced fixing moment will be distributed according to the stiffness of each element in the following manner :

i) The cantilever ring

The cantilever ring shall be solved according to the equations given in IV-2 for the following five cases of loading and the projection shall be considered equal to the loaded length in each case as shown in figure VI-32. It will be assumed that the Poison's ratio for concrete :

$$\nu = 1/6 = 0.167$$

Case 1 **P = 1.15 t**

According to 8 - a , page 80

$\beta = \frac{b}{a} = \frac{4.75}{3.5} = 1.357$

$\beta^2 = 1.84$ $\ln \beta = 0.305$ so that

$k_3 = \beta^2 \frac{1 + (1+\nu)\ln\beta}{1 - \nu + (1+\nu)\beta^2} = 1.84 \times$

$\frac{1 + 1.167 \times 0.305}{1 - 0.167 + 1.167 \times 1.84}$

$= 0.836$

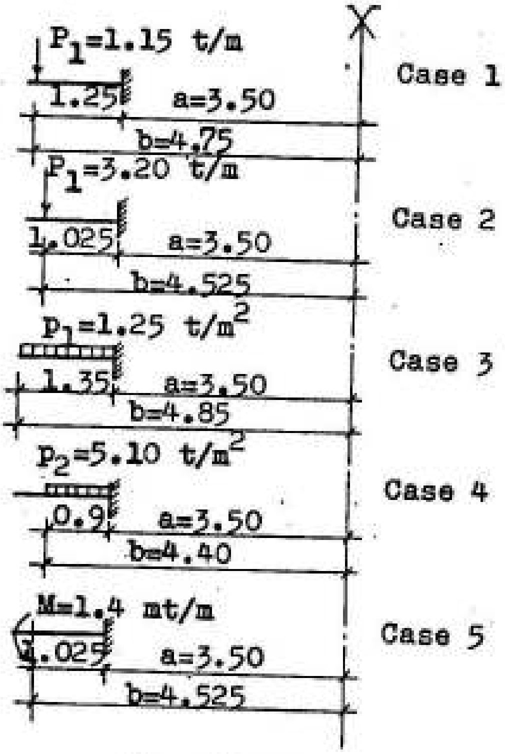


Fig. VI-32

At the fixed edge $\rho = 1$ and the radial moment is given by :

$M_r = - \frac{P\,a}{2}\, \beta \left[- 1 + (1+\nu)\, k_3 + (1-\nu)\, k_3 . \frac{1}{\rho^2} - (1+\nu) \ln \rho \right]$

$= - \frac{1.15 \times 3.5}{2} \times 1.357\ (- 1 + 1.167 \times 0.836 + 0.833 \times 0.836\ ) =$

$= 1.84$ mt/m

at the free edge $\rho = \beta$ and $M_r = 0$

The tangential moments M_t are :

$\rho = 1$, $M_t = - \frac{P\,a}{2}\, \beta \left[- \nu + (1+\nu)\, k_3 - (1-\nu)\, k_3 \right]$

$= - \frac{1.15 \times 3.5}{2} \times 1.357\ (- 0.167 + 1.167 \times 0.836 - 0.833 \times$

$\times 0.836 = - 0.30$ mt

$\rho = \beta$ $M_t = - \frac{P\,a}{2}\, \beta \left[- \nu + (1+\nu)\, k_3 - (1-\nu) \frac{k_3}{\beta^2} - (1+\nu) \ln \beta \right]$

$= - \frac{1.15 \times 3.5}{2} \times 1.357\ (- 0.167 + 1.167 \times 0.836 - 0.833 \times$

$\times \frac{0.836}{1.84} - 1.167 \times 0.305) = -0.199$ mt

Case 2 $\underline{P = 3.2\ t}$ (8 - a , page 80)

$\beta = \frac{4.525}{3.5} = 1.293$ $\beta^2 = 1.67$ and $\ln \beta = 0.256$ $k_3 = 0.779$

$\rho = 1,$ $M_r = - 4.04$ mt and $\rho = \beta,$ $M_r = 0$

$\rho = 1,$ $M_t = - 0.67$ mt and $\rho = \beta$ $M_t = -0.398$mt

Case 3 $\underline{p = 1.25\ t/m^2}$ (7 - a , page 78)

$\beta = \frac{4.85}{3.50} = 1.386$ $\beta^2 = 1.92$ and $\ln \beta = 0.326$ so that

$$k_1 = \beta^2 . \frac{(1 - \nu)\ \beta^2 + (1 + \nu)\ (1 + 4\ \beta^2 \ln \beta)}{1 - \nu + (1 + \nu)\ \beta^2}$$

$$= 1.92 \times \frac{0.833 \times 1.92 + 1.167\ (1 + 4 \times 1.92 \times 0.326)}{1 - 0.167 + 1.167 \times 1.92} = 3.55$$

$$\rho = 1, \quad M_r = \frac{p\ a^2}{16} \left[(1 + \nu)\ (1 - k_1) + 4\ \beta^2 - (3 + \nu) - (1 - \nu)\ k_1 \right]$$

$$= \frac{1.25 \times 3.50^2}{16} \left[1.16\ (1 - 3.55) + 4 \times 1.92 - 3.167 - 0.833 \times 3.55 \right] = -1.36 \text{ mt}$$

$\rho = \beta,$ $M_r = 0$

$$\rho = 1, \quad M_t = \frac{p\ a^2}{16} \left[(1 + \nu)\ (1 - k_1) + 4\ \nu\ \beta^2 - (1 + 3\ \nu) + (1 - \nu) k_1 \right]$$

$$= \frac{1.25 \times 3.5^2}{16} \left[1.16\ (1 - 3.55) + 4 \times 0.167 \times 1.9^2 + 1.5 + 0.833 \times 3.55 \right] = -0.22 \text{ mt}$$

$$\rho = \beta, \quad M_t = \frac{p\ a^2}{16} \left[(1 + \nu)\ (1 - k_1) + 4\ \nu\ \beta^2 - (1 + 3\ \nu)\ \frac{k_1}{\beta^2} + 4\ (1 + \nu)\ \beta^2 \ln \beta \right]$$

$$= \frac{1.25 \times 3.5^2}{16} \left[1.167\ (1 - 3.55) + 4 \times 0.167 \times 1.92 - 1.5 \times 1.92 + 0.833 \times \frac{3.55}{1.92} + 4 \times 1.167 \times 1.92 \times 0.326 \right] = - 0.106 \text{ mt}$$

<u>Case 4</u>   <u>$p = 5.1\ t/m^2$</u>   (7 - a, page 78)

$\beta = \frac{4.40}{3.5} = 1.257$ $\beta^2 = 1.58$ and $\ln \beta = 0.228$ $k_1 = 2.46$

$\rho = 1,$ $M_r = -2.35$ mt   " $\rho = \beta$ $M_r = 0$

$\rho = 1,$ $M_t = -0.39$ mt   " $\rho = \beta$ $M_t = -0.156$ mt

<u>Case 5</u>   <u>$M = 1.40\ mt/m$</u>   (9 - a, page 82)

$\beta = \frac{4.525}{3.50} = 1.293$ $\beta^2 = 1.67$ so that

$$k_5 = \frac{\beta^2}{1 - \nu + (1 + \nu)\beta^2} = \frac{1.67}{0.833 + 1.167 \times 1.67} = 0.6$$

$$\rho = 1, \quad M_r = -Mk_5 (1 + \nu + 1 - \nu) = -2\ Mk_5 = -2 \times 1.4 \times 0.6$$
$$= -1.68 \text{ mt}$$

$$\rho = \beta, \quad M_r = -Mk_5 \left[1 + \nu + (1 - \nu)\frac{1}{\beta^2}\right] = -1.4 \times 0.6 \left(1.167 + \frac{0.833}{1.67}\right)$$
$$= -1.40 \text{ mt}$$

$$\rho = 1, \quad M_t = -Mk_5 \left[1 + \nu - (1 - \nu)\right] = M_r \nu = -1.68 \times 0.167$$
$$= -0.28 \text{ mt}$$

$$\rho = \beta, \quad M_t = -Mk_5 \left[1 + \nu - (1 - \nu) \cdot \frac{1}{\beta^2}\right] = -1.4 \times 0.6 \left(1.167 - \frac{0.833}{1.67}\right)$$
$$= -0.56 \text{ mt}$$

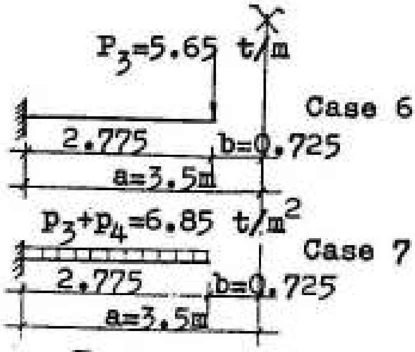
ii) <u>The hollow circular slab</u>

The hollow circular slab shall be solved for the two cases of loading shown in figure VI-33.

<u>Case 6</u>   <u>$P = 5.65\ t/m$</u>   (8-a page 80)

$\beta = \frac{0.725}{3.50} = 0.207$ $\beta^2 = 0.0428$

$\ln \beta = -1.573$ so that



$$k_3 = \beta^2 \; \frac{1 + (1+\nu)\ln\beta}{1 - \nu + (1+\nu)\beta^2}$$

$$= 0.0428 \; \frac{1 + 1.167 \times -1.573}{0.833 + 1.167 \times 0.0428} = 0.0405$$

$$\rho = 1, \quad M_r = \frac{P\,a}{2}\,\beta\left[-1 + (1+\nu)\,k_3 + (1-\nu)\,k_3\right]$$

Fig. VI-33

$$= \frac{5.65 \times 3.50}{2} \times 0.207\,(-1 - 1.167 \times 0.0405 - 0.833 \times 0.0405) = -2.22 \text{ m t}$$

$\rho = \beta, \quad M_r = 0$ and $\rho = \frac{1}{2}, \quad M_r = -0.77$ m t

$$\rho = 1, \quad M_t = \frac{P\,a}{2}\,\beta\left[-\nu + (1+\nu)\,k_3 - (1-\nu)\,k_3\right]$$

$$= \frac{5.65 \times 3.5}{2} \times 0.207\,(-0.167 - 1.167 \times 0.0405 + 0.833 \times 0.0405) = -0.37 \text{ m t}$$

$$\rho = \beta, \quad M_t = \frac{P\,a}{2}\,\beta\left[-\nu + (1+\nu)\,k_3 - (1-\nu)\,\frac{k_3}{\beta^2} - (1+\nu)\ln\beta\right]$$

$$= \frac{5.65 \times 3.5}{2} \times 0.207\,\left(-0.167 - 1.167 \times 0.0405 + 0.833 \times \frac{0.0405}{0.0428} + 1.167 \times 1.573\right) = +4.07 \text{ mt}$$

<u>Case 7</u> $\underline{p = 6.85 \text{ t/m}^2}$ (7 - a, page 78)

$\beta = 0.207$ $\beta^2 = 0.0428$ $\ln\beta = -1.573$ so that

$$k_1 = \beta^2 \cdot \frac{(1-\nu)\,\beta^2 + (1+\nu)\,(1 + 4\nu\beta^2 \ln\beta)}{1 - \nu + (1+\nu)\,\beta^2}$$

$$= 0.0428 \times \frac{0.833 \times 0.0428 + 1.167\,(1 - 4 \times 0.0428 \times 1.573)}{0.833 \times 1.167 \times 0.0428} = 0.043$$

$$\rho = 1, \quad M_r = \frac{p\,a^2}{16}\left[(1+\nu)\,(1 - k_1) + 4\,\beta^2 - (3+\nu) - (1-\nu)\,k_1\right]$$

$$M_r = \frac{6.85 \times 3.5^2}{16}(1.167 \times 0.957 + 4 \times 0.0428 - 3.167 - 0.833 \times 0.043)$$
$$= - 10.07 \text{ mt}$$

$\rho = \beta$, $M_r = 0$ and $\rho = \frac{1}{2}$ $M_r = + 1.126$ mt

$$\rho = 1,\quad M_t = \frac{p\,a^2}{16}\left[(1+\nu)(1 - k_1) + 4\nu\beta^2 - (1 + 3\nu) + (1 - \nu)\,k_1\right]$$

$$= \frac{6.85 \times 3.5^2}{16}(1.167 \times 0.957 + 4 \times 0.167 \times 0.0428 - 1.50 + 0.833$$
$$\times 0.043) = - 1.68 \text{ mt}$$

$$\rho = \beta,\quad M_t = \frac{p\,a^2}{16}\left[(1+\nu)(1 - k_1) + 4\nu\beta^2 - (1 + 3\nu)\beta^2 + (1 - \nu)\frac{k_1}{\beta^2} + 4(1+\nu)\beta^2 \ln\beta\right]$$

$$= \frac{6.85 \times 3.5^2}{16}(1.167 \times 0.957 + 4 \times 1.167 \times 0.0428 - 1.5 \times 0.0428 +$$
$$+ 0.833 \times \frac{0.043}{0.0428} - 4 \times 1.167 \times 0.0428 \times 1.573)$$
$$= + 8.42 \text{ mt}$$

Fixed end moments

The fixed end moments of the two elements at the support are therefore :

For the cantilever ring

$$\bar{M} = -1.84-4.04-1.36-2.35-1.68 = - 11.27 \text{ mt}$$

For the hollow circular slab

$$\bar{M} = - 2.22 - 10.07 = - 12.29 \text{ mt}$$

The unbalanced fixed end moment shall be distributed by the moment distribution method according to the relative stiffness of the cantilever ring and the hollow circular slab. Knowing that the stiffness is the moment causing an angle of rotation at the support equal to unity, it can be determined for each of the two elements in the following manner : (Refer to case 10 page 83).

$\beta = 0.207$ $\beta^2 = 0.0428$ $k_7 = 1.0447$

$\rho = 1,$ $M_r = 0.8$ mt/m and $\rho = \beta,$ $M_r = 0$

$\rho = \frac{1}{2},$ $M_r = 0.694$ mt

$\rho = 1,$ $M_t = M\,\beta^2\,k_7\,(\frac{1}{\beta^2} + 1)$

$= 0.8 \times 0.0428 \times 1.0447\,(\frac{1}{0.0428} + 1) = + 0.88$ mt

$\rho = \beta$ $M_t = 2\,M\cdot k_7 = 2 \times 0.8 \times 1.0447 = + 1.67$ mt

The final values of the radial and tangential bending moments can now be determined by superposition as shown in the following tables.

i) Moments in Cantilever Ring

Case	Radial moment M_r in mt/m		Tangential moment M_t in mt/m	
	$\rho = 1$	$\rho = \beta$	$\rho = 1$	$\rho = \beta$
1	- 1.84	0	- 0.30	- 0.199
2	- 4.04	0	- 0.67	- 0.398
3	- 1.36	0	- 0.22	- 0.106
4	- 2.35	0	- 0.39	- 0.156
5	- 1.68	- 1.40	- 0.28	- 0.560
a	- 0.22	0	+ 0.69	+ 0.479
Total	-11.49	- 1.40	- 1.17	- 0.94

ii) Moments in Hollow Circular Slab

Case	Radial moment M_r in mt/m			Tangential moment M_t in mt/m	
	$\rho = 1$	$\rho = ½$	$\rho = \beta$	$\rho = 1$	$\rho = \beta$
6	- 2.22	- 0.77	0	- 0.37	+ 4.07
7	- 10.07	+ 1.13	0	- 1.68	+ 8.42
b	+ 00.80	+ 0.69	0	+ 0.88	+ 1.67
Total	- 11.49	+ 1.05	0	- 1.17	+14.16

iii) <u>Bending Moment Diagrams</u> (Fig. VI-36

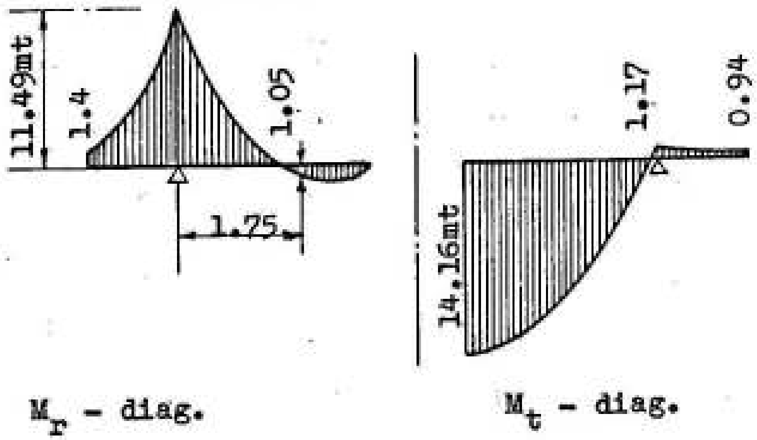


Fig. VI-36

c) <u>Design</u>

The floor of the tank is to be designed for the following internal forces :

Max. radial tension in floor = 3.45 t/m

Max. radial bending moment = - 11.49 mt/m at supporting shaft

Max. tangential " " = + 14.16 mt/m at interm. hole

For section at the supporting shaft :

M_r = 11.49 mt/m (with tension on water side) and N = 3.45 t/m tension

$t = \sqrt{M_r/3} + 2 = \sqrt{11490/3} + 2 = 64$ cms taken 70 cms

$e = M_r/N = 11.49/3.45 = 3.33$ ms $e_s = 333 - \frac{70}{2} + 4 = 304$ cms

$M_s = N.\ e_s = 3.45 \times 3.04 = 10.45$ mt/m

$d = k_1 \sqrt{M_s}$ i.e. $66 = k_1 \sqrt{10450}$ giving $k_1 = 0.645$

$$A_s = \frac{M_s}{k_2\ d} + \frac{N}{\sigma_s} = \frac{10450}{1300 \times 66} + \frac{3450}{1400} = 12.13 + 2.47 = 14.6 \text{ cm}^2$$

choose φ 22 @ 20 cms for the top and φ 13 @ 20 cms for the bottom radial reinforcement.

For the section at the intermediate shaft :

M_t = 14.16 mt/m (with tension on air side) , we have

$64 = k_1 \sqrt{14160}$ giving $k_1 = 0.54$ and $\sigma_c = 30.5$ kg/cm^2

$$A_s = \frac{14160}{1285 \times .64} = 17.2 \text{ cm}^2 \qquad \text{chosen } \varphi\, 19 \text{ @ } 12.5 \text{ cms}$$

The circular bottom reinforcement decreases as the distance from the center increases. The circular top reinforcement is chosen φ 13 @ 20 cms as shown in figure VI.44

2) The Foundation Slab

The foundation slab shall be assumed as a circular plate with an overhanging cantilever ring simply supported on the shaft and subject to the soil pressure acting upwards.

a) Loads

Total weight of tank	= 260	ton
" " " water	= 300	"
" " " shaft	= 333	"
Weight of interm. floors	= 103	"

Wind pressure w on cylindrical surfaces can be assumed = 0.45 the normally specified values acting on vertical surfaces i.e. w = 45 kg/m²

Hence

$W_1 = 9.7 \times 63 \times 45/1000 \cong 2.75$ tons

$W_2 = 7.2 \times 29.3 \times 45/1000 \cong 9.5$ tons

Total vertical load = 996 ton

Bending moment due to wind at Sec. II - II

$$M_w = 2.75 \times \left(\frac{6.3}{2} + 29.3\right) + 9.5 \times \frac{293}{2} =$$

$$= 228 \text{ mt}$$

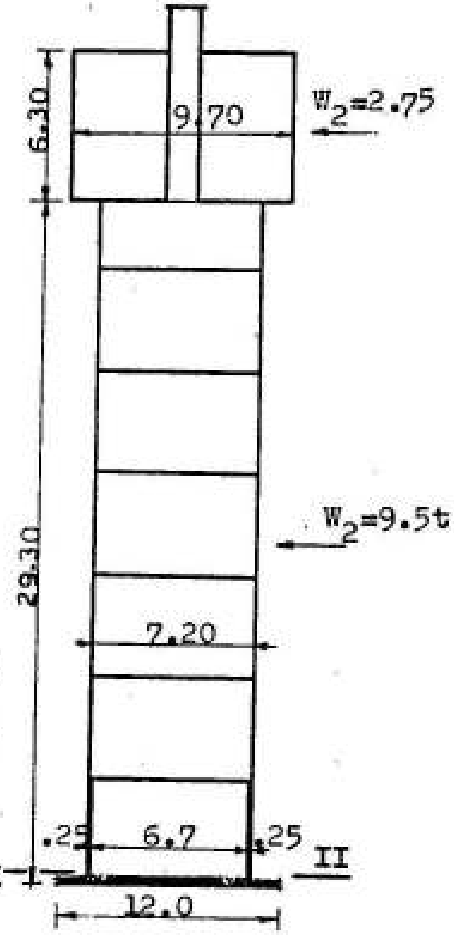


Fig. VI-37

b) Check of Stresses in Shaft at Foundation Slab

The stresses in the shaft shall be checked for the following two cases :

1) Tank full + Wind pressure $P = 996$ t, $M = 228$ mt

Net area of cross-section of shaft

$A_o = \pi \times 6.95 \times 0.25 - (0.8 + 1.35)\, 0.25 = 5.44 - 0.54 \quad = 4.9 \ m^2$

Gross section modulus : $\frac{\pi}{32} \left(\frac{7.2^4 - 6.7^4}{7.2}\right) \quad = 9.16 \ m^3$

Section modulus of openings (window and door)

$(0.80 + 1.35)\, 0.25 \times 3.475^2 / 3.6 \quad = 1.80 \ m^3$

Net section modulus $Z_o = 9.16 - 1.80 \quad = 7.36 \ m^3$

Therefore, the stresses in the shaft are given by :

$$\sigma_{\frac{1}{2}} = -\frac{P}{A_o} \pm \frac{M}{Z_o} = -\frac{996}{4.9} \pm \frac{228}{7.36} = -203 \pm 31 \ t/m^2 \quad \text{i.e.}$$

$$\underline{\sigma_1 = -23.4} \quad \text{and} \quad \underline{\sigma_2 = -17.2} \quad kg/cm^2$$

2) <u>Tank empty + Wind pressure</u> $P = 696$ t, $M = 228$ mt

$$\sigma_{\frac{1}{2}} = -\frac{696}{4.9} \pm \frac{228}{7.36} = -142 \pm 31 \ t/m^2 \quad \text{i.e.}$$

$$\underline{\sigma_1 = -17.3} \quad \text{and} \quad \underline{\sigma_2 = -11.1} \quad kg/cm^2$$

c) <u>Pressure Under Foundation Slab</u>   (Fig. VI-38)

Own weight of foundation slab :

$$2.5 \left[\frac{\pi}{4} \times 12.0^2 \times 0.8 - \frac{\pi}{4} (12.0^2 - 7.6^2) \times \frac{0.5}{2} \right] = 184 \ t$$

Total load = 996 + 184 $P = 1180$ t

Wind moment $M = 228$ mt

Area of foundation – base $= \pi \times 12.0^2 / 4 \quad = 113 \ m^2$

Section modulus $z = \pi \times 12.0^3 / 32 \quad = 169.5 \ m^3$

$$\sigma_{\frac{1}{2}} = -\frac{1180}{113} \pm \frac{228}{169.5} = -10.44 \pm 1.34 \ ton/m^2 \quad \text{i.e.}$$

$$\sigma_1 = -1.18 \quad \text{and} \quad \sigma_2 = -0.92 \ kg/cm^2$$

d) Bending Moments

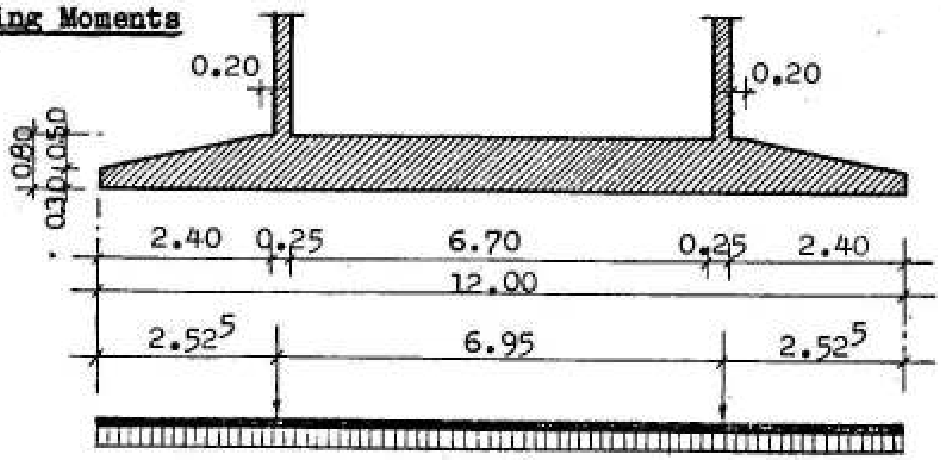


Fig. VI-38

In order to simplify the calculations, the internal forces in the foundation slab shall be determined for a net upward uniform pressure $p = 10\ t/m^2$.

Again here, we determine the radial and tangential bending moments for the cases shown in fig. VI - 39 from the equations given in IV - 2 .

Proceeding in the same way as before we get the following results.

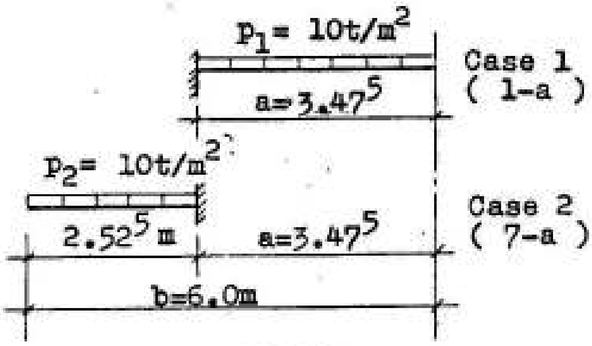


Fig. VI-39

Case	M_r in mt/m		M_t in mt/m	
	$\rho = 1$	$\rho = 0$	ρ_1	ρ_o
1	- 15.14	+ 7.71	- 2.50	+ 7.71
	$\rho = 1$	$\rho = \beta$	$\rho = 1$	$\rho = \beta$
2	- 42.27	0	- 7.12	- 4.54

To redistribute the fixed end moments (- 42.27 mt for the circular plate and - 15.14 mt for the cantilever ring) one has to determine the stiffness of each of the two elements.

i) <u>Stiffness of circular slab</u> Refer to page 77

For $\rho = 1$

$$\varphi = \frac{M\,a}{D\,(1 + \nu)} = 1 \qquad \text{thus}$$

$$M = \frac{1.167\ D}{3.475} = 0.3357\ D$$

ii) <u>Stiffness of cantilever ring</u> Refer to page 83

For $\rho = 1$

$$\varphi = \frac{M\,a}{D} \times \frac{k_7}{1 + \nu}\left(1 + \frac{1 + \nu}{1 - \nu}\,\beta^2\right) = 1 \qquad \text{in which}$$

$\beta = 1.727$, $\beta^2 = 2.98$ and $k_7 = \frac{1}{1 - \beta^2} = \frac{1}{1 - 2.98} = -\,0.521$

Therefore

$$\frac{M \times 3.475}{D} \times -\frac{0.521}{1.167}\left(1 + \frac{1.167}{0.833} \times 2.98\right) = 1 \qquad \text{or}$$

$$M = -\,0.1246\ D$$

Neglecting the effect of the variation of the depth of the cantilever ring on its flexural rigidity, then

Relative stiffness of the cantilever ring $= \frac{0.1246}{0.1246 + 0.3357} = 0.27$

" " " " circular slab $= \frac{0.3357}{0.1246 + 0.3357} = 0.73$

The connecting moment

	Cantilever ring	Circular slab
Fixed end moment	- 42.27	+ 15.14
Balancing moment	+ 7.33	+ 19.80
	- 34.94	34.94

<u>Effect of balancing moment</u> (Fig VI-40)

<u>Case a</u> 6 - page 77

For $\rho = 1$ and $\rho = 0$

$M_r = M_t = M = -19.80$ mt

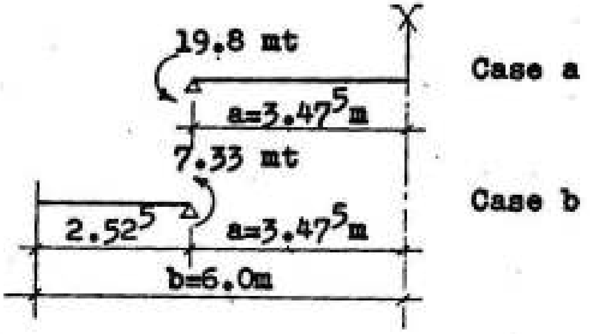


Fig. VI-40

<u>Case b</u> 10 - page 83

$\beta = 1.727$ $\beta^2 = 2.98$

$$k_7 = \frac{1}{1-\beta^2} = \frac{1}{1-2.98} = -0.521$$

For $\rho = 1$, $M_r = M = +7.33$ mt and for $\rho = \beta$, $M_r = 0$

For $\rho = 1$, $M_t = M\,k_7\,(1+\beta^2) = 7.33 \times (-0.521)(1+2.98) = -15.20$ mt

For $\rho = \beta$, $M_t = 2\,M\,k_7 = -2 \times 7.33 \times 0.521 = -7.64$ mt

<u>Final values of the radial and tangential moments</u>

i) <u>Circular Slab</u>

Case	Radial moment M_r in mt/m		Tangential moment M_t in mt/m	
	$\rho = 1$	$\rho = 0$	$\rho = 1$	$\rho = 0$
1	- 15.14	+ 7.71	- 2.50	+ 7.71
a	- 19.80	- 19.80	- 19.80	- 19.80
Total	- 34.94	- 12.09	- 22.30	- 12.09

ii) <u>Cantilever ring</u>

Case	Radial moment M_r in mt/m		Tangential moment M_t in mt/m	
	$\rho = 1$	$\rho = \beta$	$\rho = 1$	$\rho = \beta$
2	- 42.27	0	- 7.12	- 4.54
b	+ 7.33	0	- 15.20	- 7.64
Total	- 34.94	0	- 22.32	- 12.18

iii) **Bending moment diagrams :** (Fig. VI-41)

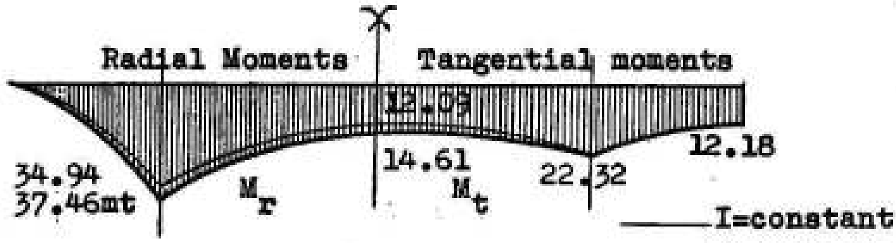


Fig. VI-41

Effect of variation of moment of inertia on radial moments

The effect of the variatiàn of the moment of inertia of the cantiliver ring on the flexural rigidity D and the corresponding effect on the radial moment can be determined according to Markus* in the following manner : (Fig. VI-42).

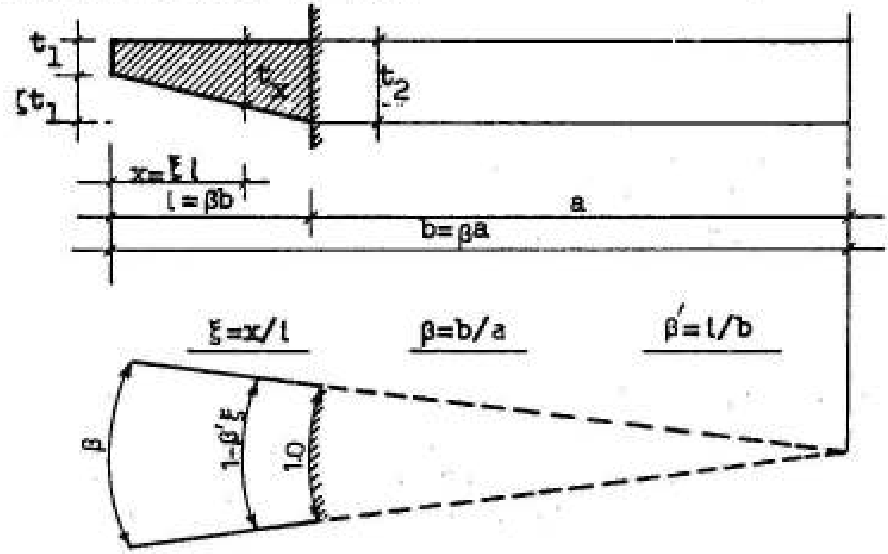


Fig. VI-42

Referring to data given in figure VI-42, we get :

$$\beta' = \frac{l}{b} = \frac{b - a}{b} = 1 - \frac{1}{\beta}$$

$t_1 + \zeta t_1 = t_2$ or $1 + \zeta = t_2 / t_1$ i.e

$$\zeta = \frac{t_2}{t_1} - 1$$

---

* Markus : "Theorie und Berechnung rotationssymmetrischer Bauwerke " Werner - Verlag Düsseldorf

According to Markus, one can prove that :

$$D_x = D_1 (1 - \beta' \xi) (1 + \zeta \xi)^3$$

Hence, at free edge 1, we have :

$$D_1 = \frac{E t_1^3}{12 (1 - \nu^2)}$$

and at fixed edge 2, where $x = l$ and $\xi = 1$, we have :

$$D_2 = \frac{E t_1^3}{12 (1 - \nu^2)} (1 - \beta') \left[1 + (\frac{t_2}{t_1} - 1) \right]^3 \qquad \text{or}$$

$$D_2 = \frac{E t_2^3}{12 (1 - \nu^2)} \cdot \frac{1}{\beta}$$

Therefore

$$D_2 = \frac{E \times 0.8^3}{12 (1 - 0.167^2)} \times \frac{3.475}{6.00} = 2.541 \times 10^{-2} E$$

and the stiffness of the cantilever ring is :

$$M = 0.1246 \times 2.541 \times 10^{-2} E = 0.3166 \times 10^{-2} E$$

Further, the stiffness of the circular slab is given by :

$$M = 0.3357 D, \text{ in which } D = \frac{E t^3}{12 (1 - \nu^2)} = \frac{E \times 0.8^3}{12 (1 - 1.167^2)} = 4.39 \times 10^{-2} E$$

thus $$M = 0.3357 \times 4.39 \times 10^{-2} E = 1.4737 \times 10^{-2} E$$

Hence

Relative stiffness of cantilever ring $= \frac{0.3166}{0.3166 + 1.4737} = 0.177$

" " " circular slab $= \frac{1.4737}{0.3166 + 1.4737} = 0.823$

The connecting moment

	Cantilever ring	Circular slab
Fixed end moment	- 42.27	+ 15.14
Balancing moment	+ 4.81	+ 22.32
Connecting moment	- 37.46	+ 37.46 mt

FLOCCULATOR ø26.00m.

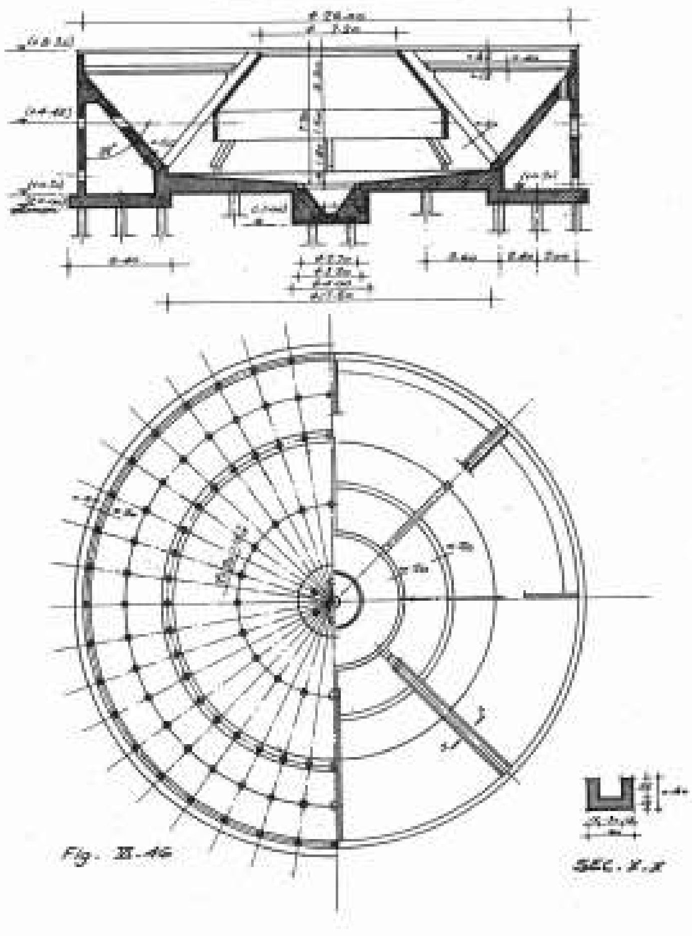


Fig. III.46

Effect of balancing moment   Fig VI-43

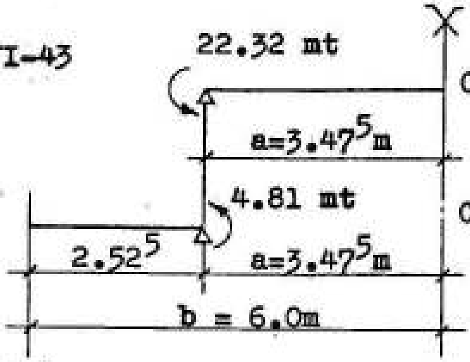


Fig. VI-43

The variation of the fluxural rigidity affects mainly the radial moments, thus :

Case a   6   page 77

For $\rho = 1$ and $\rho = 0$ $M_r = M = -22.32$ mt

Case b   10   page 83

for $\rho = 1$ $M_r = M = + 4.81$ mt   and

For $\rho = \beta$ $M_r = 0$

Therefore, the radial and tangential moments at the center of the plate is given by $M_r = M_t = + 7.71 - 22.32 = 14.61$ mt. The final moments may be assumed as shown by the heavy line in figure VI-41.

e) Design

Section at Support $M_r = 37.46$ mt , $M_t = 22.32$ mt

Assuming total thickness of slab = 80 cms   then

Theoretical depth $d = 80 - 4 = 76$ cms   and

for the radial direction, we have

$d = k_1 \sqrt{M_r}$ or $76 = k_1 \sqrt{37460}$ giving $k_1 = 0.394$

assuming $\sigma_s = 1400$ kg/cm^2 , $n = 15$ & $x\,\sigma = 0.15$

then $\sigma_c = 43$ kg/cm^2 ,   and $k_2 = 1250$

so that $A_s \cong \frac{M}{k_2 d} = \frac{37460}{1250 \times 76} = 39.4$ cm^2 chosen φ 22 @ 10 cm

for the tangential direction, we get

$$A_s \cong \frac{22320}{1300 \times 74} = 23.2 \text{ cm}^2$$ chosen φ 22 @ 15 cm

Both radial and tangential bottom reinforcements may be reduced at the center of the circular slab to :

$$A_s = \frac{14610}{1300 \times 75} = 14.9 \text{ cm}^2$$ chosen φ 22 @ 25 cm

FLOCCULATOR ϕ26.00m

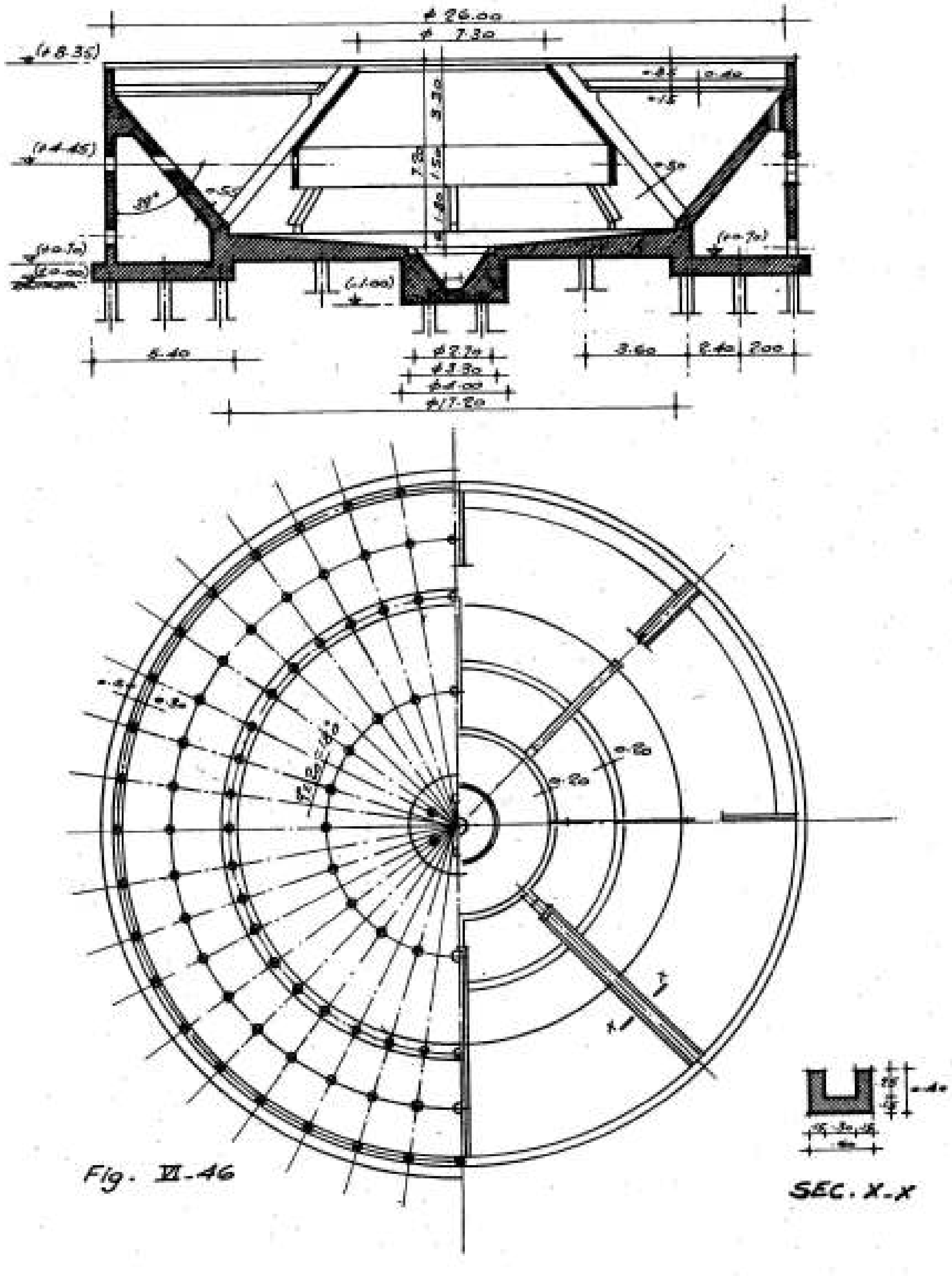


Fig. VI-46

However, the foundation slab is reinforced by a rectangular mesh Ø 22 mm @ 10 cms in its lower surface and by a mesh Ø 13 mm @ 20 cms in its upper surface.

The details of reinforcement of the water tower shown in figure IV-3a designed by the Misr Concrete Development Company is given in Fig. VI-44.

VI-4 A Treated Water Tank and a Flocculator

We give in Figs. VI-45 and VI-46 two examples of circular tanks that have been recently designed by the author for "Abu Quir Urea Plant for Fertilizers and Chemical Industries".

The soil at the site of the plant is very weak; it contains dark clays and peat for big depths that most of the units of the plant , including the tanks, are supported on mechanically driven cast-in-situ piles about 25 ms long and having a maximum capacity of 40 tons per pile.

Fig. VI-45 shows a covered treated-water-tank 23 ms diameter and about 8 ms deep. The roof slab is supported on nine interior columns and on the wall along its exterior edge. The joint between the roof and the walls is sliding (refer to Fig. III-1), so that no stresses are created in the wall due to temperature changes of the roof.

Fig. VI-46 shows a flocculator 26 ms diameter and 7 ms deep.

The main point of interest in these two tanks is the distribution of the piles which is chosen such that each pile is fully utilized and their arrangement gives a convenient thickness for the floors satisfying the requirements of the internal forces, water-tightness and utility of the tanks.

VII. DESIGN OF RECTANGULAR TANKS

VII.1. DEEP TANKS RESISTING HYDROSTATIC PRESSURE HORIZONTALLY.

In closed rectangular tanks with sliding base, the full water pressure is resisted horizontally. Also in deep tanks where H/L_1

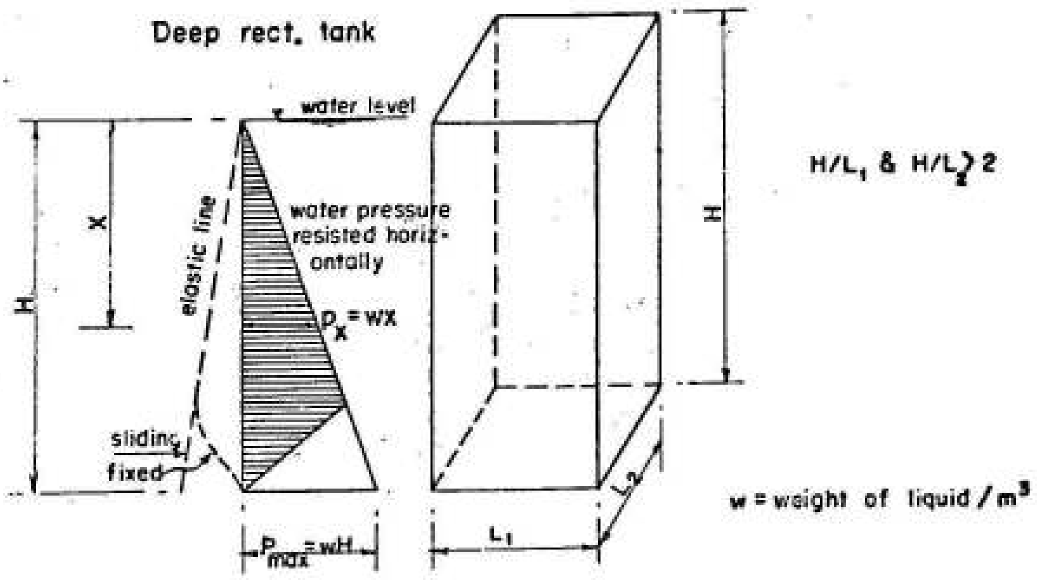
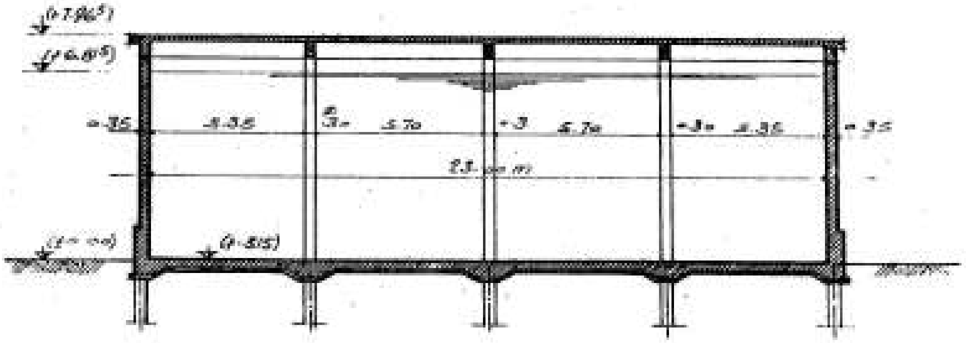


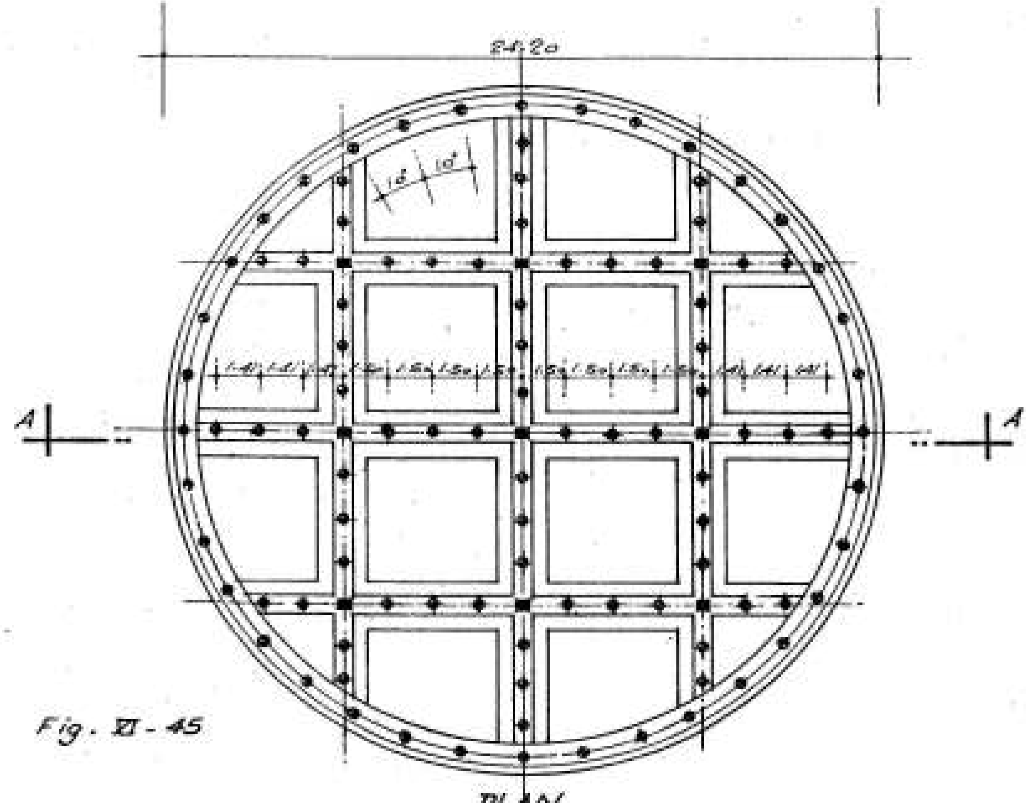
Fig. VII-1

and $H/L_2 > 2$ the effect of the fixation of the wall to the floor will be limited to a small part at the base of the wall, the rest of the wall will resist the water pressure in the horizontal direction by closed frame action (Fig. VII.I).

TREATED WATER TANK



SEC. A-A



PLAN

Fig. XI - 45

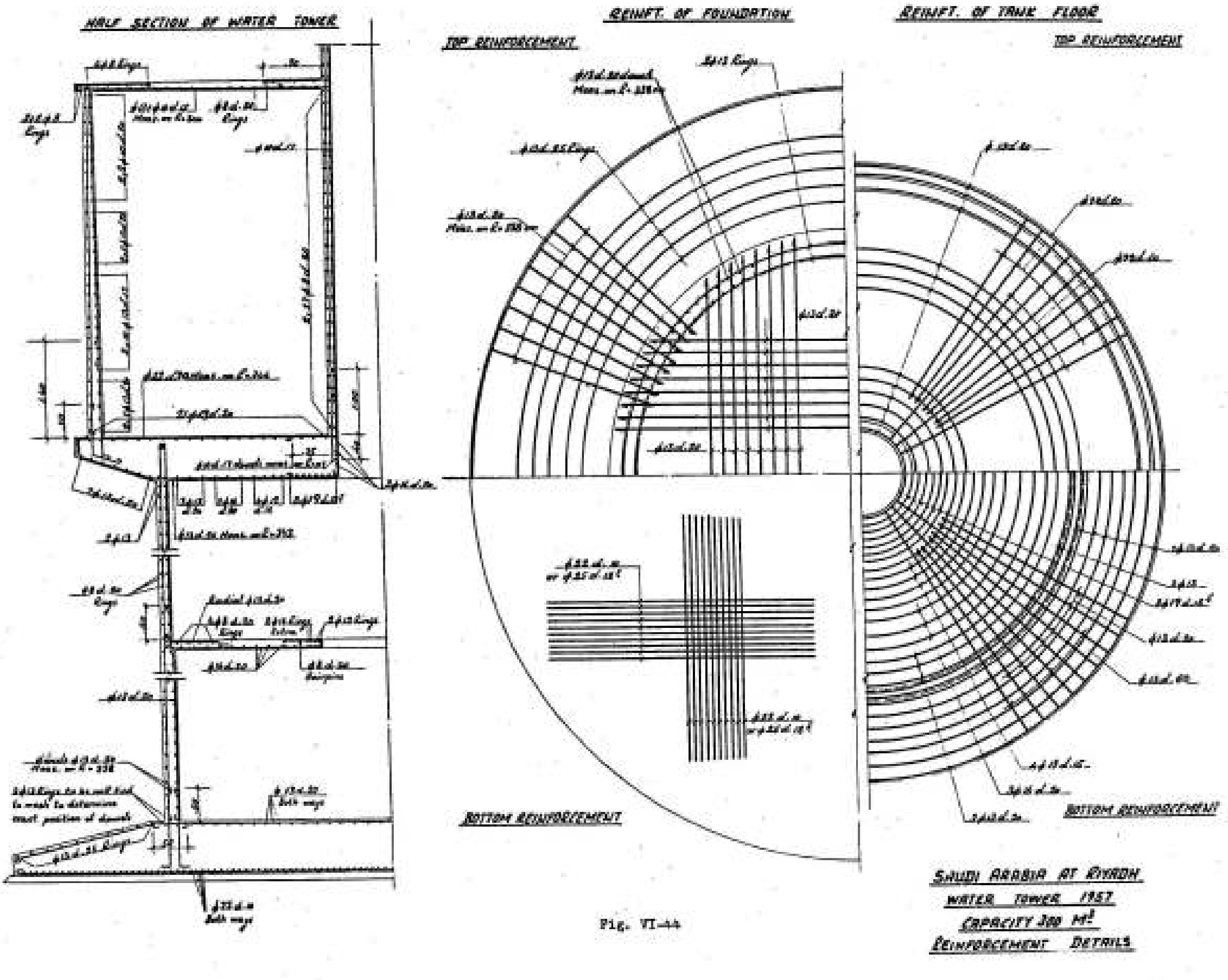

HALF SECTION OF WATER TOWER
REINFT. OF FOUNDATION
TOP REINFORCEMENT
REINFT. OF TANK FLOOR
TOP REINFORCEMENT
BOTTOM REINFORCEMENT
BOTTOM REINFORCEMENT
SAUDI ARABIA AT RIYADH
WATER TOWER 1957
CAPACITY 300 M³
REINFORCEMENT DETAILS

Fig. VI-44

a) Square Section (Fig. VII.2)

Due to symmetry of square sections subject to internal pressure the angles of rotation θ of each side at its two ends must be equal to zero; i.e. each side behaves as if it were totally fixed at both ends, and for a uniformly distributed horizontal pressure p, we get (Fig. VII.2).

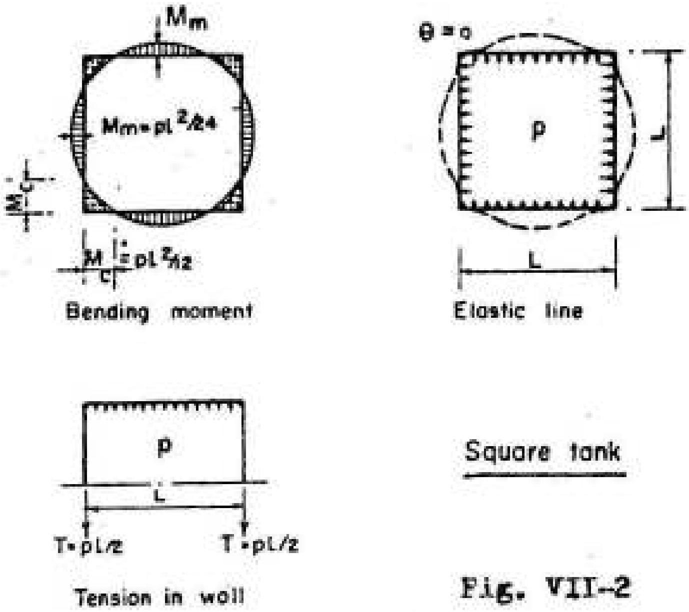


Fig. VII-2

Bending moment at middle of any side :

$$M_m = p L^2 / 24 \qquad (1\text{-a})$$

Bending moment at any of the corners :

$$M_c = p L^2 / 12 \qquad (1\text{-b})$$

Each wall will further be subject to a tensile force given by :

$$T = p L / 2 \qquad (1\text{-c})$$

giving a case of eccentric tension.

For any tank of regular section (Fig. VII.3) subject to uniform internal pressure p, we get similarly :

$\theta = 0$ and

$$\left.\begin{aligned} M_m &= p\,L^2 / 24 \\ M_c &= p\,L^2 / 12 \\ T &= p\,D / 2 \end{aligned}\right\} \qquad (2)$$

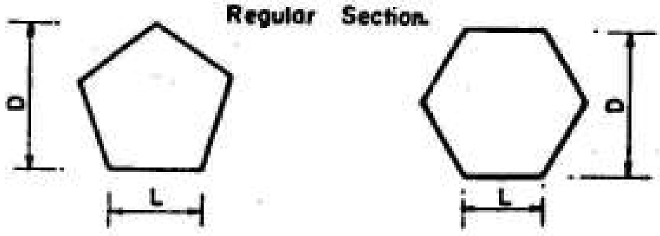


Fig. VII-3

b) <u>Rectangular Section</u> (Fig. VII.4)

Due to symmetry of loading, the corners of the cross-section will not move, and in order to determine the connecting moment M_c in the corner; the equation of three moments can be applied; thus :

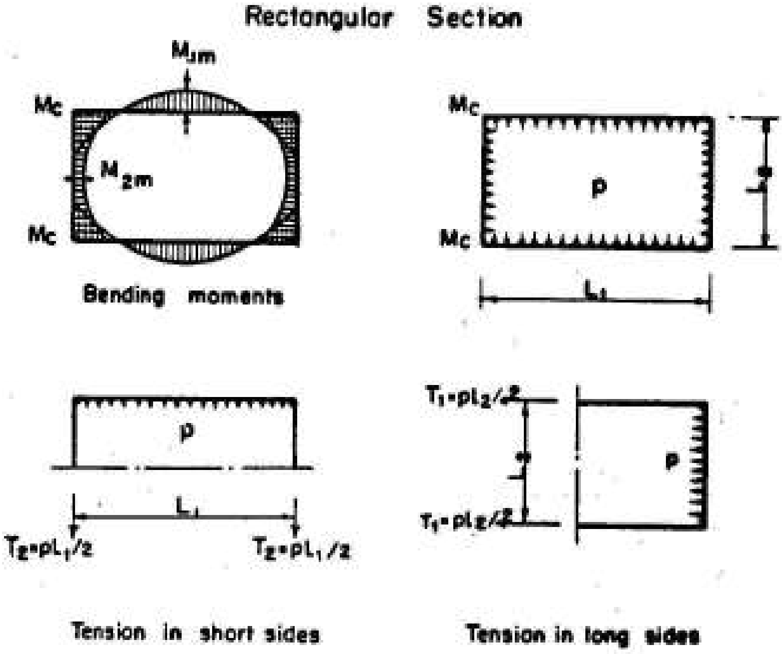


Fig. VII-4

if the moment of inertia of the two sides L_1 and L_2 is the same then

$$M_c L_1 + 2 M_c (L_1 + L_2) + M_c L_2 = - 6 \left(p \frac{L_1^3}{24} + p \frac{L_2^3}{24}\right) \qquad \text{or}$$

$$3 M_c (L_1 + L_2) = - \frac{p}{4} (L_1^3 + L_2^3) \qquad \text{i.e}$$

$$\left.\begin{aligned} M_c &= - \frac{p}{12} \cdot \frac{L_1^3 + L_2^3}{L_1 + L_2} \\ \text{or} \quad & \\ M_c &= - \frac{p}{12} (L_1^2 - L_1 L_2 + L_2^2) \end{aligned}\right\} \qquad (3a)$$

and accordingly we get :

$$M_{1m} = p \frac{L_1^2}{8} + M_c = \frac{p}{24} (L_1^2 + 2 L_1 L_2 - 2 L_2^2) \qquad (3b)$$

$$M_{2m} = p \frac{L_2^2}{8} + M_c = \frac{p}{24} (L_2^2 + 2 L_1 L_2 - 2 L_1^2) \qquad (3c)$$

For $L_1 = L_2 = L$, then

$M_c = - p L^2 / 12$ and $M_{1m} = M_{2m} = p L^2 / 24$

Assuming further :

$M_{1m} = \alpha p L_2^2 / 12$, $M_{2m} = \beta p L_2^2 / 12$ and $M_c = \gamma p L_2^2 / 12$

we get :

L_1 / L_2	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
α	0.50	0.71	0.92	1.15	1.38	1.63	1.88	2.15	2.42	2.71	3.00
β	0.50	0.39	0.27	0.11	0.06	$\overline{0.25}$	$\overline{0.46}$	$\overline{0.69}$	$\overline{0.94}$	$\overline{1.21}$	$\overline{1.50}$
γ	$\overline{1.00}$	$\overline{1.11}$	$\overline{1.24}$	$\overline{1.40}$	$\overline{1.56}$	$\overline{1.75}$	$\overline{1.96}$	$\overline{2.10}$	$\overline{2.44}$	$\overline{2.71}$	$\overline{3.00}$

If L_1 varies much from L_2 ; it is not economic to calculate with constant I, and assuming :

$$\varkappa = \frac{I_1}{L_1} \cdot \frac{L_2}{I_2}$$

the equation of three moments can be given in the form :

$$\frac{I_1}{L_1} \quad M_c \; \frac{L_1}{I_1} + 2\, M_c \left(\frac{L_1}{I_1} + \frac{L_2}{I_2} \right) + M_c \; \frac{L_2}{I_2} \quad = -\, 6 \left(p\, \frac{L_1^3}{24\, I_1} + p\, \frac{L_2^3}{24\, I_2}\right) . \; \frac{I_1}{L_1}$$

or

$$M_c + 2\, M_c\, (1 + \varkappa) + M_c \quad = -\, \frac{p}{4} \left(L_1^2 + \varkappa\, L_2^2 \right)$$

$$M_c \quad = -\, \frac{p}{12} \; \frac{L_1^2 + \varkappa\, L_2^2}{1 + \varkappa} \qquad (4)$$

Each wall is further subject to a tensile force given by :

$$T_1 \quad = p\, L_2 \,/\, 2 \qquad \text{for wall } L_1$$

and

$$T_2 \quad = p\, L_1 \,/\, 2 \qquad \text{for wall } L_2$$

c) Determination of Internal Forces by the Tension Line Method

As the walls of any closed section are subject to bending moments and axial tension, it is possible to represent the internal forces by a tension line.

For a closed frame of constant moment of inertia, subject to uniformly distributed internal pressure, the tension line is composed of a series of parabolas and can be assumed as a circle without any appreciable error.

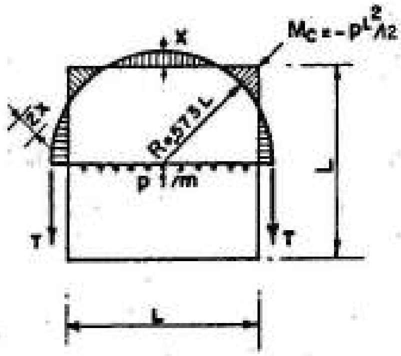


Fig. VII-5

The radius of the tension line can be determined if we study the case of a square section as that shown in Fig VII.5 Thus,

$2\,T = p\,2\,R \quad \text{or} \quad T = p\,R$

$M_m = T.\ \ x = p\,R\,x = p\,L^2 \,/\, 24$

$$R\,x = L^2 \,/\, 24$$

But $x = R - L/2$ then

$$R\,(R - L/2) = L^2 / 24 \qquad \text{or} \qquad R^2 - R\,L/2 - L^2/24 = 0$$

i.e.
$$R = \frac{L/2 \pm \sqrt{L^2/4 + 4\,L^2/24}}{2} = 0.573\ L$$

which means that the area A_t enclosed by the tension line is given by:

$$A_t = \pi\,(0.573\ L)^2 = 1.03\ L^2 \qquad (5)$$ i.e.

area enclosed by tension line = 1.03 area of section of tank.

This method can however be used for rectangular sections, Figure VII.6, as well as sections of irregular shape, Fig. VII.7, as can be proved from the following example :

Assume a rectangular tank with $L_1 = 2\ L_2$ subject to internal pressure p, Fig. VII.6

Area of cross-section $A = 2\ L_2^2$

Area enclosed by tension line :

$A_t = 1.03\ A = 1.03 \times 2\ L_2^2 = 2.06\ L_2^2$

$R = \sqrt{2.06\ L_2^2 / \pi} = 0.81\ L_2$

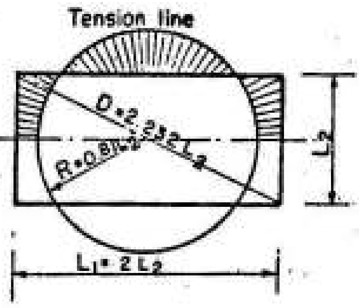


Fig. VII-6

So that

$T = p\,R = 0.81\ p\ L_2$

We have further :

Length of diagonal $D = \sqrt{L_2^2 + (2\ L_2)^2} = \sqrt{5\ L_2^2} = 2.232\ L_2$

Therefore, the corner connecting moment M_c can be given by :

$M_c = T\,(R - D/2) = 0.81\ p\ L_2\ (0.81\ L_2 - 2.232\ L_2/2) = 0.248\ p\ L_2^2$

against

$M_c = \gamma\ p\ L_2^2 / 12 = 3\ p\ L_2^2 / 12 \qquad = 0.250\ p\ L_2^2$

and

$M_{1m} = p\, L_1^2 \,/\, 8 - M_c = p.\, 4\, L_2^2 \,/\, 8 \;- 0.248\, p\, L_2^2 \qquad = 0.250\, p\, L_2^2$

against

$M_{1m} = \alpha\, p\, L_2^2 \,/\, 12 \;= 3\, p\, L_2^2 \,/\, 12 \qquad = 0.250\, p\, L_2^2$

$M_{2m} = p\, L_2^2 \,/\, 8 - M_c = 0.125\, p\, L_2^2 \;- 0.248\, p\, L_2^2 \qquad = - 0.123\, p\, L_2^2$

against

$M_{2m} = \beta\, p\, L_2^2 \,/\, 12 = - 0.15\; p\, L_2^2 \,/\, 12 \qquad = - 0.125\, p\, L_2^2$

This example shows that the method gives a very good approximate solution for the determination of the internal forces in isolated tanks of one cell. Its use simplifies the determination of the internal forces in tanks of special cross-section as that shown in Fig. VII.7 in which the area enclosed by the tension line is 1.03 the area of the cell & its center coincides on the center of gravity of the cross-section of the cell.

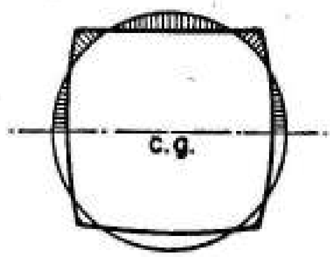


Fig. VII-7

As the liquid pressure increases with the depth, a trapezoidal vertical section for the wall gives a convenient form.

In deep tanks, the main steel reinforcement is horizontal; however it is essential to arrange vertical secondary reinforcements having a minimum cross-section area of 20% of the main horizontal steel to fix the main reinforcements in position and to resist the shrinkage and temperature stresses.

In case of walls fixed to the floor slab, the fixing moment at the foot of the wall may be estimated from the relation :

$$M_f \approx - p_{max}\, L^2 \,/\, 24 \qquad (6a)$$

The corresponding reaction is given by :

$$R \approx 0.3\, p_{max} L \qquad (6b)$$

Therefore, the maximum internal forces in an open square water tank 4 x 4 ms and 10 ms deep with fixed base can be calculated as follows :

Assuming that the maximum horizontal internal forces take place at a depth $x = 0.75\ H$ then :

The max. horizontal fixing moment is :

$$M_f = -\ p_x\ L^2/12 = -\ 0.75 \times 10 \times 4^2/12 = 10 \quad mt$$

The corresponding tension in the wall is :

$$T = p_x\ L/2 = 0.75 \times 10 \times 4/2 = 15 \quad t$$

The fixing moment at the foot of the wall in the vertical direction is according to equation 6a given by :

$$M_f = -\ p_{max} L^2/24 = -\ 10 \times 4^2/24 = 6.66 \quad mt$$

The corresponding reaction = tension in floor , is according to equation 6b given by :

$$R = 0.3\ p_{max} L = 0.3 \times 10 \times 4 = 12 \quad t$$

It is however generally sufficient to arrange vertical corner reinforcements of the same order as that used horizontally (Fig. VII.8).

It is further recommended to add a mesh of min. 5 ∅ 8 mm/m on the com - pression side of the walls and to avoid the use of bent bars in vertical walls.

The details of reinforcements of a horizontal section of a deep square tank may be done in the manner shown in figure VII.9.

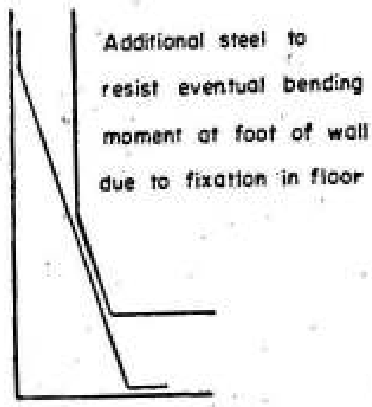
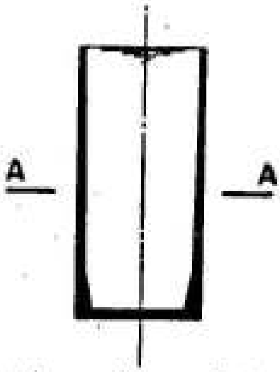
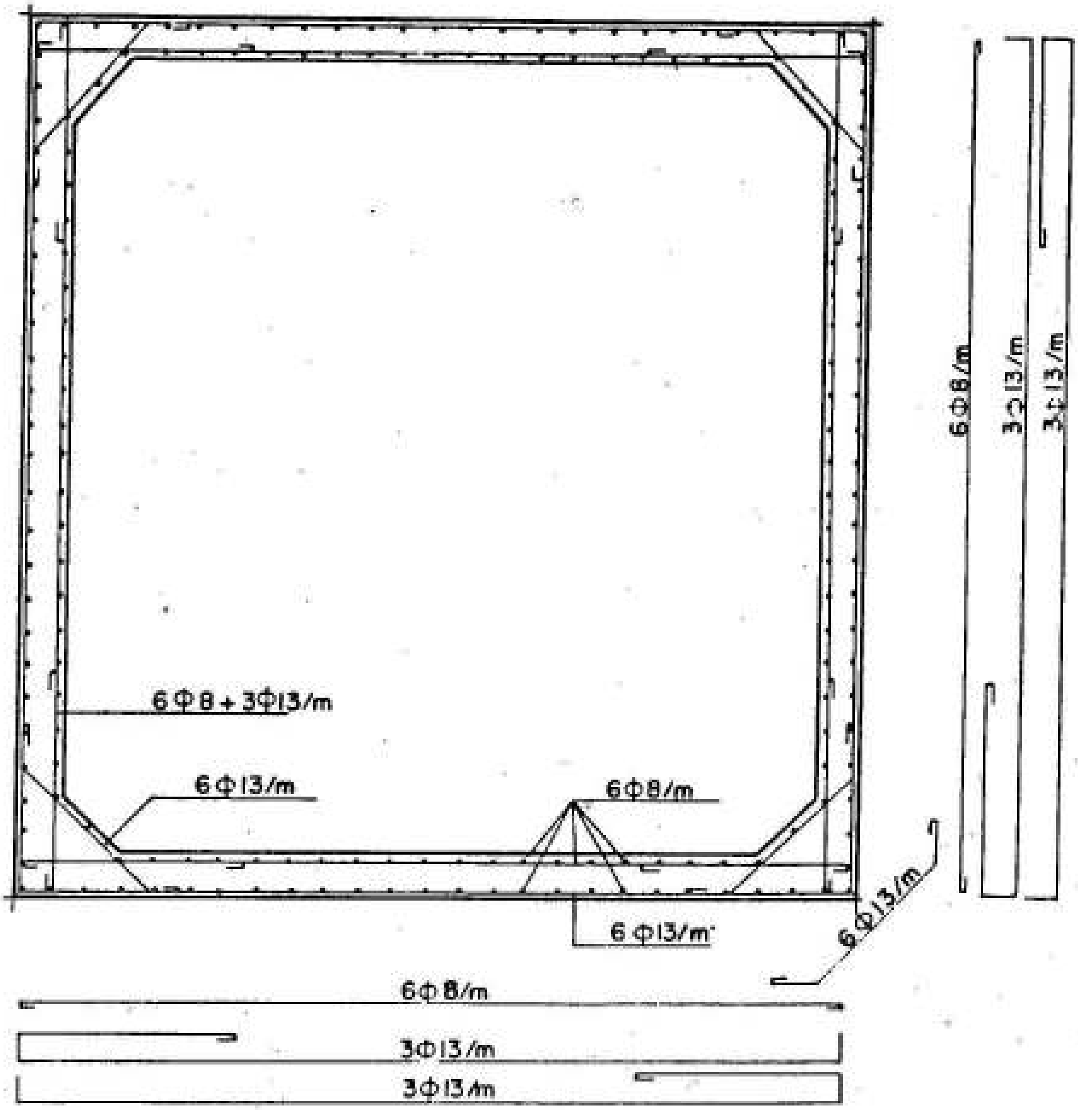


Fig. VII-8



Vertical section of tank



Details of horizontal section A - A

Fig. VII-9

VII.2. WALLS RESISTING HYDROSTATIC PRESSURE IN VERTICAL DIRECTION

a) Cantilever Walls (Fig. VII.10)

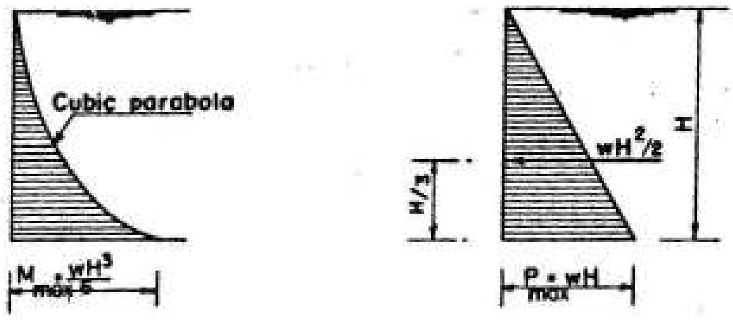


Fig. VII-10

Walls fixed to the floor may act as simple cantilevers such wall give a convenient thickness (max. ≈ 40 cms.) for a depth of water smaller than 3.0 ms.

The max. bending moment at the foot of the wall is given by :

$$M_{max.} = w\ H^3 / 6 \qquad (7a)$$

Reaction at base = Tension in floor, is given by :

$$R_{max.} = w\ H^2 / 2 \qquad (7b)$$

The convenient cross-section of the wall is trapezoidal with minimum thickness at top (15 - 20 cms) and maximum at bottom , or rectangular - of constant thickness - for small depths and eventually provided with a convenient haunch at its foot.

For a cantilever wall of a water tank 3 ms deep, we get :

$M_{max} = w\ H^3 / 6 = 1 \times 3^3 / 6 = 4.5$ m t

$t_{max} = \sqrt{M / 3} = \sqrt{4500 / 3} = 39$ cms

chosen 40 cms.

The wall can be chosen trapezoidal with a minimum thickness of 15 cms at the top, and a max. thickness of 30 cms at the bottom, and provided with a haunch 10 x 45 cms. at its base as shown in figure VII.11.

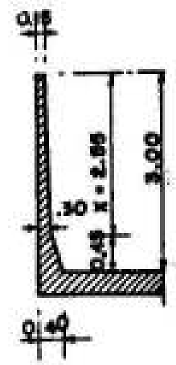


Fig. VII-11

Checking the section at beginning of haunch we get :

$M = w\,x^3 / 6 = 1 \times 2.55^3 / 6 = 2.76$ m t

Required $t = \sqrt{M/3} = \sqrt{2760 / 3} \approx 30$ cms.

It has to be noted that free cantilevers of height H and supported on the two sides of their length L must be treated as slabs supported on three sides if their length L is smaller than four times their height H.

Exact values of internal forces in walls free at top and fixed on the other three sides subject to hydrostatic pressure will be given later for ratios of H/L ($= L_y / L_x$) varying between 0.25 and 1.5.

The fixing moments at the vertical edges of cantilever walls fixed at these edges and subject to hydrostatic pressure are to be considered in the design of water tanks because their values are relatively big.

To give an idea about the values of the exact bending moments we consider a wall of depth H = 3 ms. and length L = 12 ms. free at top and fixed on the other three sides subject to hydrostatic pressure.

Max. fixing moment at foot of wall ;

$M_f = - w\,H^3 / 6.9 = - 1 \times 3^3 / 6.9 = - 3.9$ m t

Max. fixing moment at top of vertical edges (in horizontal direction):

$M_f = - w\,H^3 / 8.8 = - 1 \times 3^3 / 8.8 = - 3.06$ m t

The fixing moment at mid-height of vertical edges (in horiz.direction)

$M_f = - w\,H^3 / 18.2 = - 1 \times 3^3 / 18.2 = - 1.48$ m t

b) Walls Simply Supported at Top and Fixed at Bottom

Walls acting as one way slabs simply supported at top and fixed at bottom, and resisting the full hydrostatic pressure in the vertical direction only give a convenient solution for depths of water $H \leqslant 4.5$ ms. (Fig. VII.12).

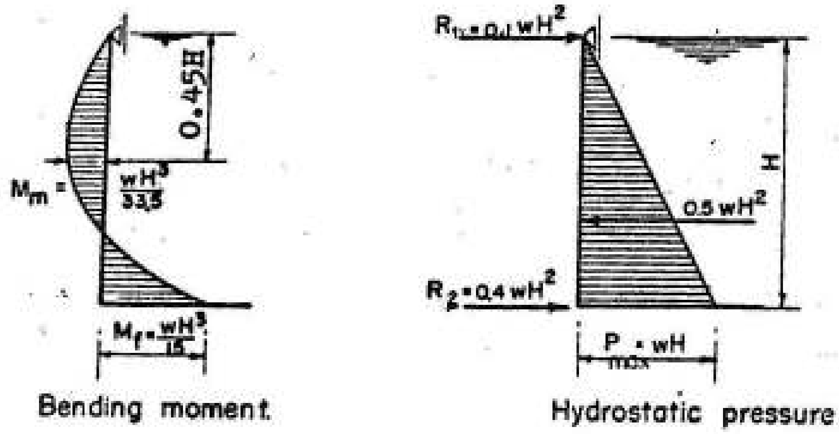


Fig. VII.12

For a wall of constant thickness t :

The max. fixing moment at base :

$$M_f = -\, w\, H^3 \,/\, 15 \tag{8a}$$

The max. field moment at 0.45 H from top

$$M_m = +\, w\, H^3 \,/\, 33.5 \tag{8b}$$

The reaction at top edge

$$R_1 = 0.10\; w\; H^2 \tag{8c}$$

The reaction at the bottom edge

$$R_2 = 0.40\; w\; H^2 \tag{8d}$$

In such walls , it is generally more convenient to chose a wall of constant thickness sufficient to resist the field moment M_m , the bigger value of the fixing moment can be resisted by making a haunch at the foot of the wall. If the wall is chosen trapezoidal with a bigger thickness at the bottom, the fixing moment is increased and the field moment is decreased* as shown in the following table :

$\vartheta = t_2 / t_1$	1.0	1.5	2.0	2.5	3.0
$M_f = w H^3$:	15	13.2	12	11.2	10.7
λ	1.0	1.14	1.25	1.34	1.40

in which : $\vartheta = \frac{t_2}{t_1} = \frac{\text{thickness of wall at bottom}}{\text{" " " " top}}$

$$\lambda = \frac{\text{real } M_f \text{ considering variation of } t}{M_f \text{ for wall of constant } t}$$

The table shows that if the variation of the thickness is neglected serious errors in M_f might take place. Such errors have a bad effect on the design especially bacause M_f causes tension on the water side.

The sedimentation tank shown in figure VII.13 gives a typical example for such walls.

In this tank, the walls, 4.0 ms. deep, are fixed to the floor at their bottom edge and supported on a horizontal beam at their top edge. In order to have adequate supports for this beam, ties are arranged in both directions at distances of 5.0 ms. at the upper surface of the tank. Columns at the points of intersection of the ties prevent their excessive saging.

The top beam behaves as a continuous beam spanning between the ties and loaded by the top reaction of the wall, ($p = R_1$). For equal spans L, we get :

---

* Refer to : " Theory of Elastically Restrained Beams ". by M. Hilal, Published by Fouad I University, 1944.

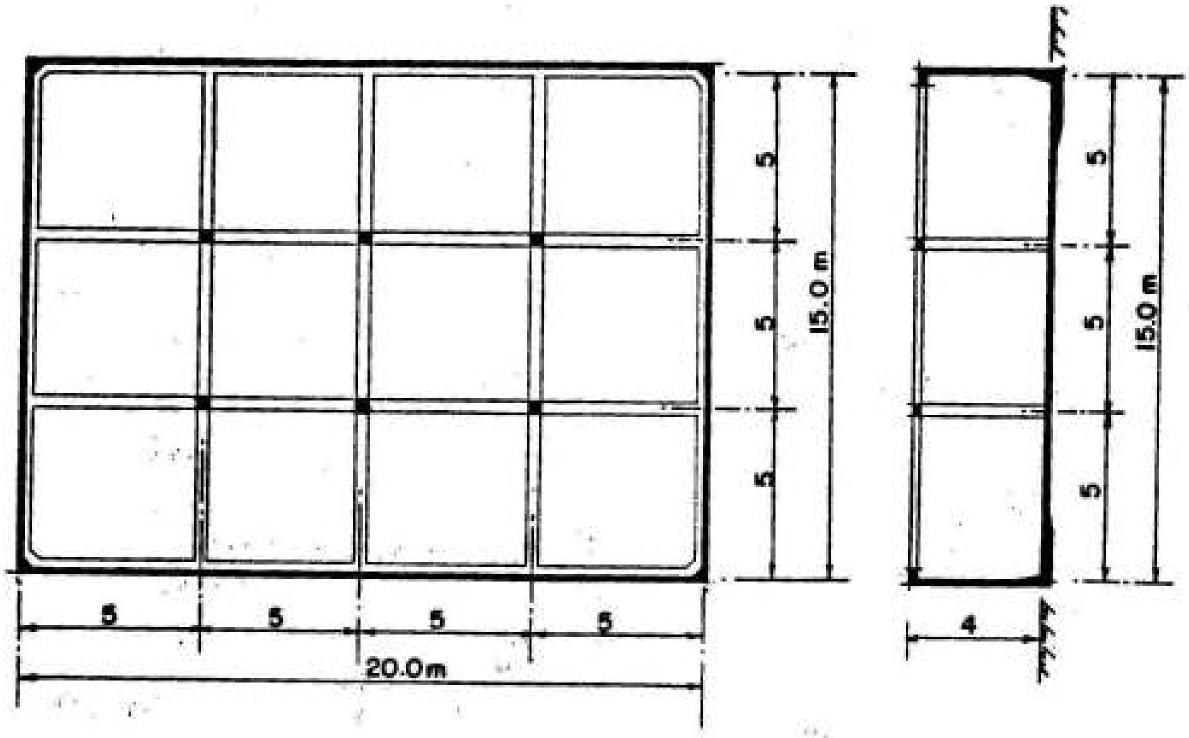


Fig. VII-13

$$\left.\begin{aligned} &\text{Field moment} \quad & M_m &= + p\,L^2 / 24 \\ &\text{Connecting "} \quad & M_c &= - p\,L^2 / 12 \\ &\text{Tension in each side} \quad & T &= p\,L / 2 \end{aligned}\right\} \qquad (9)$$

The fixing moment at the vertical edges of one way slabs simply supported at the top and fixed at the other three sides, subject to hydrostatic pressure may be estimated by :

$$M_f = w\,H^3 / 27 \qquad (10a)$$

and lies at the middle of the height of the wall.
The corresponding reaction is given by :

$$R = 0.27\,w\,H^2 \qquad (10b)$$

The necessary provisions are to be taken to resist these values safely.

c) Walls Fixed at Top and Bottom

If the wall in the previous example were fixed at top & bottom the bending moments and reactions due to a hydrostatic pressure $p = w\,H$, for the case of constant thickness, are given by : (Fig. VII.14).

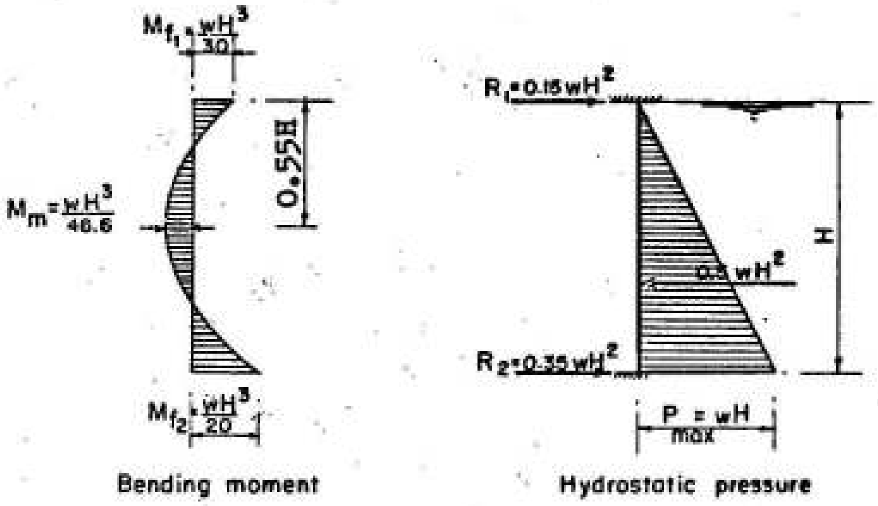


Fig. VII-14

Fixing moment at top :

$$M_{f1} = -\, w\,H^3 / 30 \qquad (11a)$$

Fixing moment at bottom :

$$M_{f2} = -\, w\,H^3 / 20 \qquad (11b)$$

Max. field moment at 0.55 H from top

$$M_m = +\, w\,H^3 / 46.6 \qquad (11c)$$

Reaction at top edge :

$$R_1 = 0.15\, w\,H^2 \qquad (11d)$$

Reaction at bottom edge :

$$R_2 = 0.35\, w\,H^2 \qquad (11e)$$

For a trapezoidal wall, we get :

$\vartheta = t_2 / t_1$	1.0	1.5	2.0	2.5	3.0
$M_{f1} = w H^3$	30	38.7	52.6	63.6	76.5
$M_{f2} = w H^3$	20	16.7	14.7	13.7	12.8
λ_{base}	1.0	1.20	1.36	1.46	1.56

Neglecting the effect of the variation of the wall thickness, the possible error in the magnitude of the fixing moment at the base may be very serious.

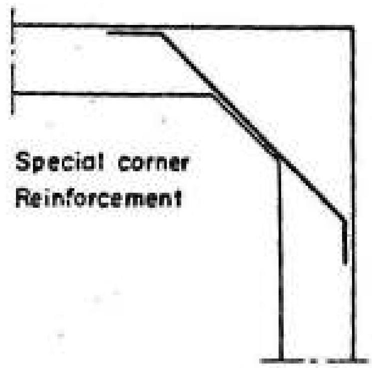
The fixing moment at the vertical edges of one way slabs fixed at all four sides and subject to hydrostatic pressure may be estimated from the relation :

$$M_f = - w H^3 / 33.8 \qquad (12a)$$

and lies at 0.4 H from the base of the wall. The corresponding reaction is given by :

$$R = 0.25 w H^2 \qquad (12b)$$

In walls of tanks resisting the water pressure in the vertical direction, it is essential to arrange horizontal secondary reinforcements having a minimum cross-sectionnal area 20% of the main vertical steel (and not less than 5 ∅ 8 mm/m) to fix the vertical reinforcements in position and to resist the temperature and shrinkage stresses as well as the eventual tensile stresses due to the reactions of the cross walls. The horizontal corner reinforcements may be calculated from the given relations. It is however generally



Sec. Plan at the corner of two walls

Fig. VII-15

sufficient to arrange corner reinforcements of the same order as the max. used vertically.(Figure VII.15)

d) Walls Continuous with Roof & Floor

The walls of a tank may however be continuous with the roof and the floor or with one of them as shown in the tank given in Fig. VII.16.

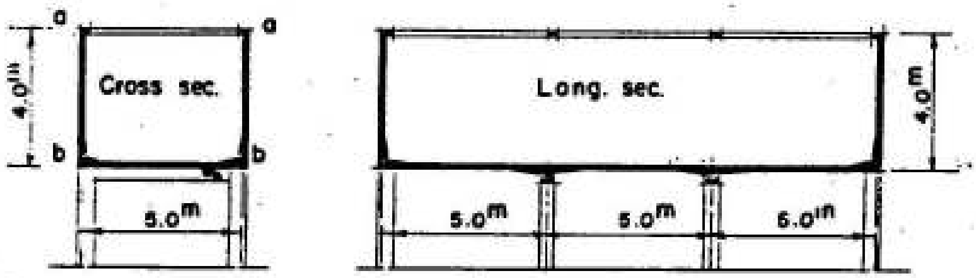


Fig. VII-16

In this tank the longitudinal walls are simply supported on the top horizontal beam and continuous with the floor slab. In order to be able to assume that the top beam is a rigid support, ties are arranged every 5 ms.

The effect of continuity can be calculated by any of the known methods of the theory of structures; e.g. the method of virtual work the moment distribution method, the equation of three moments .. etc.

VII.3. WALLS AND FLOORS RESISTING HYDROSTATIC PRESSURE IN TWO DIRECTIONS

a) The Strip Method :

This method gives an approximate solution for rectangular flat plates of constant thickness supported on four sides and subject to uniform or hydrostatic pressure. The solution is based on simplified assumptions generally adopted in structural problems.

Walls and floors supported on the four sides and having a ratio of length to breadth smaller than two are treated in this method as two way slabs.

In rectangular slabs with side lengths L_x and L_y subject to uniform loads (e.g. floor slabs), the load may be distributed in the two directions L_x and L_y according to the known coefficients of Grashoff. Due to the torsional resistance of the slab,the field (positive) moment may be reduced according to coefficients of Marcus as follows :

$$\text{reduced } M_x = r_x M_x = \left[1 - \frac{5}{6} \left(\frac{L_x}{L_y}\right)^2 \frac{M_x}{M_{xo}} \right] M_x \qquad \text{and}$$

$$\text{reduced } M_y = r_y M_y = \left[1 - \frac{5}{6} \left(\frac{L_y}{L_x}\right)^2 \frac{M_y}{M_{yo}} \right] M_y$$

in which M_x and M_y are the field moments if the effect of the torsional resistance is neglected, and

$$M_{xo} = p\, L_x^2 / 8 \qquad \& \qquad M_{yo} = p\, L_y^2 / 8$$

where

p = total load on floor slab per m^2

For a slab totally fixed on all four sides; the reduction factors r can be extracted from the following table :

L_x / L_y	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2.0
$r_x = r_y$	.86	.86	.87	.88	.89	.90	.91	.91	.92	.93	.94

It has been found that the load distribution on two way slabs subject to triangular load may approximately be assumed the same as in case of uniform laads using the coefficients <u>of Grashoff</u> i.e.

$$p = p_v + p_h$$

in which

p = the hydrostatic pressure at any depth

p_v = part of the pressure resisted in the vertical direction.

p_h = " " " " " " " horizontal " .

As the wall is rigidly connected to the floor, no pressure can be resisted horizontally in the lowest strip and accordingly one can assume that the pressure resisted horizontally is the triangle a e b only and the small triangle e c b is resisted vertically (Figure VII. 17). The position of point e varies according to ratio of sides of slab; it lies at about 3/4 H from top in deep tanks where $\frac{H}{L} \approx 2$ and at about $\frac{H}{2}$ in shallow tanks where $\frac{H}{L} \approx \frac{1}{2}$. It is however recommended to design the middle part of the wall (say the middle half) for a load equal to 3/4 p_h.

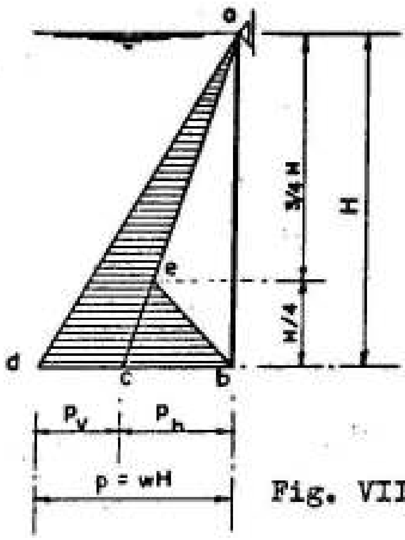


Fig. VII-17

The bending moments and reactions in the vertical direction due to p_v (triangle a c d) can be determined as shown in VII.2-b; whereas the bending moments and reactions due to the small triangle e c b, for a wall of constant thickness simply supported at top and fixed at bottom, can be determined as follows : Fig. VII.18

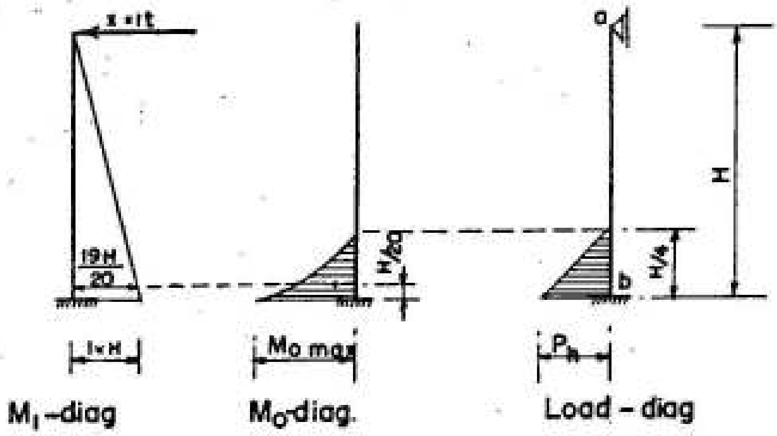


Fig. VII-18

Using the method of virtual work and choosing the simple cantilever as a main system, we get :

Max. bending moment of main system is :

$$M_{o\ max.} = p_h\ (H/4)^2\ /\ 6 \quad = p_h\ H^2\ /\ 96$$

$$\text{Area of } M_o - \text{diagram} = \frac{1}{4} M_o \text{ max.} \frac{H}{4} = p_h \frac{H^3}{1536}$$

Its center of gravity lies at 1/5 (H/4) i.e.

$$E\ I\ \delta_o = \int M_o\ M_1\ d\ H = -\ p_h \frac{H^3}{1536}\ \frac{19\ H}{20} = -\ ph \frac{19\ H^4}{30720}.$$

and

$$E\ I\ \delta_1 = \int M_1^2\ d\ H \quad = \quad \frac{H\ .\ H}{2} . \frac{2\ H}{3} = \quad \frac{H^3}{3}$$

i.e. the statically indeterminate reaction X at a is given by :

$$X = -\ \frac{\delta_o}{\delta_1} = p_h \frac{19\ H^4}{30720}\ \frac{3}{H^3} \qquad = p_h\ \frac{H}{540}$$

The fixing moment at the fixed end b is therefore :

$$M_f = p_h \frac{H}{540}\ H\ -\ ph \frac{H^2}{96} \qquad = -\ p_h \frac{H^2}{117}$$

i.e. the total fixing moment at b for a wall of constant thickness due to the pressures resisted vertically (triangles a c d + e b c) is given by :

$$M_f = -\ (p_v \frac{H^2}{15} + p_h \frac{H^2}{117}) \qquad (13a)$$

and the corresponding reaction at a is

$$R_a = p_v \frac{H}{10} + p_h \frac{H}{540} \qquad (13b)$$

If the wall were rigidly connected to the floor, the effect of continuity can be considered by any of the known methods of the theory of elasticity. The max. bending moments in the horizontal direction for the middle half of the wall can be determined for the maximum pressure on the horizontal strip at 3/4 H i.e. for max. $p_h = 3/4\ p_h$.

Example :

It is required to design the water tank shown in figure VII.16.

1) Internal Forces in Cross-Sections of Tank (Fig. VII.19)

The longitudinal wall of the tank (4 x 15 m) is a one way slab resisting the water pressure in the vertical direction only.
The floor is a series of two way slabs (5 x 5 m) resisting half weight of water and floor in each direction. Assuming the wall simply supported at a and totally fixed at b and subject to hydrostatic pressure with :

$$p_{max} = w\ H = 4\ t/m^2 \quad \text{at} \quad b\ ,$$

the fixing moment at b will be :

$$M_f = -\ w\ H^3\ /\ 15 = -\ 1 \times 4^3\ /\ 15 = -\ 4.27\ m\ t$$

The load on the floor = weight of water + own weight of floor (~ 0.7 t/m²)
or

$$= 4 + 0.7 = 4.7\ t/m^2 \qquad \text{i.e.}$$

Load in each direction $p = 4.7\ /\ 2 = 2.35\ t/m^2$

Assuming the floor slab totally fixed at both ends, the fixing moment at b will be :

$$M_f = -\ p\ L^2\ /\ 12 \qquad = -\ 2.35 \times 5^2\ /\ 12 = -\ 4.9\ m\ t$$

The unbalanced moment = 4.9 - 4.27 = 0.63 m t is to be distributed on wall and floor slab according to their relative stiffness, but its value being small, the connecting moment M_b may be assumed as the average of the two values, i.e.

$$M_b = -\ \frac{4.27 + 4.9}{2} \approx -\ 4.6\ m\ t$$

Due to symmetry, the connecting moment at b can be determined from one equation of three moments; thus

$2\ M_b\ (H + L) + M_b\ L = -\ 6\ (r_1 + r_2)$

in which

r_1 = the elastic reaction of the triangular load on wall

$= p_{max}\ H^3 / 45$, and

r_2 = the elastic reaction of the load on the floor

$= p\ L^3 / 24$, therefore

$$2\ M_b\ (4 + 5) + 5\ M_b = -\ 6\ \left(\frac{4 \times 4^3}{45} + \frac{2.35 \times 5^3}{24} \right) \quad \text{or}$$

$$25\ M_b = -\ 6\ (\ 5.8 + 12.3\ ) = -\ 108.6 \quad \text{i.e}$$

$$\underline{M_b = -\ 4.7\ \text{m t}}$$

Accordingly, the field moment in the floor , neglecting its torsional resistance is given by :

$$M_m = p\ \frac{L^2}{8} - M_b = \frac{2.35 \times 5^2}{8} - 4.7 = 2.65\ \text{m t}$$

Due to the torsional resistance of the slab, the final reduced field moment will be :

$$M_{mr} = r\ M_m = 0.86 \times 2.65 \quad \text{or}$$

$$\underline{M_{mr} = 2.28\ \ \text{m t}}$$

The max. field moment in the wall takes place at point of zero shear, thus :

Reaction at top edge of wall may either be calculated by super-posi - tion and is given by :

$$R_a = 0.1\ w\ H^2 - \frac{M_b - M_f}{H} = 0.1 \times 1 \times 4^2 - \frac{4.7 - 4.27}{4} = 1.5\ \text{t/m}$$

or from the equation of moments about corner b :

$$R_a\ H - p_{max}\ H^2 / 6 = M_b \quad \text{or}$$

$$R_a \times 4 - 4 \times 4^2 / 6 = -\ 4.7 \quad \text{or}$$

$$\underline{R_a = 1.5\ \text{t/m}}$$

If the point of zero shear lies at x meters from top end, then

$R_a = 1.5 = \frac{x^2}{2}$ or $\underline{x = 1.73 \text{ ms}}$

The max. field moment in the wall is therefore given by :

$M_m = R_a\, x - p_x \frac{x^2}{6} = \frac{x^2}{2} x - \frac{x^3}{6} = \frac{x^3}{3}$ i.e.

$M_m = 1.73^3 / 3$ or $\underline{M_m = 1.73 \text{ m t}}$

The reaction at the bottom edge of wall = tension in floor = water pressure - top reaction

$R_b = p_{max}\, H/2 - R_a = 4^2/2 - 1.5 = 6.5$ t/m

Therefore : Tension in floor = $\underline{6.5 \text{ t/m}}$

We have further :

Reaction of floor = Tension in wall

Average reaction of floor = 4.7 x 2.5/2 = 5.90 t/m

This reaction is parabolically distributed in each span and the max. ordinate at the middle is 1.5 times the average value.

Therefore, the max. tension in the wall = 1.5 x 5.90 = 8.85 t/m .

The loads, bending moments and reactions are shown in figure VII.19 .

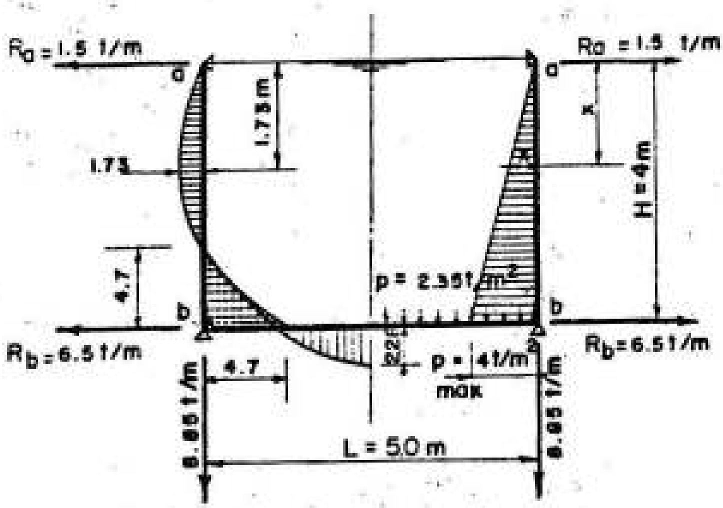


Fig. VII-19

2) Internal Forces in Longitudinal Section of Tank

The side wall with H = 4 ms and L = 5 m is a two way slab. The load will be distributed according to Grashoff, Fig. VII.20 , thus :

For L/H = 5/4 = 1.25 , we get :

$\alpha = 0.7$ and $\beta = 0.3$ i.e.

Load in vertical direction :

$p_v = \alpha\, p_{max} = 0.7 \times 4 = 2.8\ t/m^2$

Load in horizontal direction :

$p_h = \beta\, p_{max} = 0.3 \times 4 = 1.2\ t/m^2$

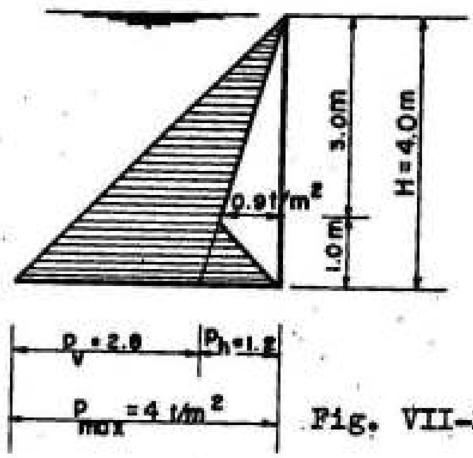


Fig. VII-20

Assuming that the thickness of wall and floor is the same and applying the equation of three moments at b and c, fig. VII.21, we get :

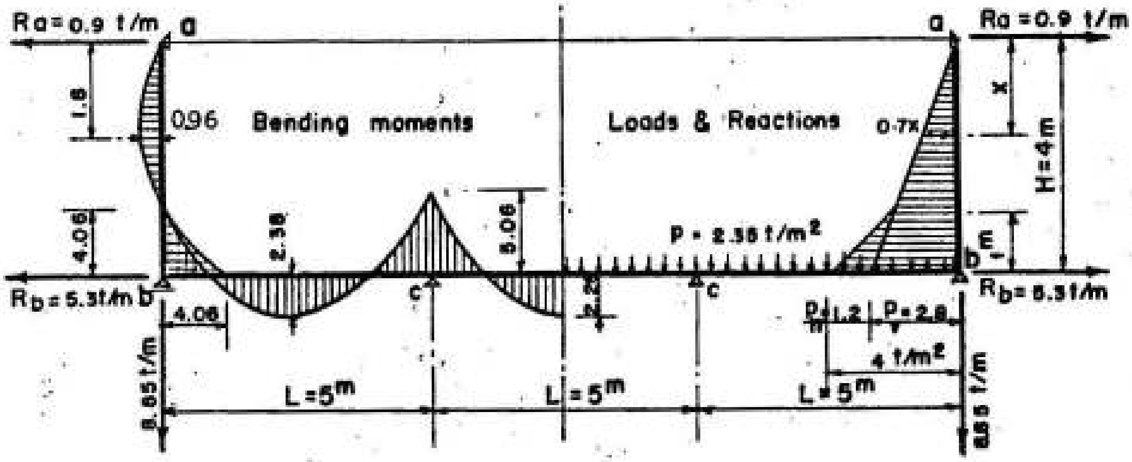


Fig. VII-21

$$\text{sec. b : } 2\,M_b\,(H + L) + M_c\,L = -6\left(p_v\frac{H^3}{45} + p_h\frac{H^3}{350} + p\frac{L^3}{24}\right)$$

$$\text{or}\quad 2\,M_b\,(4 + 5) + 5\,M_c = -6\left(\frac{2.8 \times 4^3}{45} + \frac{1.2 \times 4^3}{350} + \frac{2.35 \times 5^3}{24}\right)$$

$$\text{or}\quad 18\,M_b + 5\,M_c = -98.6 \qquad \text{(a)}$$

$$\text{sec. c : } M_b\,L + 2\,M_c \times 2\,L + M_c\,L = -6\left(2\,p\frac{L^3}{24}\right) = -\frac{2.35 \times 5^3}{2}$$

$$\text{or}\quad 5\,M_b + 25\,M_c = -147 \qquad \text{(b)}$$

Equations (a) and (b) give :

$\underline{M_b = -4.06 \text{ m t}}$ and $\underline{M_c = -5.06 \text{ m t}}$

Max. moment in b c taking its torsional resistance in consideration is given by :

$$M_{m1} = 0.86 \left[\frac{2.35 \times 5^2}{8} - \frac{4.06 + 5.06}{2} \right] = 0.86\ (7.32 - 4.56)$$

or $\underline{M_{m1} = 2.38 \text{ m t}}$

Field moment in c - c is :

$$M_{m2} = \frac{2.35 \times 5^2}{8} - 5.06 = 7.32 - 5.06$$

or $\underline{M_{m2} = 2.26 \text{ m t}}$

N.B. The torsional resistance of span c - c has been neglected in order to have regular reinforcements for the floor slab.

Average reaction at outside support of floor slab is :

$R = 0.5 \times 2.5 \times 4.7 = 5.9$ t/m

Max. reaction = max. tension in wall = 1.5 R = 1.5 x 5.9

Therefore $\underline{\max .\ R = 8.85 \text{ t/m}}$

Reaction at a is determined from the equation of moments about b , thus :

$$R_a \times 4 - 2.8 \times \frac{4^2}{6} - 1.2 \times \frac{1^2}{6} = -4.06$$

or

$\underline{R_a = 0.9 \text{ t/m}}$

The reaction at the base of the wall = the tension in the floor and is given by :

$$R_b = 2.8 \times \frac{4}{2} + 1.2 \times \frac{1}{2} - 0.9$$

or

$\underline{R_b = 5.3 \text{ t/m}}$

The max. field moment M_{ma} in the wall takes place at the point of zero shear which lies at x ms from top, thus :

$0.7 \frac{x^2}{2} = R_a = 0.9$ or $x = 1.6$ ms and

$M_m = 0.9 \times 1.6 - 0.7 \times \frac{1.6^3}{6} = 1.44 - 0.48$

or $\underline{M_m = 0.96 \text{ m t}}$

The loads, bending moments and reactions are shown in figure VII.21.

3) Internal Forces in Horizontal Section of Tank

The section shown in figure VII.22 at mid-height of tank will be calculated for a horizontal load $p = 0.9$ t/m^2 acting on the cross wall. (Refer to fig. VII.20).

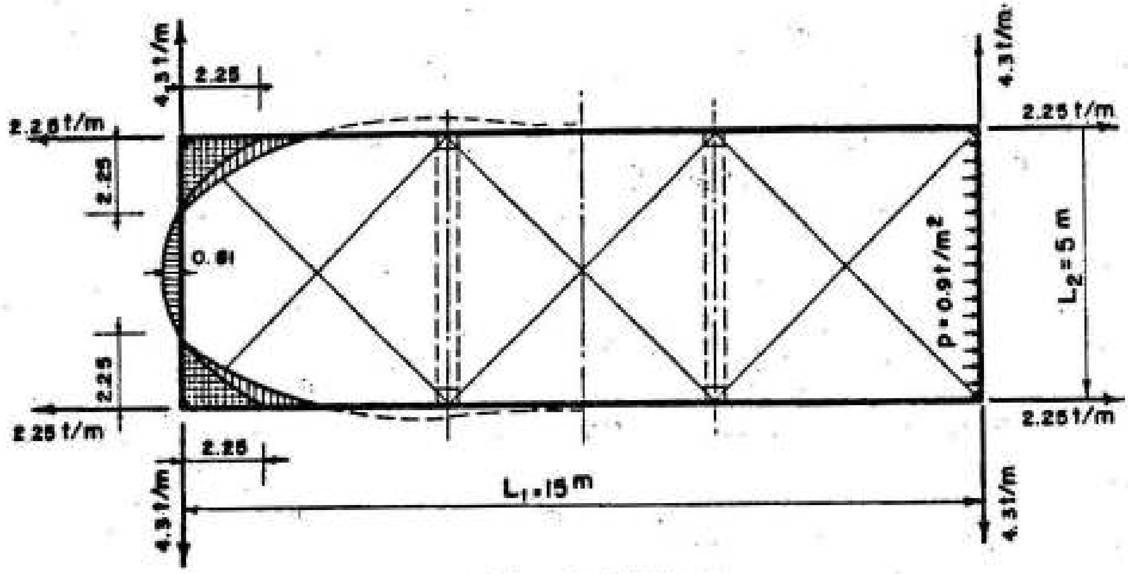


Fig. VII-22

Assuming that the wall L_2 is totally fixed at both ends, then the fixing moment M_f is given by :

$M_f = - p L_2^2 / 12 = - 0.9 \times 5^2 / 12 = - 1.88$ m t

The longitudinal wall L_1 is a one way slab resisting the full water pressure in the vertical direction, but due to its rigid connection to the cross-walls, bending moments will be created at the edges in the horizontal direction. Their magnitude may be estimated according to (10a) from the relation :

$M_f = - w H^3 / 27 = - 1 \times 4^3 / 27 = - 2.37$ m t

for a wall totally fixed at its vertical edges.

The difference of : $\Delta M = 2.37 - 1.88 = 0.49$ m t will be distributed between the two walls according to their relative stiffness, thus:

Stiffness of long. wall $\propto$ 1/15

" " cross wall $\propto$ 1/ 5 = 3/15

The distribution factors are therefore :

Long. wall 1/4 and cross wall 3/4

Accordingly, the connecting moment at the corner c will be :

$$\underline{M_c = 2.25 \text{ m t}}$$

and the horizontal field moment is :

$$M_m = p\, L_2^2/8 - M_c = \frac{0.9 \times 5^2}{8} - 2.25 = 2.82 - 2.25 = 0.57 \text{ m t}$$

The field moment should however be not smaller than the reduced moment of a totally fixed slab i.e.

$$\text{min. } M_m = 0.87\, p\, L_2^2/24 = 0.87 \times 0.9 \times 5^2/24$$

or

$$\underline{M_m = 0.81 \text{ m t}}$$

Reaction of cross wall = Tension in long. wall and is given by :

$$R = 0.9 \times 5/2 = 2.25 \text{ t/m}$$

Reaction of long. wall = Tension in cross wall and according to equation (10b) is given by :

$R = 0.27\, w\, H^2 = 0.27 \times 1 \times 4^2$ at mid-height ,

or

$$\underline{R = 4.3 \text{ t/m}}$$

The loads, bending moments and reactions are shown in figure VII.22 .

4) Internal Forces in Top Horizontal Beam and Ties

The load on the top horizontal beam is equal to the top reaction R_a of the walls (Fig. VII.23)

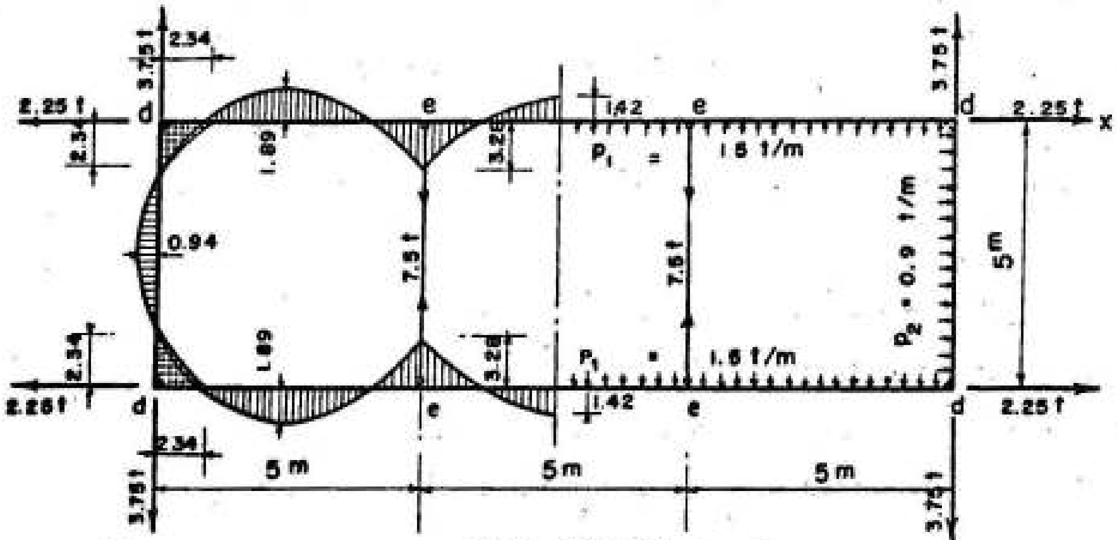


Fig. VII-23

Applying the equation of three moments at d and e, we get :

$$5\,M_d + 20\,M_d + 5\,M_e = -6\left(\frac{0.9 \times 5^3}{24} + \frac{1.5 \times 5^3}{24}\right)$$

or $\quad 5\,M_d + M_e = -15$ and

$$5\,M_d + 20\,M_e + 5\,M_e = -6\left(2 \times \frac{1.5 \times 5^3}{24}\right)$$

or $\quad M_d + 5\,M_e = -18.75$ giving

$$\underline{M_d = -2.34} \text{ m t} \quad \text{and} \quad \underline{M_e = -3.28} \text{ m t}$$

Therefore, the field moment M_m in the different spans is given by :

Span d - d : $M_m = \dfrac{0.9 \times 5^2}{8} - 2.34 = 0.48$ m t

but min. $M_m = p_2 \dfrac{L^2}{24}$ or

$$\text{min. } M_m = \frac{0.9 \times 5^2}{24} = \underline{0.94} \text{ m t}$$

Span d - e : $M_m = \frac{1.5 \times 5^2}{8} - \frac{2.34 + 3.28}{2} = \underline{1.89}$ m t

Span e - e : $M_m = \frac{1.5 \times 5^2}{8} - 3.28 = \underline{1.42}$ m t

Tension in span d-d :

$T_2 = 1.5 \times 5/2 \quad = 3.75$ t

Tension in span d-e and e-e :

$T_1 = 0.9 \times 5/2 \quad = 2.25$ t

Tension in ties :

$T = 1.5 \times 5 \quad = 7.5$ t

5) <u>Design of the Different Elements of the Tank</u>

The thickness of the walls and floors is to be determined for the max. field moments and the corresponding normal forces, haunches are provided to resist the bigger values of the connecting moments causing tension on the water side.

In order to have sufficient water-tightness, the walls and floor are assumed 25 cms thick.

A.) <u>Design of Longitudinal Walls</u>

Section of max. field moment $\underline{M_m = 1730}$ kgm.

The tensile stresses caused by the tensile force of 8850 kg/m acting at the foot of the wall reduce in the upper sections of the wall and vanish at the top. The tensile force at position of maximum moment is therefore given by :

$N = 8850 \times \frac{1.73}{4}$ or $N = 3850$ kg/m

The tensile stresses in this section being on the air side, it is designed as ordinary reinforced concrete in stage II thus :

Eccentricity $e = M/N = 1730/3850 = 0.45$ ms

Eccentricity to tension steel :

$$e_s = e - \frac{t}{2} + \text{cover} = 45 - \frac{25}{2} + 3 = 35.5 \text{ cms}$$

Moment about tension steel :

$$M_s = N\ e_s = 3850 \times 0.355 = 1370 \text{ kgm.}$$

$d = k_1 \sqrt{M_s}$ or $22 = k_1 \sqrt{1370}$ giving $k_1 = 0.53$

for $\sigma_s = 1400$ kg/cm^2 and $\alpha = 0$, we get :

$\sigma_c = 31$ kg/cm^2 and $k_2 = 1285$ so that :

$$A_s = \frac{M_s}{k_2\ d} + \frac{N}{\sigma_s} = \frac{1370}{1285 \times .22} + \frac{3850}{1400}$$

or

$A_s = 4.9 + 2.75 = 7.65$ cm^2 6 ø 13 mm/m

Section at foot wall : <u>M = 4700</u> kgm <u>N = 8850</u> kgs

The tensile stresses in this section being on the water side , they should be smaller than the tensile strength of concrete in bending, thus :

$$t = \sqrt{\frac{M}{3}} + 2 \text{ cms} = \sqrt{\frac{4700}{3}} + 2 = 40 + 2 = 42 \text{ cms}$$

The thickness is chosen 45 cms and the wall will be provided with a convenient haunch.

$$e = \frac{M}{N} = \frac{4700}{8850} = 0.53 \text{ ms}$$

$$e_s = 53 - \frac{45}{2} + 3 = 32.5 \text{ cms}$$

$$M_s = M\ e_s = 8850 \times 0.3250 = 2880 \text{ kgm}$$

$$42 = k_1 \sqrt{2880} \text{ giving } k_1 = 0.78$$

For $\sigma_s = 1400$ kg/cm^2 σ_c = low and $k_2 = 1300$, so that :

$A_s = \frac{2880}{1300 \times .42} + \frac{8850}{1400} = 5.3 + 6.3 = 11.6 \ cm^2$ 6 Ø 16 mm/m

In the horizontal direction, the middle part of the longitudinal wall will be designed for a tensile force :

$$\underline{N = 2250} \ kgs$$

the bending moment is small and can be neglected, therefore , the horizontal steel reinforcement is given by :

$A_s = N/\sigma_s = 2250 / 1400 = 1.6 \ cm^2$

choose min. steel of 5 Ø 8 mm/m ($A_s = 2.5 \ cm^2$)

The corner, at the vertical edges, is to be designed for :

M = 2250 kgm   and   N = 2250 kgs (tension)

The tensile stresses being on the water side then,

$$t = \sqrt{M/3} + 1.5 \ cms = \sqrt{2250/3} + 1.5 = 27.5 + 1.5 = 29 \ cms$$

The vertical edges of the walls will be provided by a haunch 15 x 15 cms, the theoretical thickness at the corner will be :

$$t = 25 + \frac{15}{3} = 30 \ cms.$$

Proceeding in the same way as before, we get the required area of steel reinforcement, which is given by : $A_s = 7.3 \ cm^2$ 6 Ø 13 mm/m

B) Design of Cross Walls

The internal forces, chosen thickness and reinforcements are shown in the following table :

sense	Section	M m t	N tons	Side of tensile stresses	t cm	A_s cm^2	Rfmt
Vert.	middle	0.96	3.54*	air	25	5.32	3 Ø 13 + 3 Ø 10
"	foot	4.06	8.85	water	45	10.60	6 Ø 16
Horiz.	middle	0.81	4.30	air	25	4.45	6 Ø 10
"	vert.edge	2.25	4.30	water	30	7.95	6 Ø 13

* This value gives the tension at position of max. field moment and equals 8.85 x / 1.6 / 4

C) <u>Design of Floor Slab</u>

Sense	Section	M m t	N tons	Side of ten - sile stresses	t cm	A_s cm^2	Rfmt mm/m
Cross	middle	2.28	6.5	air	25	10.55	6 Ø 16
"	corner	4.70	6.5	water	45	10.95	6 Ø 16
Long.	middle	2.38	5.3	air	25	10.50	6 Ø 16
"	corner	4.06	5.3	water	45	9.35	6 Ø 16
"	interm. sup.	5.06	5.3	water	45	11.20	6 Ø 16

D) <u>Design of Cross Beam</u> L = 4.9 ms

The beam is assumed 30 x 80 cms

The load being triangular, the equivalent load for bending moment = water and floor load + own weight

$p = 0.67 \times 5 \times 4.7 + 0.3 \times 0.55 \times 2.5 = 15.7 + 0.40 = 16.1$ t/m

$M = p\,L^2 / 8 = 16.1 \times 4.9^2 / 8 = 48.2$ m t

Effective breadth of flange B $= L/3 = 4.9/3 = 1.63$ ms

$d = k_1 \sqrt{M/B}$ or $73 = k_1 \sqrt{\frac{48200}{1.63}}$ i.e. $k_1 = 0.425$

Using high grade steel with $\sigma_s = 2000$ kg/cm^2, we get :

$\sigma_c = 49.5$ kg/cm^2 and $k_2 = 1820$ kg/cm^2

Therefore ,

$A_s = M/k_2.d = 48200/1820 \times 0.73 = 37$ cm^2 choose 10 Ø 22

The average load on the beam is given by :

$p = 0.5 \times 5 \times 4.7 + 0.3 \times 0.55 \times 2.5 = 10.9 + 0.4 = 11.3$ t/m

The shearing force is therefore :

$Q = p\,L/2 = 11.3 \times 4.9/2 = 27.5$ t and the max. shear stress is :

$$\tau = \frac{Q}{0.87\ b\ d} = \frac{27500}{0.87 \times 30 \times 73}$$

$$= 14.5 \quad kg/cm^2$$

The corresponding diagonal tension will be resisted by four branch stirrups Ø 8 mm @ 20 cms and bent bars. The shear stress resisted by the stirrups :

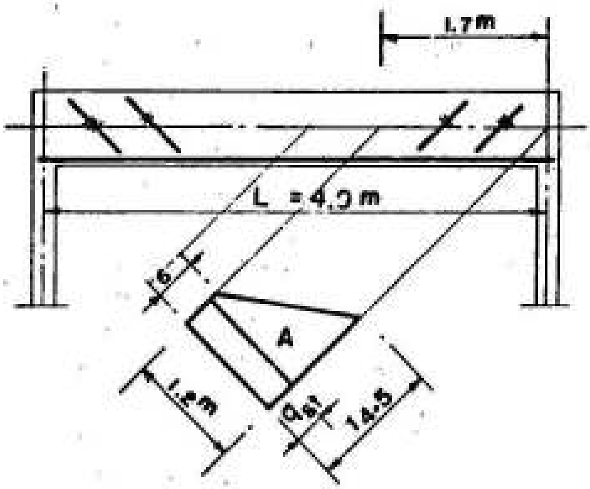


Fig. VII-24

$$\tau_{st} = \frac{n\ A_s\ \sigma_s}{e\ b}$$

in which

n	=	number of branches of stirrup	=	4
A_s	=	area of cross section of one branch	=	0.5 cm^2
e	=	spacing of stirrups	=	20 cm
b	=	breadth of web of beam	=	30 cm

i.e.

$$\tau_{st} = \frac{4 \times 0.5 \times 1400}{20 \times 30} = 4.67 \quad kg/cm^2$$

The bent bars resist the remaining area A of the diagonal tension diagram x b . Fig. VII.24, thus :

$$A_{sb} = \left(\frac{14.5 + 6}{2} - 4.67 \right) \frac{120 \times 30}{2000} = 10.0\ cm^2 \qquad \text{chosen} \quad 5\ \text{Ø}\,22$$

E) Walls as Deep Beams

The walls will act as deep beams (given later) supported on the columns.

Load p = water and floor load + own weight of wall

$$= 0.67 \times 2.5 \times 4.7 + 0.25 \times 4 \times 2.5 = 7.9 + 2.5 = 10.4 \quad t/m.$$

The max. tension at the bottom of a simple deep beam is :

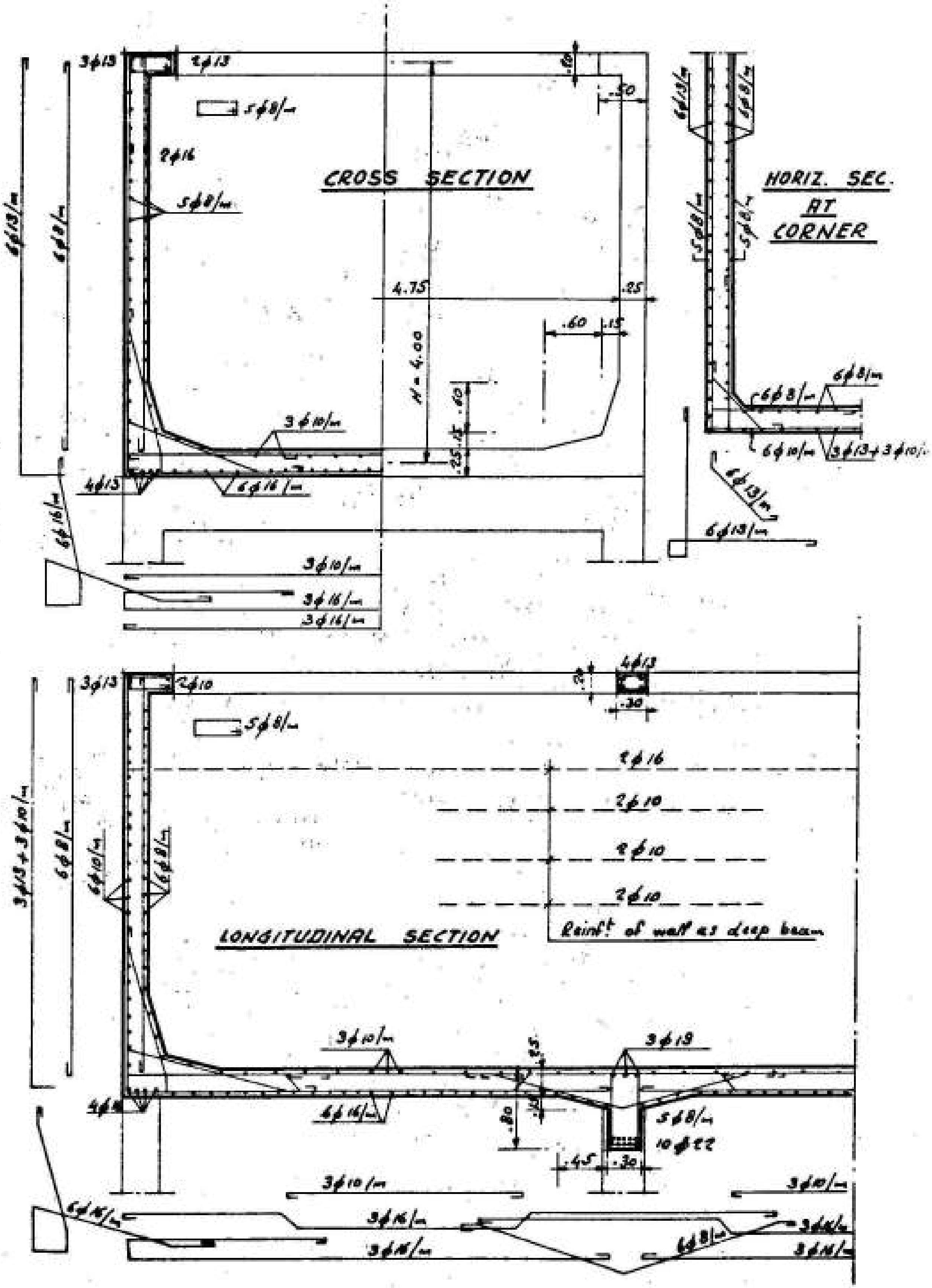

CROSS SECTION
HORIZ. SEC. AT CORNER
LONGITUDINAL SECTION
Reinf. of wall as deep beam

Fig. VII-25

$$T = 0.2\ p\ L = 0.2 \times 10.4 \times 5 = 10.4 \text{ ton}$$

$$A_s = T/\sigma_s = 10.4 / 1.4 = 7.4 \text{ cm}^2 \qquad \text{chosen } 4\ \varnothing\ 16$$

The max. tension at the bottom of the continuous wall at center line of spans is given by :

$$T = 0.13\ p\ L = 0.13 \times 10.4 \times 5 = 6.8 \text{ ton}$$

$$A_s = T / \sigma_s = 6.8 / 1.4 = 4.9 \text{ cm}^2 \qquad \text{chosen } 4\ \varnothing\ 13$$

The tension over intermediate supports is

$$T = 0.2\ p\ L \quad \text{giving } A_s = 7.4 \text{ cm}^2 \quad \text{or} \quad 2\ \varnothing\ 16 \ + \ 6\ \varnothing\ 10$$

F) <u>Design of Top Horizontal Beam and Ties</u>

It is recommended to avoid the use of bent bars in this beam, this means that the shear stresses must be low. Such a provision can be satisfied if the depth of the beam is chosen sufficiently big.

Assuming the beam to be 20 x 50 cms, the reinforcements at the different sections will be as follows. (See fig. VII.23).

Section at	M m t	N t	A_s cm^2	Rft
c in long. wall	3.28	2.25	6.5	2 Ø 13
d in cross wall	2.34	3.75	5.2	2 Ø 16
middle of d.e	1.89	2.25	3.94	
" " e.e	1.42	2.25	3.1	3 Ø 13
" " d.d	0.94	3.75	2.9	

The tie is chosen 20 x 30 cms, reinforced by $A_s = \frac{7.5}{1.4} = 5.3 \text{ cm}^2$ or

4 Ø 13.

The details of the tank are shown in figure VII-25.

b) General Theory of Flat Plates

The exact distribution of the internal forces in a slab subject to distributed - eventually concentrated - loads under different edge conditions is dealt with in text books of theory of elasticity e.g. " Theory of Plates and Shells " and " Theory of Elasticity " by Timoshenko.

For a rectangular plate subject to a distributed load p and having any given boundary conditions, the internal forces and stresses in any direction can be obtained mathematically according to the following assumptions and basic equations : (Fig. VII.26).

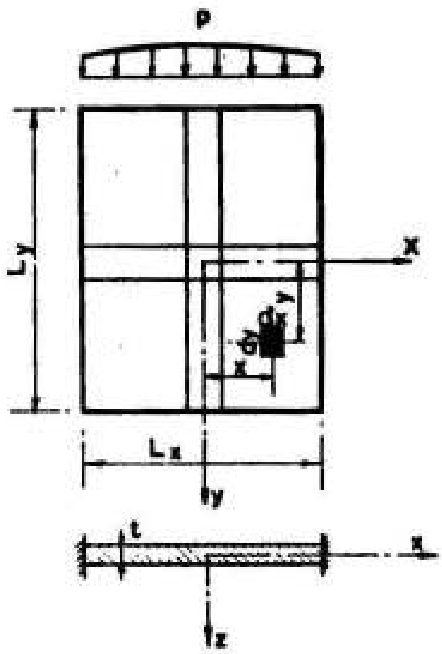


Fig. VII-26

$\nu = 1/m = 1/5 - 1/6$ = Poissons ratio

$D = \frac{E\ t^3}{12\ (1-\nu^2)}$ = Flexural rigidity of a plate of thickness t.

z = Lateral displacement of plate.

M_x , M_y = Bending moments per unit length of sections of a plate in directions x and y respectively.

M_{xy}, M_{yx} = Twisting moment per unit length of a section of a plate .

Q_x , Q_y = Shearing forces parallel to z - axis per unit length of sections of a plate perpendicular to x and y axes respectively.

R_x , R_y = Reactions per unit length of plate, acting on sections perpendicular to x and y axes respectively.

Considering the equilibrium of the elementary prism d x , d y , t, we get :

$$M_x = -D\left(\frac{\partial^2 z}{\partial x^2} + v\frac{\partial^2 z}{\partial y^2}\right)$$

$$M_y = -D\left(\frac{\partial^2 z}{\partial y^2} + v\frac{\partial^2 z}{\partial x^2}\right)$$

$$M_{xy} = M_{yx} = D\,(1 - v^2)\,\frac{\partial^2 z}{\partial x\,\partial y}$$

$$Q_x = \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} = -D\left(\frac{\partial^3 z}{\partial x^3} + \frac{\partial^3 z}{\partial x\,\partial y^2}\right)$$

$$Q_y = \frac{\partial M_y}{\partial y} - \frac{\partial M_{xy}}{\partial x} = -D\left(\frac{\partial^3 z}{\partial y^3} + \frac{\partial^3 z}{\partial y\,\partial x^2}\right)$$

which give the values of the bending moments, the torsional moments and the shearing forces at any section of a flat plate in terms of a deflection z.

The load p may be assumed as distributed on the two directions x and y so that :

p_x = part of the load transmitted in the direction of the x - axis
p_y = " " " " " " " " " " y - axis
and

$$p = p_x + p_y$$

knowing that :

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + p = 0$$ then

these two last equations can be identical if :

$$p_x = -\frac{\partial Q_x}{\partial x} \quad \text{and} \quad p_y = -\frac{\partial Q_y}{\partial y}$$

Substituting for Q_x and Q_y their values given before, we get :

$$p_x = D\left(\frac{\partial^4 z}{\partial x^4} + \frac{\partial^4 z}{\partial x^2\,\partial y^2}\right)$$ and

$$p_y = D\left(\frac{\partial^4 z}{\partial y^4} + \frac{\partial^4 z}{\partial x^2\,\partial y^2}\right)$$

Adding these two equations , we get :

$$\frac{\partial^4 z}{\partial x^4} + 2 \frac{\partial^4 z}{\partial x^2 \, \partial y^2} + \frac{\partial^4 z}{\partial y^4} = \frac{p}{D} \qquad (14)$$

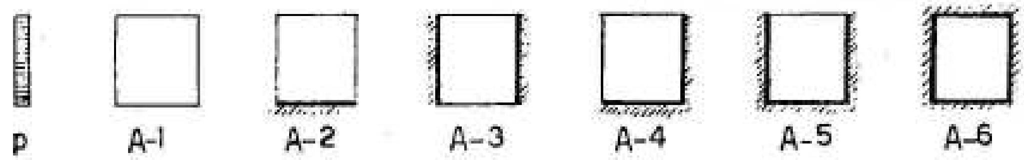
which gives the differential equation of the elastic surface of a plate loaded perpendicular to its plane.

The previous investigation shows that the determination of the internal forces in a flat plate by the mathematical theory of elasticity requires the solution of a partial differential equation of the fourth degree.

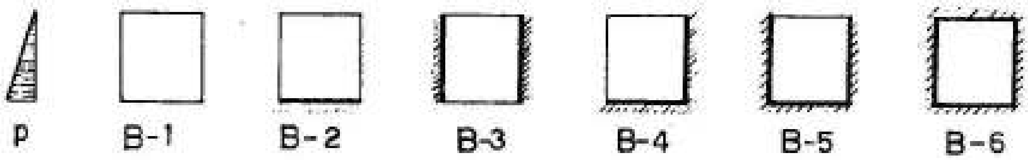
Internal Forces in Flat Plates by Czerny :

The internal forces in the following individual typical cases of rectangular plates subject to uniform and triangular loads have been determined by Czerny.

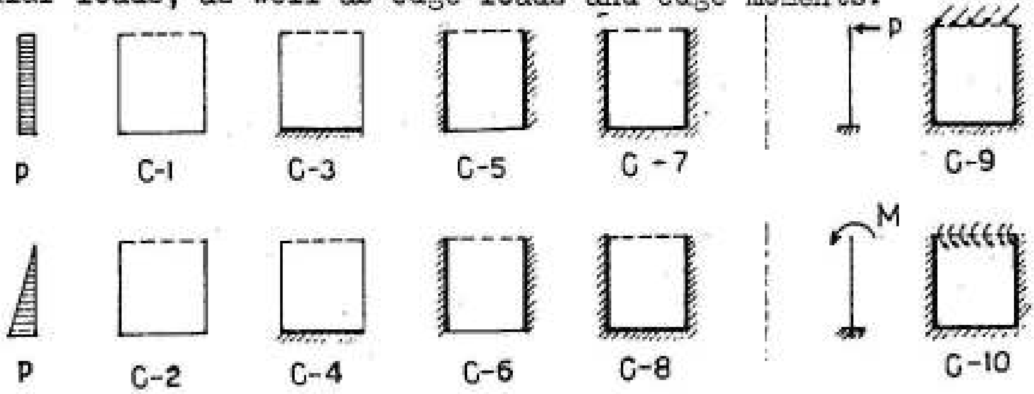
A) Slabs supported on four sides and subject to uniform load



B) Slabs supported on four sides and subject to triangular load.



C) Slabs supported on three sides and subject to uniform and triangular loads, as well as edge loads and edge moments.



We give in the following tables the internal forces in the shown cases. Reactions of case C are given in table C.11.

Notations :

Loading : Uniform or triangular

Spans : L_x and L_y

Supports: ---- Free

—— Hinged

▨ Fixed

Bending Moments :

In direction x M_x

In direction y M_y

At fixed end $M_{min.}$

Torsional Moment M_{xy}

Reactions :

In direction x R_x

In direction y R_y

At fixed end R

Shearing Forces

In direction x Q_x

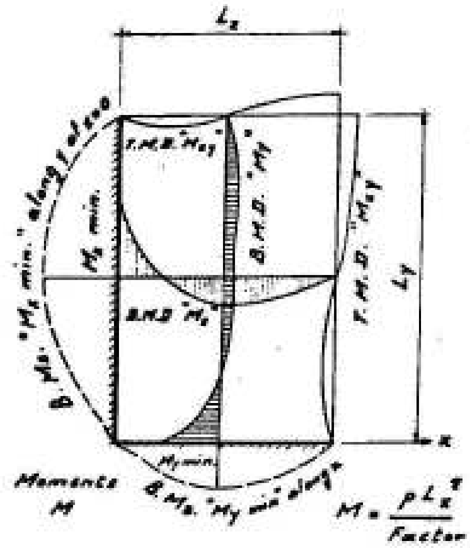
In direction y Q_y

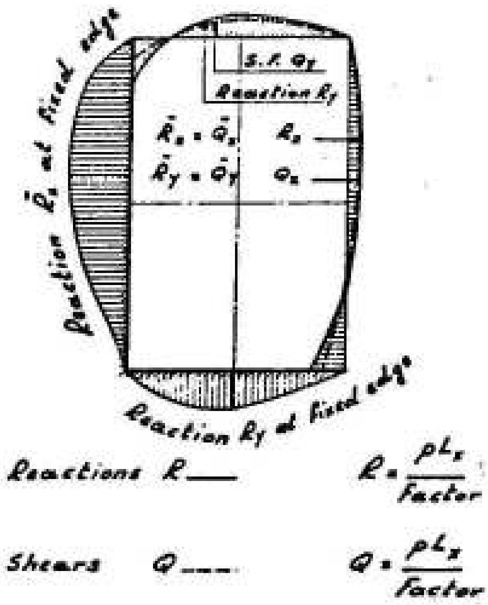
At fixed end Q = R

E = Modulus of Elasticity

t = Thickness of Slab

m = Deflection at mid-point of slab

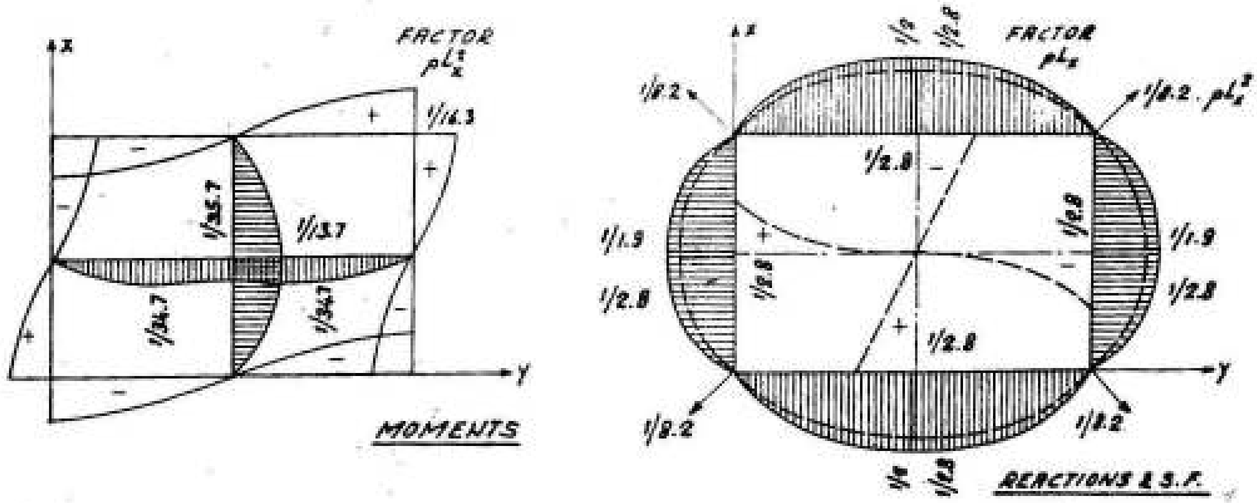




A) SLABS SUPPORTED ON FOUR SIDES AND SUBJECT TO UNIFORM LOADS.

A-1) SLAB SIMPLY SUPPORTED ON ALL FOUR SIDES.

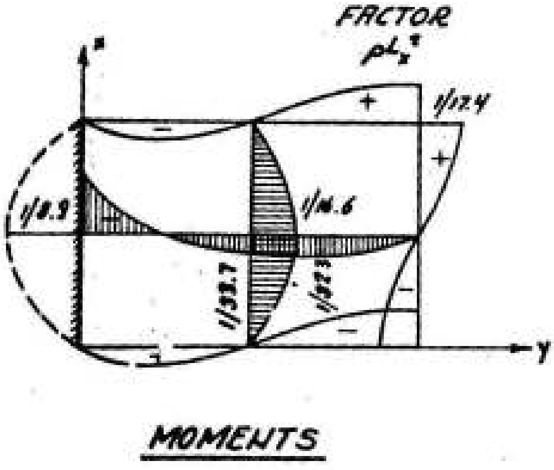
L_y / L_x	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
$M_x m = pL_x^2 :$	27.2	22.4	19.1	16.8	15.0	13.7	12.7	11.9	11.3	10.8	10.4
M_y max. $= pL_x^2 :$	27.2	27.9	29.1	30.9	32.8	34.7	36.1	37.3	38.5	39.4	40.3
M_{xy} corner $= \pm pL_x^2 :$	21.6	19.7	18.4	17.5	16.8	16.3	15.9	15.6	15.4	15.3	15.1
R corner $= pL_x^2 :$	10.8	9.9	9.2	8.8	8.4	8.2	8.0	7.8	7.7	7.7	7.6
$R_x m = pL_x :$	2.2	2.1	2.0	2.0	2.0	2.0	1.9	1.9	1.9	1.9	1.9
$R_y m = pL_x :$	2.2	2.1	2.0	2.0	1.9	1.9	1.9	1.9	1.8	1.8	1.8
$\delta_m = \frac{pL_x^4}{Et^3}$	.049	.058	.068	.073	.085	.093	.100	.106	.112	.117	.122



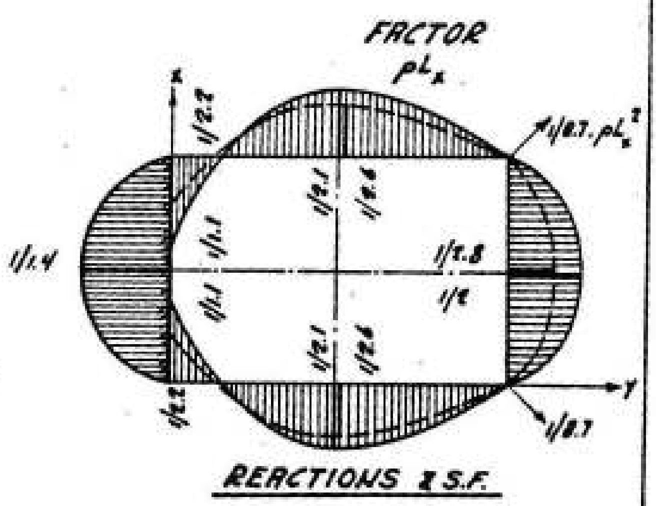
$L_y / L_x = 1.5$

A-2) SLAB FIXED ON ONE OF THE SHORTER SIDES AND SIMPLY SUPPORTED ON THE OTHER THREE SIDES.

L_y / L_x	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	200
$M_{xm} = pL_x^2 :$	41.2	31.9	25.9	21.7	18.8	16.6	15.0	13.8	12.8	12.0	11.4
$M_{y\,min} = -pL_x^2 :$	11.9	10.9	10.1	9.6	9.2	8.9	8.7	8.5	8.4	8.3	8.2
$M_{y\,max} = pL_x^2 :$	29.4	28.8	28.9	29.7	30.8	32.3	33.6	34.9	36.2	37.5	38.8
M_{xy} corners $= \pm pL_x^2 :$	26.2	23.2	21.0	19.4	18.3	17.4	16.8	16.3	15.9	15.6	15.4
R corner $= pL_x^2 :$	13.1	11.6	10.5	9.7	9.1	8.7	8.4	8.1	7.9	7.8	7.7
$R_{xm} = pL_x :$	3.0	2.4	2.3	2.2	2.1	7.1	2.0	2.0	2.0	2.0	2.0
$R'_{ym} = pL_x :$	1.7	1.6	1.6	1.5	1.5	1.4	1.4	1.4	1.4	1.4	1.4
$R_{ym} = pL_x :$	2.5	2.3	2.2	2.1	2.0	2.0	1.9	1.9	1.9	1.9	1.8
$\delta_m = \frac{pL_x^4}{Et^3} \cdot$	.033	.042	.051	.060	.069	.077	.085	.093	.099	.106	.111



MOMENTS

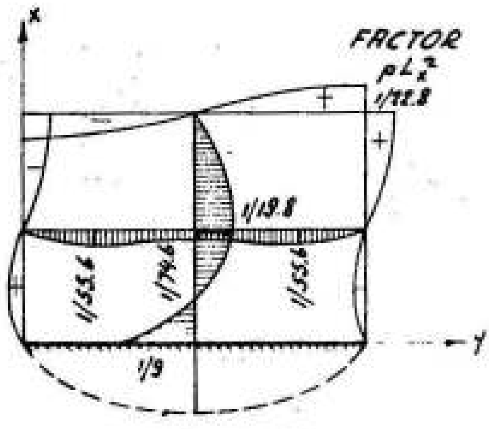


REACTIONS & S.F.

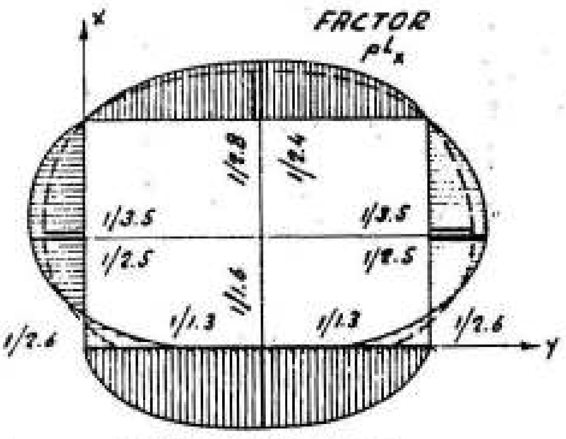
$L_y / L_x = 1.5$

A-2') SLAB FIXED ON ONE OF THE LONGER SIDES AND SIMPLY SUPPORTED ON THE OTHER THREE SIDES.

L_y/L_x	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
$M_x min = -pL_x^2$:	11.9	10.9	10.2	9.7	9.3	9.0	8.8	8.6	8.4	8.3	8.3
$M_x m = pL_x$:	31.4	27.3	24.5	22.4	21.0	19.8	19.0	18.3	17.8	17.4	17.1
$M_y max = pL_x^2$:	41.2	45.1	48.8	51.8	54.3	55.6	56.8	57.8	58.6	59.0	59.2
M_{xy} corner $= \pm pL_x^2$:	26.2	24.9	24.0	23.5	23.0	22.8	22.6	22.5	22.4	22.4	22.4
R corner $= pL_x^2$:	13.1	12.4	12.0	11.7	11.5	11.4	11.3	11.2	11.2	11.2	11.2
$R'_x m = pL_x$:	1.7	1.7	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6	1.6
$R_x m = pL_x$:	2.5	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.5	2.5	2.5
$R_y m = pL_x$:	2.6	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
$\delta_m = \frac{pL_x^4}{Et^3}$	.033	.038	.042	.046	.049	.051	.053	.055	.056	.058	.059



MOMENTS

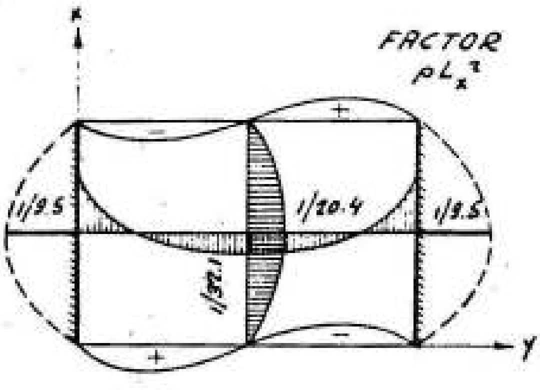


REACTIONS & S.F.

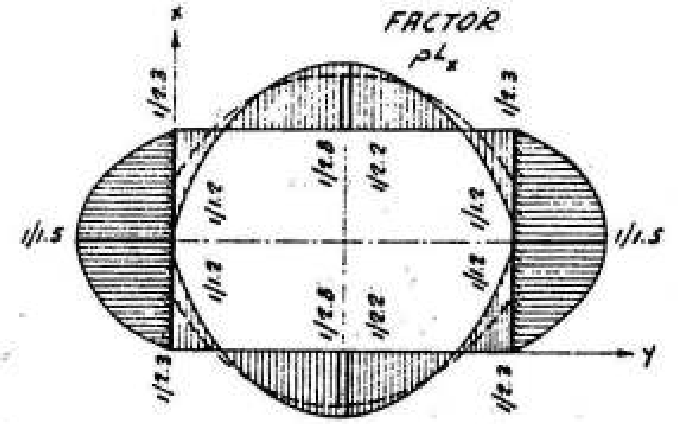
$L_y / L_x = 1.5$

A-3) SLAB FIXED ON THE TWO SHORTER SIDES AND SIMPLY SUPPORTED ON THE OTHER TWO SIDES.

L_y / L_x	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
$M_x m = pL_x^2 :$	63.3	46.1	35.5	28.5	23.7	20.4	17.9	16.0	14.6	13.4	12.5
$M_y min = -pL_x^2 :$	14.3	12.7	11.5	10.7	10.0	9.5	9.2	8.9	8.7	8.5	8.4
$M_y max = pL_x^2 :$	35.1	32.9	31.7	31.2	31.4	32.1	33.3	34.9	37.1	39.7	42.4
$R_x m = pL_x :$	3.0	2.7	2.5	2.4	2.3	2.2	2.1	2.1	2.0	2.0	2.0
$R_y m = \pm pL_x :$	1.9	1.8	1.7	1.6	1.6	1.5	1.5	1.4	1.4	1.4	1.4
$\delta m = \frac{pL_x^4}{EI^3} \cdot$	.023	.030	.038	.047	.055	.064	.072	.080	.088	.095	.101



MOMENTS

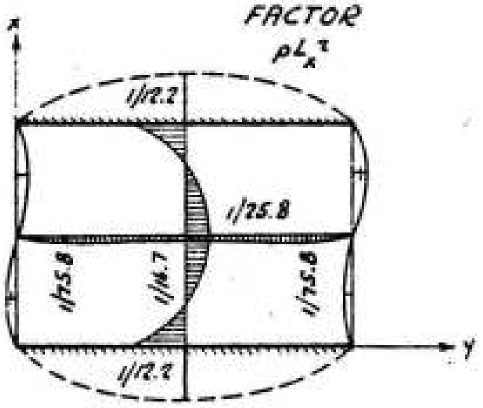


REACTIONS & S.F.

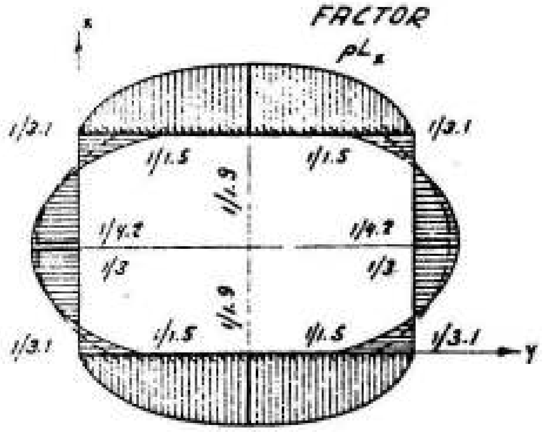
$L_y / L_x = 1.5$

A-3) SLAB FIXED ON THE TWO LONGER SIDES AND SIMPLY SUPPORTED ON THE OTHER TWO SIDES.

L_y / L_x	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
M_x min $= -pL_x^2$:	14.3	13.5	13.0	12.6	12.3	12.2	12.0	12.0	12.0	12.0	12.0
$M_x m = pL_x^2$:	35.1	31.7	29.4	27.8	26.6	25.8	25.2	24.7	24.4	24.3	24.1
$M_y max = pL_x^2$:	61.7	67.2	71.5	73.5	74.6	75.8	77.0	77.0	77.0	77.0	77.0
$R_x m = t\,pL_x$:	1.9	1.9	1.9	1.9	1.9	1.9	1.9	1.9	1.9	2.0	2.0
$R_y m = pL_x$:	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0	3.0
$\delta_m = \frac{pL_x^4}{Et^3}$ ×	.023	.025	.027	.028	.029	.030	.030	.031	.031	.031	.031



MOMENTS

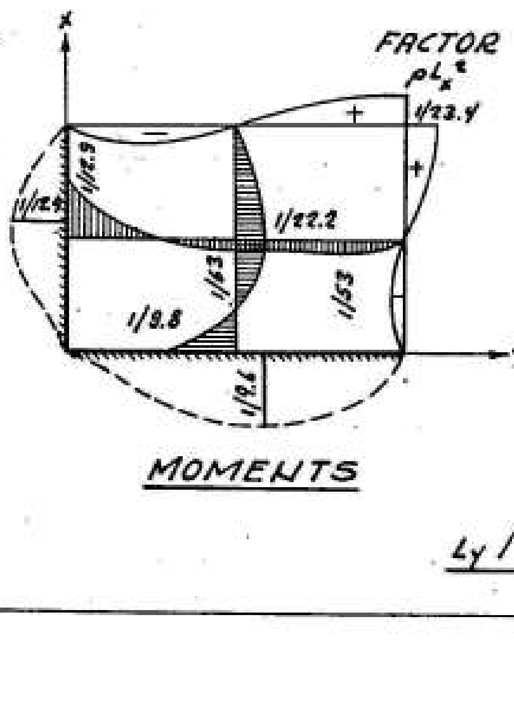


REACTIONS & S.F.

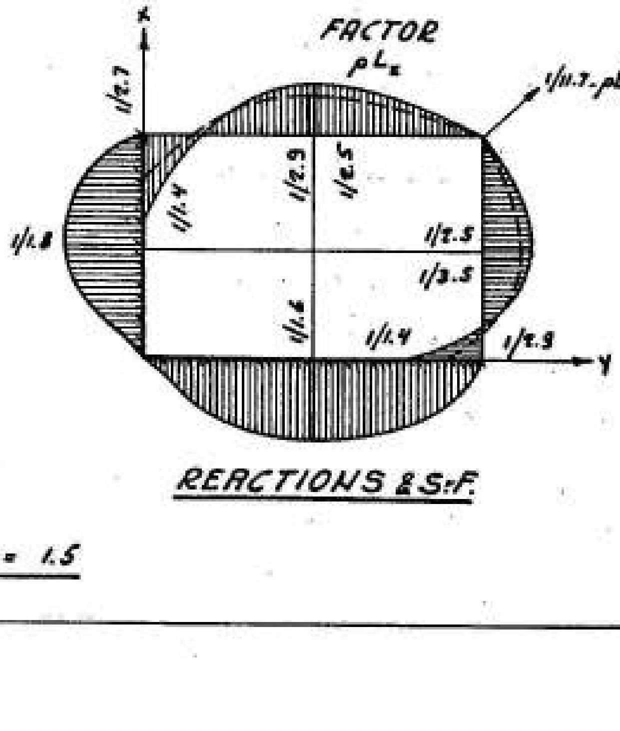
$L_y / L_x = 1.5$

A-4) SLAB FIXED ON TWO ADJACENT SIDES AND SIMPLY SUPPORTED ON THE OTHER TWO SIDES.

L_y / L_x	1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
$M_x min = -pL_x^2$:	14.3	12.7	11.5	10.7	10.0	9.6	9.2	8.9	8.7	8.5	8.4
$M_x m = pL_x^2$:	42.7	35.1	30.0	26.5	24.1	22.2	21.0	19.9	19.1	18.4	17.9
$M_y min = -pL_x^2$:	14.3	13.6	13.1	12.8	12.6	12.4	12.3	12.2	12.2	12.2	12.2
$M_y max = pL_x^2$:	40.2	42.0	44.0	47.6	51.0	53.0	54.8	56.3	57.7	59.0	60.2
$R'_x m = pL_x$:	2.0	1.8	1.8	1.7	1.7	1.6	1.6	1.6	1.6	1.6	1.6
$R_x m = pL_x$:	2.8	2.6	2.6	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
$R'_y m = pL_x$:	2.0	1.9	1.9	1.9	1.8	1.8	1.8	1.8	1.8	1.8	1.8
$R_y m = pL_x$:	2.8	2.7	2.6	2.5	2.5	2.5	2.5	2.5	2.5	2.5	2.5
$\delta_m = \frac{pL_x^4}{Et^3}$ ·	.025	.030	035	.039	.043	.046	.048	.051	.053	.055	.056



MOMENTS

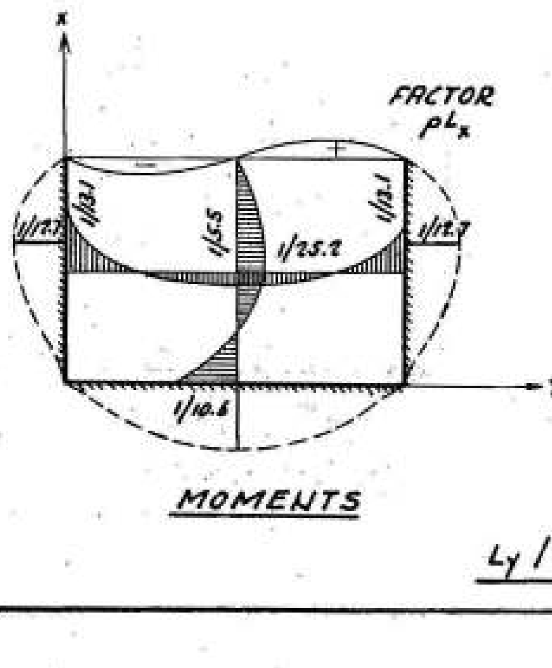


REACTIONS & S.F.

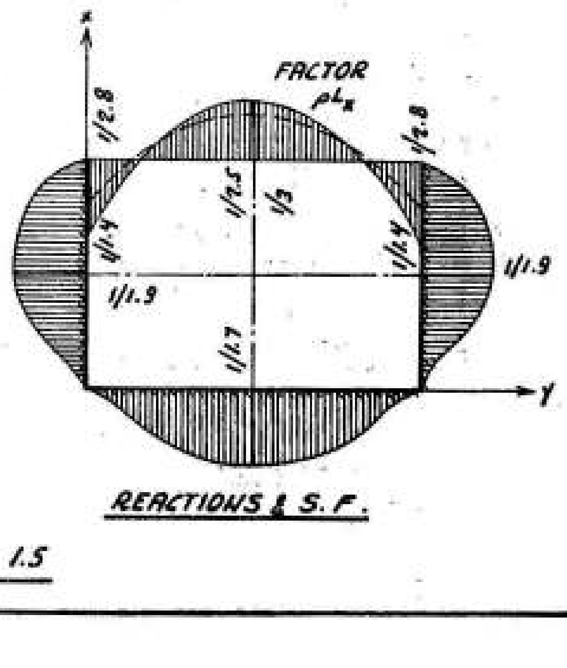
$L_y / L_x = 1.5$

A-5) SLAB SIMPLY SUPPORTED ON ONE LONGER SIDE AND FIXED ON THE OTHER THREE SIDES.

L_y / L_x		1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
M_x min.	$= -pL_x^2$:	18.3	15.4	13.5	12.2	11.2	10.6	10.1	9.7	9.4	9.0	8.8
M_x m	$= pL_x^2$:	59.5	46.1	37.5	31.8	28.0	25.2	23.3	21.7	20.5	19.5	18.7
M_y min	$= -pL_x^2$:	16.2	14.8	13.9	13.3	13.0	12.7	12.6	12.5	12.4	12.3	12.3
M_y m	$= pL_x^2$:	44.1	43.7	44.8	46.9	50.3	55.0	61.6	70.4	79.6	89.8	101
R'_x m	$= pL_x$:	2.2	2.0	1.9	1.8	1.8	1.7	1.7	1.7	1.7	1.6	1.6
R_x m	$= pL_x$:	3.0	2.8	2.7	2.6	2.5	2.5	2.5	2.5	2.5	2.5	2.5
R'_y m	$= \pm pL_x$:	2.1	2.0	1.9	1.9	1.9	1.9	1.8	1.8	1.8	1.8	1.8
δm	$= \frac{pL_x^4}{Et^3}$:	.019	.024	.028	.033	.037	.041	.044	.047	.050	.052	.054



MOMENTS

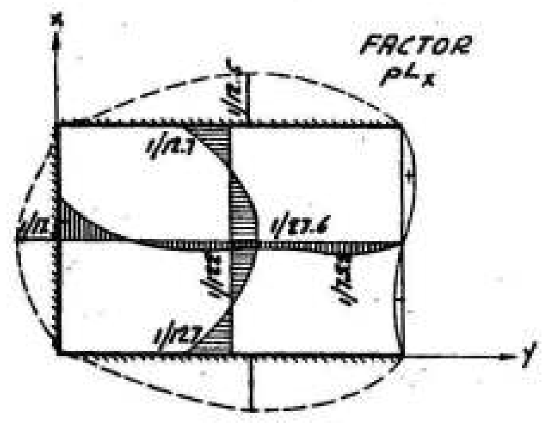


REACTIONS & S.F.

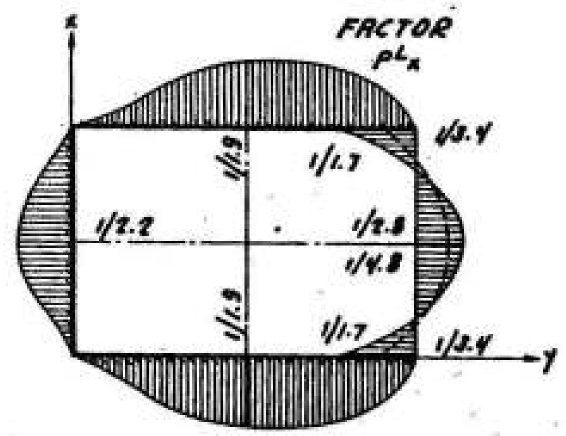
$L_y / L_x = 1.5$

A-5') SLAB SIMPLY SUPPORTED ON ONE SHORTER SIDE AND FIXED ON THE OTHER THREE SIDES.

L_y / L_x		1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
M_x min	$= -pL_x^2$:	16.2	14.8	13.9	13.2	12.7	12.5	12.3	12.2	12.1	12.0	12.0
M_x m	$= pL_x^2$:	44.1	37.9	33.8	31.0	29.0	27.6	26.5	25.7	25.1	24.7	24.6
M_y min	$= -pL_x^2$:	18.3	17.7	17.5	17.5	17.5	17.5	17.5	17.5	17.5	17.5	17.5
M_y max	$= pL_x^2$:	55.9	60.3	66.2	69.0	72.0	75.2	78.7	82.5	86.8	91.7	97.0
R'_x m	$= \pm pL_x$:	2.1	2.0	2.0	1.9	1.9	1.9	1.9	1.9	1.9	2.0	2.0
R'_y m	$= pL_x$:	2.2	2.1	2.1	2.1	2.1	2.2	2.2	2.2	2.2	2.2	2.2
R_y m	$= pL_x$:	3.0	2.9	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8	2.8
δ_m	$= \frac{pL_x^4}{Et^3}$ ·	.019	.021	.024	.025	.027	.028	.029	.030	.030	.031	.031



MOMENTS

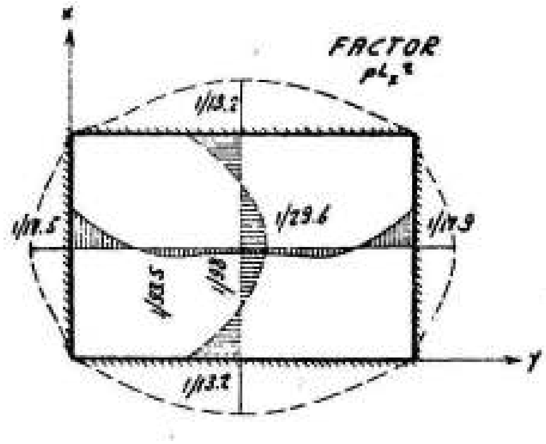


REACTIONS & S.F.

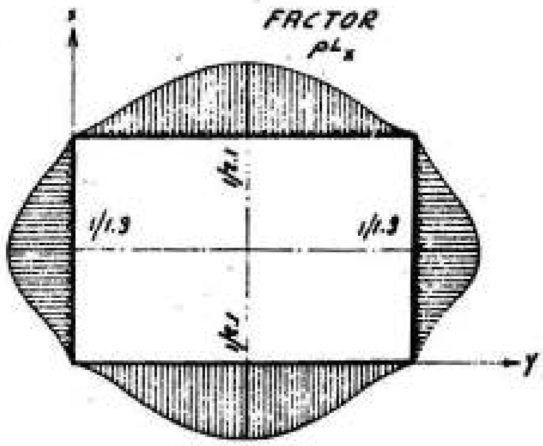
$L_y / L_x = 1.5$

A-6) SLAB FIXED ON ALL FOUR SIDES.

L_y / L_x		1.00	1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
M_x min	$= pL_x^2$:	19.4	17.1	15.5	14.5	13.7	13.2	12.8	12.5	12.3	12.1	12.0
M_x m	$= pL_x^2$:	56.8	46.1	39.4	34.8	31.9	29.6	28.1	26.9	26.0	25.4	25.0
M_y min	$= -pL_x^2$:	19.4	18.4	17.9	17.6	17.5	17.5	17.5	17.5	17.5	17.5	17.5
M_y max	$= pL_x^2$:	56.8	60.3	65.8	73.6	83.4	93.5	98.1	101	103	105	105
R_x m	$= \pm pL_x$:	2.2	2.1	2.0	2.0	1.9	1.9	1.9	1.9	1.9	1.9	1.9
R_y m	$= \pm pL_x$:	2.2	2.2	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1	2.1
δ_m	$= \frac{pL_x^4}{EL^3}$.	.015	.018	.021	.023	.025	.026	.028	.029	.029	.030	.030



MOMENTS



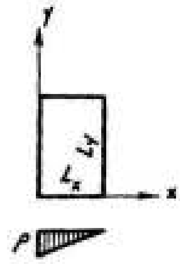
REACTIONS & S.F.

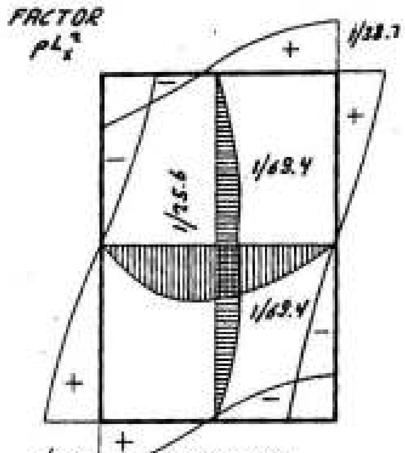
$L_y / L_x = 1.5$

B - SLABS SUPPORTED ON FOUR SIDES & SUBJECT TO TRIANGULAR LOADS.

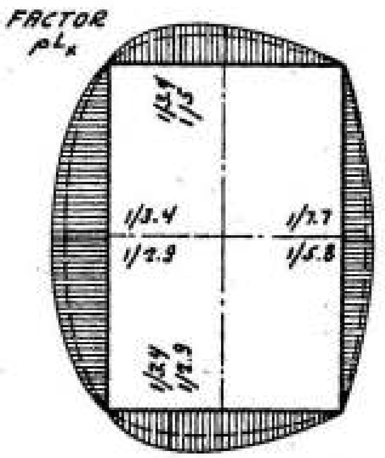
B-1) SLAB SIMPLY SUPPORTED ON ALL FOUR SIDES.

POSITION		L_y/L_x	1.0	1.1	1.2	1.3	1.4	1.5	2.0
$y = 0.5L_y$	M_x max	$= pL_x^2 :$	45.5	38.9	34.2	30.8	27.9	25.6	19.8
	at	$\frac{x}{L_x} =$	0.3	0.32	0.34	0.36	0.37	0.38	0.4
$y = 0.5L_y$	M_x m	$= pL_x^2 :$	54.3	44.6	38.2	33.5	30.1	27.5	20.8
$x = 0.5L_x$	M_y max	$= pL_x^2 :$	54.3	55.9	58.1	61.7	65.8	69.4	80.6
	at	$\frac{y}{L_y} =$	0.5	0.5	0.5	0.5	0.4 & 0.6	0.3 & 0.7	0.2 & 0.8
$x = 0.5L_x$	M_y m	$= pL_x^2 :$	54.3	55.9	58.1	61.7	66.2	71.4	115
$x = 0$	M_{xy} corner	$= \pm pL_x^2 :$	35.8	33.3	31.4	30.0	29.1	28.2	26.6
$x = L_x$			54.0	48.5	44.8	42.2	40.2	38.7	35.5
$x = 0$	Q_x m	$= pL_x :$	4.0	3.8	3.7	3.6	3.5	3.4	3.2
	R_x m	$=$	3.2	3.1	3.0	3.0	3.0	2.9	2.9
$x = L_x$	Q_x m	$= -pL_x :$	11.1	10.0	9.2	8.6	8.1	7.7	6.6
	R_x m	$= pL_x :$	7.0	6.6	6.3	6.1	5.9	5.8	5.7
$y = 0$ & $y = L_y$	Q_y max	$= \pm pL_x :$	5.2	5.1	5.1	5.0	5.0	5.0	4.9
	at	$\frac{x}{L_x} =$	0.3	0.3	0.3	0.3	0.3	0.3	0.3
$y = 0$ & $y = L_y$	R_y max	$= pL_x :$	3.8	3.6	3.6	3.5	3.4	3.4	3.3
	at	$\frac{x}{L_x} =$	0.3	0.3	0.3	0.3	0.3	0.3	0.3
$x = 0.5L_x$, $y = 0.5L_y$	δ_m	$= \frac{pL_x^4}{Et^3} :$	.024	.029	.034	.038	.043	.046	.061





MOMENTS

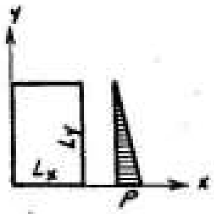


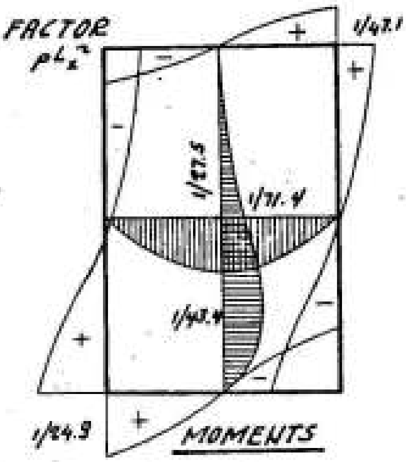
REACTIONS & S.F.

$L_y/L_x = 1.5$

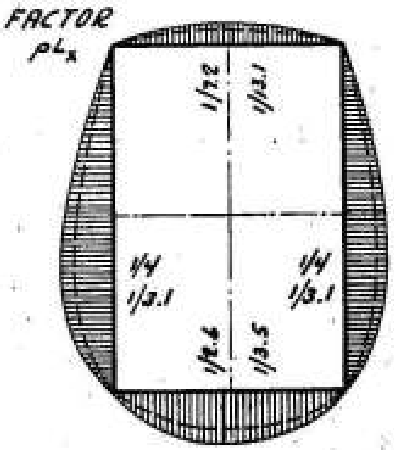
B-1') SLAB SIMPLY SUPPORTED ON ALL FOUR SIDES.

POSITION	L_y/L_x	1.0	1.1	1.2	1.3	1.4	1.5	2.0
$y=0.5L_y$	$M_x\,max \doteq M_{x\,m} = pL_x^2:$	54.3	44.6	38.2	33.5	30.1	27.5	20.7
$x=0.5L_x$	$M_y\,max = pL_x^2:$	45.5	44.2	43.5	42.9	42.7	43.1	43.1
	at $\frac{y}{L_y} =$	0.30	0.28	0.26	0.24	0.22	0.20	0.16
$x=0.5L_x$	$M_{y\,m} = pL_x^2:$	54.3	55.8	58.1	61.7	66.2	71.4	115
$y=0$	M_{xy} corner $= \pm pL_x^2:$	35.8	32.4	29.8	27.7	26.2	24.9	21.3
$y=L_y$		54.0	50.7	48.8	47.6	47.1	47.1	52.4
$x=0$ & $x=L_x$	$Q_x\,max = \pm pL_x:$	5.2	4.9	4.6	4.3	4.2	4.0	3.5
	at $\frac{y}{L_y} =$	0.3	0.3	0.3	0.3	0.3	0.3	0.3
$x=0$ $x=L_x$	$R_x\,max = pL_x:$	3.8	3.6	3.4	3.3	3.2	3.1	2.8
	at $\frac{y}{L_y} =$	0.3	0.3	0.3	0.3	0.3	0.3	0.2
$y=0$	$Q_y\,max = pL_x:$	4.0	3.9	3.7	3.6	3.5	3.5	3.2
	$R_y\,max = pL_x:$	3.2	3.0	2.9	2.7	2.6	2.6	2.3
$y=L_y$	$Q_y\,max = -pL_x:$	11.1	11.4	11.7	12.1	12.6	13.1	16.4
	$R_y\,max = pL_x:$	7.0	6.9	6.9	7.0	7.1	7.2	8.5
$x=0.5L_x$ $y=0.5L_y$	$\delta m = \frac{pL_x^4}{Et^3}\cdot$	.024	.029	.034	.038	.043	.046	.061





MOMENTS

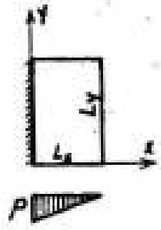


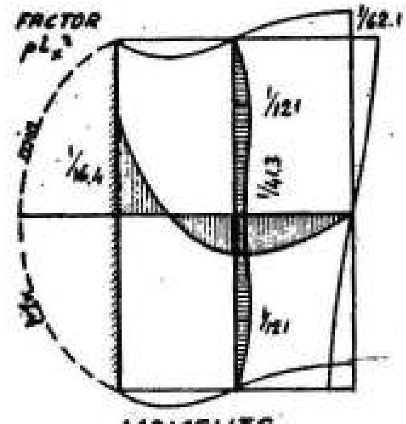
REACTIONS & S.F

$L_y/L_x = 1.5$

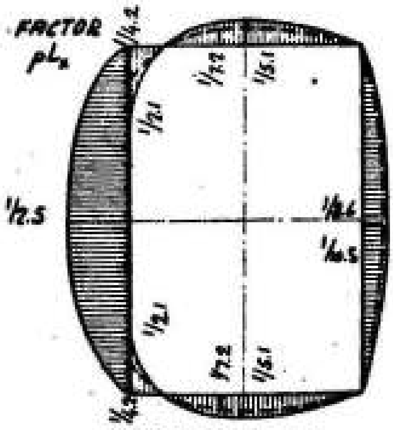
B-2) SLAB FIXED ON A LONGER SIDE AND SIMPLY SUPPORTED ON THE OTHER THREE SIDES.

POSITION	L_y / L_x	1.0	1.1	1.2	1.3	1.4	1.5	2.0
$y = 0.5 L_y$	M_x min $= -pL_x^2$:	20.7	19.1	18.2	17.4	16.8	16.4	15.3
$y = 0.5 L_y$	M_x max $= pL_x^2$:	63.6	56.2	51.0	46.7	42.8	41.3	35.7
	at $\frac{x}{L_x} =$	0.46	0.48	0.5	0.52	0.53	0.54	0.55
$y = 0.5 L_y$	$M_{xm} = pL_x^2$:	64.5	56.5	51.0	46.9	44.1	41.8	36.5
$x = 0.5 L_x$	M_y max $= pL_x^2$:	92.1	99.0	109	112	116	121	128
	at $\frac{y}{L_y} =$	0.5	0.4 & 0.6	0.3 & 0.7	0.2 & 0.8	0.2 & 0.8	0.2 & 0.8	0.1 & 0.9
$x = 0.5 L_x$	$M_{ym} = pL_x^2$:	92.1	100	111	127	145	167	400
$x = L_x$	M_{xy} corners $\pm pL_x^2$:	73.0	69.0	65.8	64.1	62.9	62.1	61.0
$x = 0$	$\bar{Q}_{xm} = \bar{R}_{xm} = pL_x$:	2.6	2.6	2.5	2.5	2.6	2.5	2.6
$x = L_x$	$Q_{x m} = -pL_x$:	13.2	12.3	11.6	11.1	10.8	10.5	10.0
	$R_{x m} = pL_x$:	8.3	8.6	8.5	8.5	8.5	8.6	9.1
$y = 0$ & $y = L_y$	Q_y max $= \pm pL_x$:	7.1	7.2	7.2	7.2	7.2	7.2	7.3
	at $\frac{x}{L_x} =$	0.4	0.4	0.4	0.4	0.4	0.4	0.4
$y = 0$ & $y = L_y$	R_y max $= pL_x$:	5.2	5.2	5.2	5.2	5.2	5.1	5.1
	at $\frac{x}{L_x} =$	0.4	0.4	0.4	0.4	0.5	0.5	0.5
$x = 0.5 L_x$ $y = 0.5 L_y$	$\delta_m = \frac{pL_x^4}{Et^3}$ ·	.015	.017	.019	.021	.022	.023	.026





MOMENTS

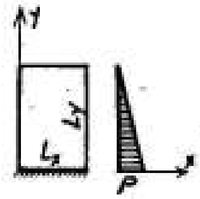


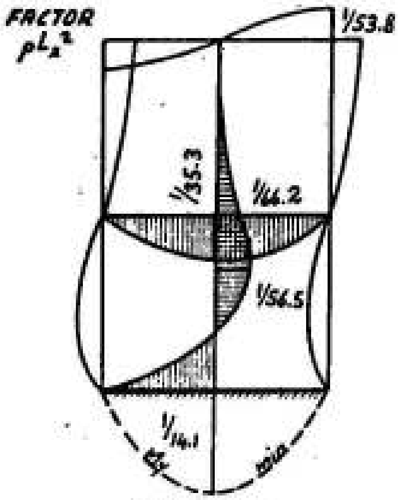
REACTION & S. F.

$L_y / L_x = 1.5$

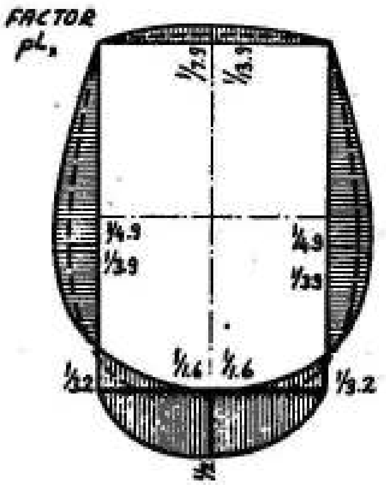
B-2) SLAB FIXED ON A SHORTER SIDE AND SIMPLY SUPPORTED ON THE OTHER THREE SIDE.

POSITION	L_y/L_x	1.0	1.1	1.2	1.3	1.4	1.5	2.0
$x = 0.5L_x$	M_y min $= pL_x^2$:	20.7	18.6	17.0	15.8	14.8	14.1	11.8
$y = 0.5L_y$	M_x max $\simeq M_{xm} = pL_x^2$:	90.1	69.4	55.8	46.7	40.2	35.3	23.6
$x = 0.5L_x$	M_y max $= pL_x^2$:	63.6	60.6	59.2	57.8	56.8	56.5	56.5
	at $\frac{y}{L_y}$ =	0.46	0.44	0.42	0.4	0.38	0.36	0.28
$x = 0.5L_y$	M_{ym} $= pL_x^2$:	64.5	62.1	60.7	62.1	63.7	66.2	92.6
$y = L_y$	M_{xy} corner $= \pm pL_x^2$:	73.0	65.3	60.6	57.1	54.9	52.8	54.9
$x = 0$ & $x = L_x$	Q_x max $= \pm pL_x$:	7.1	6.5	6.0	5.6	5.2	4.9	4.1
	R_x max $= pL_x$:	5.2	4.8	4.5	4.2	4.0	3.9	3.3
	at $\frac{y}{L_y}$ =	0.4	0.4	0.4	0.4	0.4	0.4	0.3
$y = 0$	$\bar{Q}_{ym} = \bar{R}_{ym} = pL_x$:	2.6	2.4	2.3	2.2	2.1	2.0	1.8
$y = L_y$	Q_{ym} $= -pL_x$:	13.2	13.0	13.1	13.2	13.5	13.9	16.7
	R_{ym} $= pL_x$:	8.9	8.4	8.1	7.9	7.9	7.9	8.8
$x = 0.5L_x$ $y = 0.5L_y$	δ_m $= \frac{pL_x^4}{Et^3}$:	.015	.020	.024	.028	.032	.037	.054





MOMENTS

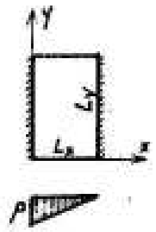


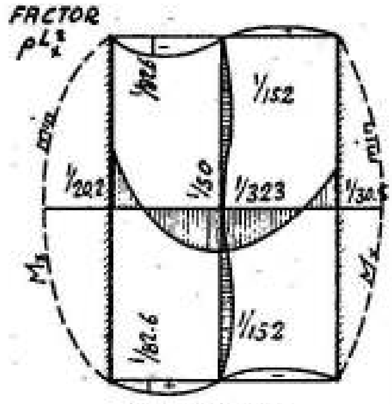
REACTION & S. F.

$L_y / L_x = 1.5$

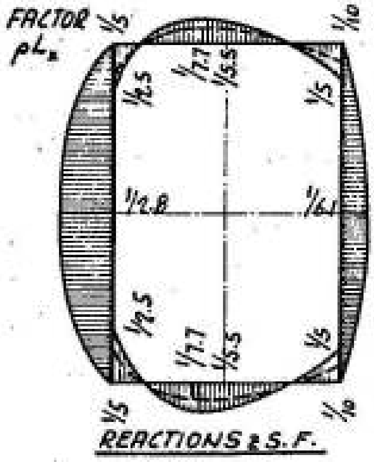
B-3) SLAB FIXED ON THE TWO LONGER SIDES AND SIMPLY SUPPORTED ON THE OTHER TWO SIDES.

POSITION		L_y/L_x	1.0	1.1	1.2	1.3	1.4	1.5	2.0
$y=0.5L_y$, $x=0$	M_x min	$=-pL_x^2:$	23.2	22.2	21.5	20.9	20.6	20.2	20.0
$y=0.5L_y$, $x=L_x$	M_x min	$=-pL_x^2:$	37.6	34.8	33.1	31.9	31.2	30.5	30.0
$y=0.5L_y$	M_x max	$=pL_x^2:$	65.8	60.6	56.8	54.1	51.3	50.0	46.7
	at	$\frac{x}{L_x}=$	0.42	0.43	0.44	0.45	0.45	0.45	0.46
$y=0.5L_y$	M_{xm}	$=pL_x^2:$	70.4	63.3	58.8	55.6	53.2	51.5	48.1
$x=0.5L_x$	M_y max	$=pL_x^2:$	124	133	143	147	149	152	154
	at	$\frac{y}{L_y}=$	0.20 & 0.70	0.26 & 0.74	0.20 & 0.80	0.20 & 0.80	0.17 & 0.83	0.15 & 0.85	0.12 & 0.88
$x=0.5L_x$	M_{ym}	$=pL_x^2:$	127	149	179	213	256	323	1111
$y=0$ & $y=L_y$	M_{xy}	$=\pm pL_x^2:$	85.5	83.3	82.6	82.6	82.6	82.6	83.3
	at	$\frac{x}{L_x}=$	0.16	0.16	0.16	0.16	0.16	0.16	0.16
$x=0$	$\acute{Q}_{xm}=\acute{R}_{xm}$	$=pL_x:$	2.8	2.8	2.8	2.8	2.8	2.8	2.8
$x=L_x$	$\acute{Q}_{xm}=\acute{R}_{xm}$	$=pL_x:$	6.5	6.2	6.1	6.1	6.1	6.1	6.3
$y=0$ & $y=L_y$	Q_y max	$=\pm pL_x:$	7.5	7.6	7.6	7.7	7.7	7.7	7.7
	R_y max	$=pL_x:$	5.5	5.4	5.4	5.5	5.5	5.5	5.5
	at	$\frac{x}{L_x}=$	0.4	0.4	0.4	0.4	0.4	0.4	0.4
$x=0.5L_x$, $y=0.5L_y$	δ_m	$\frac{pL_x^4}{Et^3}=$	.012	.013	.013	.014	.015	.015	.016





MOMENTS

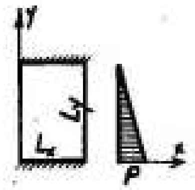


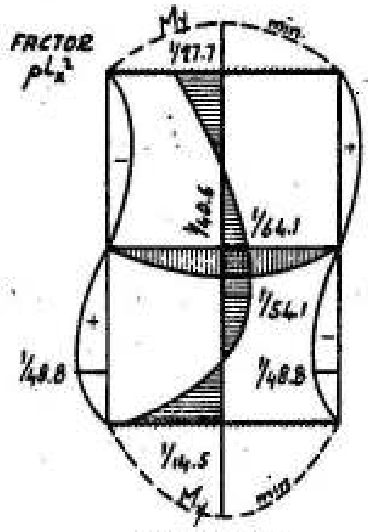
REACTIONS & S.F.

$L_y/L_x=1.5$

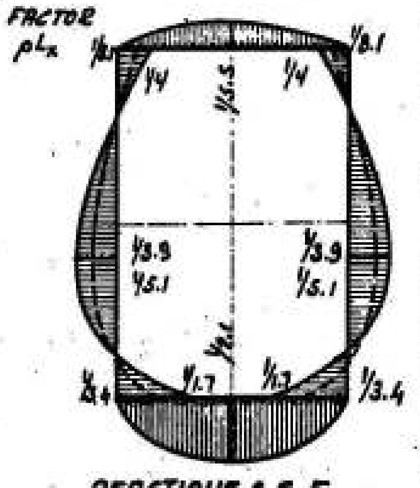
B-3) SLAB FIXED ON THE TWO SHORTER SIDES AND SIMPLY SUPPORTED ON THE OTHER TWO SIDES.

POSITION		L_y/L_x	1.0	1.1	1.2	1.3	1.4	1.5	2.0
$x = 0.5L_x$, $y = 0$	M_y min	$= -pL_x^2$:	23.2	20.4	18.3	16.7	15.5	14.5	11.9
$y = L_y$	M_y min	$= -pL_x^2$:	37.6	33.7	31.2	29.5	28.3	27.7	28.5
$y = 0.5L_y$	M_x max	$= pL_x^2$:	124	91.7	70.9	56.8	47.6	40.6	25.0
	at	$\frac{x}{L_x}$ =	0.3 & 0.7	0.5	0.5	0.5	0.5	0.5	0.5
$x = 0.5L_x$	M_y max	$= pL_x^2$:	66.8	61.3	57.5	55.5	54.3	54.1	55.2
	at	$\frac{y}{L_y}$ =	0.42	0.40	0.39	0.38	0.37	0.36	0.30
$x = 0.5L_x$	M_y m	$= pL_x^2$:	70.4	65.8	63.2	62.5	62.5	64.1	85.5
$x = 0$ & $x = L_x$	M_{xy} max	$= \pm pL_x^2$:	85.5	71.9	62.9	56.8	51.5	48.8	36.3
	at	$\frac{y}{L_y}$ =	0.16	0.15	0.15	0.15	0.15	0.13	0.10
$x = 0$ & $x = L_x$	Q_x max	$= \pm pL_x$:	7.6	6.8	6.3	5.8	5.4	5.1	4.1
	R_x max	$= pL_x$:	5.5	5.0	4.6	4.3	4.1	3.9	3.3
	at	$\frac{y}{L_y}$ =	0.4	0.4	0.4	0.4	0.4	0.4	0.3
$y = 0$	$\bar{Q}_{ym} = \bar{R}_{ym}$	$= pL_x$:	2.8	2.6	2.4	2.3	2.2	2.1	1.8
$y = L_y$	$\bar{Q}_{ym} = \bar{R}_{ym}$	$= -pL_x$:	6.5	6.0	5.7	5.5	5.5	5.5	5.9
$x = 0.5L_x$	δ_m	$\frac{pL_x^4}{Et^3}$:	.012	.015	.019	.023	.028	.032	.051





MOMENTS

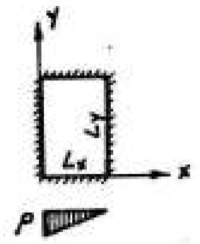


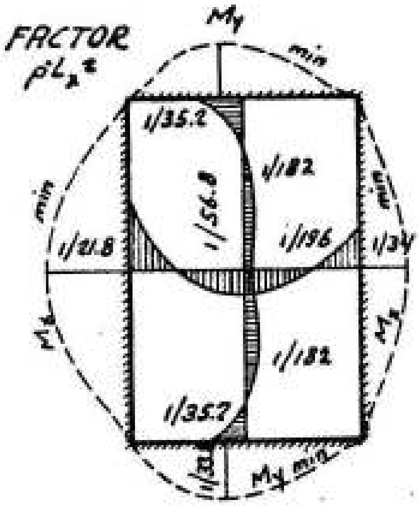
REACTIONS & S.F.

$L_y/L_x = 1.5$

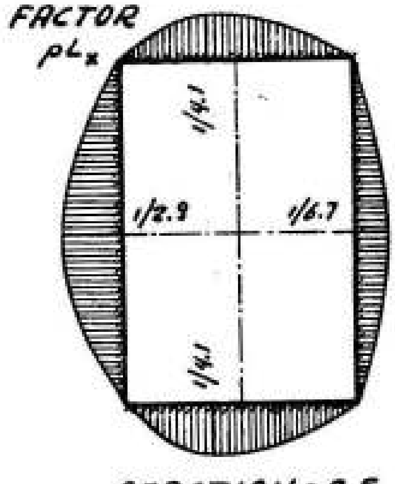
B-6) SLAB FIXED ON ALL FOUR SIDES.

POSITION		L_y/L_x	1.0	1.1	1.2	1.3	1.4	1.5	2.0
$x=0$, $y=0.5L_y$	M_x min	$=-pL_x^2$:	30.0	26.7	24.7	23.3	22.2	21.8	20.2
$x=L_x$	M_x min	$=-pL_x^2$:	56.2	47.2	41.7	38.0	35.5	34.0	30.4
$y=0$	M_y min	$=-pL_x^2$:	36.9	36.0	35.1	34.6	34.4	33.8	33.8
$y=L_y$	at x/L_x	$=0.4$							
$y=0.5L_y$	M_x max	$=pL_x^2$:	98.0	82.6	73.0	65.8	60.6	56.8	48.5
	at	$\frac{x}{L_x}=$	0.34	0.38	0.40	0.41	0.42	0.44	0.45
$y=0.5L_y$	$M_x m$	$=pL_x^2$:	114	91.7	78.7	69.9	63.7	59.2	50.0
$x=0.5L_x$	M_y max	$=pL_x^2$:	114	120	132	147	167	182	213
	at	$\frac{y}{L_y}=$	0.5	0.5	0.5	0.5	0.40 & 0.60	0.34 & 0.66	0.23 & 0.77
$x=0.5L_x$	$M_y m$	$=pL_x^2$:	114	120	132	147	170	196	526
$y=0.5L_y$	$-\bar{Q}_x m$ } $\bar{R}_x m$	$=pL_x$:	3.1	2.9	2.9	2.9	2.9	2.9	2.9
	at x	$=0$							
$y=0.5L_y$	$-\bar{Q}_x m$ } $\bar{R}_x m$	$=pL_x$:	8.3	7.4	6.7	6.7	6.7	6.7	6.7
	at x	$=L_x$							
$y=0$ &	$\pm\bar{Q}_y$ max } $\bar{R}_y$ max	$=pL_x$:	4.1	4.1	4.1	4.1	4.1	4.1	4.1
$y=L_y$	at	$\frac{x}{L_x}=$	0.3	0.3	0.4	0.4	0.4	0.4	0.4
$y=0.5L_x$, $y=0.5L_y$	δ_m	$\frac{pL_x^4}{Et^3}$ -	.008	.009	.010	.012	.012	.013	.015





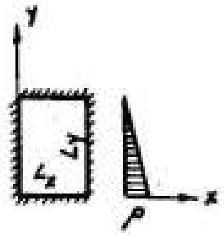
MOMENTS



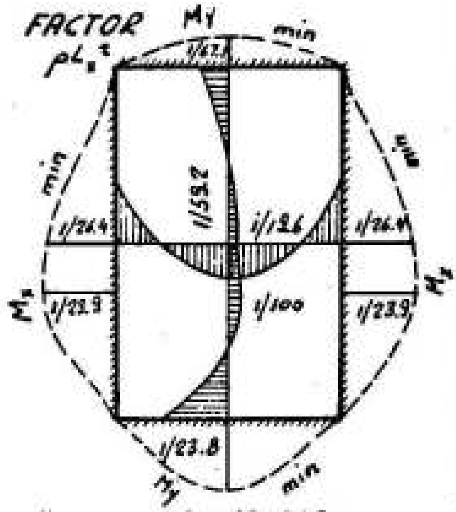
REACTION & S.F.

$L_y/L_x = 1.5$

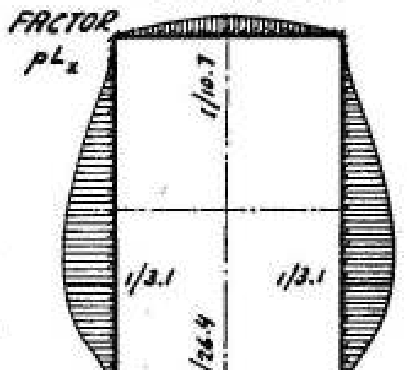
B-6) SLAB FIXED ON ALL FOUR SIDES.



POSITION	L_y/L_x	1.0	1.1	1.2	1.3	1.4	1.5	2.0
x = 0 & x = L_x	M_x min = $-pL_x^2$:	36.9	33.1	28.8	27.5	25.6	23.9	20.2
	at $\frac{y}{L_y}$ =	0.42	0.40	0.40	0.40	0.38	0.37	0.33
y = 0, x = 0.5L_x	M_y min = $-pL_x^2$:	30.0	27.5	26.1	25.0	24.1	23.8	21.9
y = L_y	M_y min = $-pL_x^2$:	56.2	55.5	57.1	59.5	62.9	67.1	92.6
y = 0.5L_y	M_x max ≈ M_{xm} pL_x^2 :	114	91.7	78.7	69.9	63.7	59.2	50.0
x = 0.5L_x	M_y max = pL_x^2 :	98.0	98.0	98.0	99.0	99.0	100	100
	at $\frac{y}{L_y}$ =	0.37	0.35	0.33	0.33	0.33	0.32	0.24
x = 0.5L_x	M_{ym} = pL_x^2 :	114	121	132	147	170	196	526
x = 0 & x = L_x	$\pm \bar{Q}_x$ max = $\bar{R}_x$ max } = pL_x : at y/L_y = 0.3	4.1	3.9	3.6	3.4	3.3	3.1	2.8
y = 0, x = 0.5L_x	$\bar{Q}_y$ = $\bar{R}_y$ = pL_x :	3.1	2.9	2.8	2.8	2.7	2.6	2.5
y = L_y	$-\bar{Q}_y$ = $\bar{R}_y$ = pL_x :	8.3	8.4	8.8	9.4	10.0	10.7	14.3
x = 0.5L_x, y = 0.5L_y	δ_m = $\frac{pL_x^4}{Et^3}$ ·	.008	.009	.010	.012	.012	.013	.015



MOMENTS

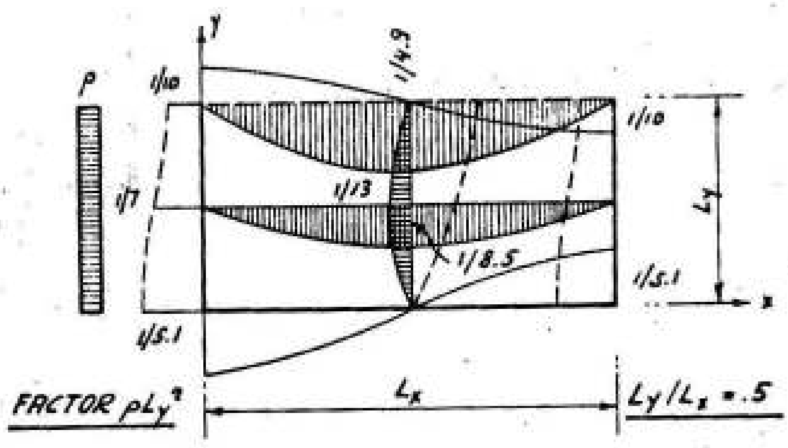


REACTIONS & S.F.

$L_y / L_x = 1.5$

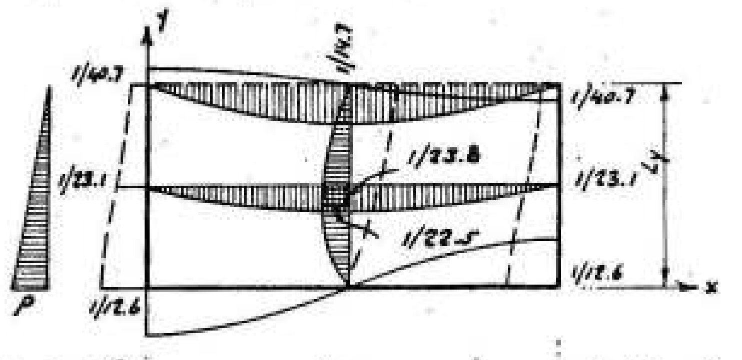
C-1) SLAB SIMPLY SUPPORTED ON THREE SIDES AND SUBJECT TO UNIFORM LOAD.

POSITION	L_y/L_x	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$x=0.5L_x$	$M_y m = pL_y^2$:	8.4	8.9	10.5	13.0	16.5	21.3	27.6	35.7	46.2	59.4	75.9	96.5	122	154
$x=0.5L_x$	$M_y max = pL_y^2$:	8.4	8.9	10.5	13.0	16.4	20.9	26.6	33.5	41.7	50.4	60.0	70.4	81.7	93.8
	at $\frac{y}{L_y}$ =	0.5	0.5	0.5	0.48	0.45	0.42	0.4	0.38	0.33	0.28	0.26	0.25	0.24	0.22
$y=0.5L_y$	$M_x m = pL_y^2$:	7.9	7.9	8.0	8.5	9.1	10.0	11.0	12.3	13.7	15.2	17.0	18.8	20.9	23.0
$y=L_y$	$M_x max = M_x m = pL_y^2$:	4.0	4.1	4.4	4.9	5.5	6.3	7.3	8.5	9.8	11.3	12.9	14.7	16.7	18.9
$y=0$	M_{xy} corner $= \pm pL_y^2$:	2.2	2.7	3.8	5.1	6.6	8.3	10.3	12.7	15.3	18.3	21.6	25.2	29.1	33.3
$y=0.5L_y$	$M_{xy} m = \pm pL_y^2$:	2.6	3.2	4.9	7.0	9.8	13.4	18.1	24.1	31.8	41.5	53.8	69.2	88.4	112
$y=L_y$	M_{xy} corner $= \pm pL_y^2$:	2.9	3.8	6.3	10.1	15.8	24.5	37.6	57.2	86.5	129	193	286	422	618
$x=0.5L_x$, $y=L_y$	$\delta_m = \frac{pL_x^4}{Et^3}$.	.022	.030	.048	.067	.085	.100	.113	.120	.126	.132	.137	.148	.153	.164



C-2) SLAB SIMPLY SUPPORTED ON THREE SIDES AND SUBJECT TO TRIANGULAR LOAD

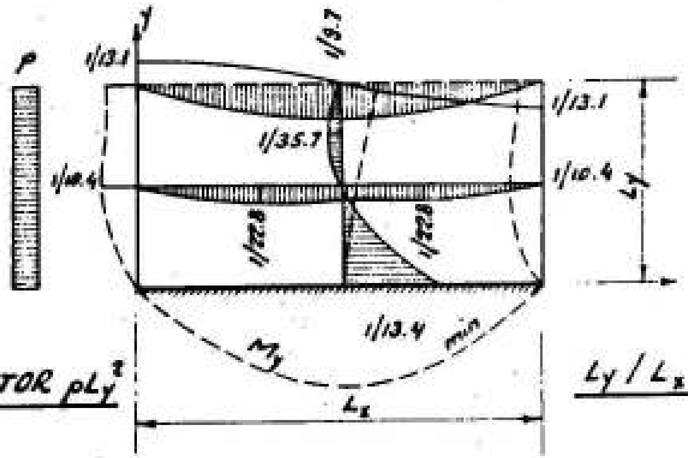
POSITION		L_y/L_x	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$x=0.5L_x$	$M_y m$	$= pL_y^2 :$	16.7	17.5	20.2	24.0	29.5	36.6	45.8	57.4	71.9	89.7	112	138	171	210
$x=0.5L_x$	M_y max	$= pL_y^2 :$	16.2	16.9	19.1	22.5	26.7	31.6	37.6	43.8	51.3	60.5	70.2	79.8	89.9	101
	at	$\frac{y}{L_y} =$	0.4	0.4	0.4	0.4	0.35	0.32	0.3	0.28	0.25	0.23	0.22	0.2	0.19	0.18
$x=0.5L_x$	M_y min	$= -pL_y^2 :$										4566	1647	1175	1016	960
	at	$\frac{y}{L_y} =$										0.9	0.9	0.9	0.9	0.9
$y=0.5L_y$	$M_x m$	$= pL_y^2 :$	23.5	23.2	23.2	23.8	25.0	26.8	28.9	31.4	34.3	37.4	40.9	44.7	48.6	52.9
$y=L_y$	$M_x m$	$= pL_y^2 :$	12.1	12.3	13.2	14.7	16.8	19.6	23.0	27.3	32.4	38.5	45.6	54.0	63.6	74.7
$x=0.5L_x$	M_x max	$= pL_y^2 :$	12.1	12.3	13.2	14.7	16.8	19.6	23.0	27.3	31.7	35.9	40.1	44.5	48.6	52.9
	at	$\frac{y}{L_y} =$	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	0.8	0.65	0.6	0.55	0.5	0.5
$y=0$	M_{xy} corner	$= \pm pL_y^2 :$	6.4	7.5	10.0	12.6	15.5	18.7	22.2	26.0	30.1	34.4	39.1	44.0	49.0	54.5
$y=0.5L_y$	$M_{xy} m$	$= pL_y^2 :$	±7.9	±10.1	±15.6	±23.1	±33.8	±49.4	±72.7	±109	±171	±285	±544	±1558	±84483	±1274
$y=L_y$	M_{xy} corner	$= pL_y^2 :$	±9.4	±12.8	±22.7	±40.7	±77.3	±168	±559	∓1678	∓527	∓394	∓360	∓359	∓374	∓402
$x=0.5L_x$ $y=L_y$	δm	$= \frac{pL_x^4}{Et^3} \cdot$	.007	.010	.016	.021	.027	.031	.034	.037	.038	.039	.039	.039	.038	.038



FACTOR pL_y^2 L_x L_y/L_x = .5

C-3) SLAB SIMPLY SUPPORTED ON TWO SIDES, FIXED ON ONE SIDE AND SUBJECT TO UNIFORM LOAD.

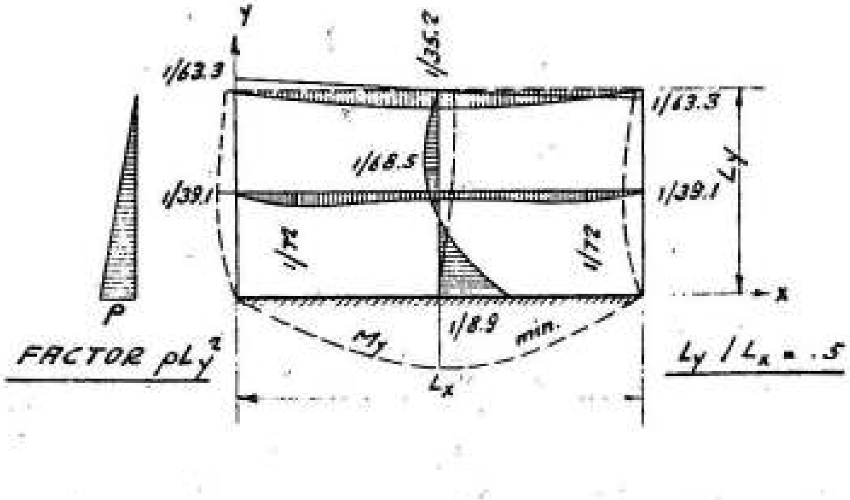
POSITION		L_y/L_x	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$x=0.5L_x$	M_y min	$=-pL_y^2$:	2.3	2.4	2.9	3.4	4.1	4.9	6.0	7.1	8.5	10.0	11.8	13.7	15.8	18.1
	M_y m	$=pL_y^2$:	-12.7	-17.2	-43.6	∞	82.0	57.0	52.2	54.0	59.2	67.5	78.7	92.9	110	132
	M_y max	$=pL_y^2$:	170	90	46.8	35.7	33.3	35.4	39.7	45.4	54.5	66.4	79.7	93.7	110	127
	at	$\frac{y}{L_y}=$	0.88	0.85	0.79	0.75	0.71	0.68	0.65	0.62	0.6	0.57	0.53	0.5	0.46	0.43
$y=L_y$	M_x m	$=pL_y^2$:	26.4	18.0	11.8	9.7	9.0	9.1	9.6	10.4	11.4	12.6	14.0	15.7	17.6	19.6
	M_x max	$=pL_y^2$:	14.3	13.3	11.4	9.7	9.0	9.1	9.6	10.4	11.4	12.6	14.0	15.7	17.6	19.6
	at	$\frac{x}{L_x}=$	0.15 & 0.85	0.2 & 0.8	0.3 & 0.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$y=0.5L_y$	M_x m	$=pL_y^2$:	73.2	52.3	31.7	24.0	20.6	18.9	18.6	18.9	19.7	20.7	22.0	23.5	26.2	27.2
	M_x max	$=pL_y^2$:	27.5	27.3	25.6	22.8	20.6	18.9	18.6	18.9	19.7	20.7	22.0	23.5	26.2	27.2
	at	$\frac{x}{L_x}=$	0.15 & 0.85	0.15 & 0.85	0.2 & 0.8	0.3 & 0.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$y=L_y$	M_{xy} corner	$=\pm pL_y^2$:	8.7	9.0	10.5	13.1	17.1	23.2	32.2	45.4	64.5	92.3	133	191	274	394
$x=0.5L_x$, $y=L_y$	δ_m	$=\frac{pL_x^4}{Et^3}$.	.005	.009	.020	.035	.051	.070	.087	.098	.109	.120	.131	.142	.153	.164



FACTOR pL_y^2 $L_y/L_x = .5$

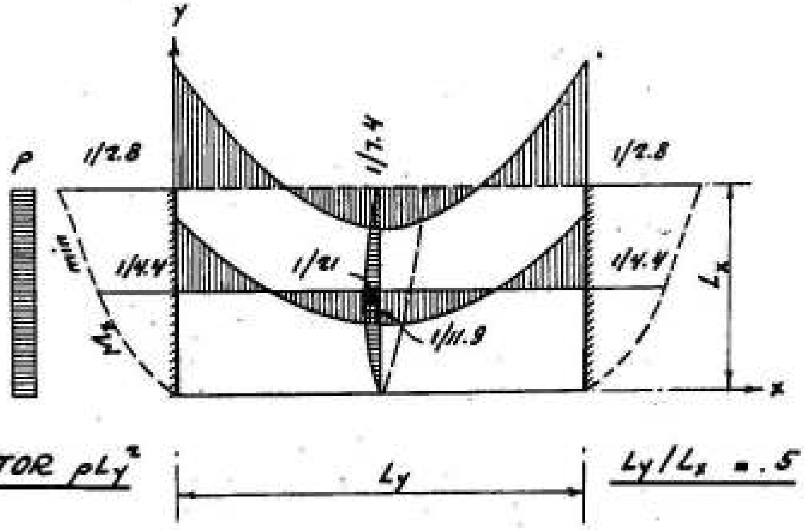
C-4) SLAB SIMPLY SUPPORTED ON TWO SIDES, FIXED ON ONE SIDE AND SUBJECT TO TRIANGULAR LOAD.

POSITION		L_y/L_x	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$x = 0.5L_x$	M_y min	$= -pL_y^2$:	6.6	6.9	7.8	8.9	10.3	11.7	13.5	15.4	17.5	19.8	22.2	24.8	27.5	30.5
	M_y m	$= pL_y^2$:	-114	-299	175	87.8	71.4	68.3	70.9	72.3	86.7	99.2	115	134	158	186
	M_y max	$= pL_y^2$:	220	135	84.0	68.5	64.2	65.1	68.6	76.5	85.5	95.7	107	119	132	145
	at	$\frac{y}{L_y} =$	0.75	0.75	0.63	0.62	0.58	0.53	0.49	0.46	0.45	0.43	0.4	0.38	0.36	0.35
$y = L_y$	M_x m	$= pL_y^2$:	94.7	66.0	42.8	35.2	32.7	32.9	34.7	37.9	42.6	47.6	54.2	62.0	71.1	81.9
	M_x max	$= pL_y^2$:	43.5	48.4	41.2	35.2	32.7	32.9	34.7	37.9	42.6	47.6	54.2	62.0	71.1	81.9
	at	$\frac{x}{L_x} =$	0.15 & 0.85	0.2 & 0.8	0.3 & 0.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$y = 0.5L_y$	M_x m	$= pL_y^2$:	252	184	110	80.5	66.0	59.2	56.0	54.9	55.0	56.3	58.2	60.7	63.6	67.0
	M_x max	$= pL_y^2$:	650	75.0	76.8	72.0	64.8	59.2	56.0	54.9	55.0	56.3	58.1	60.6	63.6	66.9
	at	$\frac{x}{L_x} =$	0.1 & 0.9	0.15 & 0.85	0.15 & 0.85	0.2 & 0.8	0.3 & 0.7	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
$y = L_y$	M_{xy} corner	$= \pm pL_y^2$	37.0	39.1	47.3	63.3	90.1	143	258	618	15152	1122	678	560	524	521
$x = 0.5L_x$, $y = L_y$	δm	$= \frac{pL_x^4}{Et^3}$ ·	.001	.003	.006	.010	.015	.019	.024	.026	.031	.033	.034	.035	.035	.035



C-5) SLAB FIXED ON TWO SIDES, SIMPLY SUPPORTED ON ONE SIDE AND SUBJECT TO UNIFORM LOAD

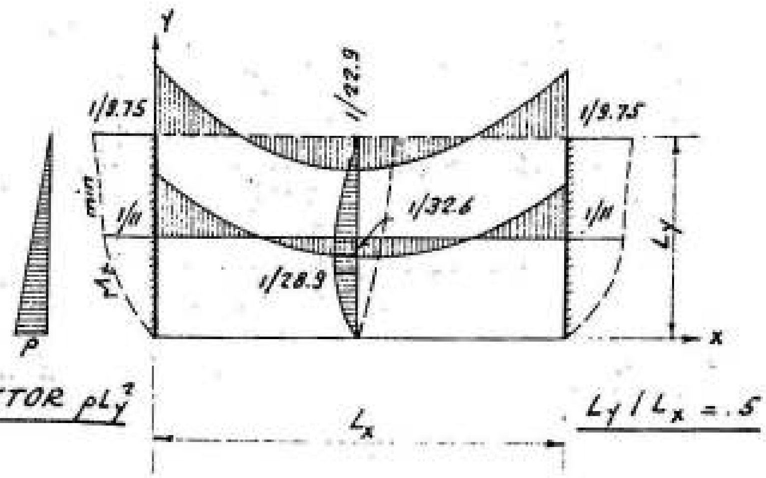
POSITION	L_y/L_x	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$y = L_y$	$M_x\ min = -pL_y^2:$	1.0	1.3	1.9	2.8	3.8	5.2	6.8	8.7	10.8	13.2	15.7	18.6	21.6	24.9
	$M_x\ m = pL_y^2:$	4.4	4.6	5.8	7.4	9.6	12.2	15.4	19.1	23.4	28.2	33.5	39.4	45.8	52.6
$y = 0.5L_y$	$M_x\ min = -pL_y^2:$	2.0	2.5	3.4	4.4	5.6	7.0	8.6	10.3	12.2	14.4	16.8	19.4	22.3	25.4
	$M_x\ m = pL_y^2:$	8.1	8.4	9.9	11.9	14.4	17.3	20.5	24.0	28.2	32.8	37.5	43.0	48.8	55.4
$x = 0.5L_x$	$M_y\ m = pL_y^2:$	9.6	11.4	15.3	21.6	30.5	43.2	61.1	85.0	122	171	240	331	462	652
	$M_y\ max = pL_y^2:$	9.6	11.9	15.2	21.0	28.3	36.6	48.2	61.2	75.8	90.9	104	118	133	150
	at $\frac{y}{L_y} =$	0.5	0.48	0.47	0.43	0.38	0.33	0.28	0.25	0.22	0.19	0.16	0.13	0.1	0.1
$x = 0.5L_x$, $y = L_y$	$\delta_m = \frac{pL_x^4}{Et^3}\cdot$	.015	.020	.025	.030	.033	.034	.034	.034	.034	.034	.034	.034	.034	.034



FACTOR pL_y^2 $L_y/L_x = .5$

C-6) SLAB FIXED ON TWO SIDES, SIMPLY SUPPORTED ON ONE SIDE AND SUBJECT TO TRIANGULAR LOAD.

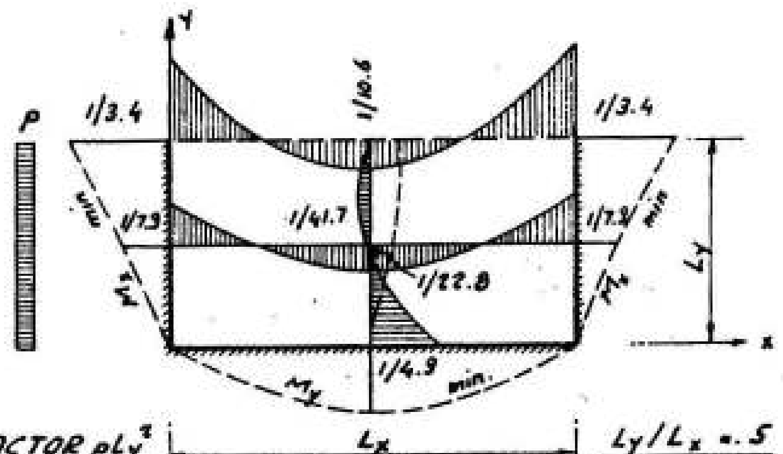
POSITION	L_y/L_x	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$y = L_y$	M_x min $= -pL_y^2$:	3.2	4.1	6.3	9.8	14.8	21.8	32.3	46.1	64.6	88.0	117	150	187	230
	M_x m $= pL_y^2$:	13.0	13.8	17.6	22.9	30.5	40.7	55.2	74.2	98.0	129	166	210	262	324
$y = 0.5L_y$	M_x min $= -pL_y^2$:	5.7	6.7	8.8	11.0	13.3	16.0	18.9	22.3	26.0	29.9	34.6	39.5	45.1	51.3
	M_x m $= pL_y^2$:	24.3	25.3	28.3	32.6	37.7	43.4	49.6	56.3	63.6	72.2	81.4	91.5	102	115
$x = 0$, $x = L_x$	M_x min $= -pL_y^2$:	5.7	6.7	8.8	11.0	13.2	16.0	18.6	21.6	24.6	27.9	31.4	34.7	39.2	43.5
	at $\frac{y}{L_y} =$	1.0	1.0	1.0	1.0	0.57	0.5	0.46	0.42	0.39	0.36	0.34	0.33	0.32	0.31
$x = 0.5L_x$	M_y m $= pL_y^2$:	18.3	20.2	25.7	33.6	44.6	60.1	81.6	111	152	209	288	390	532	750
	M_y max $= pL_y^2$:	17.7	19.1	23.5	28.9	35.9	44.4	54.2	65.5	78.4	93.5	113	132	153	175
	at $\frac{y}{L_y} =$	0.4	0.38	0.35	0.33	0.29	0.28	0.25	0.23	0.21	0.2	0.19	0.18	0.16	0.15
$x = 0.5L_y$	M_y min (at $\frac{y}{L_y} = .88$) $= pL_y^2$	—	—	—	—	—	—	—	-1386	-1060	-957	-936	-1001	-1148	-1350
$x = 0.5L_x$, $y = L_y$	$\omega = \frac{pL_x^4}{Et^3}$ ·	.005	.005	.008	.009	.009	.003	.009	.008	.007	.007	.006	.006	.005	.005



FACTOR pL_y^2 $L_y/L_x = .5$

C-7) SLAB FIXED ON THREE SIDES AND SUBJECT TO UNIFORM LOAD.

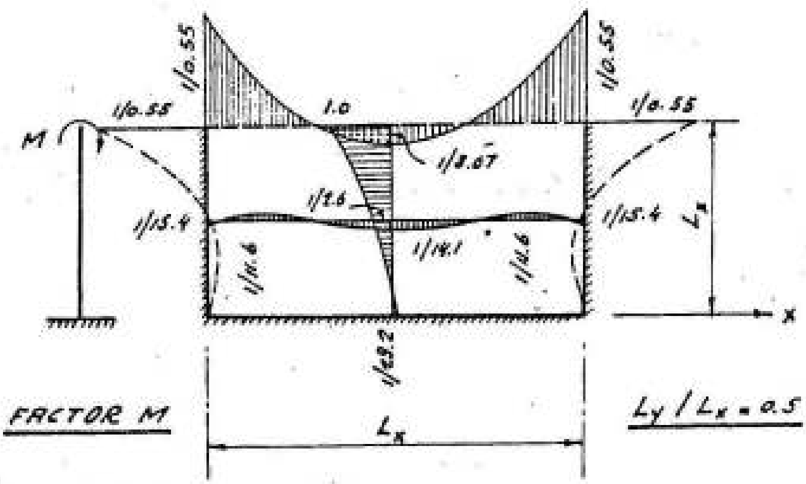
POSITION	L_y/L_x	0.25	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5
$x=0.5L_x$	M_y min $= -pL_y^2 :$	2.4	2.7	3.6	4.9	6.7	8.8	11.3	14.3	17.6	21.3	25.3	29.7	34.4	39.5
$y=0.5L_y$	M_x min $= -pL_y^2 :$	6.8	6.9	7.3	7.9	8.8	10.0	11.4	13.1	15.0	17.1	19.5	22.1	25.0	28.1
$y=L_y$	M_x min (max) $= -pL_y^2 :$	2.2	2.3	2.7	3.4	4.5	5.8	7.4	9.4	11.6	14.1	16.9	19.9	23.2	26.7
$x=0.5L_x$	M_{ym} $= pL_y^2 :$	-16.1	-26.7	-26.8	88.1	62.9	63.8	73.4	89.6	113	145	190	250	333	446
$x=0.5L_x$	M_y max $= pL_y^2 :$	104	62.5	43.2	41.7	48.0	58.5	72.7	89.6	109	132	157	184	213	245
	at $\frac{y}{L_y} =$	0.88	0.8	0.75	0.7	0.65	0.6	0.55	0.5	0.45	0.4	0.35	0.32	0.3	0.28
$y=0.5L_y$	M_{xm} $= pL_y^2 :$	57.1	37.9	25.4	22.8	23.2	24.9	27.5	30.7	34.3	38.4	43.0	48.1	53.6	59.7
$y=L_y$	$M_{xm} = M_x$ max $: pL_y^2 :$	13.2	13.9	10.7	10.6	11.9	13.9	16.7	20.2	24.3	29.0	34.4	40.3	46.7	53.7
$x=0.5L_x$, $y=L_y$	δ_m $= \frac{pL_x^4}{Et^3}.$	.004	.007	.015	.021	.027	0.30	0.32	0.33	.034	.035	.035	.035	.034	.034



FACTOR pL_y^2 L_x $L_y/L_x = .5$

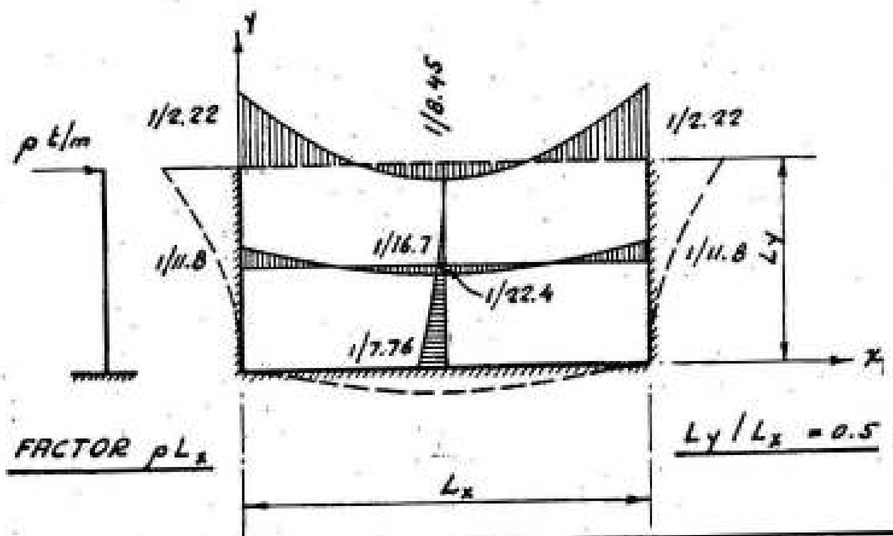
C-10) SLAB FIXED ON THREE SIDES AND SUBJECT TO UNIFORM EDGE MOMENT M ml/m.

POSITION x/L_x	POSITION y/L_y		L_y/L_x	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	1.10	1.20	1.30	1.40	1.50
0;1	1	M_x min	= M :	0.61	0.57	0.54	0.55	0.56	0.57	0.60	0.62	0.64	0.67	0.69	0.72	0.74	0.77
0;1	0.5	M_x	= M :	$\bar{5}.98$	$\bar{8}.29$	$\bar{4}.76$	15.4	7.91	6.17	5.59	5.56	5.94	6.67	7.55	8.66	10.2	12.4
0;1	VARIABLE	M_x max	= M :	3.00	69.0	23.2	12.2	7.82	6.03	5.10	4.72	4.52	4.42	4.37	4.33	4.31	4.31
		at	y/L_y =	0.14	0.18	0.29	0.39	0.47	0.54	0.59	0.63	0.66	0.69	0.72	0.74	0.76	0.77
0.5	1	$M_x m$	= M :	5.40	4.00	3.14	3.07	3.18	3.29	3.40	3.44	3.48	3.49	3.50	3.52	3.53	3.54
0.5	0.5	$M_x m$	= M :	16.30	12.3	10.6	14.1	22.4	∞	$\overline{36}.1$	$\overline{21}.3$	$\overline{18}.5$	$\overline{17}.5$	$\overline{17}.9$	$\overline{18}.7$	$\overline{20}.4$	$\overline{23}.4$
VARIABLE	0.5	M_x min	= M :		7.00	8.2	11.6	18.5	29.5	27.8	$\overline{21}.3$	18.5	17.5	17.9	18.7	20.4	23.4
		at	x/L_x =		.05 .95	.08 .92	.13 .87	.15 .85	.21 .79	.27 .73	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.5	0	M_y	= M :	$\bar{1}.52$	$\bar{2}.08$	$\bar{6}.05$	29.2	8.03	6.90	7.55	9.00	12.7	18.5	27.8	41.7	66.7	115
0.5	0.5	$M_y m$	= M :	1.32	1.52	2.01	2.60	3.36	4.39	6.07	8.62	13.1	21.7	40.0	100	∞	130
0.5	1	δ =	$\frac{ML_x^2}{Et^3}$.	.273	.325	.363	.363	.353	.343	.333	.333	.325	.325	.316	.308	.308	.300



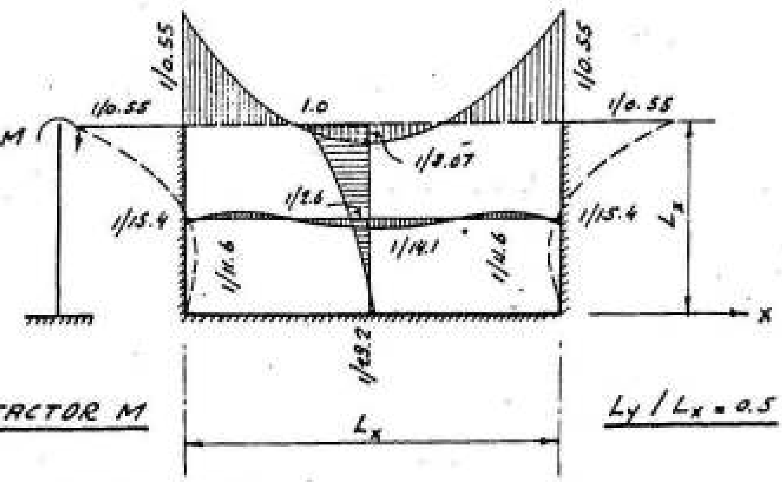
C-9) SLAB FIXED ON THREE SIDES AND SUBJECT TO UNIFORM EDGE LOAD p t/m.

POSITION x/L_x	POSITION y/L_y	L_y/L_x	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	1.10	1.20	1.30	1.40	1.50
0;1	1	M_x min = $-pL_x$:	3.96	3.17	2.47	2.22	2.14	2.12	2.14	2.16	2.19	2.21	2.24	2.27	2.31	2.34
0;1	0.5	M_x min = $-pL_x$:	15.0	13.0	11.6	11.8	13.0	15.3	19.2	25.0	33.5	46.3	66.3	98.0	155	250
0.5	1	M_x m = pL_x:	31.2	18.8	10.8	8.45	7.62	7.35	7.21	7.19	7.18	7.19	7.20	7.21	7.23	7.25
0.5	0.5	M_x m = pL_x:	93.6	53.2	28.5	22.4	21.4	22.5	26.1	31.2	38.6	48.3	63.0	83.4	111	150
0.5	0	M_y min = $-pL_x$:	5.03	4.95	5.72	7.76	11.2	18.7	32.9	64.1	130	270	—	—	—	—
0.5	0.5	M_y m = $-pL_x$:	11.8	12.2	14.3	16.7	19.0	21.0	23.6	26.8	30.7	36.0	42.6	51.0	67.1	76.5
0.5	VARIABLE	M_y min = $-pL_x$: at	—	—	—	—	—	20.0	19.7	19.5	19.7	20.0	20.3	20.6	20.8	20.9
		y/L_y =	—	—	—	—	—	0.63	0.68	0.73	0.76	0.78	0.80	0.82	0.84	0.85
0.5	1	$\delta_m = \frac{pL_x^3}{Et^3}$:	.046	.067	.092	.109	.120	.126	.126	.133	.133	.126	.126	.126	.126	.126



C-10) SLAB FIXED ON THREE SIDES AND SUBJECT TO UNIFORM EDGE MOMENT M ml/m.

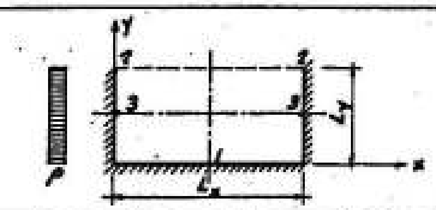
POSITION x/Lx	POSITION y/Ly		Ly/Lx	0.25	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00	1.10	1.20	1.30	1.40	1.50
0;1	1	M_x min	=..M:	0.61	0.57	0.54	0.55	0.56	0.57	0.60	0.62	0.64	0.67	0.69	0.72	0.74	0.77
0;1	0.5	M_x	= M:	5.98	8.29	4.76	15.4	7.91	6.17	5.59	5.56	5.94	6.67	7.55	8.66	10.2	12.4
0;1	VARIABLE	M_x max	= M:	3.00	69.0	23.2	12.2	7.82	6.03	5.10	4.72	4.52	4.42	4.37	4.33	4.31	4.31
		at	y/Ly=	0.14	0.19	0.29	0.39	0.47	0.54	0.59	0.63	0.66	0.69	0.72	0.74	0.76	0.77
0.5	1	M_xm	= M:	5.40	4.00	3.14	3.07	3.18	3.29	3.40	3.44	3.48	3.49	3.50	3.52	3.53	3.54
0.5	0.5	M_xm	= M:	16.90	12.3	10.6	14.1	29.4	∞	36.1	21.3	18.5	17.5	17.9	18.7	20.4	23.4
VARIABLE	0.5	M_x min	=-M:		7.00	8.2	11.6	18.5	29.5	27.8	21.3	18.5	17.5	17.9	18.7	20.4	23.4
		at	x/Lx=		.05 .95	.08 .92	.13 .87	.15 .85	.21 .79	.27 .73	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.5	0	My	= M:	1.52	2.08	6.05	29.2	8.03	6.90	7.55	9.00	12.7	18.5	27.8	41.7	66.7	115
0.5	0.5	Mym	=-M:	1.32	1.52	2.01	2.60	3.36	4.39	6.07	8.62	13.1	21.7	40.0	100	∞	130
0.5	1	δ =	$\frac{ML_x^2}{Et^3}$.	.273	.325	.363	.363	.353	.343	.333	.333	.325	.325	.316	.308	.308	.300



FACTOR M   Ly / Lx = 0.5

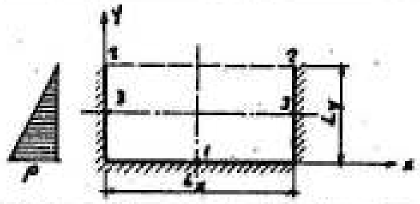
C-11) REACTIONS OF SLABS FIXED ON THREE SIDES AND SUBJECT TO UNIFORM AND TRIANGULAR LOADS.*

a- UNIFORM LOAD.



POSITION x/L_x	POSITION y/L_y	L_y/L_x	0.60	0.70	0.80	0.90	1.00	1.25	1.50
0.5	0	$R_1 = pL_y$:	1.44	1.70	1.95	2.20	2.50	3.22	4.02
0;1	1	$R_2 = pL_y$:	0.80	0.98	1.17	1.37	1.59	2.20	2.85
0;1	0.5	$R_3 = pL_y$:	2.00	2.00	2.07	2.17	2.30	2.53	3.05

b- TRIANGULAR LOAD.



POSITION x/L_x	POSITION y/L_y	L_y/L_x	0.60	0.70	0.80	0.90	1.00	1.25	1.50
0.5	0	$R_1 = pL_y$:	2.40	2.70	2.90	3.14	3.33	4.05	4.87
0;1	1	$R_2 = pL_y$:	6.50	8.70	11.6	15.8	22.2	69.5	-2.50
0;1	0.5	$R_3 = pL_y$:	4.40	4.40	4.50	4.60	4.80	5.35	6.13

* TIMOSHENKO : "PLATES AND SHELLS" FOR $\nu = 1/m = 1/6$.

d) <u>Direct Application of Czerny's-Tables in Tanks</u>.

The tables of Czerny can be directly used in some typical problems of rectangular tanks as can be seen from the following example.

It is required to design a 160 m^3 capacity open square water tank supported on four columns.

The clear height of the tank will be assumed 4.3 ms. then:

$$\text{Clear area} = 160/4.3 = 37.2 \text{ m}^2$$

$$\text{Clear side length} = \sqrt{37.2} = 6.1 \text{ m}$$

To reduce the internal forces in the floor, four beams are arranged so that its span is 4.5 ms only. The chosen dimensions of the tank will be as shown in figure VII.27

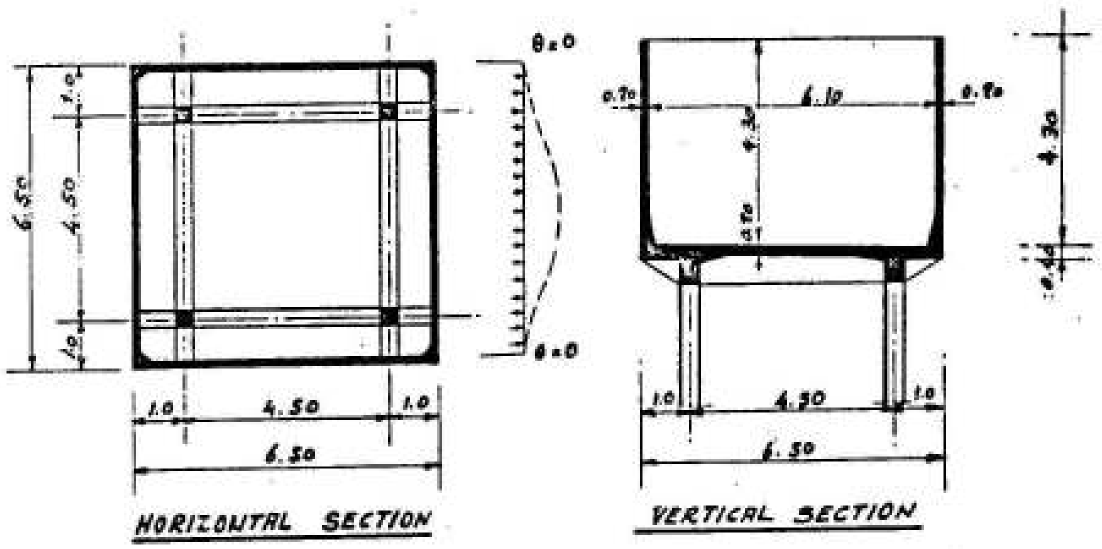


Fig. VII-27

Each of the walls of the tank has :

a theoretical height of $H = 4.50$ ms and ,

a theoretical length of $L = 6.30$ ms

The walls can be assumed fixed at their vertical edges due to symmetry in shape and loading ($\theta = o$) and fixed to the floor due to the continuity with a short span of relatively big thickness.

The floor slab 4.5 x 4.5 ms can be assumed as fixed on all four sides for the same reason.

Max. water pressure on walls $p_{max} = 4.3\ t/m^2$

Load on intermediate part of floor slab p_1

= weight of water + own weight of floor slab (assumed 20 cms. thick)

or $p_1 = 4.3 + 0.2 \times 2.5 = 4.3 + 0.5 = 4.8\ t/m^2$

Load on outside part of floor slab (assumed 40 cms. thick)

$$p_2 = 4.3 + 0.4 \times 2.5 = 4.5 + 1.0 = 5.3\ t/m^2$$

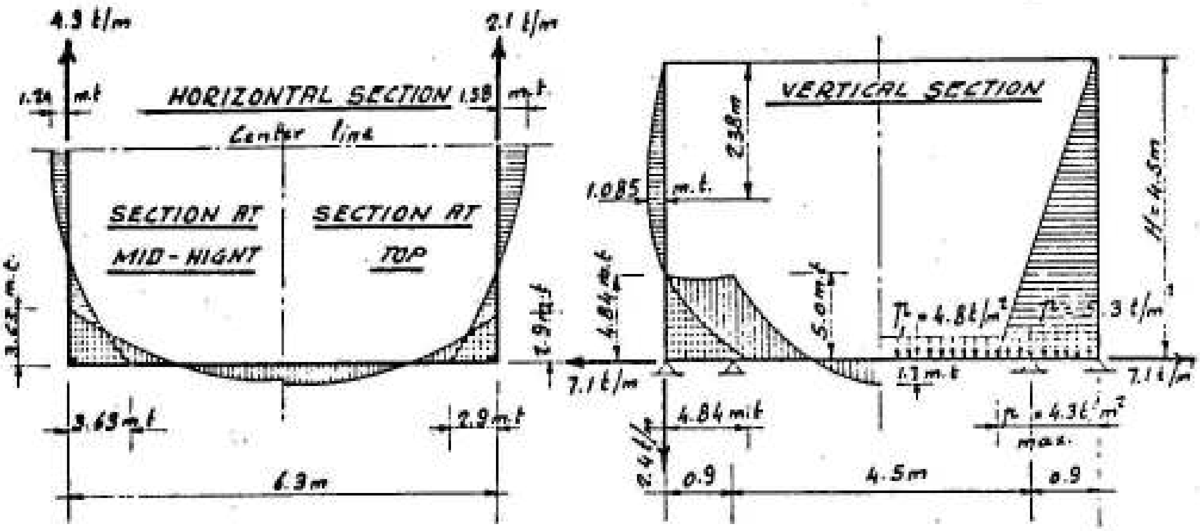


Fig. VII-28

The internal forces in the walls, free at top and fixed on the other three sides can be calculated according to table C-8 as follows : (Fig. VII.28).

$H/L = 4.5 / 6.3 = 0.715$ $\qquad p_{max}H^2 = 4.3 \times 4.5^2 = 87$ mt.

Bending moments along middle vertical axis are therefore :

Max. fixing moment at base :

$$M_y\text{min.} = - p_{max.}H^2/18 = - 87/18 = - 4.84 \text{ m t}$$

Max. field moment at 0.47 H from base :

$$M_y\text{max.} = p\ H^2/80 = 87/80 = 1.085 \text{ m t}$$

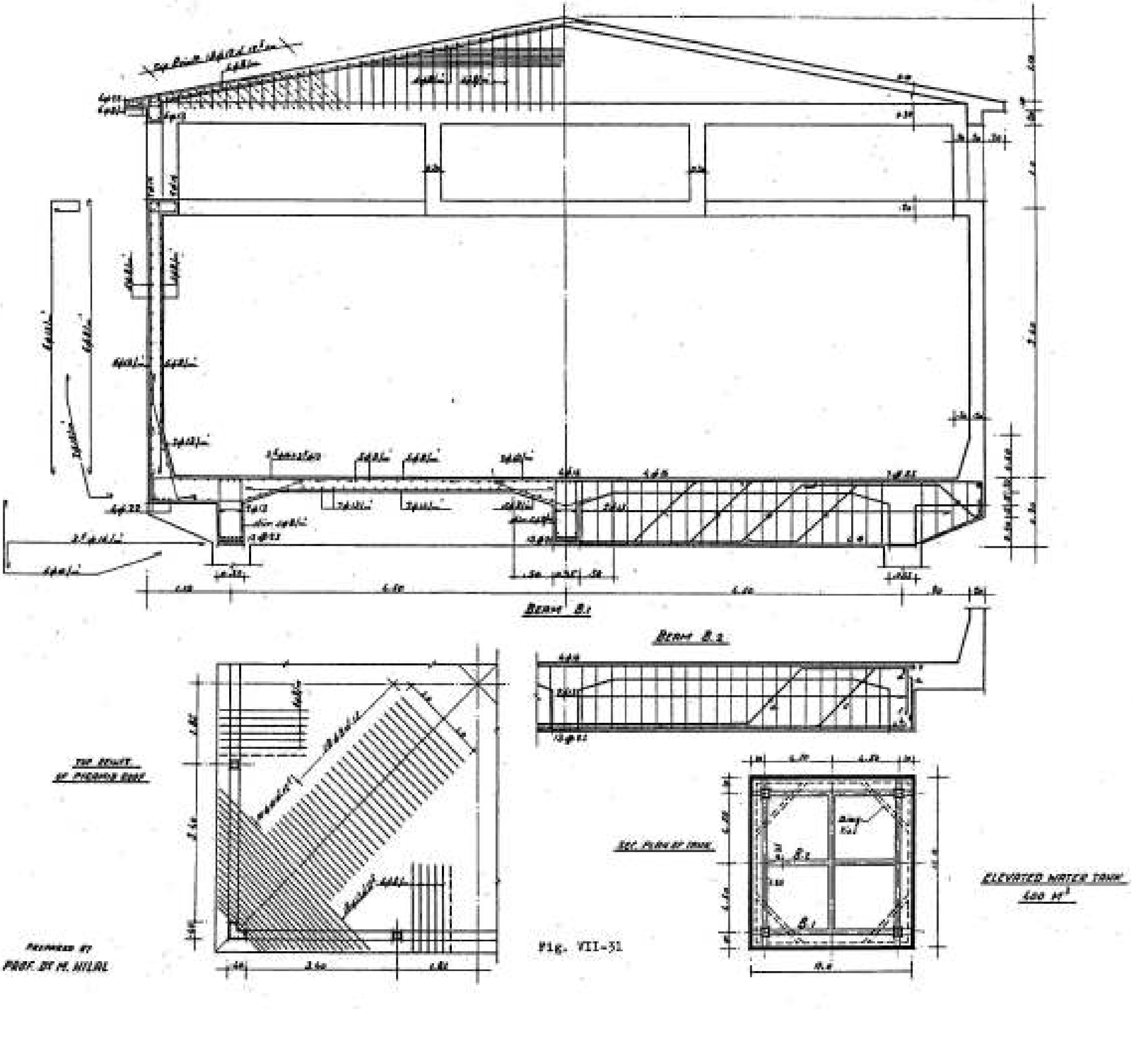

BEAM B.1
BEAM B.2
ELEVATED WATER TANK
PREPARED BY
PROF. DR M. HILAL

Fig. VII-31

The floor slab 4.5 x 4.5 ms can be assumed as fixed on all four sides for the same reason.

Max. water pressure on walls $p_{max} = 4.3\ t/m^2$

Load on intermediate part of floor slab p_1

= weight of water + own weight of floor slab (assumed 20 cms. thick)

or $p_1 = 4.3 + 0.2 \times 2.5 = 4.3 + 0.5 = 4.8\ t/m^2$

Load on outside part of floor slab (assumed 40 cms. thick)

$p_2 = 4.3 + 0.4 \times 2.5 = 4.5 + 1.0 = 5.3\ t/m^2$

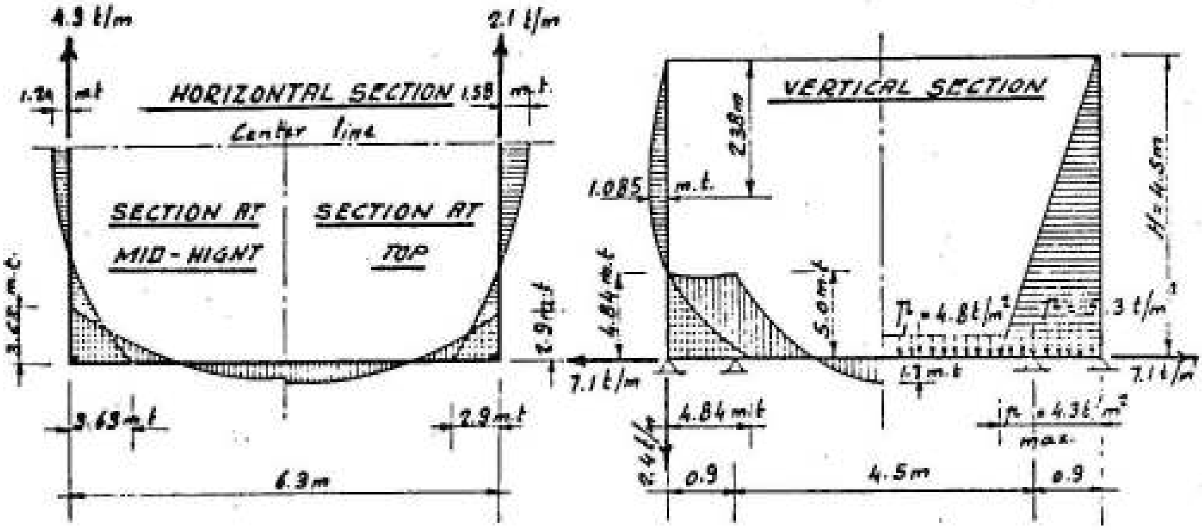


Fig. VII-28

The internal forces in the walls, free at top and fixed on the other three sides can be calculated according to table C-8 as follows : (Fig. VII.28).

$H/L = 4.5 / 6.3 = 0.715$ $p_{max}H^2 = 4.3 \times 4.5^2 = 87$ mt.

Bending moments along middle vertical axis are therefore :

Max. fixing moment at base :

$$M_y min. = - p_{max.} H^2/18 = - 87/18 = - 4.84 \text{ m t}$$

Max. field moment at 0.47 H from base :

$$M_y max. = p\ H^2/80 = 87/80 = 1.085 \text{ m t}$$

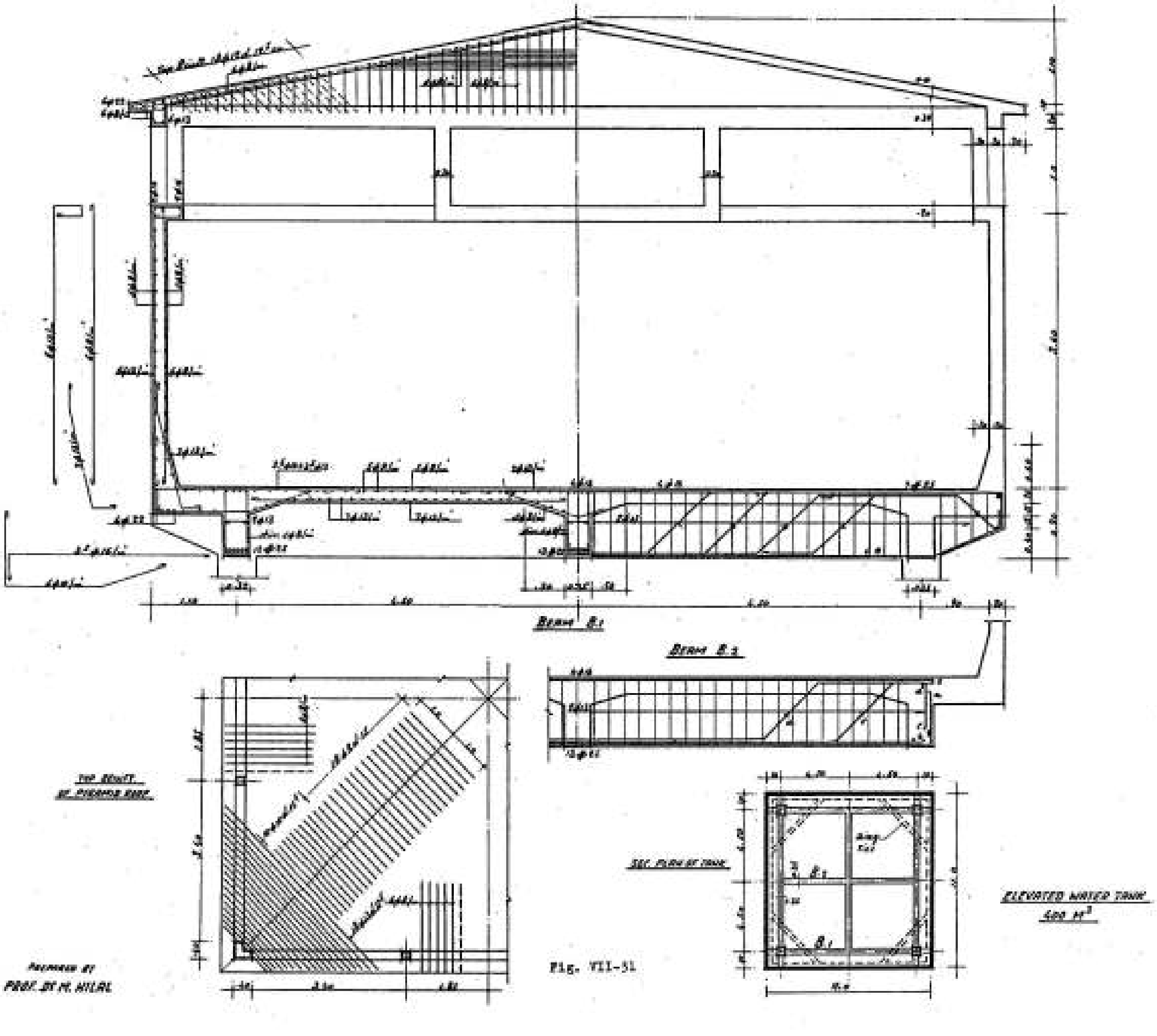

BEAM B1
BEAM B2
TOP REINF. OF PYRAMID ROOF
SEC. PLAN OF TANK
ELEVATED WATER TANK
400 M³
PREPARED BY
PROF. DR M. HILAL

Fig. VII-31

Bending moments in horizontal direction are :

Section at top surface :

Max. fixing moment

$$M_x min. = - p_{max.} H^2/30 = - 87/30 = - 2.90 \text{ m t}$$

Max. field moment

$$M_m max. = p_{max.} H^2/55 = 87/55 = \underline{1.58} \text{ m t}$$

Section at 0.6 H from bottom

Max. fixing and field moments :

$$M_x min. = - p_{max.} H^2/24 = - 87/24 = - \underline{3.63} \text{ m t}$$

$$M_m max. = p_{max.} H^2/72.4 = 87/72.4 = \underline{1.20} \text{ m t}$$

The reactions can be calculated from the table of Timoshenko C-11b , thus : for H/L = 0.715 we get :

Reaction at middle of base = max. tension in floor

$$R_1 = pH/2.73 = 4.3x4.5/2.73 = 7.1 \text{ t/m}$$

Reaction at top edge of wall = horizontal tension in side walls

$$R_2 = pH/9.14 = 4.3x4.5/9.14 = 2.1 \text{ t/m}$$

Reaction at mid. height of wall = horizontal tension in side walls

$$R_3 = pH/4.42 = 4.3x4.5/4.42 = 4.9 \text{ t/m}$$

Tension in wall at its base

$$R_4 = p_2 \times 0.9/2 = 5.3 \times 0.45 = 2.40 \text{t/m}$$

The internal forces in the floor slab, fixed on all four sides and subject to uniform load $p_1 = 4.8$ t/m^2 can be calculated according to table A-6 as follows :

$L_y = L_x = L = 4.5$ m $\qquad p_1 L^2 = 4.8 \times 4.5^2 = 97$ m t

Max. fixing moment

$$M_f max. = - p L^2/19.4 = - 97/19.4 = - \underline{5.00} \text{ m t}$$

Max. field moment

$$M_m max. = p L^2/56.8 = 97/56.8 = \underline{1.70} \text{ m t}$$

The vertical and horizontal sections of the tank are to be calculated for the internal forces shown in figure VII.28.

The bending moments and shearing forces in the floor beams can be calculated as follows : (Fig. VII.29).

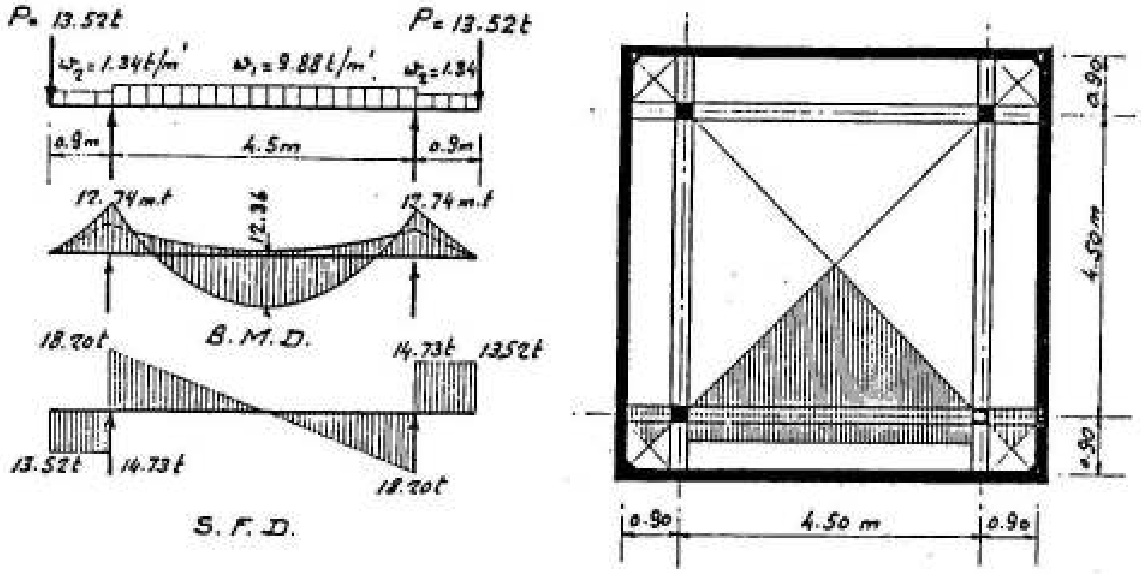


Fig. VII-29

Each of the four supporting beams can be considered as a simple beam 4.5 ms. span with double cantilevers 0.90 ms. each. The beam is assumed 30 x 60 cms.

Load on intermediate span.

$w_1 = 0.3 \times 0.4 \times 2500 + 0.67 \times \frac{4.5}{2} \times 4800 + 0.45 \times 5300$ or

$w_1 = 300 + 7200 + 2380 = 9880$ kg/m

Load on outside cantilevers :

$w_2 = 0.3 \times 0.4 \times 2500 \times \frac{1}{2} + \frac{0.45}{2} \times 5300 = 150 + 1190 = 1340$ kg/m

Concentrated load :

P = own weight of wall = $0.2 \times 4.5 \times 2500 \times \frac{6.3}{2}$ = 7100 kg

+ uniform floor load = $0.45 \times 5300 \times \frac{4.5}{2}$ = 5350 kg

+ triangular floor load = $\frac{0.45}{2} \times 5300 \times 0.9$ = 1070 kg

i.e. total P = 13520 kgs.

Bending moments in horizontal direction are :

Section at top surface :

Max. fixing moment

$$M_x min. = - p_{max.} H^2/30 = - 87/30 = - 2.90 \text{ m t}$$

Max. field moment

$$M_m max. = p_{max.} H^2/55 = 87/55 = \underline{1.58} \text{ m t}$$

Section at 0.6 H from bottom

Max. fixing and field moments :

$$M_x min. = - p_{max.} H^2/24 = - 87/24 = - \underline{3.63} \text{ m t}$$

$$M_m max. = p_{max.} H^2/72.4 = 87/72.4 = \underline{1.20} \text{ m t}$$

The reactions can be calculated from the table of Timoshenko C-11b , thus : for H/L = 0.715 we get :

Reaction at middle of base = max. tension in floor

R_1 = pH/2.73 = 4.3x4.5/2.73 = 7.1 t/m

Reaction at top edge of wall = horizontal tension in side walls

R_2 = pH/9.14 = 4.3x4.5/9.14 = 2.1 t/m

Reaction at mid. height of wall = horizontal tension in side walls

R_3 = pH/4.42 = 4.3x4.5/4.42 = 4.9 t/m

Tension in wall at its base

R_4 = p_2 x 0.9/2 = 5.3 x 0.45= 2.40t/m

The internal forces in the floor slab, fixed on all four sides and subject to uniform load $p_1 = 4.8 \text{ t/m}^2$ can be calculated according to table A-6 as follows :

$L_y = L_x = L = 4.5$ m $\qquad p_1 L^2 = 4.8 \times 4.5^2 = 97$ m t

Max. fixing moment

$$M_f max. = - p L^2/19.4 = - 97/19.4 = - \underline{5.00} \text{ m t}$$

Max. field moment

$$M_m max. = p L^2/56.8 = 97/56.8 = \underline{1.70} \text{ m t}$$

Therefore, the max. cantilever moment is given by :

$$M_{max.} = 13.52 \times 0.9 + 1.34 \times \frac{0.9^2}{2} = 12.2 + 0.54 = 12.74 \quad m\ t$$

and the max. field moment is :

$$M_m max. = 9.88 \times \frac{4.5^2}{8} - 12.74 = 25.0 - 12.74 = 12.36 \quad m\ t$$

The loads, bending moments and shearing forces are shown in figure VII.29.

One can see that the choice of the dimensions of the tank was convenient for the following reasons :

1) The max. fixing moment in the wall (- 4.84 m t) is approx. equal to the max. fixing moment in the floor (- 5.00 m t).
2) The max. moments are accumulated at the corner between walls and floor, so that only a small part of the slabs needs a relatively big thickness (~ 40 cms).
3) The max. field moment in the wall (1.58 m t) is approx. equal to the max. field moment in the floor (1.70 mt).
4) The thickness of slabs is generally governed by their field moments and as their values are small, only the minimum thickness necessary for water-tightness (~ 20 cms) is sufficient.
5) The cantilever part has been chosen such that it gives approximately equal cantilever and field moments in the supporting beams.

The same idea can be adopted for bigger capacities as shown in figure VII.30 in which the capacity of the tank is :

$4.00 \times 10.60 \times 10.60 \approx 450 \quad m^3.$

The floor slabs are again here fixed at the intermediate panelled frames, because they are arranged on the axes of symmetry.

Each of the floor panelled frames has a span of 9 ms. and carries, in addition to its own weight, a floor load of $p = \frac{4.5}{2} \times 4.5 = 10$ t/m on the intermediate span.

The walls may however be supported at their top edge on cover slabs or horizontal beams as those shown in the example. Such beams can be assumed as supports for the walls if they are supported at convenient distances (smaller than ca 6 ms.). In the shown example diagonal ties may give a convenient solution.

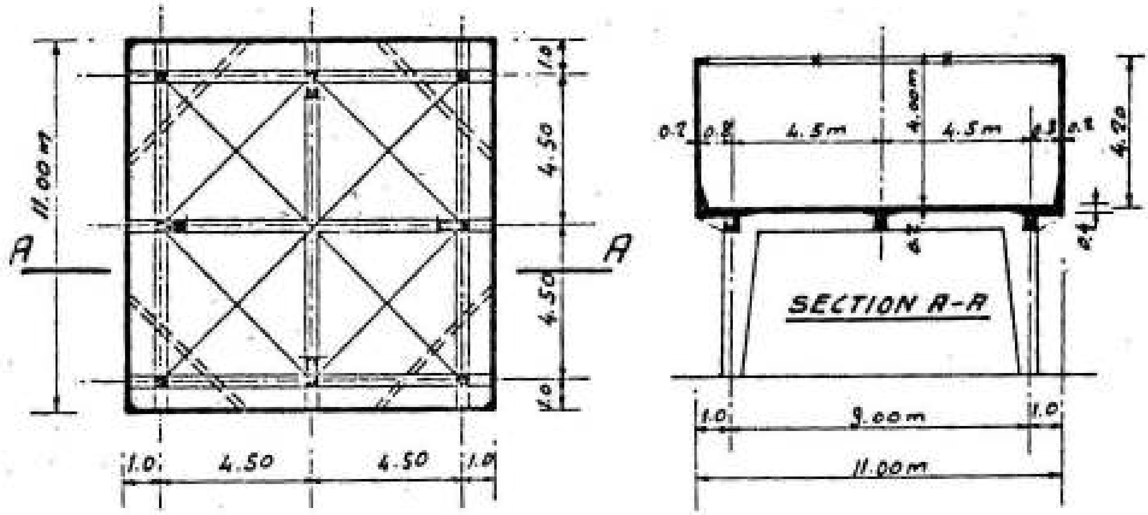


Fig. VII-30

For the case of walls simply supported at top and fixed to the floor slab at bottom, and resisting the full water pressure in the vertical direction, the height of wall H and the span of square floor slabs L giving the same fixing moment in wall and floor can be determined as follows :

Max. hydrostatic pressure on wall $p_{max.} \cong 0.95\ H$

Load on floor slab including own weight $p \cong 1.10\ H$

Fixing moment of wall = connecting moment of floor or

$$0.95\ H^3 / 15 = 1.10\ H\ L^2 / 19.4 \quad \text{or}$$

$$\underline{L = 1.06\ H}$$

Figure VII.31 shows the concrete dimensions and details of reinforcements of a covered reinforced concrete square tank = 400 m^3

capacity supported on four columns. The general arrangement of the supporting elements of the tank follows the given recommendations. The supporting elements of the floor are composed of four simple girders supported on the columns ; each of them is provided with two cantilever arms supporting the walls as deep beams. The square floor enclosed between the main girders is subdivided by two panelled simple beams. Each of the four panels of the floor can be assumed as totally fixed at its edges. The walls being 3.6 x 10.80 m, they behave as one way slabs simply supported on the top horizontal beam and totally fixed to the floor. They resist the water pressure in the vertical direction only. In this manner, the bigger values of the bending moments in a vertical section of the tank are accumulated at the corner between the walls and the floor. However, the local horizontal bending moments along the vertical corners of the tank - equations 10 - are to be taken in consideration. In order to avoid vertical supports inside the tank, the top horizontal beam of the wall is supported on diagonal ties shown dotted-. The roof is a reinforced concrete square pyramid supported on twelve posts arranged on the top of the walls at the corners and the third points of each side. The windows between the posts allow for good airation of the water in the tank.

VII.4. COUNTERFORTED WALLS :

In order to reduce the thickness of walls of deep rectangular tanks, it may be of advantage to support them on counterforts as can be seen in the following example of a diving pool 5 x 20 x 20 ms . (Fig. VII.32).

It is obvious that , in this case , it is not possible to arrange any supports at the upper surface of the water, and, if the walls are constructed without any supports, i.e. free at top and totally fixed at the other three sides, we get for water pressure, according to Czerny's table C-8, the following :

If $L_y/L_x = H / L = 5/20 = 0.25$

and $p\,H^2 = 5 \times 5^2 = 125$ m t

the max. fixing moments at foot of wall will be :

$M_y$min. $= - p\,H^2/6.9 = - 125/6.9 = - 18.1$ m t and

the max. fixing moment between the walls at their top surface :

$M_x$min. $= - p\,H^2/8.8 = - 125/8.8 = - 14.2$ m t

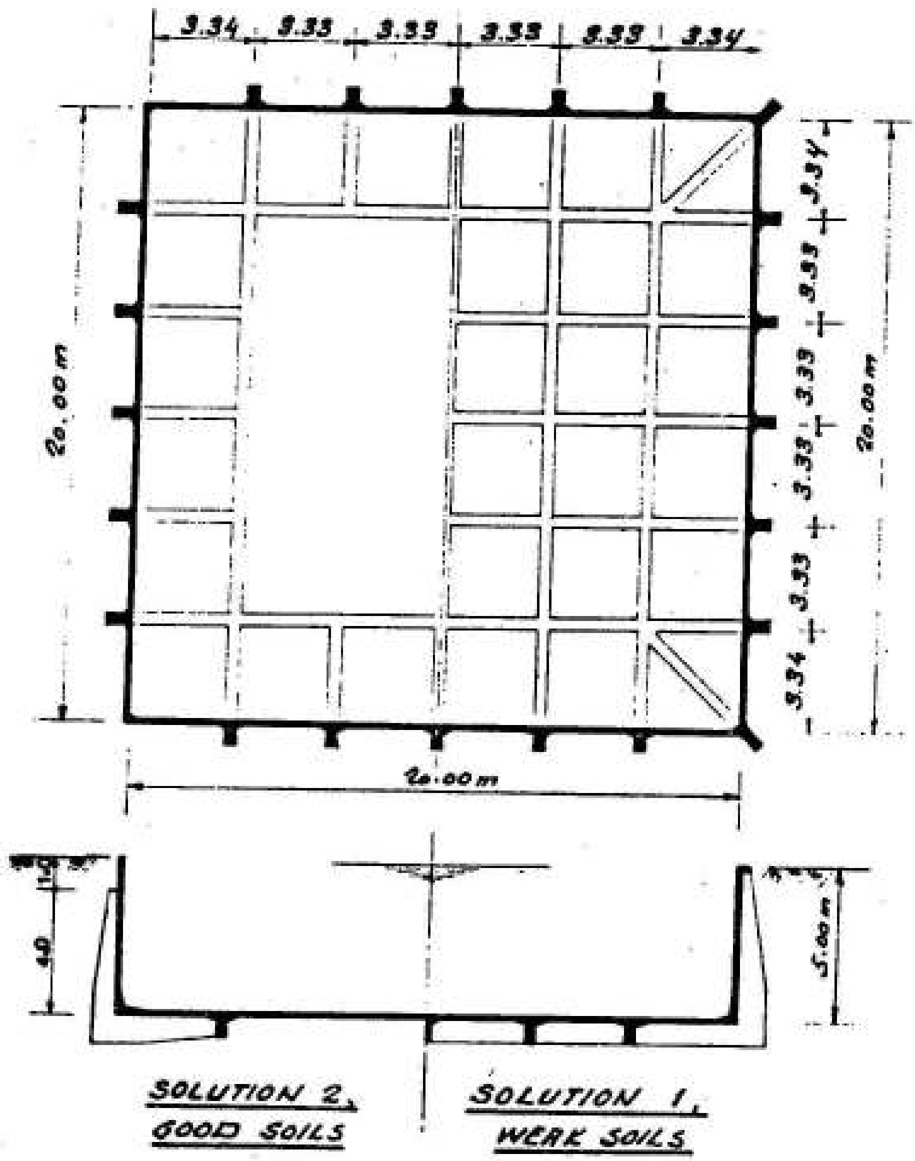


Fig. VII-32

Both moments cause tensile stresses on the water side, so that, the thickness required for the wall at its base is given by :

$t_{max} = \sqrt{M / 3} = \sqrt{18100 / 3} = 77.5$ cms

and the corresponding tension steel will be :

$= 0.26\ t_{max} = 0.26 \times 77.5 = 20\ cm^2$ 7 ϕ 19 mm/m

Both values t and A_s are relatively big :

We give in the following some trials for convenient solutions of the wall.

1st. Trial : Wall Supported on Four Sides :

As a first trial, we arrange horizontal beams at the top edge of the wall and vertical counterforts at say 3.33 ms. as shown in solution 1 of figure VII.32. In this manner, the walls behave as two way slabs simply supported at top and fixed on the other three sides. The internal forces can be calculated from Czerny's table B-5' as follows :

For $L_y / L_x = H / L = 5/3.33 = 1.5$

& $p\ L_x^2 = 5 \times 3.33^2 = 55.5$ m t

we get, the bending moments :

$M_y min = - p\ L_x^2 / 23.5 = - 55.5 / 23.5 = - 2.33$ mt at base

$M_y max = p\ L_x^2 / 104 = 55.5 / 104 = 0.535$ mt at 0.27 H from base

$M_x min = - p\ L_x^2 / 23.7 = - 55.5 / 23.7 = - 2.34$ mt at 0.38 H "

$M_x max = p\ L_x^2 / 57.2 = 55.5 / 57.2 = 0.97$ mt at mid height

and the reactions :

$\bar{R}_{ym} max$ = max. tension in floor slab

$= p\ L_x / 2.7 = 5 \times 3.33/2.7 = 6.2$ t/m

R_{ym} = load at middle of top beam

$= p\ L_x / 17.4 = 5 \times 3.33/17.4 = 0.96$ t/m

R_{ys} = load at support of top beam

$= -p\ L_x\ /\ 9.4 = -\ 5 \times 3.33\ /\ 9.4 = -\ 1.78$ t/m'

The reaction of the top beam and the horizontal reaction of the wall give the loads acting on the counterfort, thus :

If we assume that the load distribution on the top beam is parabolic, its reaction on the top edge of the counterfort will be nil (≈ 0) because the negative value of the load at the support of the beam (1.78 t/m) is approximately equal to double the maximum positive value at the middle (0.96 t/m).

The maximum reaction of the wall on the counterfort is given by :

$R_x max. = 2\ p\ L_x\ /\ 3.1 = 2 \times 5 \times 3.33\ /\ 3.1 = 11$ t/m at 0.3 H from base.

In order to calculate the bending moments and reaction at base of counterfort, one may replace the load diagram acting on the counterfort by a polygon as shown in Fig. VII.33 which shows the internal forces in the wall and supporting beams.

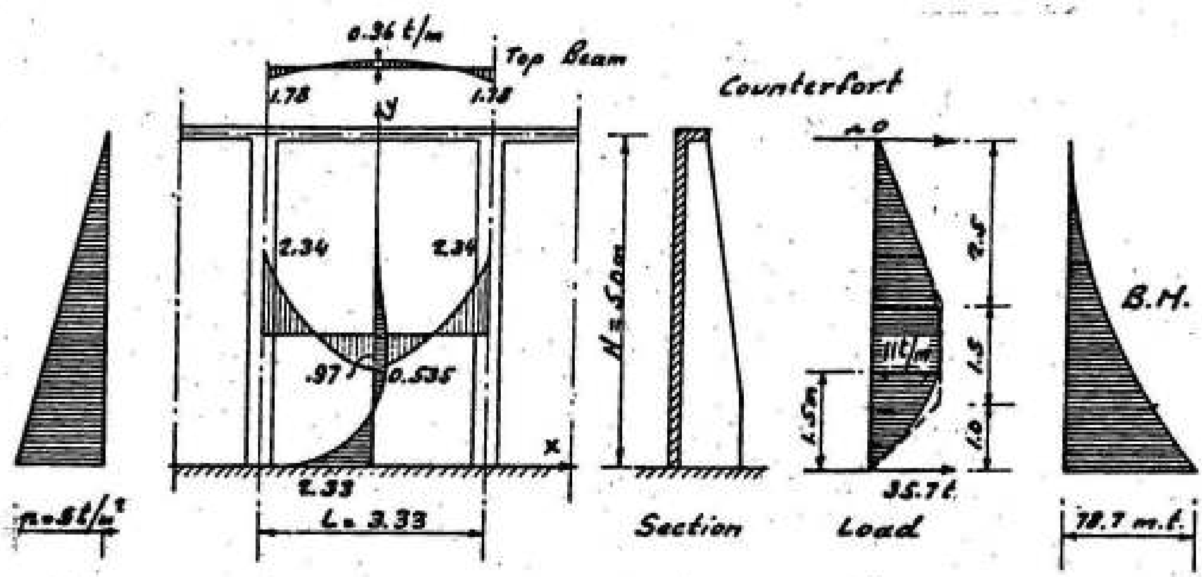


Fig. VII-33

The figure shows further, the general arrangement of the counterforts in elevation and section, the bending moments in the wall slab in the vertical and horizontal directions, and the internal forces and reactions of the counterfort.

Reaction of counterfort at its base :

$$R = 11 \times \frac{2.5}{2} + 11 \times 1.5 + 11 \times \frac{1}{2} \qquad = 33.7 \text{ ton}$$

Maximum bending moment of counterfort :

$$M_{max.} = \frac{11 \times 2.5}{2} \left(\frac{2.5}{3} + 2.5 \right) + 11 \times 1.5 \times 1.75 + \frac{11 \times 1}{2} \times \frac{2}{3} = 78.7 \text{ mt}$$

<u>2nd. trial</u> : <u>Wall Supported on Three Sides</u> :

We have seen, in the previous trial, that the loads on the top beam are so small that it does not give an effective support for the wall. Further studies have shown that so long as the height H of a wall subject to hydrostatic pressure is bigger than its span L, the internal forces are not affected if we dispense with the top beam, in which case the wall will be free at top and totally fixed on the other three sides. The bending moments can be determined from Czerny's table C-8 as follows :

For $L_y / L_x = H / L = 5 / 3.33 = 1.5$

and $p L_y^2 = 5 \times 5^2 = 125$ m t

We get the bending moments :

$M_y$min. = $- p L_y^2/53.5 = - 125/53.5 = - 2.34$ m t at base

$M_y$max. = $p L_y^2/242 = 125/242 = 0.516$ m t at 0.2 H from base

$M_x$min. = $- p L_y^2/54.2 = - 125/54.2 = - 2.32$ m t at 0.36 H " "

$M_x$max. = $p L_y^2/125 = 125/125 = 1.00$ m t at 0.40 H " "

The reactions of the slab can be calculated from table C-11 , thus :

R_1 = max. tension in floor-slab

= $p L_y/4.82 = 5 \times 5/4.82 = 5.2$ t/m

$2R_2$ = load on an intermediate counterfort at its top edge

$= - 2\ p\ L_y\ /\ 250 = - 2 \times 5 \times 5\ /\ 250 = - 0.2$ t/m

$2R_3$ = load on an intermediate counterfort at mid-height

$= 2\ p\ L_y\ /\ 6.13 = 2 \times 5 \times 5/6.13 = 8.15$ t/m

The maximum load on the counterfort lies at ~ 0.3 H from base, so that the load diagram can be drawn as shown in figure VII.34 ; its ordinates being the same as those of the previous case, the bending moment of the counterfort and the reaction at its lower edge will be the same (Fig. VII.34).

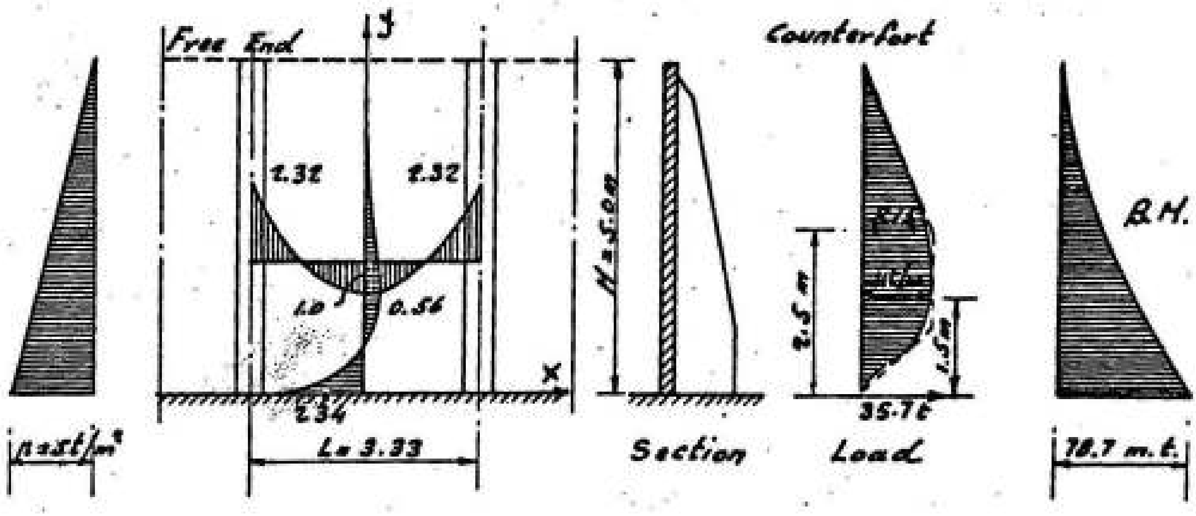


Fig. VII-34

Comparing the values of the internal forces obtained for this trial, we find that they are nearly the same as those of the first trial.

For such internal forces, it is sufficient to construct the walls with a thickness of 20 cms only at the middle of the spans and ≈ 30 cms at the supports.

The maximum section of the counterfort may be chosen 45 x 110cms and the corresponding reinforcement ≈ 9 ϕ 25 .

3rd. Trial : Cantilevering Wall Supported on Three Sides :

In this new trial, the height of the counterfort will be reduced to 4.00 m. In this manner, the lower part of the wall will be fixed on three sides and free at its upper edge where we have 1 meter freecantilever; it is subject to a trapezoidal hydrostatic pressure with :

$p_1 = 1.00\ t/m^2$ at its top edge and

$p = p_1 + p_2 = 1.00 + 4.00 = 5.00\ t/m^2$ at its lower edge in addition to a bending moment M_o given by :

$M_o = p_1\ C^2/6 = 1.00^3 / 6 = 0.166\ mt/m$

and a shearing force $Q_o = 1.00^2/2 = 0.50\ t/m$ acting at its free edge as shown in figure VII.35 .

The bending moments in the lower part of the slab can be calculated from the following tables :

Table C-7 for the uniform load $p_1 = 1.00\ t/m^2$

Table C-8 " " triangular " $p_2 = 4.00\ t/m^2$

Table C-9 for the shear $Q_o = 0.5\ t/m$ acting on the free edge

Table C-10 for the moment $M_o = 0.166\ mt/m$ " " " " "

The results are given in table 1.

The reactions due to M_o are so small that they can be neglected, while the reactions due to Q_o will be directly transmitted to the edges of the counterforts (at points d) and are given by :

$$R_d = 0.5 \times 3.33 = 1.67\ t$$

The reactions of the lower part of the wall due to the uniform pressure $p_1 = 1.0\ t/m^2$ and the triangular pressure $p_2 = 4.0\ t/m^2$ can be calculated from table C-11 as shown in table 2.

R_b given in the table indicates the maximum tension in the floor slab.

Table 1

Load	Uniform Load $P_1= 1.0\ t/m^2$ Table C-7		Triang. Load $P_2= 4.0\ t/m^2$ Table C-8		Edge Load $Q_o= 0.5\ t/m$ Table C-9		Edge Moment $M_o=0.166\ mt/m$ Table C-10		Total
B.M.	$M= \frac{P_1 L_y^2}{m} = \frac{1x4^2}{m}$		$M= \frac{P_1 L_y^2}{m} = \frac{4x4^2}{m}$		$M=\frac{Q_o L_x}{m}=\frac{.5x3.33}{m}$		$M= \frac{M_o}{m} = \frac{0.166}{m}$		B.M.
M	m	M	m	M	m	M	m	M	M
		mt		mt		mt		mt	mt
M_{xa}	34.4	0.465	177	0.361	7.2	0.232	3.5	0.048	1.106
M_{xd}	- 16.9	-0.950	- 130	-0.492	-2.42	-0.746	- 0.69	-0.242	-2.430
M_{xm}	43.0	0.372	98.3	0.650	63.0	0.027	-17.9	-0.009	1.040
M_{xc}	- 19.5	-0.820	- 41.5	-1.540	-66.3	-0.025	7.55	0.022	-2.363
M_{ya}	∞	0	∞	0	∞	0	- 1.00	-0.166	-0.166
M_{ym}	190	0.084	238	0.269	-42.6	-0.039	-40	-0.004	0.310
M_{yb}	- 25.3	-0.632	-37.4	-1.710	-	-	115	0.001	-2.340

Table 2

Loading	Uniform Load $P_1 = 1.0\ t/m^2$ Table C-11a		Triangular Load $P_2 = 4.0\ t/m^2$ Table C-11b		Total	
Reaction	$R = \frac{P_1 L_y}{m} = \frac{1x4}{m}$		$R = \frac{P_2 L_y}{m} = \frac{4x4}{m}$		Reaction	
	m	R = 4/m	m	R = 16/m		
		t		t	t	
R_b	3.06	1.30	4.04	3.96	5.26	
2 R_d	2.08	2 x 1.92	60	2 x 0.27	4.38	+1.67= 6.05t
2 R_c	2.48	2 x 1.16	4.76	2 x 3.35	9.02	

The maximum load on the counterfort lies at 0.3 H from base and can be estimated diagramatically as shown in figure VII-35 in which the load is replaced by a polygon of the given ordinates.

Accordingly, the reaction at the foot of the counterfort is given by:

$$R = \frac{6.05 + 9.02}{2} \times 2.5 + 10 \times 1.0 \times 0.5 = 33.9 \text{ t}$$

and its maximum moment is given by $M = 65.25$ mt

The internal forces in the wall and in an intermediate counterfort are shown in figure VII-35.

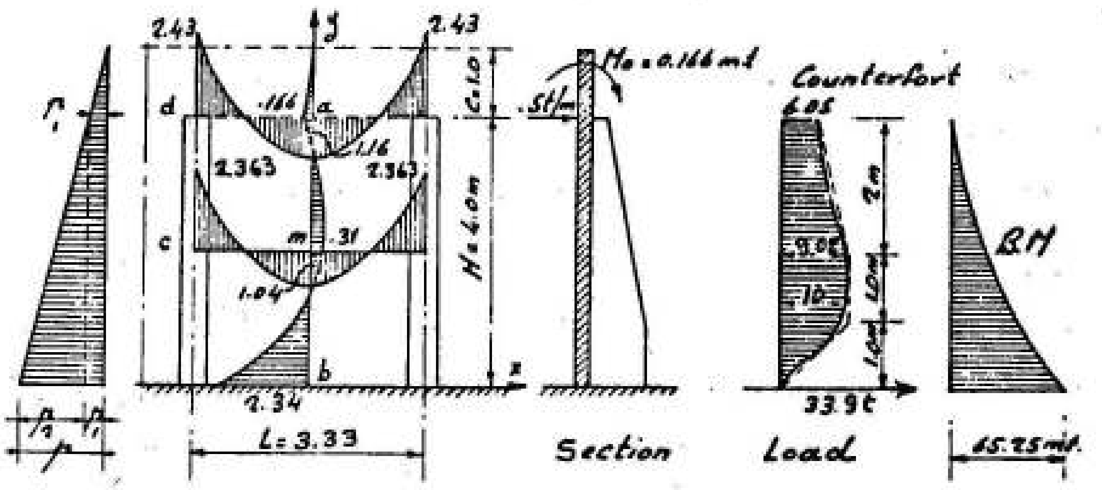


Fig. VII-35

It is clear from the previous investigations that the choice of a convenient and simple solution as that shown in the third trial , requires a thorough knowledge of the distribution of the internal forces in flat plates under different loads for the various edge conditions.

In all the shown cases , the counterforts behave as simple cantilevers subject to the reactions of the wall and the top beam , if any, giving maximum bending moments at their base. The tensile stresses in the countrrfort due to the hydrostatic pressure are on the water side, and it is therefore essential to prevent tension cracks from being developed. In order to satisfy this requirement,the sections of the counterfort will be designed as reinforced concrete rectangular sections because their flange lies in the tension zone and, in spite of that, the depth of the different sections must be chosen such that the maximum tensile stresses in the concrete are smaller than its tensile strength, i.e. no cracks are allowed in the flange of the beam. In this last case, we are allowed to assume the sections as plain concrete (or eventually reinforced concrete) T - sections with breadth of flange:

$$B = 6\ t_s + 2\ b_s + b_o^{*}$$

and smaller than H/3 . This means , the sections are to be treated as reinforced concrete rectangular sections in stage II and at the same time as plain concrete T-sections in stage I.

The walls must be designed for the full hydrostatic pressure assuming earth removed , and the full earth plus any ground water pressure + effect of surcharge , if any , with tank empty. In this last case , the wall sections behave as T both in stage I and stage II.

Corner counterforts may be arranged as shown in solution 1 , in which case , they resist the horizontal reactions of the two adjoining panels of the walls. If they are not arranged, as shown in solution 2 , the walls will be subject to horizontal forces , equal to the horizontal reactions of the end panels of walls , over their whole length.

The fixing moments and bottom reactions of counterforts must be transmitted to floor beams , (or eventually thick floor slabs). It is recommended to extend the floor beams over the whole width of

---

*- t_s = thickness of flange b_s = breadth of haunches if any
b_o = breadth of counterfort.

the floor if it rests on relatively weak soils - solution 1 of figure VII-32 - and, to extend them on a short distance of the floor slab if it rests on stiff soils as shown in solution 2 of the same figure.

It is generally not recommended to use bent bars in vertical slabs and beams of counterforted walls even in cases where such walls are subject to earth or water pressure from one side only because bent bars are liable to move from their position during concreting operations.

Figure VII-36 shows the concrete dimentions and details of reinforcements of the third trial shown in figure VII-35 for solution 2 of figure VII-32 assuming that the internal forces due to earth pressure are 0.65 the values due to water pressure.

It has to be noted here that no bent bars are used in the different elements of the pool and that high grade steel stressed to 2000 kg/cm^2 in tension is used as reinforcement for the counterforts.

The required concrete dimensions and reinforcements of the different elements are as follows:

1.) Wall and floor slabs:

Thickness 20 cms provided with haunches 10 x 30 cms at the supporting beams.

Tension steel 7 Ø 13 mm/m for water pressure and 7 Ø 10 mm/m for earth pressure.

2.) Counterforts:

Cross-section: max. 45 x 110 cm and min. 45 x 60 cm.

Tension steel: 9 ϕ 25 mm on water side and 6 25 mm on earth side.

3.) Intermediate floor beams:

Cross-section: 45 x 60 cm.

Reinforcement: 3 ϕ 25 mm top and bottom.

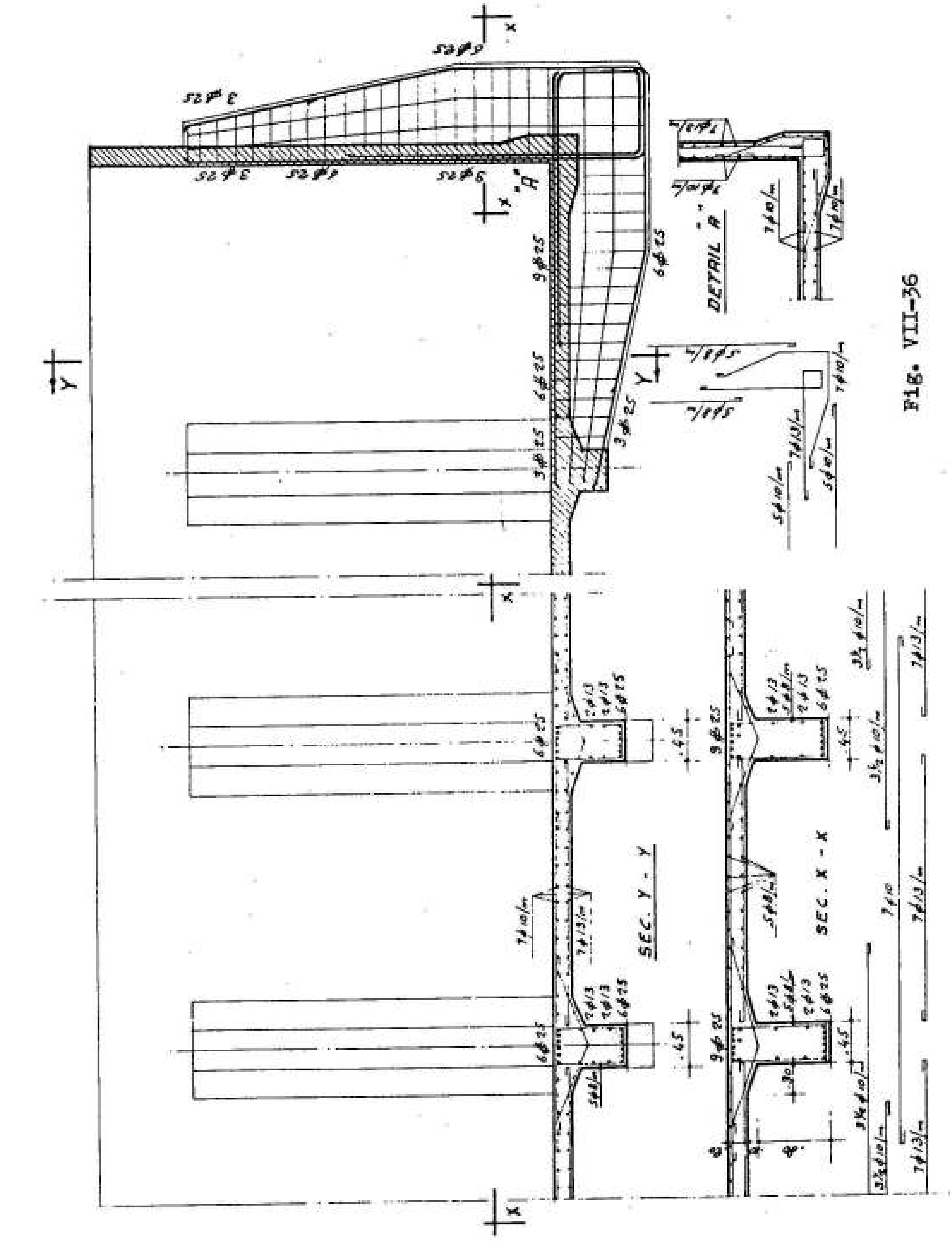


Fig. VII-36

VIII. T A N K S D I R E C T L Y B U I L T O N T H E G R O U N D

In tanks directly built on the ground, we recognize three different cases, namely :

a) Tanks on fill or soft-weak soils,
b) Tanks on rigid foundation and,
c) Tanks on compressible soils.

VIII.1 TANKS ON FILL OR SOFT WEAK SOILS :

The stresses on the soil due to weight of tank and water is generally low ($\sim$ 0.6 kg/cm^2 for a depth of water of 5 ms.) ; but in spite of this fact, it is not recommended to construct a tank directly on unconsolidated fill as this may expose the tank to very serious differential settlements due to the unhomogenous texure of the fill.

Soft weak clayey layers, peat and similar soils may consolidate to big values even under small stresses.

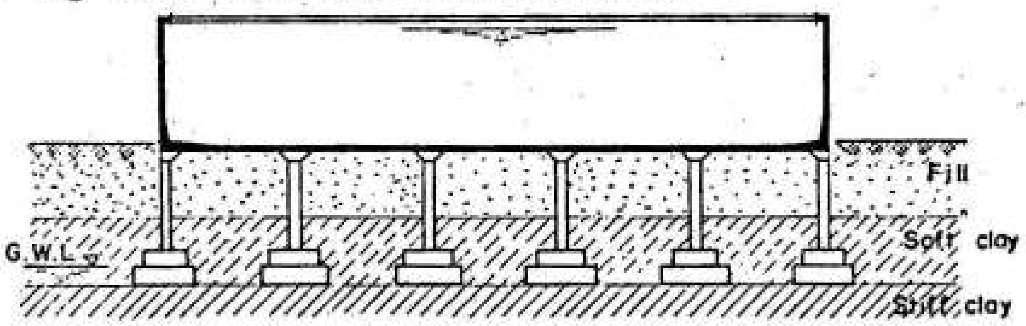


Fig. VIII-1

In such cases, it is recommended to support the tank on columns and isolated or strip footings if the stiff soil layers are at a reasonable depth from the ground surface and no big difficulties are liable to arise due to high ground water as shown in fig. VIII.1 in which the floor of the tank, acting as a flat slab, is supported on columns and isolated footings resting on the stiff clay.

In case of medium soils at foundation level, one may need a raft over the whole area as shown in figure VIII.2 . The floor of the tank and the lower raft may be arranged as flat slabs. If it is required to increase the stiffness of the tank, longitudinal and cross-girders may be arranged as shown .

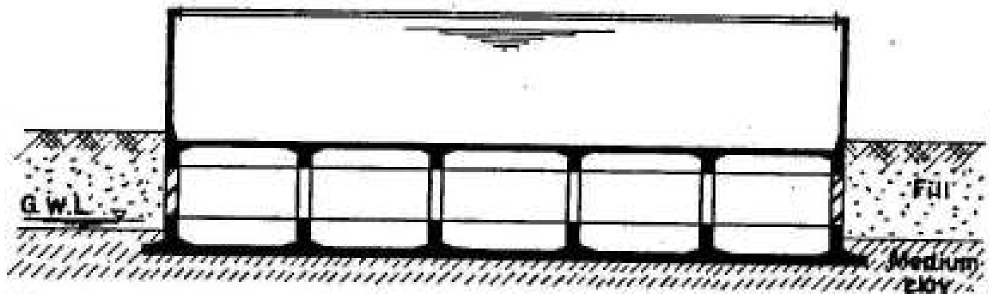


Fig. VIII-2

If the incompressible layers are deep or the ground water level is high one may support the tank on piles. It is recommended to drive the piles, where possible , to the incompressible layers and to arrange them such that we get just the floor thickness sufficient to give the necessary water-tightness (20 - 25 cms). The pile caps may act as column heads for a flat slab system as shown in figure VIII.3

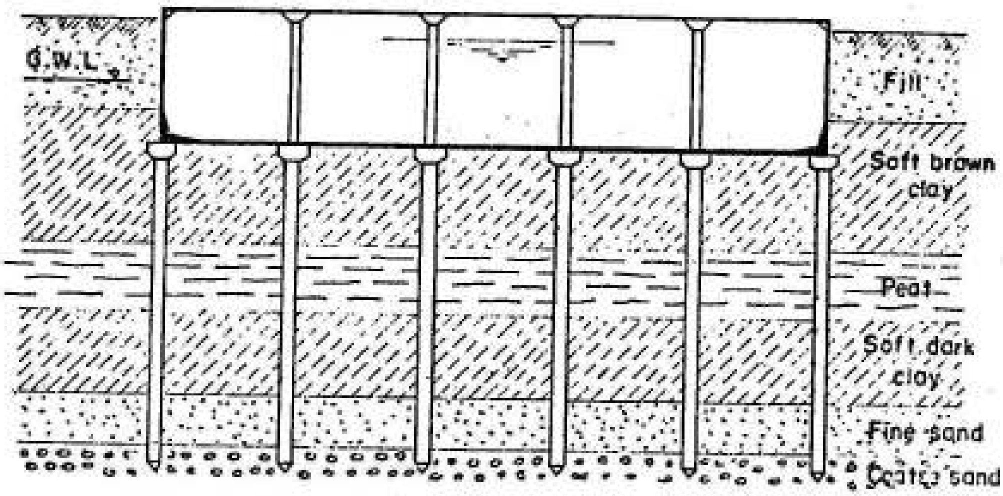
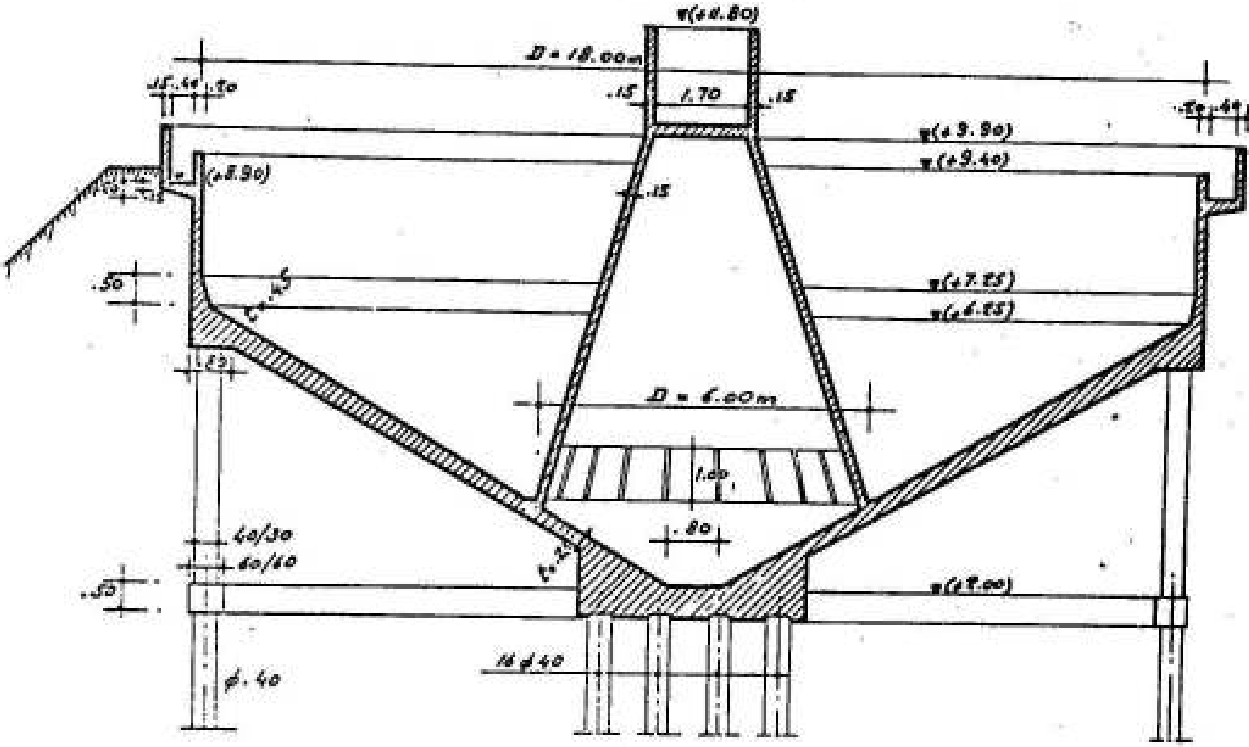
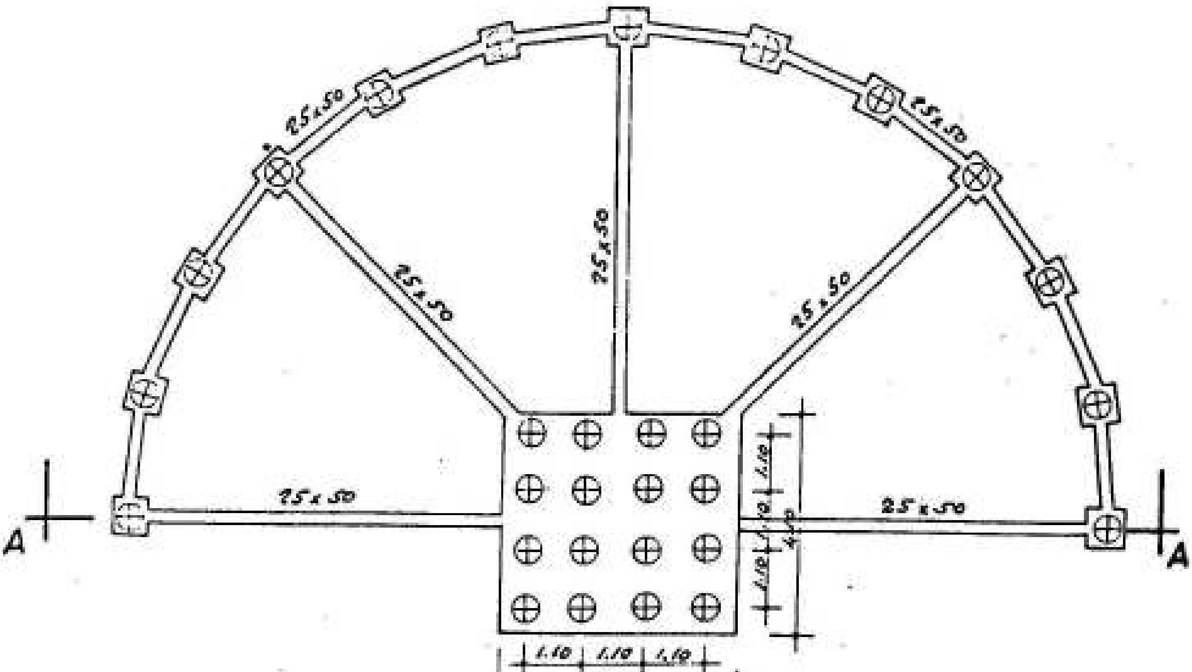


Fig. VIII-3

In this example, the soil layers being soft and compressible, the weight of the tank and liquid will be transmitted to the incompressible



SECTION ELEVATION A-A



PLAN OF FOUNDATION

WATER SUPPLY FLOCCULATOR

Fig. VIII-4

sand layers through cast in place piles.

The roof may be a flat slab or of the slab and beam type supported on columns. The columns and the floor, as a flat slab, are supported on piles, with the pile caps acting as column heads. Single piles are arranged such that each pile carries its full capacity and the required thickness of the floor slab is 20 to 25 cms. The steel tubes in which the concrete of the piles is cast are driven from the ground surface to the coarse sand layer, they are filled with concrete to the level of the floor and with sand fill to the ground surface.

We give in figure VIII.4 a circular flocculator which is to be constructed in very soft swampy soils on piles arranged in the outside perimeter below the outside vertical wall and in the center underneath the solid part shown.

VIII.2 TANKS ON RIGID FOUNDATION :

If we assume that a tank is supported on a rigid foundation,

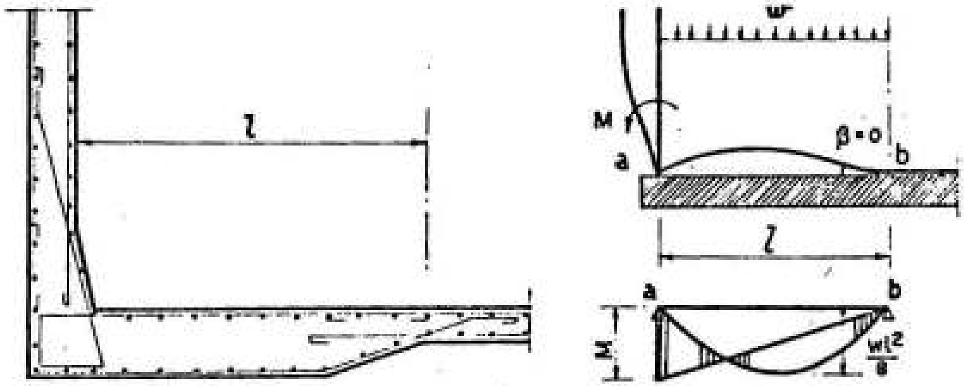


Fig. VIII-5

then the vertical reaction of the wall will be directly resisted by the area beneath it, while the bending moment M will deflect the floor in a length l beyond which no deformation or bending moments are created. The deformation due to M will be counter balanced by the weight of the liquid and the floor w. (Fig. VIII.5) accordingly, we should have at point b : $M_b = 0$ and $\beta = 0$

But $\beta = 0 = \frac{w\, l^3}{24\ E\ I} - \frac{M\, l}{6\ E\ I}$

so that $l = 2\sqrt{\frac{M}{w}}$

The part l of the floor slab is to be designed to resist the bending moments shown in figure VIII.5 plus an axial tensile force N equal to the reaction at the base of the wall. The middle part of the floor slab is to be designed to resist the tension N with a minimum thickness of 15 - 20 cms giving sufficient water-tightness. The minimum reinforcement is 5 Φ 8 mm/m in each direction on both surfaces of the slab.

VIII.3. TANKS ON COMPRESSIBLE SOILS :

Floors of tanks resting on medium clayey or sandy soils may be calculated in the following manner : (Fig. VIII.6).

We assume that the internal forces transmitted from the wall to the floor at b are distributed on the soil by the part b c = l of the floor where $l = 0.4$ to 0.6 H. The length l is chosen such that the max. stress σ_1 is smaller than the allowed soil bearing pressure, and $\sigma_2 > \sigma_1 / 2$ on clayey soils and $\sigma_2 > 0$ on sandy soils.

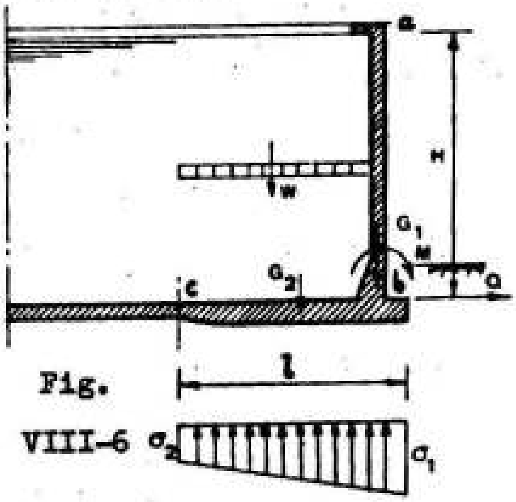


Fig. VIII-6

These limitations are recommended in order to prevent relatively big rotations of the floor at b.

The internal forces in the part bc of the floor are to be determined for the downward forces :

G_1 = weight of wall and roof (if any)

G_2 = " " floor cb

= " " water on cb

plus the bending moment M and the shearing force Q at the foot of the wall and the corresponding upward stresses σ .

The part of the floor to the left of c is subject to tension equal to Q . Its thickness is chosen 15 to 20 cms. to give the necessary watertightness, reinforced by two meshes of area A_s in any direction given by :

$$A_s = Q / \sigma_s$$

For the details of reinforcements refer to figure VIII.5. The floor of a tank directly built on stiff medium or weak compressible soils can however be treated as a floor on elastic foundation if the soil is homogenous and its elastic properties are known.

General Theory of Beams on Elastic Foundation*

In order to determine the deformation and internal forces in a floor on elastic foundation, one has to know the general theory of beams on elastic foundation. We give in the following , the outlines of the theory as may be needed for our purpose.

A beam A B supported along its entire length by an elastic medium and subjected to vertical forces p will deflect producing continuously distributed stresses σ in the supporting medium' (Fig. VIII.7).

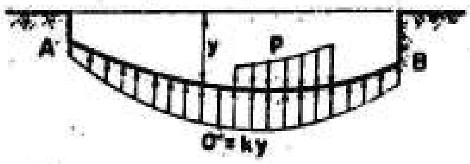


Fig. VIII-7

The fundamental assumption of elastic supports is :
" the intensity of the stress σ at any point is proportional to the deflection of the beam y at that point " ; that is :

$$\sigma = k\,y$$

The elasticity of the supporting medium is characterized by the stress σ which causes a deflection equal to unity. This constant of the supporting medium k_o kg/cm^3 is called the modulus of the foundation.

* Hetényi " Beams on Elastic Foundation "

Assume that the beam under consideration has a uniform cross section and that b is its constant width, which is supported on the foundation. A unit deflection of this beam will cause a stress b k_o in the foundation , consequently, at a point where the deflection is y the intensity of distributed reaction (per unit length of the beam) will be :

$$\sigma \text{ kg/cm} = b\, k_o\, y$$

For the sake of brevity we shall use the symbol k kg/cm^2 for b cms x k_o kg/cm^2 in the following derivations, but it is to be remembered that this k includes the effect of the width of the beam and will be numerically equal to k_o only if we deal with a beam of unit width.

Considering the equilibrium of an infinitely small element enclosed between two vertical cross-sections a distance dx apart on the beam we find: (Fig. VIII.8).

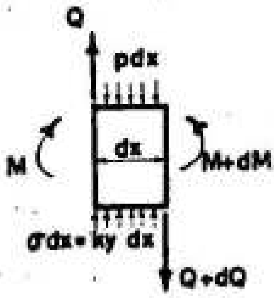


Fig. VIII-8

$$Q - (Q + dQ) + k\, y\, dx - p\, dx = 0$$

which gives :

$$\frac{dQ}{dx} = k\, y - p \qquad (a)$$

knowing that $\frac{dM}{dx} = Q$, we can write

$$\frac{dQ}{dx} = \frac{d^2M}{dx^2} = k\, y - p \quad (b)$$

We know further that :

$$\frac{d^2y}{dx^2} = - \frac{M}{E\, I} \qquad (c)$$

Differentiating this relation twice, we obtain :

$$E\, I\, \frac{d^4y}{dx^4} = - \frac{d^2M}{dx^2} \qquad (d)$$

Relations (b) and (d) give :

$$E\,I\ \frac{d^4y}{dx^4} = -\,k\,y + p \qquad (e)$$

which gives the differential equation of the elastic line of a beam on elastic foundation. Along the unloaded parts of the beam, where no distributed load is acting, $p = 0$, and equ. (e) will take the form :

$$E\,I\ \frac{d^4y}{dx^4} = -\,k\,y \qquad (f)$$

It will be sufficient to consider below only the general solution of (f), from which solutions will be obtained also for cases implied in (e) by adding to it a particular integral corresponding to p. Assuming further that :

$$n = \sqrt[4]{\frac{k}{4\,E\,I}} \qquad (g)$$

we can give the solution of the differential equation (f) in the form :

$$y = e^{nx}\,(\,A_1 \cos nx + B_1 \sin nx\,) + e^{-nx}\,(\,B_1 \cos nx + B_2 \sin nx\,) \qquad (15)$$

Here n includes the flexural regidity of the beam as well as the elasticity of the supporting medium, and is an important factor influencing the shape of the elastic line. For this reason the factor n is called the characteristic of the system, and, since its dimension is length^{-1}, the term $1/n$ is frequently referred to as the characteristic length. Consequently, nx will be an absolute number.

Differentiating equation (15), we get :

$$\left.\begin{aligned}
\frac{1}{n}\cdot\frac{dy}{dx} &= e^{nx}\left[A_1\,(\cos nx - \sin nx) + A_2\,(\cos nx + \sin nx)\right] \\
&\quad - e^{-nx}\left[B_1\,(\cos nx + \sin nx) - B_2\,(\cos nx - \sin nx)\right] \\
\frac{1}{2\,n^2}\cdot\frac{d^2y}{dx^2} &= -\,e^{nx}\,(A_1 \sin - A_2 \cos nx) + e^{-nx}(B_1 \sin nx - B_2 \cos nx) \\
\frac{1}{3\,n^3}\cdot\frac{d^3y}{dx^3} &= -\,e^{nx}\left[A_1(\cos nx + \sin nx) - A_2(\cos nx - \sin nx)\right] \\
&\quad + e^{-nx}\left[B_1(\cos nx - \sin nx) + B_2(\cos nx - \sin nx)\right]
\end{aligned}\right\}\quad (16)$$

Knowing that :

$$\frac{dy}{dx} = \tan \theta \; , \quad - E\,I \; \frac{d^2y}{dx^2} = M \quad \text{and} \quad - E\,I \; \frac{d^3y}{dx^3} = Q$$

we can obtain the general expressions for the slope θ ($\theta = \tan \theta$) of the deflection line as well as for the bending moment M and the shearing force Q . The intensity of pressure in the foundation will be found from equation (15) to be $\sigma = k\,y$.

In applying these general equations, or corresponding ones including the term depending on p, to particular cases the next step is to determine the integration constants A_1, A_2, B_1 and B_2 . These integration constants depend on the manner in which the beam is subjected to the loading and have constant values along each portion of the beam within which the elastic line and all its derivatives are continuous. Their values can be obtained from the conditions existing at the two ends of such continuous portions. Out of the four quantities (y, θ , M and Q) characterizing the condition of an end, two are usually known at each end, from which sufficient data are available for the determination of the constants A and B.

Internal Forces for Particular Cases of Loading :

In the following formulae, we have :

The characteristic value $n = \sqrt[4]{\frac{k}{4\,E\,I}}$

The foundation modulus k_o in kg/cm^3

$k = b\,k_o$ in kg/cm^2

Constant width of beam in contact with the foundation = b cm

The flexural rigidity of the beam $= E\,I$

The deflection y, slope angles θ , bending moments M, and shearing forces Q are given in the following for beams with free ends under some particular cases of loading.

The numerical values of the trigonometric and hyperbolic functions needed in the different equations are given in the appendix.

1) Equal Concentrated Forces at Both Ends :

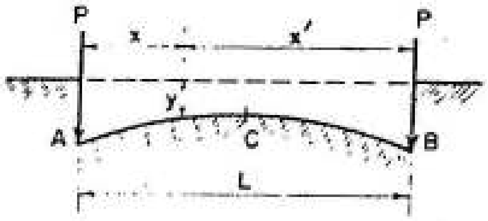


Fig. VIII-9

$$y = \frac{2 P n}{k} \cdot \frac{\cosh nx . \cos nx' + \cosh nx' . \cos nx}{\sinh nL + \sin nL} \qquad (17)$$

Deflection at the end points :

$$y_A = y_B = \frac{2 P n}{k} \cdot \frac{\cosh nL + \cos nL}{\sinh nL + \sin nl} \qquad (17a)$$

Deflection at the middle :

$$y_C = \frac{4 P n}{k} \quad \frac{\cosh \frac{nL}{2} \cos \frac{nL}{2}}{\sinh nL + \sin nL} \qquad (17b)$$

The deflection $y_C = 0$ when $nL = \pi$, 3π , 5π , ect..

Slope angles at end points A and B :

$$\theta_A = - \theta_B = - \frac{2 P n^2}{k} \cdot \frac{\sinh nL - \sin nL}{\sinh nL + \sin nL} \qquad (18)$$

The bending moment :

$$M = - \frac{P}{n} \cdot \frac{\sinh nx . \sin nx' + \sinh nx' . \sin nx}{\sinh nL + \sin nL} \qquad (19)$$

The bending moment at the middle :

$$M_C = - \frac{2 P}{n} \cdot \frac{\sinh \frac{nL}{2} \sin \frac{nL}{2}}{\sinh nL + \sin nL} \qquad (19a)$$

$M_C = 0$ when $nL = 2\pi$, 4π , 6π , etc..

M_C is negative when $nL < 2\pi$

M_C is positive when $2\pi < nL < 4\pi$

The shearing force :

$$Q = P \frac{1}{\sinh nL + \sin nL} (\sinh nx . \cos nx' - \cosh nx . \sin nx' + \cosh nx' . \sin nx - \sinh nx' . \cos nx) \qquad (20)$$

2) <u>Equal Concentrated Moments at Both Ends</u> :

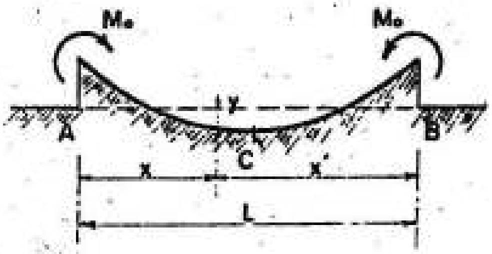


Fig. VIII-10

$$y = -\frac{2\,M_o\,n^2}{k} \cdot \frac{1}{\sinh nL + \sin nL}(\sinh nx.\cos nx' - \cosh nx.\sin nx' + \sinh nx'.\cos nx - \cosh nx'.\sin nx) \quad (21)$$

Deflection at end points :

$$y_A = y_B = -\frac{2\,M_o\,n^2}{k} \cdot \frac{\sinh nL - \sin nL}{\sinh nL + \sin nL} \quad (21a)$$

Slope at end points

$$\theta_A = -\theta_B = \frac{4\,M_o\,n^3}{k} \cdot \frac{\cosh nL - \cos nL}{\sinh nL + \sin nL} \quad (22)$$

The bending moment :

$$M = M_o \frac{1}{\sinh nL + \sin nL}(\sinh nx.\cos nx' + \cosh nx.\sin nx' + \sinh nx'.\cos nx + \cosh nx'.\sin nx) \quad (23)$$

The bending moment at the middle :

$$M_c = 2\,M_o \cdot \frac{\sinh \frac{nL}{2} \cdot \cos \frac{nL}{2} + \cosh \frac{nL}{2} \cdot \sin \frac{nL}{2}}{\sinh nL + \sin nL} \quad (23a)$$

$M_c = 0$ when $\cos \frac{nL}{2} = -\sin \frac{nL}{2}$, that is,

when $nL = \frac{3}{2}\pi \;,\; \frac{7}{2}\pi \;,\; \frac{11}{2}\pi$, etc...

The shearing force :

$$Q = 2\,M_o\,n \cdot \frac{\sinh nx.\sin nx' - \sinh nx'.\sin nx}{\sinh nL + \sin nL} \quad (24)$$

3) Concentrated Force at One End :

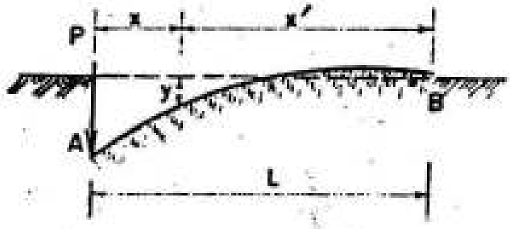


Fig. VIII-11

$$y = \frac{2Pn}{k} \cdot \frac{\sinh nL.\cos nx.\cosh nx' - \sin nL.\cosh nx.\cos nx'}{\sinh^2 nL - \sin^2 nL} \tag{25}$$

Deflection at the end points :

$$\left.\begin{aligned} y_A &= \frac{2Pn}{k} \cdot \frac{\sinh nL.\cosh nL - \sin nL.\cos nL}{\sinh^2 nL - \sin^2 nL} \\ y_B &= \frac{2Pn}{k} \cdot \frac{\sinh nL.\cos nL - \sin nL.\cosh nL}{\sinh^2 nL - \sin^2 nL} \end{aligned}\right\} \tag{25a}$$

The deflection $y_B = 0$ when $nL = \frac{5\pi}{4}, \frac{9}{4}\pi$, etc..

Slopes at the end points :

$$\left.\begin{aligned} \theta_A &= -\frac{2Pn^2}{k} \cdot \frac{\sinh^2 nL + \sin^2 nL}{\sinh^2 nL - \sin^2 nL} \\ \theta_B &= -\frac{4Pn^2}{k} \cdot \frac{\sinh nL \;.\; \sin nL}{\sinh^2 nL - \sin^2 nL} \end{aligned}\right\} \tag{26}$$

The slope $\theta_B = 0$ when $nL = \pi, 2\pi, 3\pi$, etc..

The bending moment :

$$M = -\frac{P}{n} \frac{\sinh nL.\sin nx.\sinh nx' - \sin nL.\sinh nx.\sin nx'}{\sinh^2 nL - \sin^2 nL} \tag{27}$$

The shearing force :

$$Q = -P \frac{1}{\sinh^2 nL - \sin^2 nL} \Big[\sinh nL (\cos nx.\sin nx' - \sin nx.\cosh nx') - \sin nL (\cosh nx.\sin nx' - \sinh nx.\cos nx') \Big] \tag{28}$$

4) <u>Concentrated Moment at One End</u> :

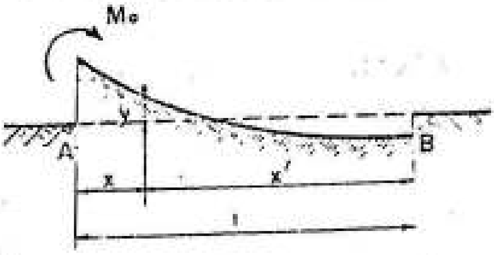


Fig. VIII-12

$$y = \frac{2 M_o n^2}{k} \cdot \frac{1}{\sinh^2 nL - \sin^2 nL} \Big[\sinh nL \, (\cos nx' . \sin nx - \sinh nx' . \cos nx) + \sin nL \, (\sinh nx . \cos nx' - \cosh nx . \sin nx') \Big] \quad (29)$$

Deflection at the end points :

$$y_A = \frac{2 M_o n^2}{k} \cdot \frac{\sinh^2 nL + \sin^2 nL}{\sinh^2 nL - \sin^2 nL}$$

$$y_B = \frac{4 M_o n^2}{k} \cdot \frac{\sinh nL . \sin nL}{\sinh^2 nL - \sin^2 nL} \quad (29a)$$

The deflection $y_B = 0$ when $nL = \pi$, 2π , 3π , etc.

Slopes at the end points :

$$\theta_A = \frac{4 M_o n^3}{k} \cdot \frac{\sinh nL . \cosh nL + \sin nL . \cos nL}{\sinh^2 nL - \sin^2 nL}$$

$$\theta_B = \frac{4 M_o n^3}{k} \cdot \frac{\sinh nL . \cos nL + \sin nL . \cosh nL}{\sinh^2 nL - \sin^2 nL} \quad (30)$$

The slope $\theta_B = 0$ when $nL = \frac{3}{4}\pi$, $\frac{7}{4}\pi$, $\frac{11}{4}\pi$, etc..

The bending moment :

$$M = M_o \frac{1}{\sinh^2 nL - \sin^2 nL} \Big[\sinh nL \, (\sinh nx' . \cos nx + \cosh nx' . \sin nx) - \sin nL \, (\sinh nx . \cos nx' + \cosh nx . \sin nx') \Big] \quad (31)$$

The shearing force :

$$Q = 2 M_o n \, \frac{\sinh nL . \sinh nx' . \sin nx + \sin nL . \sinh nx . \sin nx}{\sinh^2 nL - \sin^2 nL} \quad (32)$$

<u>Foundation Modulus k_o</u> :

The magnitude of the foundation or subgrade modulus k_o depends mainly on the following factors :

1) Kind of soil and its properties.

2) Internal friction between soil particles and their water content.

3) Size and form of loaded area.

4) Depth of foundation base from ground level.

We give in the following the approximate average values of k_o:

Peat	0.5 - 1.0	kg/cm^3
Fill of sand and gravel	1.0 - 2.0	"
Wet clayey soil	2.0 - 3.0	"
Moistured clay	4.0 - 5.0	"
Dry clay	6.0 - 8.0	"
Hard dry clay	10 kg/cm^3	
Coarse sand Coarse sand + small amount of gravel Fine gravel + small amount of sand	8 - 10	"
Middle size gravel + fine sand	10 - 12	"
Middle size gravel + coarse sand	12 - 15	"
Large size gravel + coarse sand	15 - 20	"

Examples :

1) In figure VIII.13 is shown the cross-section of an aqueduct. It is required to find the pressure distribution in the sub-soil and the moment diagram for the bottom plate.

Assume :

E_c = 200.000 kg/cm^2 and k_o = 5 kg/cm^2

Solution :

The bottom plate can be regarded as a beam on elastic foundation subject to :

i) A uniformly distributed loading p equal to the weight of the bottom plate itself plus the weight of the water,

ii) Two concentrated forces P due to the weight of the side walls.

iii) Two moments M_o due to the hydrostatic pressure.

Computing the loads of the section per meter, we get :

Own weight of bottom plate

$= 0.2 \times 2500 = 500$ kg/ m^2

Own weight of water

$= 2.2 \times 1000 = 2200$ "

Total $p = 2700$ "

Corresponding stress on soil

$= 0.27$ kg/cm^2

Own weight of wall

$P = 2.30 \times 500 = 1150$ kg/m

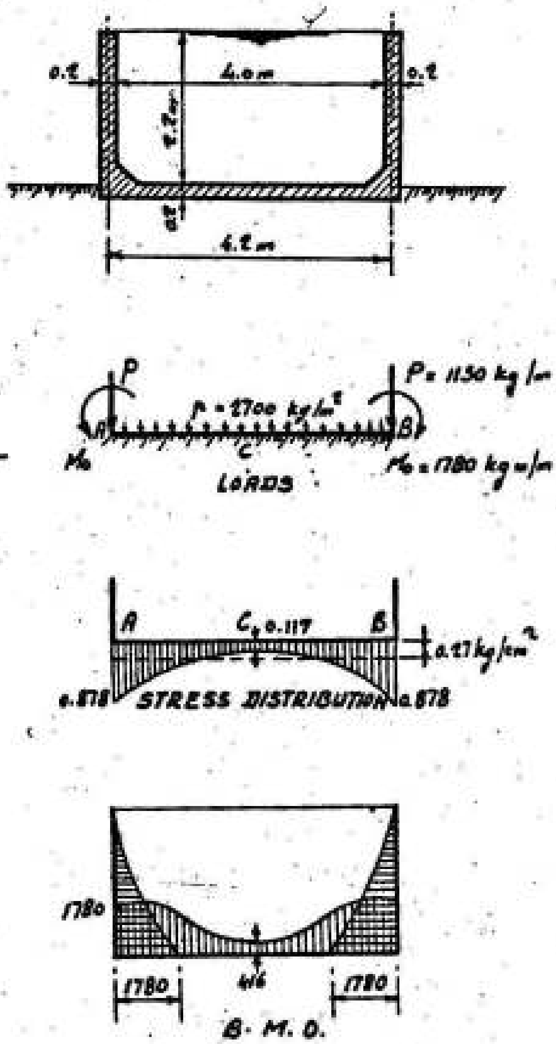


Fig. VIII-13

Moment due to water pressure $M_o = -2200 \times 2.2^2 / 6 = -1780$ kgm/m.
The deflection and moments of the floor can be calculated with the help of equations 17 , 19 and 21 , 23 thus :

$k = b\ k_o = 100 \times 5 = 500$ kg/cm^2

$I = b\ t^2/12 = 100 \times 20^3/12 = 6.67 \times 10^4\ cm^4$

$$n = \sqrt[4]{\frac{k}{4\ E\ I}} = \sqrt[4]{\frac{500}{4 \times 200\ 000 \times 6.67 \times 10^4}} \approx 0.010$$

The deflection at the edges y_A and y_B , and at the middle y_C due to the forces P = 11.5 kg/cm is, according to equation 17- a & b, given by :

$$y_A = y_B = \frac{2\,P\,n}{k}\;\frac{\cosh nL + \cos nL}{\sinh nL + \sin nL} \quad \& \quad y_C = \frac{4\,P.n}{k}\;\frac{\cosh \frac{nL}{2} + \cos \frac{nL}{2}}{\sinh nL + \sin nL}$$

in which

$nL = 0.01 \times 420 = 4.20$, $nL / 2 = 0.01 \times 420/2 = 2.1$

$\sin nL = -0.872$, $\cos nL = -0.490$, $\sin \frac{nL}{2} = 0.863$, $\cos \frac{nL}{2} = -0.505$

$\sinh nL = 33.3357$, $\cosh nL = 33.3507$, $\sinh \frac{nL}{2} = 4.0219$, $\cosh \frac{nL}{2} = 4.1443$

So that :

$$y_A = y_B = \frac{2 \times 11.5 \times .01}{500} \times \frac{33.3507 - 0.490}{33.3357 - 0.872} = 4.65 \times 10^{-4} \text{ cm}$$

and
$$y_C = \frac{4 \times 11.5 \times .01}{500} \times \frac{-4.1446 \times 0.505}{33.3357 - 0.872} = -0.59 \times 10^{-4} \text{ cm}$$

The stresses due to the forces P are therefore :

$\sigma_A = \sigma_B = k\,y_A = 500 \times 4.65 \times 10^{-4} = 0.233$ kg/cm^2

and $\sigma_C = k\,y_C = -500 \times 0.59 \times 10^{-4} = -0.030$ "

The bending moment M_c at the middle point of the bottom due to the forces P is, according to equation (19a), given by :

$$M_c = -\frac{2\,P}{n} \cdot \frac{\sinh \frac{nL}{2} \sin \frac{nL}{2}}{\sinh nL + \sin nL} \quad \text{or}$$

$$M_c = -\frac{2 \times 11.5}{0.01} \times \frac{4.0219 \times 0.863}{33.3357 - 0.872} = -246 \text{ kg cm/cm}$$

The deflection at the edge and middle points of the bottom plate due to the moments $M_o = -1780$ kgcm/cm is, according to equation (21), given by :

$$y_A = y_B = -\frac{2\,M_o\,n^2}{k} \cdot \frac{\sinh nL - \sin nL}{\sinh nL + \sin nL}$$

$$y_C = -\frac{4\,M_o\,n^2}{k} \cdot \frac{\sinh \frac{nL}{2} \cos \frac{nL}{2} - \cosh \frac{nL}{2} \sin \frac{nL}{2}}{\sinh nL + \sin nL}$$

So that :

$$y_A = y_B = \frac{2 \times 1780 \times .01^2}{500} \cdot \frac{33.3357 + 0.872}{33.3357 - 0.872} = 7.50 \times 10^{-4} \text{ cm}$$

$$y_C = \frac{4 \times 1780 \times .01^2}{500} \cdot \frac{-4.0219 \times 0.505 - 4.1443 \times 0.863}{33.3357 - 0.872} = -2.45 \times 10^{-4} \text{cm}$$

The stresses due to the moments M_o are therefore :

$$\sigma_A = \sigma_B = k\, y_A = 500 \times 7.5 \times 10^{-4} = 0.375 \quad \text{kg/cm}^2$$

$$\sigma_C = k\, y_C = -500 \times 2.45 \times 10^{-4} = -0.123 \quad "$$

The bending moment at the middle point of the bottom plate due to the edge moments M_o is, according to equation23-a, given by :

$$M_c = 2\, M_o \; \frac{\sinh \frac{nL}{2} \cos \frac{nL}{2} + \cosh \frac{nL}{2} \sin \frac{nL}{2}}{\sinh nL + \sin nL} \qquad \text{or}$$

$$M_c = -2 \times 1780 \cdot \frac{-4.0219 \times 0.505 + 4.1443 \times 0.863}{33.3357 - 0.872} = -170 \text{ kgcm/cm}$$

The total stresses and bending moments are therefore : (Fig. VIII.13)

$$\sigma_A = \sigma_B = 0.270 + 0.233 + 0.375 = 0.878 \quad \text{kg/cm}^2$$

$$\sigma_C = 0.270 - 0.030 - 0.123 = 0.117 \quad "$$

$$M_c = -246 - 170 = -416 \quad \text{kgm/m}$$

2) It is required to determine the stress distribution and bending moments of the floor of the diving pool shown in figure VII.32 for the case of water pressure assuming $E = 2000$ kg/cm^2 and $k_o = 10$ kg/cm^3. The solution will be done for a strip $B = 3.33$ ms (distance between center lines of floor panels).

In order to simplify the computations, the floor girders will be assumed with a constant average depth of 90 cms.

The floor strip is subject to : (Fig. VIII.14).

i) The weight of water and floor p where

$$p = 3.33\,(5.0 + 0.2 \times 25 + 0.45 \times 0.7 \times 2.5 \qquad = 19.1 \quad \text{t/m}$$

ii) The weight of the wall slab and counterfort P where

$$P = 3.33 \times 0.2 \times 5 \times 2.5 + 0.45 \times 0.7 \times 4.5 \times 2.5 \qquad = 11.9 \quad \text{t}$$

iii) A concentrated bending moment at the outer edge M due to the fixing moments of the counterfort and the wall given by :

$M = 65.23 + \frac{2}{3} \times 2.34 \times 3.33 = 65.23 + 5.2 \simeq 70$ mt

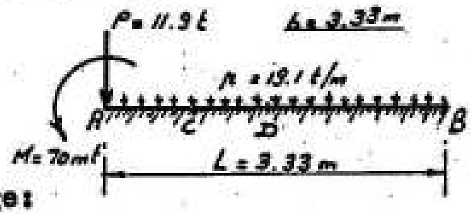


Fig. VIII-14

Area of the section : (Fig. VIII.15)

$A = 0.2 \times 3.33 + 0.7 \times 0.45 \quad = 0.981\ m^2$

Statical moment about mid-height of flange:

$S = 0.45 \times 0.7 \times 0.45 \quad = 0.142\ m^2$

Distance between center of gravity of flange and center of gravity of section is :

$$\frac{0.142}{0.981} = 0.145\ m$$

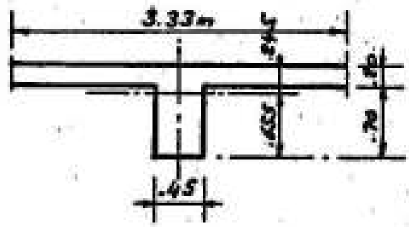


Fig. VIII-15

So that the distance of the center of gravity is equal to :

0.245 ms from top surface of the flange

0.655 ms from bottom " " " web.

The moment of inertia of the section about its center of gravity :

$$I = \frac{0.45 \times 0.7^3}{12} + 0.45 \times 0.7 \times .305^2 + 3.33 \times \frac{0.2^3}{12} + 2.33 \times 0.2 \times .145^2 = 1853 \times 10^4\ cm^4$$

The characteristic value n :

$$n = \sqrt[4]{\frac{B\ k_o}{4\ E\ I}} = \sqrt[4]{\frac{333 \times 10}{4 \times 20 \times 10^4 \times 1853 \times 10^4}} = 0.00385\ cm^{-1}$$

Therefore :

$nL = \frac{3.85}{1000} \times 333 = 1.28$ $\qquad \frac{nL}{2} = 0.64$

$\sin nL = 0.958$ $\qquad \sin \frac{nL}{2} = 0.597$

$\cos nL = 0.287$ $\qquad \cos \frac{nL}{2} = 0.802$

$\sinh nL = 1.6593$ $\qquad \sinh \frac{nL}{2} = 0.6846$

$\cosh nL = 1.9373$ $\qquad \cosh \frac{nL}{2} = 1.2119$

The deflection at the edges y_A and y_B due to the force

$P = \frac{11900}{333} = 35.7$ kg/cm is given according to equations(25a) by :

$$y_A = \frac{2\,P\,n}{k} \cdot \frac{\sinh nL.\ \cosh nL - \sin nL.\ \cos nL}{\sinh^2 nL - \sin^2 nL}$$

$$y_B = \frac{2\,P\,n}{k} \cdot \frac{\sinh nL.\ \cos nL - \sin nL\ \cosh nL}{\sinh^2 nL - \sin^2 nL}$$

So that :

$$y_A = \frac{2 \times 35.7 \times 0.00385}{3330} \cdot \frac{1.6593 \times 1.9373 - 0.958 \times 0.287}{1.6593^2 - 0.958^2} = 1.32 \times 10^{-4}\,\text{cm}$$

$$y_B = \frac{2 \times 35.7 \times 0.00385}{3330} \cdot \frac{1.6593 \times 0.287 - 0.958 \times 1.9373}{1.6593^2 - 0.958^2} = -.625 \times 10^{-4}\,\text{cm}$$

The deflection at the quarter point C and middle point D due to the force P is given, according to equation 25, by :

$$y = \frac{2\,P\,n}{k} \cdot \frac{\sinh nL.\ \cos nx.\ \cosh nx' - \sin nL.\ \cosh nx.\ \cos nx'}{\sinh^2 nL - \sin^2 nL}$$

For point C $x = L/4$ $x' = 3L/4$ $nx = 0.34$ $nx' = 1.02$

For point D $x = L/2$ $x' = L/2$ $nx = nx' = 0.68$

cos 0.34	= 0.943	cosh 0.34	= 1.0584
cos 0.68	= 0.778	cosh 0.68	= 1.2402
cos 1.02	= 0.523	cosh 1.02	= 1.5669

So that :

$y_C = 0.865 \times 10^{-4}$ cm $\qquad y_D = 0.306 \times 10^{-4}$ cm

The bending moments at points C and D due to the concentrated load P can be calculated from equation 27, thus :

$$M = -\frac{P}{n} \cdot \frac{\sinh nL.\ \sin nx.\ \sinh nx' - \sin nL.\ \sinh nx.\ \sin nx'}{\sinh^2 nL - \sin^2 nL}$$

in which :

For point C $nx = 0.34$ $nx' = 1.02$

For point D $nx = nx' = 0.68$

sin 0.34	= 0.334	sinh 0.34	= 0.3466
sin 0.68	= 0.629	sinh 0.68	= 0.7336

sin 1.02 = 0.852 sinh 1.02 = 1.2063

So that :

M_c = - 1940 kgcm/cm M_D = - 1630 kgcm/cm

The deflection at the edges y_A and y_B due to the moment

$M = -\dfrac{70 \times 10^5}{333} = -21000$ kgcm/cm is given, according to equations 29-a, by the relations :

$$y_A = -\frac{2\,M\,n^2}{k} \cdot \frac{\sinh^2 nL + \sin^2 nL}{\sinh^2 nL - \sin^2 nL}$$

$$y_B = \frac{4\,M\,n^2}{k} \cdot \frac{\sinh nL \cdot \sin nL}{\sinh^2 nL - \sin^2 nL}$$

So that :

$$y_A = \frac{2 \times 21000}{3330} \cdot \left(\frac{3.85}{1000}\right)^2 \cdot \frac{1.6593^2 + 0.958^2}{1.6593^2 - 0.958^2} = 3.76 \times 10^{-4} \text{ cm}$$

$$y_B = -\frac{4 \times 21000}{3330} \left(\frac{3.85}{1000}\right)^2 \cdot \frac{1.6593 \times 0.958}{1.6593^2 - 0.958^2} = -3.23 \times 10^{-4} \text{ cm}$$

The deflection at the quarter point C and middle point D due to M can be calculated from equation 29 giving :

$y_C = 1.73 \times 10^{-4}$ cm $y_D = -0.143 \times 10^{-4}$ cm

The bending moments at C and D due to M can be calculated from equation 31 giving :

M_c =-19500 kgcm/cm M_D = - 10950 kgcm/cm

The stress due to the uniform load p is given by :

$$\sigma = \frac{19100}{100 \times 333} = 0.573 \text{ kg/cm}^2$$

The corresponding deflection y is therefore :

$y = \sigma / k = 0.573/3330 = 1.72 \times 10^{-4}$ cm

The deflections, stresses and bending moments are as shown in the following table : Fig. VIII.16a

Case	Point	A	C	D	B
deflection y due to	p	1.720×10^{-4}	1.720×10^{-4}	1.720×10^{-4}	1.720×10^{-4} cm
	P	1.320×10^{-4}	0.865×10^{-4}	0.30[illegible] $\times 10^{-4}$	-0.625×10^{-4} cm
	M	3.760×10^{-4}	1.730×10^{-4}	-0.143×10^{-4}	-3.230×10^{-4} cm
	p+P+M	6.800×10^{-4}	4.315×10^{-4}	1.883×10^{-4}	-2.125×10^{-4} cm
Stress $\sigma = ky$ $= 3330\ y$		2.266	1.438	0.628	-0.708 kg/cm^2
B.M. due to	P	0	− 1940	− 1630	0 kgcm/cm
	M	− 21000	− 19500	− 10950	0 " "
	P + M	− 21000 $= M_o$	− 21440 $= 1.02\ M_o$	− 12580 $= 0.6\ M_o$	0 " "

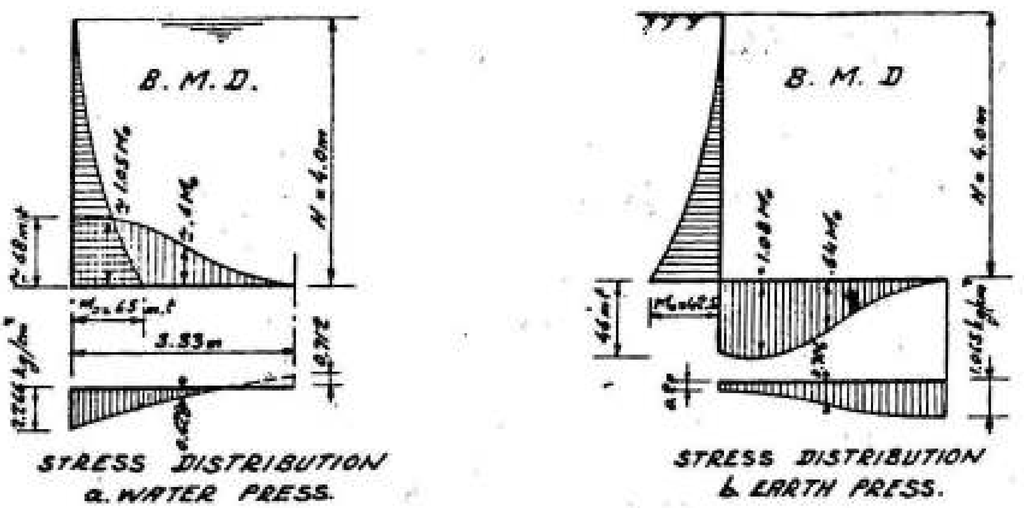


Fig. VIII-16

The maximum bending moment can therefore be estimated by 1.05 M_o, that is , for a maximum bending moment in the counterfort M_o = 65 m t, the maximum bending moment in the floor beam = 1.05 x 65 = 68 m t

Assuming the bending moment due to earth pressure is 0.65 that of water pressure and of opposite sense, the deflections , stresses bending moments of the floor beam will be as shown in the following table : Fig. VIII.16b.

Case	Point	A	C	D	B
Deflection y due to	p	1.720×10^{-4}	1.720×10^{-4}	1.720×10^{-4}	1.720×10^{-4} cm
	P	1.320×10^{-4}	0.865×10^{-4}	0.306×10^{-4}	-0.625×10^{-4} cm
	M	-2.440×10^{-4}	-1.120×10^{-4}	0.093×10^{-4}	2.100×10^{-4} cm
	p+P+M	0.600×10^{-4}	1.465×10^{-4}	2.119×10^{-4}	3.195×10^{-4} cm
Stress $\sigma = ky$ $= 3330\ y$		0.20	0.488	0.706	1.065 kg/cm^2
B.M. due to	P	0	- 1940	- 1630	0
	M	- 13700	- 12700	- 7100	0
	P + M	- 13700 $= M_o$	- 14640 $= 1.07\ M_o$	- 8730 $= 0.64\ M_o$	0

IX. BUNKERS AND SILOS

IX.1 DEFINITIONS

Bunkers and silos are containers used for storing dry materials such as cement, corn, coal etc.. They are generally filled from the top and emptied from the bottom.

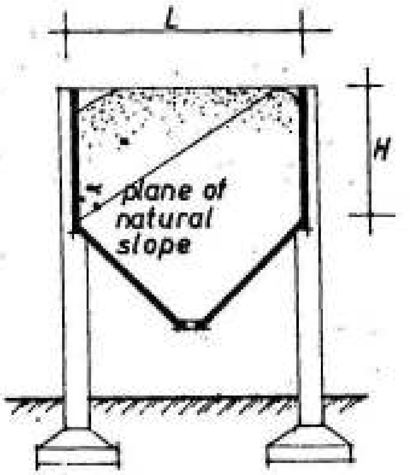


Fig. IX-1 BUNKER

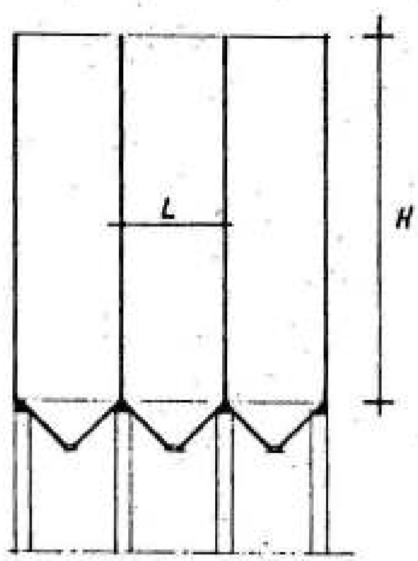


Fig. IX-2 SILO

In bunkers, figure IX-1, the plane of natural repose of the filling material must not intersect the opposite wall i.e.

$H \tan\alpha < L$ practically $H < 1.5\, L_{max}$.

whereas in silos, figure IX-2, the depth is big in proportion to the section, and the plane of natural repose generally intersects the opposite wall i.e.

$H \tan\alpha > L$ practically $H > 1.5\, L_{max}$

IX.2 LAYOUT OF SILO CELLS

Silos are generally composed of a series of square, hexagonal octagonal or circular deep cells as shown in fig. IX-3. The dead zones

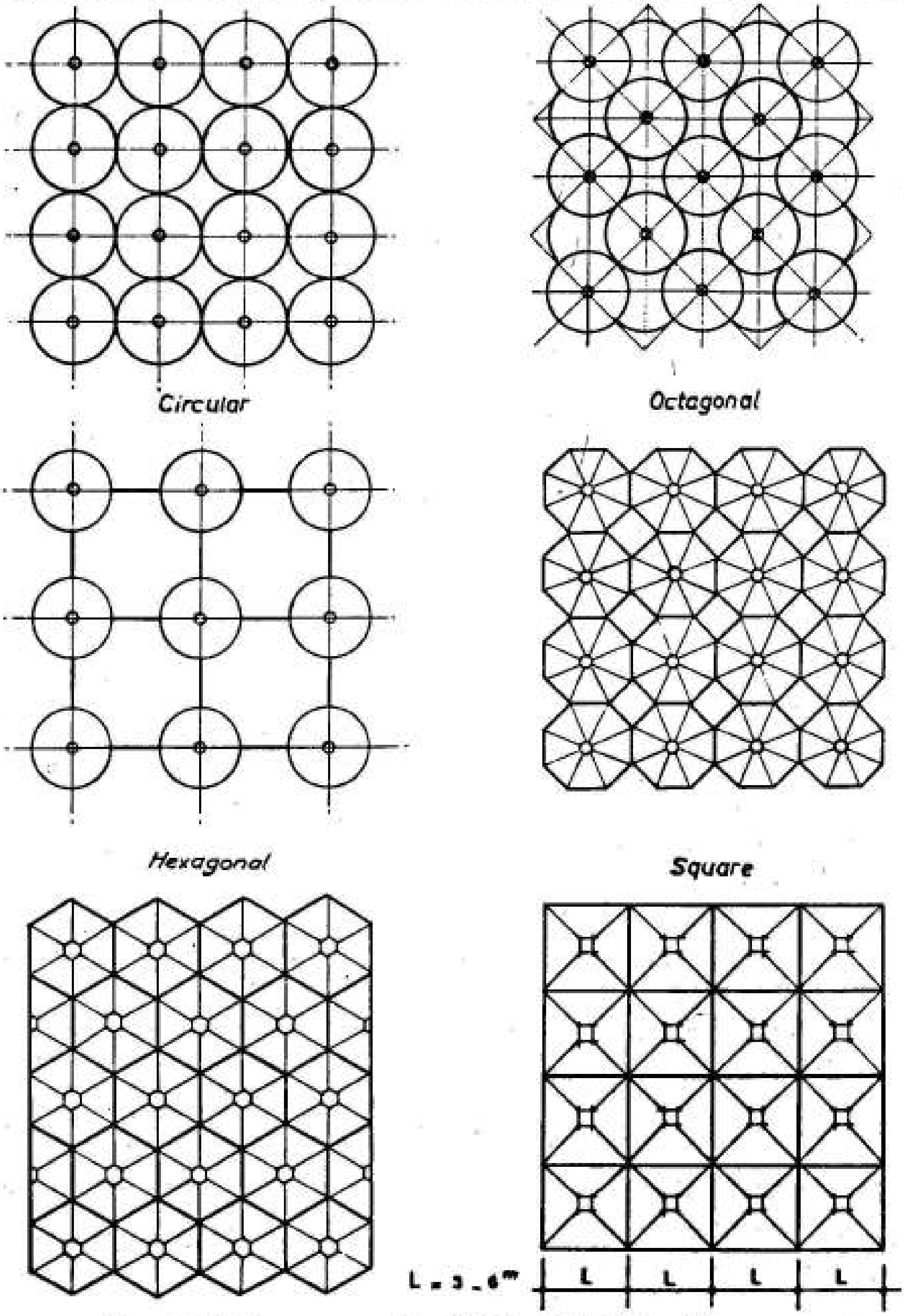


Fig. IX-3 Arrangement of Silo-Cells in Plan between octagonal or circular cells do not exist in case of square, rectangular or hexagonal cells ; these zones may however be used for ventilation, piping, lifts etc..

The bending moments in walls of circular cells are nill and in walls of octagonal cells are smaller than in walls of hexagonal or rectangular cells.

In order to reduce the bending moments in exterior walls, the cells may be constructed in the form shown in figure IX-4 .

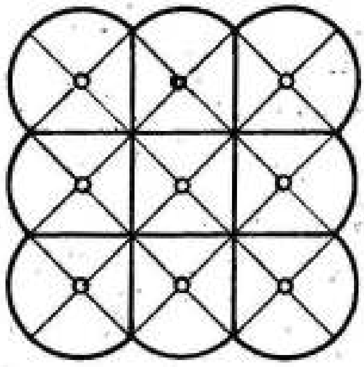


Fig. IX-4 CELLS WITH CURVED OUTSIDE WALL.

Silos are generally provided with hoppers, the slope of their sides is to be chosen 5% more than the natural slope of the filling material in order to empty the cells without difficulty. The sloping sides of the hoppers may be of reinforced concrete, but due to the difficulties encountered in determining the stress distribution and in the shuttering and reinforcements, it may be easier to make a flat floor for the cells and to make the sloping sides of the hopper by using lean plain concrete as shown in figure IX-5, in which the

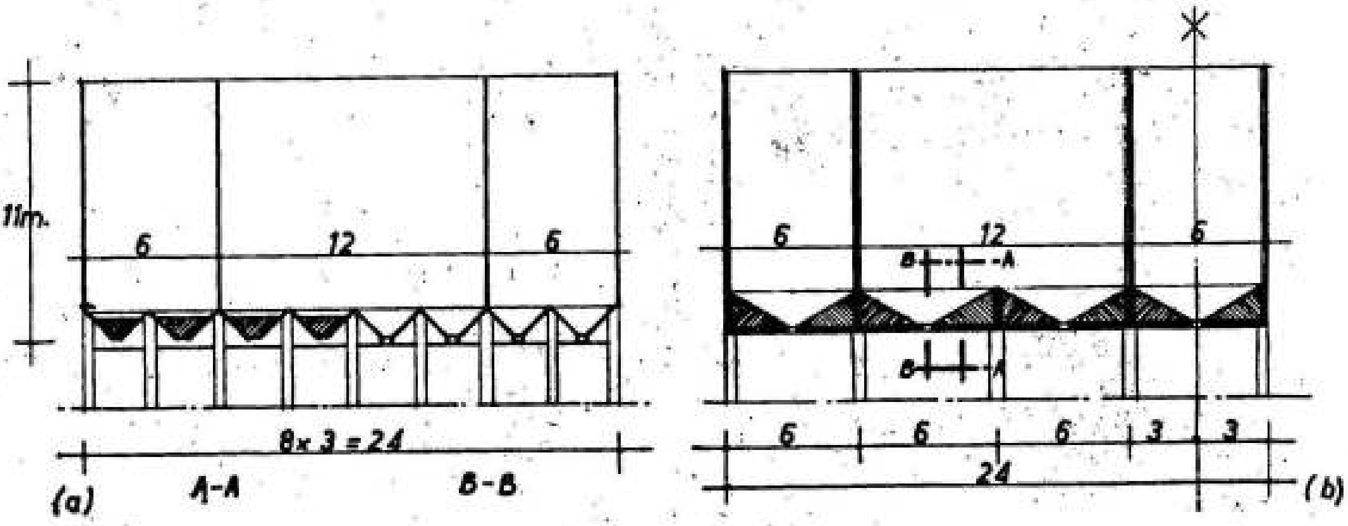


Fig. IX-5

silos are 24 meters wide, 42 meters long and 11 meters deep and composed of two small cells 6 x 6 m. and two inside big cells 12 x 12 m.

The thickness of the walls is generally chosen to follow the values of the internal forces, which are small at the top and increase with the depth i.e. the walls are chosen of a trapezoidal shape with maximum thickness at the base. Recently, big silos are constructed by using sliding forms which take a relatively short time in construction. In order to be able to use such forms, the choice of a constant wall thickness is essential.

If a bunker or a silo is used for storing a material which sticks to the concrete such as cement or flour, one may cover the inside surface of the hopper and part of the walls at the base with steel or zinc plates or with timber planks.

IX.3 DETERMINATION OF PRESSURE INTENSITIES ACCORDING TO CLASSIC THEORY

IX.3-1 Bunkers

When calculating the side pressure on the walls and floor of a bunker, it is allowed due to the small depth of the bunker, to neglect the friction between the filling material and the walls.

The side pressure can be calculated according to the known theories of earth pressure. Thus, assuming.

γ = weight of filling material per cubic meter

p_1 = vertical pressure intensity at any depth x

p_2 = horizontal " " " " " x

ρ = angle of internal friction of filling material

ρ' = angle of friction between filling material and walls,

the vertical and horizontal pressure intensities can be calculated from the relations:

$$p_1 = \gamma H$$

$$p_2 = H \gamma \tan^2 (45 - \rho/2)$$

Therefore, the max. cantilever moment is given by :

$$M_{max.} = 13.52 \times 0.9 + 1.34 \times \frac{0.9^2}{2} = 12.2 + 0.54 = 12.74 \text{ m t}$$

and the max. field moment is :

$$M_m max. = 9.88 \times \frac{4.5^2}{8} - 12.74 = 25.0 - 12.74 = 12.36 \text{ m t}$$

The loads, bending moments and shearing forces are shown in figure VII.29.

One can see that the choice of the dimensions of the tank was convenient for the following reasons :

1) The max. fixing moment in the wall (- 4.84 m t) is approx. equal to the max. fixing moment in the floor (- 5.00 m t).

2) The max. moments are accumulated at the corner between walls and floor, so that only a small part of the slabs needs a relatively big thickness (~ 40 cms).

3) The max. field moment in the wall (1.58 m t) is approx. equal to the max. field moment in the floor (1.70 mt).

4) The thickness of slabs is generally governed by their field moments and as their values are small, only the minimum thickness necessary for water-tightness (~ 20 cms) is sufficient.

5) The cantilever part has been chosen such that it gives approximately equal cantilever and field moments in the supporting beams.

The same idea can be adopted for bigger capacities as shown in figure VII.30 in which the capacity of the tank is :

$4.00 \times 10.60 \times 10.60 \approx 450 \text{ m}^3$.

The floor slabs are again here fixed at the intermediate panelled frames, because they are arranged on the axes of symmetry.

Each of the floor panelled frames has a span of 9 ms. and carries, in addition to its own weight, a floor load of $p = \frac{4.5}{2} \times 4.5 = 10$ t/m on the intermediate span.

then

$$\frac{dp_1}{dx} = \gamma - k\,p_1 \qquad \text{and} \qquad \frac{dx}{dp_1} = \frac{1}{\gamma - k\,p_1}$$

giving

$$x = -\frac{1}{k}\log(\gamma - k\,p_1) + C$$

But for $x = 0$, $p_1 = 0$, thus :

$$0 = -\frac{1}{k}\log\gamma + C \qquad \text{or} \qquad C = \log\gamma / k$$

Substituting this value in the equation of x , we get

$$-\ k\,x = \log(\gamma - k\,p_1) - \log\gamma = \log\frac{\gamma - k\,p_1}{\gamma}$$

Therefore

$$\frac{\gamma - k\,p_1}{\gamma} = e^{-kx} = \frac{1}{e^{kx}}$$

in which $e = 2.71828$ is the base of natural logarithm.

Accordingly, we get :

$$p_1 = \frac{\gamma}{k}\left(1 - \frac{1}{e^{kx}}\right)$$

and

$$\text{max. } p_1 = \gamma / k$$

Substituting for k, we get :

$$\text{max. } p_1 = \frac{\gamma}{\frac{O}{A}\tan\rho'\,\tan^2(45 - \rho/2)}$$

We have further :

$$p_2 = p_1\ \tan^2(45 - \rho/2) = \frac{\gamma}{k}\left(1 - \frac{1}{e^{kx}}\right)\tan^2(45 - \rho/2)$$

substituting for k, we get :

$$p_2 = \gamma\left(1 - \frac{1}{e^{kx}}\right)\frac{A}{O\tan\rho'}$$

$$\text{max. } p_2 = \frac{\gamma}{\frac{O}{A}\tan\rho'} \qquad \text{at } x = \infty$$

Therefore :

$$\text{max. } p_1 = \frac{\text{max. } p_2}{\tan^2(45 - \rho/2)}$$

The theoretical horizontal pressure distribution can approximately be replaced by the broken line O A B C D shown in fig.IX.7. According to this approximation, max. p_2 is assumed to take place at a distance 2h, where

$$h = \frac{\max.\ p_2}{\gamma \tan^2 (45 - \rho/2)}$$

h is, in this manner, the depth at which the horizontal pressure p_2 calculated from the relation $p_2 = \gamma\ x \tan^2 (45 - \rho/2)$ is equal to max. p_2 calculated according to given theory of silos. Accordingly we get further :

$$p_x = \max.\ p_2 \left(\frac{0.368}{h} x + 0.264 \right)$$

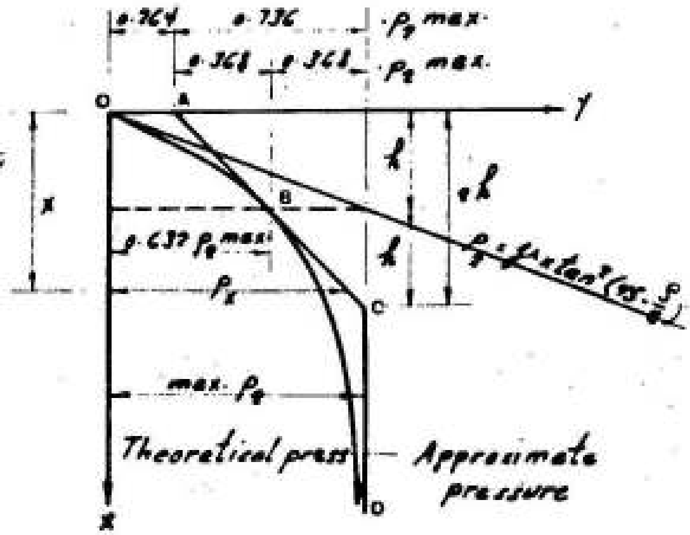


Fig. IX-7

Assuming the intensity of the upward frictional force = p_3 and $p_3 / p_1 = \varkappa$, the value of $\varkappa$ may be estimated = 0.2 - 0.25.

Results of recent researches are shown in figure IX.8

- Horizontal pressures on wall of circular cells
- Stored material : corn.
- Curve 1 : theoretical values of Janssen.
- Curve 2 : measured horizontal pressure after filling of cell.
- Curve 3 : measured pressure by emptying of cell.
- Curve 4 : measured pressure after 8 days from filling of cell.

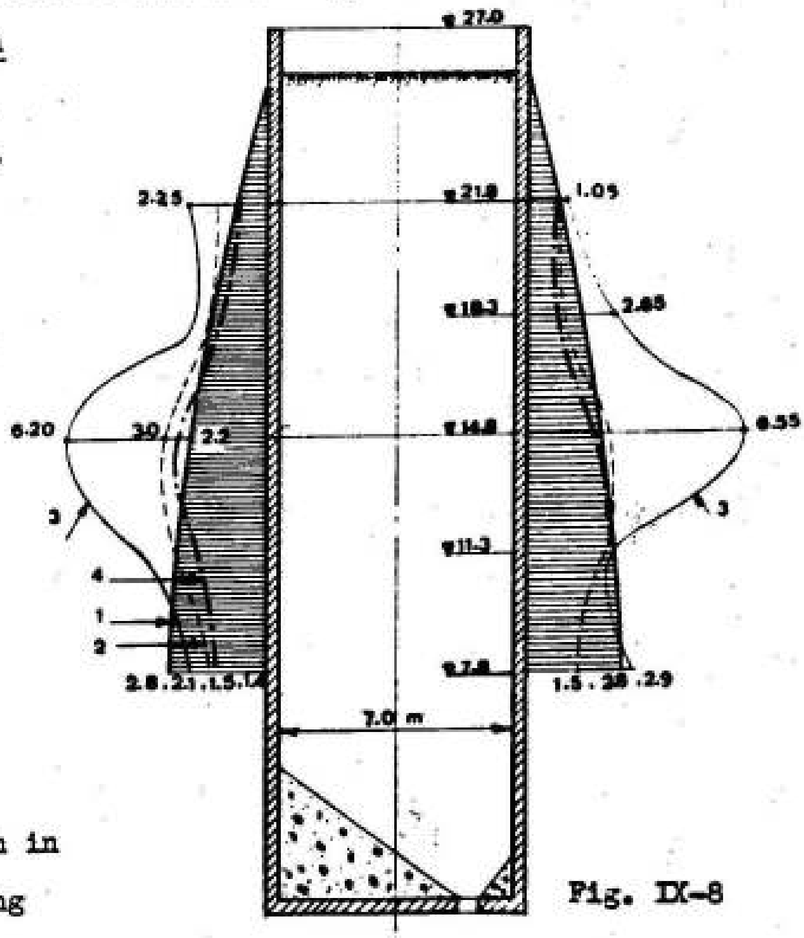


Fig. IX-8

It is clear that even in case of granular filling material, the horizontal pressure by emptying is much bigger than the values obtained from the theory of Janssen.

The German Specifications given below have given the basic guide lines for determining the pressure intensities in silo cells.

IX-4 PRESSURE INTENSITIES ACCORDING TO GERMAN SPECIFICATIONS DIN 1055 - SHEET 6 - 1965

The Janssen classic theory of silos as given in most text books leads, in some cases, to internal pressure intensities much smaller than the real actual values. Based on test results, the following rules, can be followed.

1. Definitions and Limitations

1.1 The following rules apply to silos with prismatic & cylindrical cells. The forces acting in the zone of the hoppers and silo pockets, as well as in bunkers without cross walls need further study and therefore do not follow the given rules.

1.2 Stored Filling Materials

It is assumed that the stored filling materials whether granular or dusty, are cohesionless or materials in which the cohesion is small relative to the internal friction. The given methods can also be applied to silo-cells containing pressurised materials or fermented seeds.

1.3 Internal Pressure Intensities in Silo-Cells

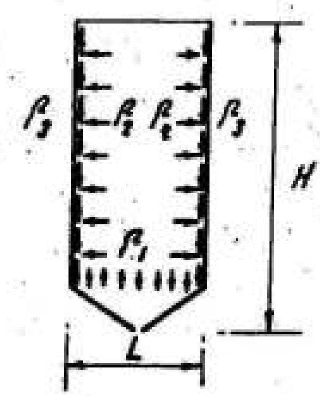
It will be assumed that : (Fig. IX.9)

p_1 = vertical pressure intensity in t/m^2

p_2 = horizontal " " " "

p_3 = Frictional forces transmitted to the walls in t/m^2

Fig. IX-9



2. Determination of Internal Pressure Intensities

2.1 Data

The form of the cross-section will be included by the relation A/O meter ; in which

A = net area of cross-section of cell in m^2

O = internal perimeter of cell in ms.

The depth x in meter is measured from the real assumed upper plane surface of the filling material. (Fig. IX.10).

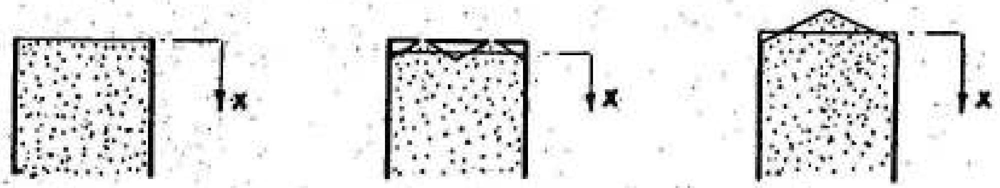


Fig. IX-10

The weight of the filling material γ in tons per cubic meter and the angle of internal friction ρ may be taken from the following table:

	Coal	Cement	Lime	Gypsum	Tabac	Sugar	Beans & Grains	Flour
γ	0.8-1.2	1.7	1.7	1.5	0.5	1.0	0.8	0.6
ρ	30°	20°	45°	25°	-	35°	30°	25°

The angle of friction ρ' between silo-wall and filling material and the coefficient of friction μ are given by :

$$\tan \rho' = p_3 / p_2 = \mu$$

ρ' is a factor of ρ . It can be assumed according to the following table:

Filling material	angle of friction ρ' in degrees	
	by filling ρ'_f	by emptying ρ'_e
coarse fill having grain-diameter > 0.20 mm	0.75 ρ	0.60 ρ
dusty fill having grain-diameter < 0.06 mm	1.00 ρ	1.00 ρ

Linear interpolation may be used for grain-diameters between 0.06 and 0.20

The relation between the horizontal and vertical pressure intensities :

$$\lambda = p_2 / p_1$$

will be assumed constant over the whole depth of the silo.

By filling , assume : $\lambda_f = 0.5$

By emptying, assume : $\lambda_e = 1.00$

2.2 Governing Cases of Loading

The biggest loads generally take place for the cases shown in the following table :

Load	Coarse Fill		Dusty Fill	
	finite depth	infinite depth	finite depth	infinite depth
Vert. pressure intensity p_1	filling	filling	filling	filling
Horiz. pressure intensity p_2	emptying	emptying	emptying	Filling = emptying
Frictional force p_3	emptying	filling= emptying	emptying	filling = emptying

2.3 Pressure Intensities at an Infinite Depth

The pressure intensities reach their maximum values at a depth equal to infinity :

By filling :

$$\max.\ p_1 = \gamma A / \lambda_f \cdot \mu_f \cdot O$$

$$\max.\ p_2 = \gamma A / \mu_f \cdot O$$

$$\max.\ p_3 = \gamma A / O$$

By emptying :

$$\max.\ p_1 = \gamma A / \lambda_e \cdot \mu_e \cdot O$$

$$\max.\ p_2 = \gamma A / \mu_e \cdot O$$

$$\max.\ p_3 = \gamma A / O$$

2.4 Pressure Intensities at a Finite Depth

The pressure intensities p increase with the depth x according to the relation :

$$p_x = \max.\ p\ \varphi$$

in which

$$\varphi = (1 - e^{-\frac{x}{x_o}})$$

where

$$x_o = A / \lambda_f \cdot \mu_f \cdot O \qquad \text{by filling}$$

$$x_o = A / \lambda_e \cdot \mu_e \cdot O \qquad \text{by emptying}$$

Values of φ as a factor of x / x_o

x/x_o	,0	,1	,2	,3	,4	,5	,6	,7	,8	,9
0,	0,00	0,10	0,18	0,26	0,33	0,39	0,45	0,50	0,55	0,59
1,	0,63	0,67	0,70	0,73	0,75	0,78	0,80	0,82	0,83	0,85
2,	0,86	0,88	0,89	0,90	0,91	0,92	0,93	0,93	0,94	0,94
3,	0,95	0,96	0,96	0,96	0,97	0,97	0,97	0,98	0,98	0,98

3. Factors Increasing Pressure Intensities

3.1. Arch Action of Filling Materials

The pressure intensities are much increased by the sudden failure of the material - arches - existing inside the cells. For this reason, the vertical pressure intensity on the hopper may be double but is not to be assumed in any case bigger than $\gamma\ x$.

This rule may not be applied, only if it is proved by practical experience that arching of the filling material is not liable to take place.

3.2. Eccentric Hopper-Opening

By emptying of silo-cells provided with eccentric hopper-opening, non-uniform horizontal forces acting on the perimeter are created over the whole depth of the cell.

These additional horizontal forces $p_2^{\prime}$ are to be considered in positions giving bigger internal stresses.

Assuming :

p_2 = horizontal pressure intensity of original silo - cell according to article 2

$p_2' = p_{2i} - p_2$ = an anti-symmetrical load acting on the original cell keeping the equilibrium of the frictional forces. (Fig. IX.11).

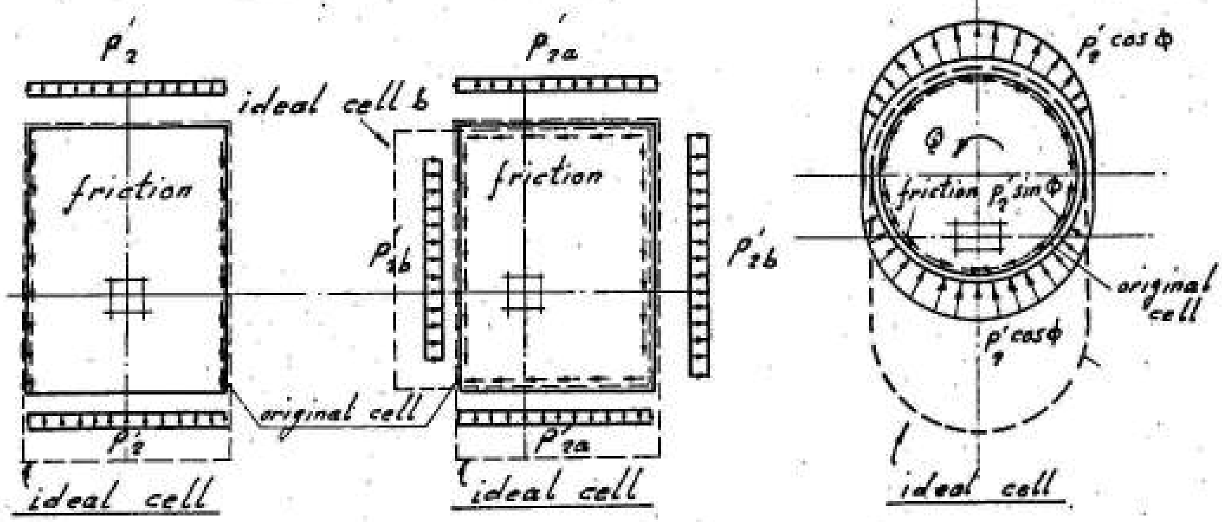


Fig. IX-11

p_{2i} = horizontal pressure intensity created by emptying an ideal symmetrical silo-cell chosen according to figure IX.11.

These additional horizontal pressure intensities may be neglected if the eccentricity of the hopper opening is smaller than d/6 or if the height of the cell H is smaller than 2d where d is the diameter of the circle that can be drawn inside the cross-section of the cell.

3.3. Filling Material under Pneumatic Pressure

The horizontal pressure intensity in silo-cells provided with piping to pressurise the filling material depends on its grain size. In cells used for storing cohesionless granular filling materials, the horizontal pressure intensity is to be increased by an amount equal to the air-pressure and is to be gradually decreased to zero from the highest pressure-opening in the cell to the upper surface of the filling material. No increase in the magnitude of the horizontal pressure intensity need to be considered for dusty filling materials.

4. Factors Decreasing Pressure Intensities

4.1. Floor of Cells

Due to the fixation of the walls to the floor, the horizontal pressure intensity may be reduced in the manner shown in figure IX.12.

4.2. Special Emptying Plant

If a silo is provided with a special emptying plant which enables the emptying of the upper layers of the filling material without moving the lower layers, the emptying pressure intensities need not be considered in the design. In this case, it is essential to take the necessary provisions to prevent the emptying of the silo from the hopper-opening.

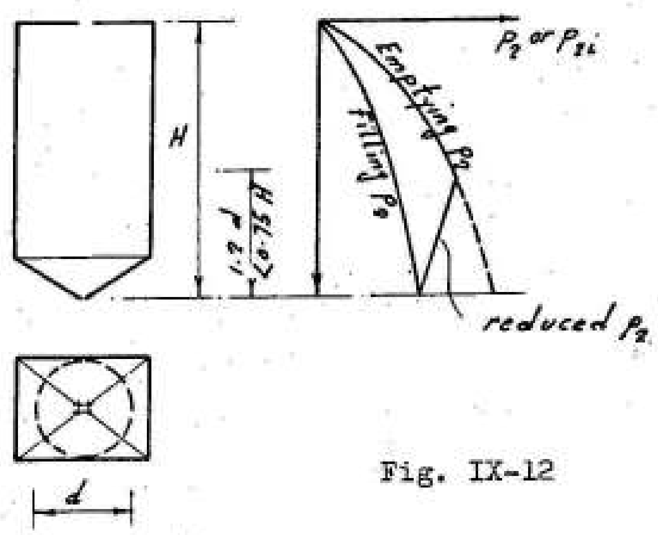


Fig. IX-12

5. Special Cases

5.1. Special Mixing Plant

If a silo-building is provided with a special pneumatic mixing plant giving a homogenious regularly distributed dusty mix, the pressure intensities are to be calculated according to article 2 provided that their magnitude is bigger than the values given by the relation

$$p_1 = p_2 = 0.6\ \gamma\ x$$

5.2 Silos for Fermented Seeds

Fermented seeds do not follow the rules of granular or dusty materials. The forces due to such materials depend on the included amount of water and on the fermentation process. They may be assumed according to the following table :

	Category I material highly fermented	Category II fermented material	Category III wet stored material
Dry weight in % of fresh weight	> 35	23 - 35	< 23
Weight to be assumed in t/m^3	0.5	0.75	1.00
Vertical pressure intensity p_1 in t/m^2	γ x	γ x	γ x
Horizontal pressure intensity p_2 in t/m^2	0.75 γ x	0.70 γ x	1.00 γ x
Friction force p_3 in t/m^2	0.16 p_2	0.14 p_2	0.10 p_2

If the stored materials given under categories I & II are wet, the silo should not be filled to more than half its depth and to provide it with a draining outlet, so that the material-juice is not more than one meter deep.

IX.5 ILLUSTRATIVE EXAMPLE

A silo-cell 5 x 5 ms. and 40 ms. deep is used for storing corn, determine the maximum pressure intensities p_1 , p_2 and p_3 during filling and emptying according to both the classic theory and the german specifications. Assume $\gamma = 0.8$ t/m^3 & $\rho = 30°$.

a) According to classic theory

area of cell $A = 5 \times 5 = 25 \ m^2$

perimeter of cell $O = 4 \times 5 = 20 \ m$

angle of friction between corn and wall ρ' according to the classic theory is assumed the same (= 25°) both during filling and emptying.

So that :

$k = \frac{O}{A} \tan \rho' \tan^2 (45 - \rho/2) = \frac{20}{25} \tan 25° . \tan^2 (45° - 15°)$ or

$k = 0.8 \times 0.466 \times 0.577^2 = 0.124$

max. $p_1 = \gamma / k = 0.80/0.124 = 6.45 \ t/m^2$

max. $p_2 = \frac{\gamma}{\frac{O}{A} \tan \rho'} = \frac{0.8}{0.8 \times 0.466} = 2.14$ ms.

max. p_2 acts at a depth 2h where

$$h = \frac{\max.\ p_2}{\gamma \tan^2 (45 - \rho/2)} = \frac{2.14}{0.8 \times 0.577^2} = 8.03 \text{ ms.}$$

i.e. the max. horizontal pressure acts at a depth of ~ 16 ms.

$$\max.\ p_3 = \max.\ p_2 \tan \rho' = 2.14 \times 0.466 = 1.00 \ t/m^2$$

b) <u>According to DIN 1055</u>

$$\gamma A/O = 0.8 \times 25/20 = 1 \ t/m^2$$

$\lambda = p_2 / p_1$, $\lambda_f = 0.5$, $\lambda_e = 1.0$

$\mu = \tan \rho' = p_3 / p_2$

By filling : $\rho_f = 0.75 \rho = 0.75 \times 30° = 22.5°$ and $\mu_f = 0.414$

By emptying: $\rho_e = 0.60 \rho = 0.60 \times 30° = 18°$ and $\mu_e = 0.325$

So that the maximum pressure intensities at an infinite depth are given by :

$$\left.\begin{aligned} \max.\ p_1 &= \gamma A / \lambda_f \cdot \mu_f \ O = 1/0.5 \times 0.414 = 4.85 \ t/m^2 \\ \max.\ p_2 &= \gamma A / \mu_f \ O = 1/0.412 = 2.42 \ t/m^2 \\ \max.\ p_3 &= \gamma A / O = 1 \ t/m^2 \end{aligned}\right\} \text{filling}$$

$$\left.\begin{aligned} \max.\ p_1 &= \gamma A / \lambda_e \cdot \mu_e \ O = 1/1.0 \times 0.325 = 3.08 \ t/m^2 \\ \max.\ p_2 &= \gamma A / \mu_e \ O = 1/0.325 = 3.08 \ t/m^2 \\ \max.\ p_3 &= \gamma A / O = 1 \ t/m^2 \end{aligned}\right\} \text{emptying}$$

The results areshown in the following table :

Pressure Intensity	t/m^2	p_1	p_2	p_3
Classic Theory	Filling & Emptying	6.45	2.14	1.00
DIN 1055	Filling	4.85	2.42	1.00
	Emptying	3.08	3.08	1.00

The table shows that the difference in this case reaches ± 50 %

IX.6 DESIGN OF WALLS AND FLOORS

IX.6.1. Design of Walls of Circular Cells

Walls of circular silo-cells are to be calculated for: a) ring tension, b) compression due to frictional forces, c) temperature stresses (if any).

a) Due to the horizontal pressure p_2, the walls of a circular cell, with diameter D and radius r, will be subject to ring tension T/meter height given by :

$$T = p_2 \, r$$

The corresponding ring reinforcement must be capable to resist the full tension i.e.

$$A_s = T/\sigma_s$$

The upper part of the walls (2-3 ms) may be reinforced by one mesh of reinforcements and the lower parts by double meshes. The reinforcements may be chosen as follows :

ϕ of rings $\leqslant$ 16 mms and $\geqslant$ 8 mms at a distance $\leqslant$ 20 cms and $\geqslant$ 10 cms.

Minimum vertical reinforcement : 4 ϕ 8 mm/m

IX.6.2. Width and Spacing of Cracks

The thickness of the wall depends on the allowed crack width which can be calculated as follows :

Assuming :

Spacing between cracks = e cms

Width of cracks = Δe cms

Ratio of tension steel in section = A_s / A_c = μ

Diameter of tension steel = ϕ

Tensile stress in steel = σ_s

Stress in steel at which cracking occurs = σ_{so}

Then

$$e = \frac{0.4\,\phi}{\mu} \quad \text{and} \quad \frac{\Delta e}{e} = \frac{\sigma_s - \sigma_{so}}{E_s}$$

σ_{so} depends on the concrete quality and the percentage of the steel in the section. It can be calculated from the relation :

$$\sigma_{so} = \left(\frac{1}{16\mu} + 1.75 \right) \sigma_{c28}$$

Assuming $\sigma_{c28} = 200$ kg/cm^2 , then

$$\underline{\sigma_{so} = \frac{12.5}{\mu} + 350}$$

For normal cases, a width of crack < 0.2 mm. may be accepted.

If the walls are to be free from cracks then the thickness of the wall may be chosen from the relation.

$$\underline{t \text{ in cms} = 0.8 \; T \text{ in tons/m}}$$

<u>Example</u> :

A circular silo cell 15 meters diameter is subject to an internal pressure $p = 5$ t/m^2 . If cold twisted steel with an allowable stress $\sigma_s = 2$ t/cm^2 is used for the reinforcement, determine the spacing and width of cracks if the wall is chosen 20 cms thick.

Ring tension	$T = p\,D/2 = 5 \times 15/2$	$= 37.5$ t/m
Ring reinforcement	$A_s = T / \sigma_s = 37.5 / 2$	$= 18.75$ cm^2

Choose 7 ϕ 13 on each face giving an area of 18.5 cm^2

Ratio of tension steel $\mu = A_s/bt = 18.5 / 100 \times 20 = .00925$

Assuming a concrete quality C200, we get

$$\sigma_{so} = \left(\frac{12.5}{.00925} + 350 \right) = 1250 + 350 = 1600 \text{ kg/cm}^2$$

Therefore

The average spacing of the cracks is :

$$e = 0.4\phi/\mu = 0.4 \times 1.3 / 0.00925 = 56 \text{ cms}$$

The average width of cracks is :

$$\Delta e = \frac{\sigma_s - \sigma_{so}'}{E_s} \cdot e = \frac{2000 - 1600}{2100\,000} \times 56 = 0.0107 \text{ cms} = 0.107 \text{ mm.}$$

In order to have a wall free from cracks, its thickness t is to be chosen $> 0.8\,T$ i.e. $t = 0.8 \times 37.5 = 30$ cms in which case

$\mu = 18.5 / 100 \times 30 = .0062$ and

$\sigma_{so} = \frac{12.5}{.0062} + 350 = 2350$ kg/cm^2 $> \sigma_s$ i.e. no cracks !

b) The horizontal sections of the wall are subject to compressive forces due to the own weight of the wall and the frictional forces from the filling materials.

The frictional force N_x at any depth x is given by

$$N_x = \frac{(\gamma x - p_{1x}) \frac{\pi}{4} D^2}{\pi D}$$

c) Eventual effect of temperature differences (as will be shown later) may be included in the design.

IX.6.3. Design of Conical Hoppers

Circular silo-cells are generally provided with conical hoppers. These hoppers are subject to meridian and ring forces due to the vertical pressure p_1 of the filling material and the own weight of the hopper* . (Fig. IX-13)

The tensile meridian force N_s at any section a - a per unit length circumference is given by :

$$N_s = \frac{\pi r_1^2 \; p_1 + G}{2 \pi r_1 \sin \varphi}$$

in which, G is the own weight of the hopper below a - a .

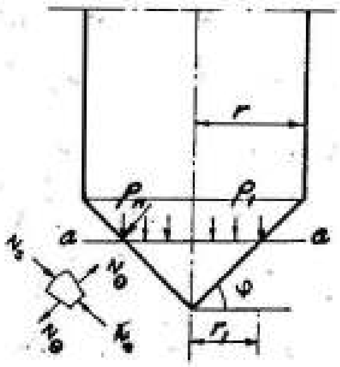


Fig. IX-13

The tensile ring force N_θ per unit length meridian is given by :

$$N_\theta = p_n \; r_1 / \sin \varphi$$

where p_n = component of the pressure of the filling material normal to the plane of the hopper. It is given by :

* Refer to "Fundamentals of Reinforced and Prestressed Concrete" , by M. Hilal.

$$p_n = p_1 \cos^2\varphi + p_2 \sin^2\varphi$$

The component of the pressure of the filling material parallel to the plane of the hopper is given by :

$$p_t = p_1 \sin^2\varphi + p_2 \cos^2\varphi$$

N_s and N_θ are zero at the bottom of the hopper and maximum at its top edge.

The reinforcements in the hopper are :

longitudianl bars : $A_{s1} = N_s / \sigma_s$

rings : $A_{s2} = N_\theta / \sigma_s$

Due to the rigid connection between the cylindrical wall and the conical floor, bending moments M and shearing forces Q are created, they may be roughly estimated by the relations : (Fig. IX.14)

$M = p_2 \; r^2 / 6$ $\qquad$ $Q = p_2 \; r / 2$

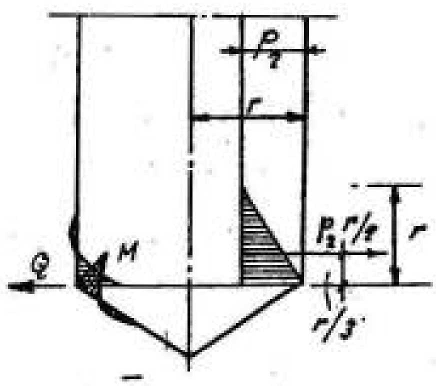


Fig. IX-14

The connecting moment M can however be determined by the moment distribution method as explained in IV.3.6

IX.6.4 Design of Rectangular Cells

The bending moments and tension in the walls of rectangular silo-cells due to the internal horizontal pressure p_2 can be determined according to the methods given in VII.1. Cells of approximately equal spans may be considered as isolated i.e. effect of continuity of different cells is neglected. Effect of continuity must however be taken in consideration if the spans of the cells differ much as for example the silos shown in figure IX.5.

Due to the rigid connection between the wall and the floor, the fixed end moment M and the shearing force Q can be roughly estimated from the relations :

$$M = p_2 L^2 / 24 \qquad \& \qquad Q = 0.3\, p_2\, L$$

IX.6.5 Width & Spacing of Cracks

The spacing and width of cracks in elements subject to simple bending can be calculated from the relations :

$$e = \frac{0.24\, \Phi}{\mu} \qquad \text{and} \qquad \frac{\Delta e}{e} = \frac{\sigma_s - \sigma_{so}}{E_s}$$

$$\sigma_{so} = \left(\frac{1}{25\mu} + 1.75 \right) \sigma_{c28}$$

Assuming $\sigma_{c28} = 200$ kg/cm^2 , then

$$\sigma_{so} = \left(\frac{8}{\mu} + 350 \right)$$

IX.6.6. Design of Floor of Rectangular Cells

If the floor of a cell is flat it may be treated as a two way slab hung to the walls, and carries it own weight and the weight of the lean concrete forming the required shape of the hopper plus the maximum vertical pressure of the filling material.

If the hopper is a reinforced concrete pyramid, it may approximately be calculated as follows :

The sides of the hoppers are subjected to the pressure of the filling material plus their wwn weight. The pressure of the filling material is resolved to the two components p_n and p_t normal and parallel to the sides of the hopper :

$$p_n = p_1 \cos^2\varphi + p_2 \sin^2\varphi$$

$$p_t = p_1 \sin^2\varphi + p_2 \cos^2\varphi$$

The triangular or trapezoidal sides of the hopper may be replaced by rectangular slabs as follows* : (Fig. IX.15).

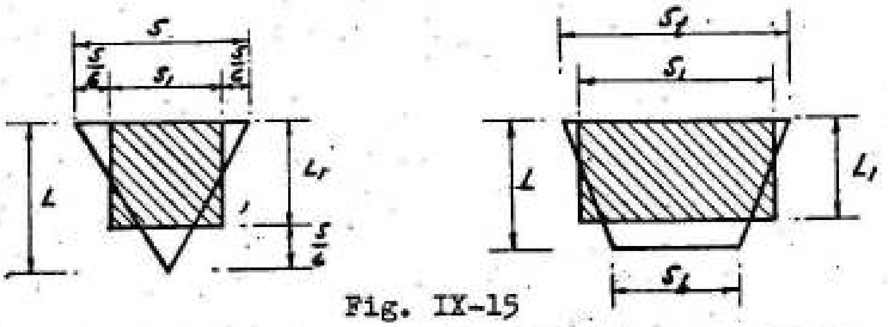


Fig. IX-15

* Refer to : Sachnowski " Stahlbeton Konstructionen ". V E B Velag Technik. Berlin

For a triangular side $S_1 = \frac{2}{3} S$ and $L_1 = L - \frac{S}{6}$

and for a trapezoidal side $S_1 = \frac{2}{3} \frac{S_t^2 + 2 S_t S_b}{S_t + S_b}$

and $L_1 = L - \frac{S_t (S_t - S_b)}{6 (S_t + S_b)}$

The load p_n is distributed on the two directions of the slab accor - ding to the ratio of its reduced sides L_1 / S_1. In horizontal direction, the different sectors of the hopper behave as closed frames subjected to bending moments and tension; in the inclined direction,each side behaves as a slab continuous with the wall at the top and simply supported at the bottom. Due to the tangential component p_t, the sides are subjected to tensile stresses and are to be treated as folded plates.

The exact values of the bending moments in isotropic triangular and trapezoidal flat plates with different edge conditions and subjected to uniform and triangular pressure are given in the text book of Bareš " Tables for the Analysis of Plates, Slabs and Diaphragms " Published by : Bauverlag. Berlin. Dr. Shaker El-Behairy in his reinforced concrete Design Handbook " has extracted the following tables from the book of Bareš .

IX.6.7 Calculation of Wall as a Beam

The shear stress in the wall due to its own weight plus the force due to friction must be less than the allowable values. Hence

Own weight of wall = L. H. t x 2.5 tons

Frictional force on the two sides of the wall = $2 \left[\frac{A}{4} (\gamma H - p_{1max})\right]$ tons

Total Q = tons

Max Shear Stress $\tau_{max} \cong \frac{Q}{2} \cdot \frac{1.5}{t.H} = \frac{3 Q}{4 t H}$.

The wall is however to be calculated as a deep beam carrying its own weight + weight of floor + weight of stored material (Refer to Chapter XI).

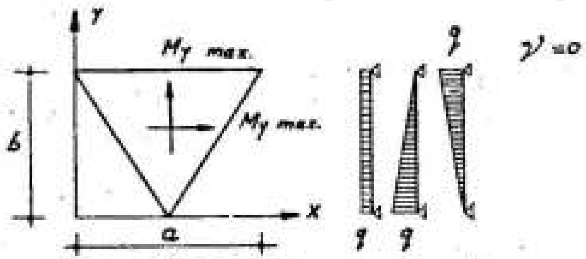
IX.7 FOUNDATIONS

The column loads in silo-buildings are generally high and the foundations are to be carefully designed and executed.

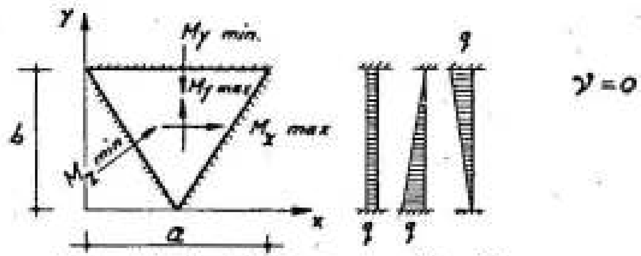
Due to the big rigidity of the concrete silos, differential settlements are generally not possible. For good soils, one may use isolated footings either plain or reinforced . For medium bearing soils, strip continuous foundations may be used; whereas for weak soils or

Bending Moments in Simply Supported and Totally Fixed Triangular Plates

due to Rectangular and Triangular Loads



Case of Loading : A B C



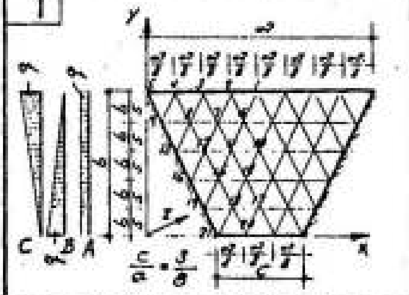
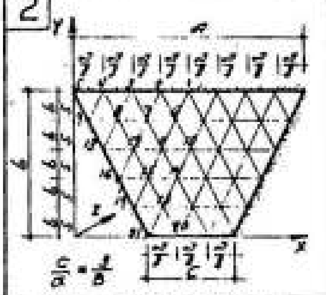
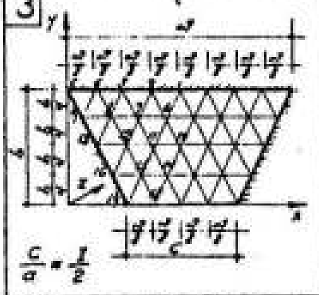
Case of loading A B C

a/b	a) Simply Supported Triangular Plate						b) Totally Fixed Triangular Plate :											
	Case A		Case B		Case C		Case A				Case B				Case C			
	M_x max	M_y max	M_x max	M_y max +ve	M_x max +ve	M_y max +ve	M_x max +ve	M_y max +ve	M_y min −ve	M_z min −ve	M_x max +ve	M_y max +ve	M_y min −ve	M_z min −ve	M_x max +ve	M_y max +ve	M_y min −ve	M_z min −ve
0.5	.0396	.0209	.0125	.0027	.0271	.0182	.0187	.0089	.0210	.0378	.0046	.0007	.0036	.0085	.0141	.0082	.0174	.0293
0.6	.0343	.0197	.0117	.0030	.0226	.0167	.0169	.0098	.0219	.0335	.0053	.0022	.0042	.0087	.0116	.0076	.0177	.0248
0.7	.0303	.0186	.0108	.0032	.0195	.0154	.0150	.0100	.0220	.0297	.0053	.0028	.0044	.0087	.0097	.0072	.0176	.0210
0.8	.0270	.0178	.0100	.0035	.0170	.0143	.0131	.0096	.0215	.0264	.0048	.0028	.0046	.0086	.0083	.0068	.0169	.0178
0.9	.0241	.0172	.0091	.0038	.0150	.0133	.0114	.0089	.0206	.0234	.0043	.0024	.0046	.0081	.0071	.0065	.0160	.0153
1.0	.0214	.0166	.0084	.0041	.0130	.0125	.0098	.0082	.0196	.0207	.0034	.0020	.0046	.0071	.0061	.0062	.0150	.0136
1.1	.0192	.0161	.0078	.0044	.0114	.0117	.0087	.0076	.0184	.0183	.0034	.0018	.0044	.0065	.0053	.0058	.0140	.0120
1.2	.0172	.0154	.0072	.0044	.0100	.0110	.0077	.0071	.0173	.0162	.0031	.0017	.0043	.0056	.0046	.0054	.0130	.0106
1.3	.0155	.0145	.0067	.0041	.0088	.0104	.0069	.0067	.0163	.0145	.0029	.0017	.0042	.0051	.0040	.0050	.0121	.0094
1.4	.0141	.0135	.0063	.0038	.0078	.0097	.0061	.0063	.0153	.0131	.0026	.0017	.0041	.0048	.0035	.0046	.0112	.0083
1.5	.0128	.0126	.0058	.0035	.0070	.0091	.0055	.0059	.0144	.0120	.0024	.0017	.0041	.0046	.0031	.0042	.0103	.0074
1.6	.0118	.0118	.0055	.0033	.0063	.0085	.0049	.0056	.0136	.0111	.0021	.0017	.0041	.0044	.0028	.0038	.0095	.0067
1.7	.0108	.0111	.0051	.0031	.0057	.0080	.0044	.0053	.0128	.0103	.0019	.0017	.0040	.0042	.0025	.0035	.0088	.0061
1.8	.0099	.0105	.0047	.0030	.0052	.0075	.0039	.0050	.0120	.0096	.0017	.0017	.0038	.0040	.0022	.0033	.0082	.0056
1.9	.0090	.0099	.0042	.0029	.0048	.0070	.0034	.0048	.0112	.0091	.0015	.0017	.0035	.0039	.0019	.0031	.0077	.0052
2.0	.0081	.0094	.0036	.0028	.0045	.0066	.0029	.0045	.0103	.0087	.0013	.0016	.0025	.0038	.0016	.0029	.0072	.0049

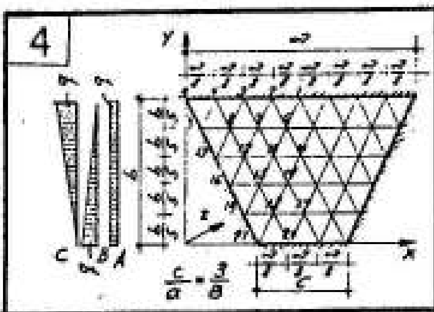
Bending Moments = coeff . qa^2

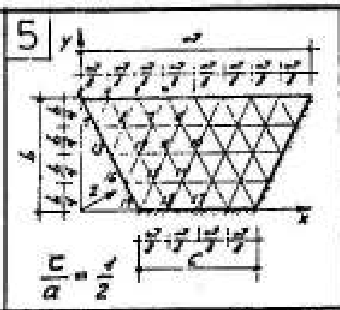
Bending Moments in Trapezoidal Plates with Different Edge Conditions for Rectangular & Triangular Loads

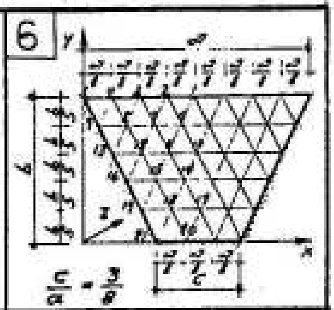
Bending moments = coeff. $q.a^2/64$, $\nu = 0.16$, $b = (a-c)\frac{\sqrt{3}}{2}$

1  2  3 

M at point	1 Shape of loading A	1 B	1 C	2 Shape of loading A	2 B	2 C	3 Shape of loading A	3 B	3 C
M_{x6}	.3446	.1349	.2097	.1417	.0547	.087	.1266	.0546	.0720
M_{x7}	.1833	.0541	.1292	.0756	.0147	.0609	.0895	.0290	.0605
M_{x8}	-.0782	-.0.56	-.0222	-.0424	-.0393	-.0031	-.0168	- 0.02	.0092
M_{x10}	.5909	.2708	.3201	.4109	.2003	.2106	.3100	.1646	.1454
M_{x11}	.4189	.1804	.2385	.2952	.1329	.1623	.2693	.1400	.1293
M_{x12}	.0382	.0545	.0163	-.0595	-.0411	.0016	.0243	-.0002	.0245
M_{x14}	.5880	.3062	.2818	.4762	.2630	.2132	.2748	.1629	.1119
M_{x15}	.0516	.0241	0.02	.0586	.0270	.0316	.1549	.0995	.0554
M_{x17}	.4731	.2747	.1984	.4066	.2492	.1574			
M_{x18}	.2373	.1528	.0845	.2192	.1458	.0734			
M_{y6}	.6225	.1742	.4483	.0981	-.0287	.1268	.1663	.0191	.1472
M_{y7}	.5223	.1435	.3788	.1348	-.0026	.1374	.1728	.0277	.1451
M_{y8}	.2996	.0771	.2225	.1253	.013	.1123	.1656	.0545	.1111
M_{y10}	.7374	.2940	.4434	.5404	.2183	.3221	.595	.3027	.2923
M_{y11}	.6331	.2529	.3802	.4742	.1919	.2823	.5102	.2583	.2519
M_{y12}	.3092	.1261	.1831	.2560	.1053	.1307	.2692	.1383	.1309
M_{y14}	.5024	.2956	.2068	.4936	.2922	.2014	.4930	.3297	.1633
M_{y15}	.1816	.1313	.0503	.2202	.1457	.0745	.2493	.1851	.0642
M_{y17}	.2903	.2520	.0383	.3294	.2671	.0623			
M_{y18}	.0882	.1263	-.0381	.1439	.1476	-.0037			
M_{z9}	-.2860	-.0922	-.1983	-.1582	-.0456	-.1126	-.1585	-.0563	-.1022
M_{z13}	-.8689	-.3303	-.5336	-.5621	-.2160	-.3461	-.5295	-.2374	-.2921
M_{z16}	-1.303	-.5935	-.7102	-.9742	-.4681	-.5061	-.7911	-.4238	-.3675
M_{z19}	-1.317	-.6956	-.6217	-1.079	-.6042	-.4750	0	0	0
M_{x9}	-.2265	-.0730	-.1535	-.1253	-.0361	-.0892	-.1250	-.0441	-.0809
M_{x13}	-.8843	-.4579	-.4264	-.4449	-.1709	-.2740	-.4190	-.1878	-.2312
M_{x16}	-1.032	-.4699	-.5623	-.7711	-.3705	-.4006	-.6261	-.3354	-.2907
M_{x19}	-1.042	-.5507	-.4922	-.8544	-.4783	-.3761	0	0	0
M_{y9}	-.1073	-.0346	-.0727	-.0593	-.0171	-.0422	-.0592	-.0209	-.0383
M_{y13}	-.4189	-0.216	-.2020	-.2108	-.0810	-.1298	-.1985	-.0890	-.1095
M_{y16}	-.4889	-.2226	-.2663	-.3653	-.1755	-.1898	-.2963	-.1586	-.1377
M_{y19}	-.4941	-.2609	-.2332	-.4047	-.2266	-.1781	0	0	0
M_{y1}				-1.150	-.4590	-.6910	-1.006	-.4600	-.5465
M_{y2}				-.9780	-.3774	-.6006	-.8890	-.3896	-.4994
M_{y3}				-.5612	-.1935	-.3677	-.5346	-.2153	-.3193
M_{y4}				-.1582	-.0456	-.1126	-.1599	-.0577	-.1022







Moment at Point	4: Shape of Loading A	4: B	4: C	5: Shape of Loading A	5: B	5: C	6: Shape of Loading A	6: B	6: C
M_{x6}	.1318	.0488	.0830	.1033	.0415	.0623	.3174	.1195	.1979
M_{x7}	.0791	.0161	.0630	.0915	.0301	.0614	.1830	.0539	.1291
M_{x8}	-.0303	-.0318	.0013	.0039	.0162	.0201	-.0596	-.0455	-.0141
M_{x10}	.3567	.1679	.1888	.2145	.1160	.0145	.5045	.2272	.2773
M_{x11}	.2738	.1201	.1537	.2077	.1047	.1030	.3853	.1618	.2239
M_{x12}	-.0439	-.0604	.0135	.0617	.0219	.0398	-.0332	-.0332	0
M_{x14}	.3777	.2041	.1736	.1525	.0815	.0510	.4658	.2367	.2291
M_{x15}	.0921	.0411	.0510	.1040	.0699	.0341	.0796	.0399	.0397
M_{x17}	.2272	.1418	.0854	.	.	.	.2596	.1538	.1058
M_{x18}	.1321	.0935	.0386	.	.	.	.1347	.0945	.0402
M_{y6}	.1285	-.0105	.1390	.2022	.0397	.1625	.6200	.1728	.4472
M_{y7}	.1466	.0022	.1444	.1957	.0408	.1549	.5169	.1805	.3764
M_{y8}	.1313	.0167	.1146	.1472	.0321	.1151	.2998	.0755	.2243
M_{y10}	.5381	.2171	.3210	.4942	.245	.249	.6688	.2841	.3347
M_{y11}	.4697	.1892	.2805	.4316	.2133	.2183	.6168	.2437	.3731
M_{y12}	.2679	.1180	.1499	.2390	.1211	.1179	.3028	.1225	.1803
M_{y14}	.4266	.2521	.1745	.1730	.1465	.0265	.4276	.2510	.1757
M_{y15}	.2553	.1296	.1257	.0743	.0848	-.0105	.1525	.1148	.0377
M_{y17}	.0388	.0931	-.0543	.	.	.	-.0937	.0618	-.1055
M_{y18}	-.0397	.0376	-.0773	.	.	.	-.1218	.0073	-.1291
M_{z9}	-.1561	-.0443	-.1118	-.1486	-.0504	-.0982	-.2762	-.0866	-.1896
M_{z13}	-.5359	-.2003	-.3356	-.4553	-.1949	-.2604	-.8190	-.3020	-.5170
M_{z16}	-.8687	-.4049	-.4638	-.5744	-.2996	-.2748	-.1583	-.5111	-.6472
M_{z19}	-.7973	-.4219	-.3754	-.2712	-.1356	-.1356	-.1110	-.5220	-.4890
M_{z21}	-.2989	-.1805	-.1184	.	.	.	-.3460	-.1980	-.1480
M_{x9}	-.1236	-.0351	-.0885	-.1176	-.0399	-.777	-.2187	-.0686	-.1501
M_{x13}	-.4240	-.1584	-.2656	-.3605	-.1543	-.2062	-.7590	-.3497	-.4093
M_{x16}	-.6876	-.3205	-.3671	-.4546	-.2371	-.2175	-.9170	-.8046	-.5124
M_{x19}	-.6575	-.3604	-.2971	-.2712	-.1356	-.1356	-.8003	-.4132	-.3871
M_{x21}	-.2989	-.1805	-.1184	.	.	.	-.3860	-.1980	-.1480
M_{y9}	-.0585	-.0166	-.0419	-.0557	-.0189	-.0368	-.1032	-.0325	-.0707
M_{y13}	-.2009	-.0751	-.1258	-.1708	-.0731	-.0977	-.3595	-.1656	-.1939
M_{y16}	-.3257	-.1518	-.1739	-.2153	-.1123	-.1030	-.4344	-.1917	-.2427
M_{y19}	-.3115	-.1707	-.1408	.	.	.	-.3794	-.1960	-.1834
M_{y20}	-.8347	-.4993	-.3354	.	.	.	-.9740	-.5510	-.4230
M_{y21}	-.2989	-.1805	-.1184	.	.	.	-.3460	-.1980	-.1418
M_{y1}	-1.070	-.411	-.6588	-.8431	-.3562	-.4869			
M_{y2}	-.9184	-.3417	-.5767	-.7470	-.3683	-.4387			
M_{y3}	-.5397	-.1806	-.3591	-.4740	-.1806	-.2934			
M_{y4}	-.1561	-.0443	-.1118	-.1486	-.0504	-.0982			
M_{y5}	0	0	0	0	0	0			

heavy loads raft or pile foundations are generally chosen. The fundamentals and details of the design of any of these systems can be done according to the known rules in foundation - design*

IX.8 <u>CONSTRUCTIONAL DETAILS</u> (Fig. IX.16)

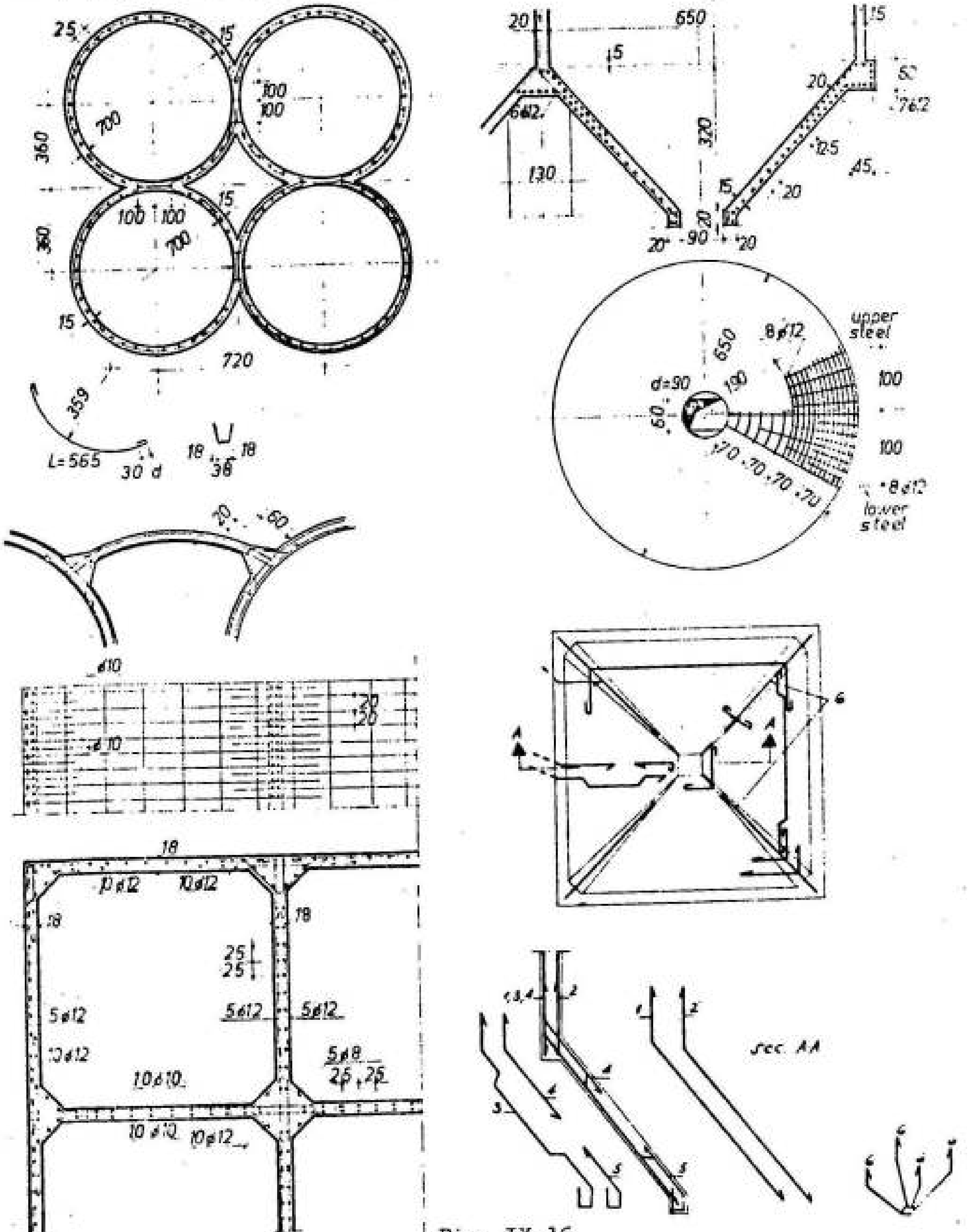


Fig. IX-16

* Refer to "Fundamentals of Reinforced and Prestressed Concrete", by M. Hilal.

IX.9 APPENDIX : CIRCULAR CEMENT SILOS

Silos used for storing cement (and flour) must be precisely treated due to the big variation that is liable to take place in its properties. Fischer* proposes the direct use of test results which give a parabolic distribution of the horizontal pressure on the walls according to the relation.

$$p_2 = \sqrt{7\,x} \quad \text{in } t/m^2 \text{ for } x \text{ in meters}$$

Leonhardt* gives the following recommendations for the design of circular cement silos :

1) Horizontal pressure on the walls

Up to 4 m silo depth : calculation according to conventional theory of earth pressure i.e.

$$p_2 = \gamma\, x \tan^2 (45 - \rho/2)$$

where $\gamma = 2\ t/m^3$, $\rho = 17.5°$ $\rho' = 0$

Above 9 m silo depth : calculation in accordance with Jansen's theory multiplying the results by a factor $K = 2$

$$p_2 = \gamma \left(1 - \frac{1}{e^{kx}} \right) \frac{A}{O \tan \rho'} \cdot K$$

with $\gamma = 1.6\ t/m^3$, $\rho = \rho' = 30°$

Between 4 and 9 m silo depth : linear interpolation between the pressure values at 4 and 9 m.

2) Vertical loads on the walls

The vertical loads acting on the walls can be determined according to the following assumptions :

a) For maximum wall load max. p_3

$\gamma = 1.6\ t/m^3$, $\rho = \rho' = 35°$ (determined from the equations of Janssen.

The high value for the angle of friction makes allowance for the arching (bridging) effect.

---

* 1- Fischer "Silos and Bunker in Stahlbeton" Der Verlag Für Bauwesen Berlin 1966.

* 2- Leonhardt "The safe design of cement silos". In german language Beton & Stahlbeton. Vol. 55 No. 13. March 1960

b) For minimum wall load.

The own weight of the wall only (empty silo) is to be considered.

3) Vertical and horizontal bending moments in circular silo walls

Horizontal bending moments may be ignored. Vertical moments due to restraint of wall at base of the silo depend on the type of construction employed, the thickness and reinforcement at the base must be sufficient to resist the induced bending moments and normal forces. Other vertical moments are to be resisted by nominal reinforcements - distributers - and are not determined by calculation.

4) Design load for floor slab

The weight of all the contents of the silo are to be assumed acting on the floor slab. The weight per cu. meter can be chosen according to fig. IX-17.

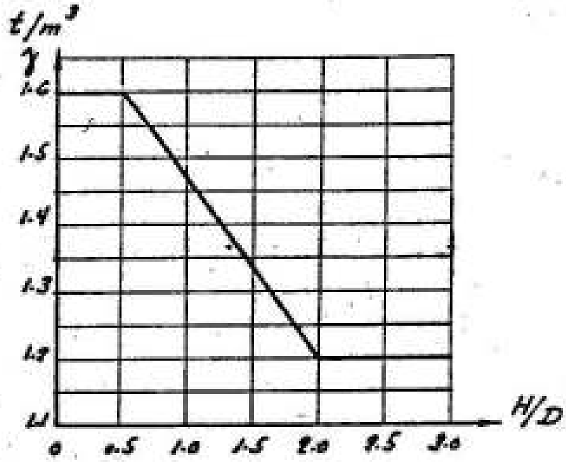


Fig. IX-17

5) Constructional precautions

Figure IX-18 shows a carefully designed circular cement silo in which a high grade concrete quality C600 was used.

5.1 Concrete quality

The minimum allowed concrete quality is C300.

5.2 Circumferential reinforcement

The full ring tension is to be resisted by thin circumferential reinforcement arranged near the external face of the wall only, spacing 4-12 cms. This close spacing of the bars is essential in order to avoid wide cracks. Splices in bars should be staggered. Non-tensioned reinforcement is adequate for the purpose.

5.3 Design of base of wall

Hinged connection or, preferably, horizontally sliding bearing embodying rubber rings or inserts of bituminous or synthetic substances of plastic consistence are recommended. Fig. IX-19a shows a rubber bearing especially suitable where pneumatic discharge with air at high

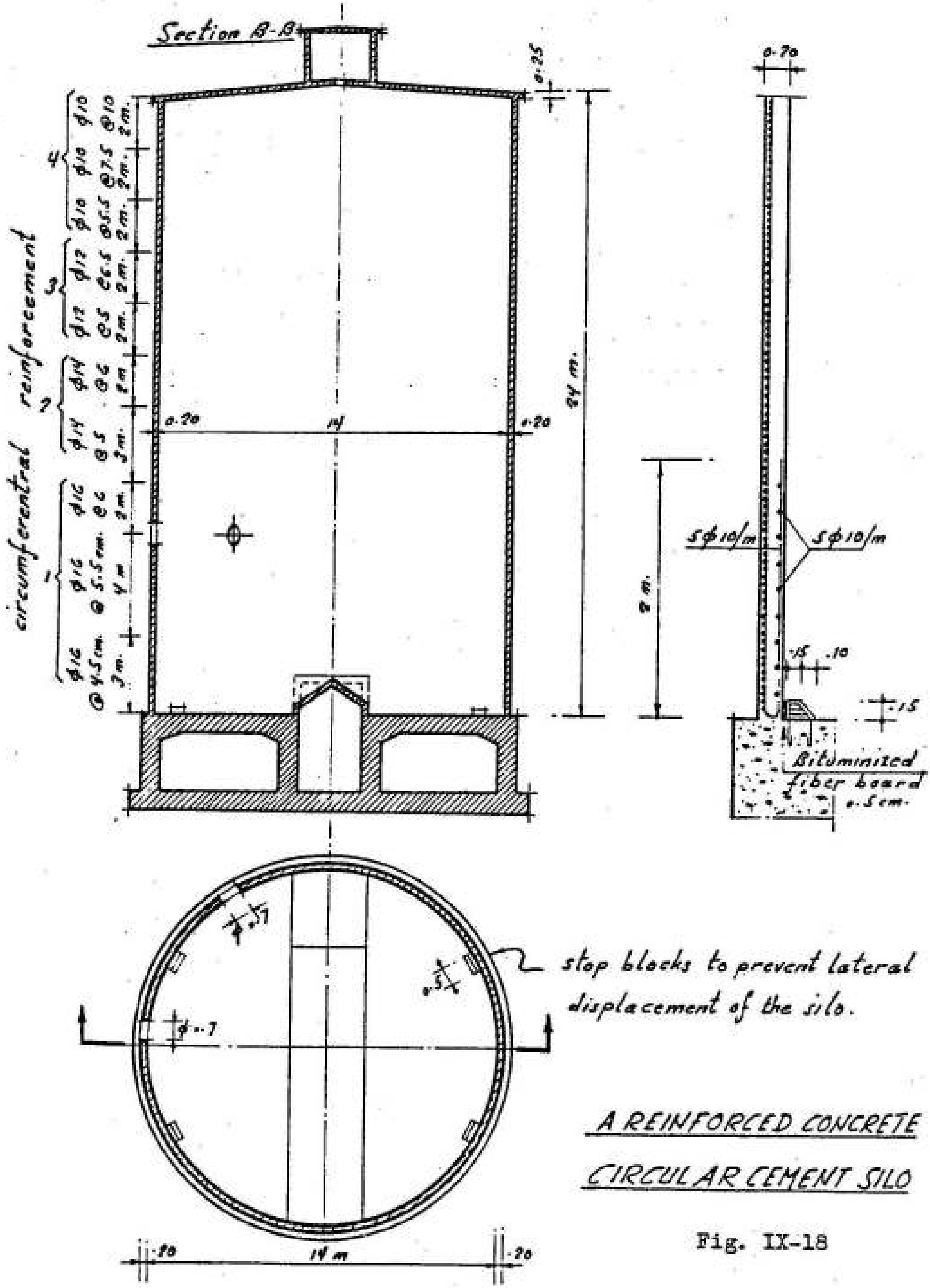

Section B-B
circumferentral reinforcement
0.20
14
0.20
24 m.
0.25
9 m.
5ϕ10/m
5ϕ10/m
.15 .10
.15
Bituminized fiber board 0.5cm.
stop blocks to prevent lateral displacement of the silo.
.20
14 m
.20
A REINFORCED CONCRETE CIRCULAR CEMENT SILO

Fig. IX-18

pressure is employed, because the walls receive hardly any vertical loads from the material in the silo but do undergo considerable deformations. Fig. IX.19 b shows another arrangement using some convenient bituminous compound.

5.4 Vertical reinforcement in wall

Not less than 5 Φ 8 mm/m, to be installed within one layer of circumferential reinforcement or between two layers (if any) in the regions where bending moments occur at the base of the wall, vertical reinforcements (duly calculated) should be provided both externally and internally.

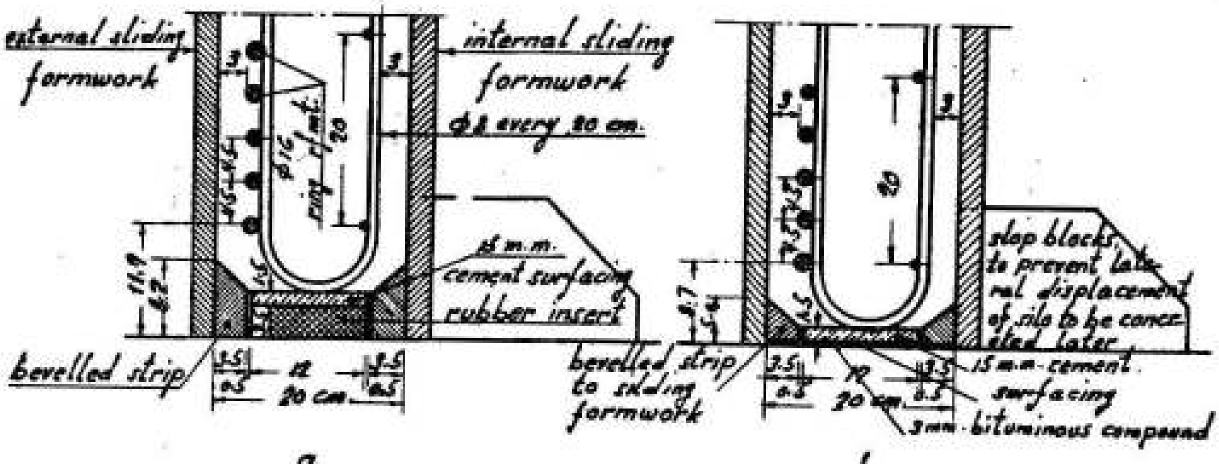


Fig. IX-19

6) Temperature stresses*

The stresses in the walls of cement silos for a difference of temperature of 30°C are to be included in the design. The temperature stresses are however liable to become serious only if the wall is made too thick and is fixed at the base.

As soon as cracks occur, the temperature stresses are considerably reduced. The floor usually does not undergo the same amount of thermal-expansion as the wall, because the floor is protected from the high temperature by a layer of residual cement which acts as an effective insulator.

* Refer to XI-3

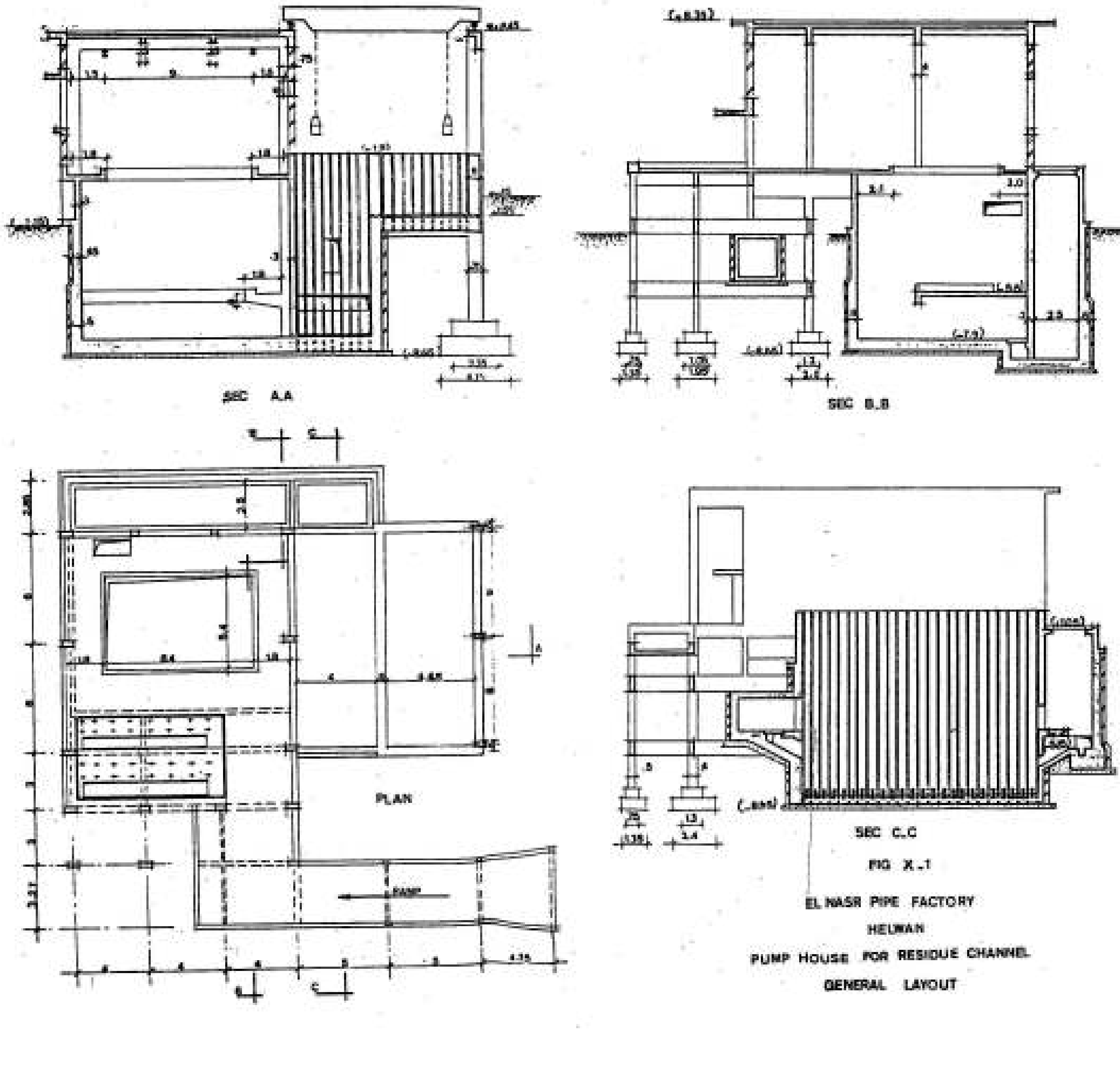


FIG X.1

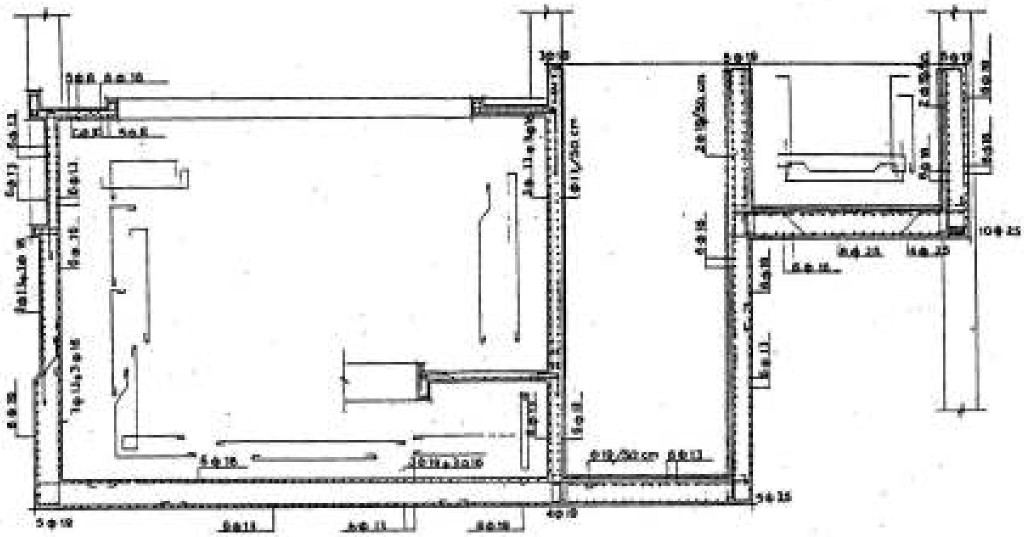
EL NASR PIPE FACTORY

HELWAN

PUMP HOUSE FOR RESIDUE CHANNEL

GENERAL LAYOUT

SEC. A A



SEC B B

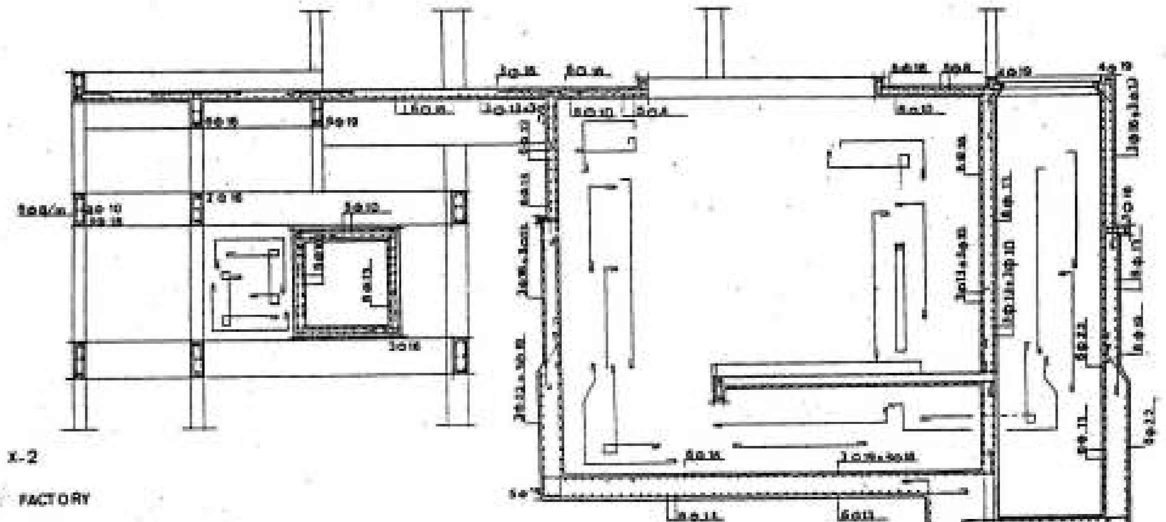


FIG X-2

EL NASR PIPE FACTORY

HELWAN

PUMP HOUSE FOR RESIDUE CHANNEL

DETAILS OF REINFORCEMENTS

SEC . A A

SEC B B

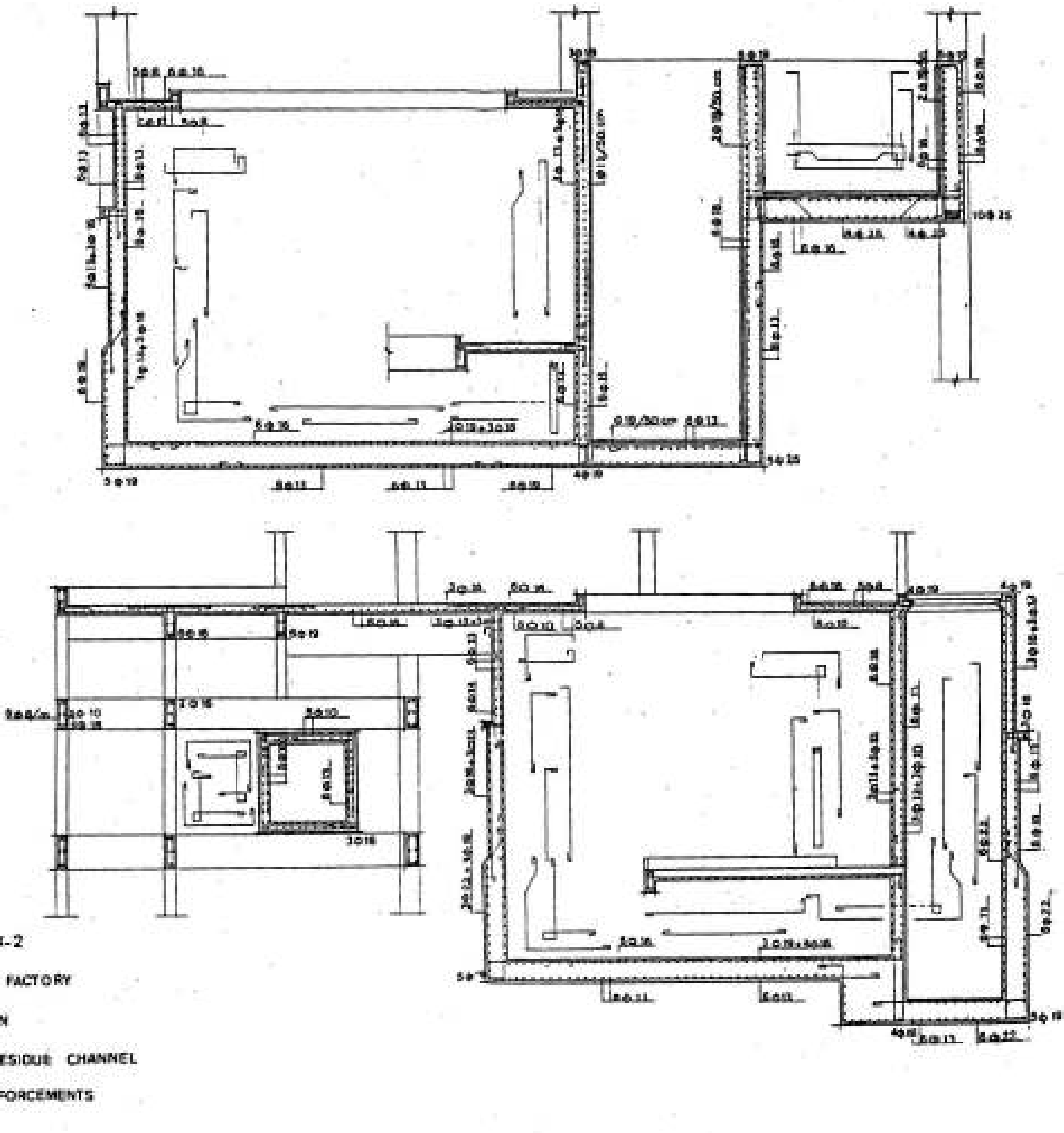


FIG X-2

EL NASR PIPE FACTORY

HELWAN

PUMP HOUSE FOR RESIDUE CHANNEL

DETAILS OF REINFORCEMENTS

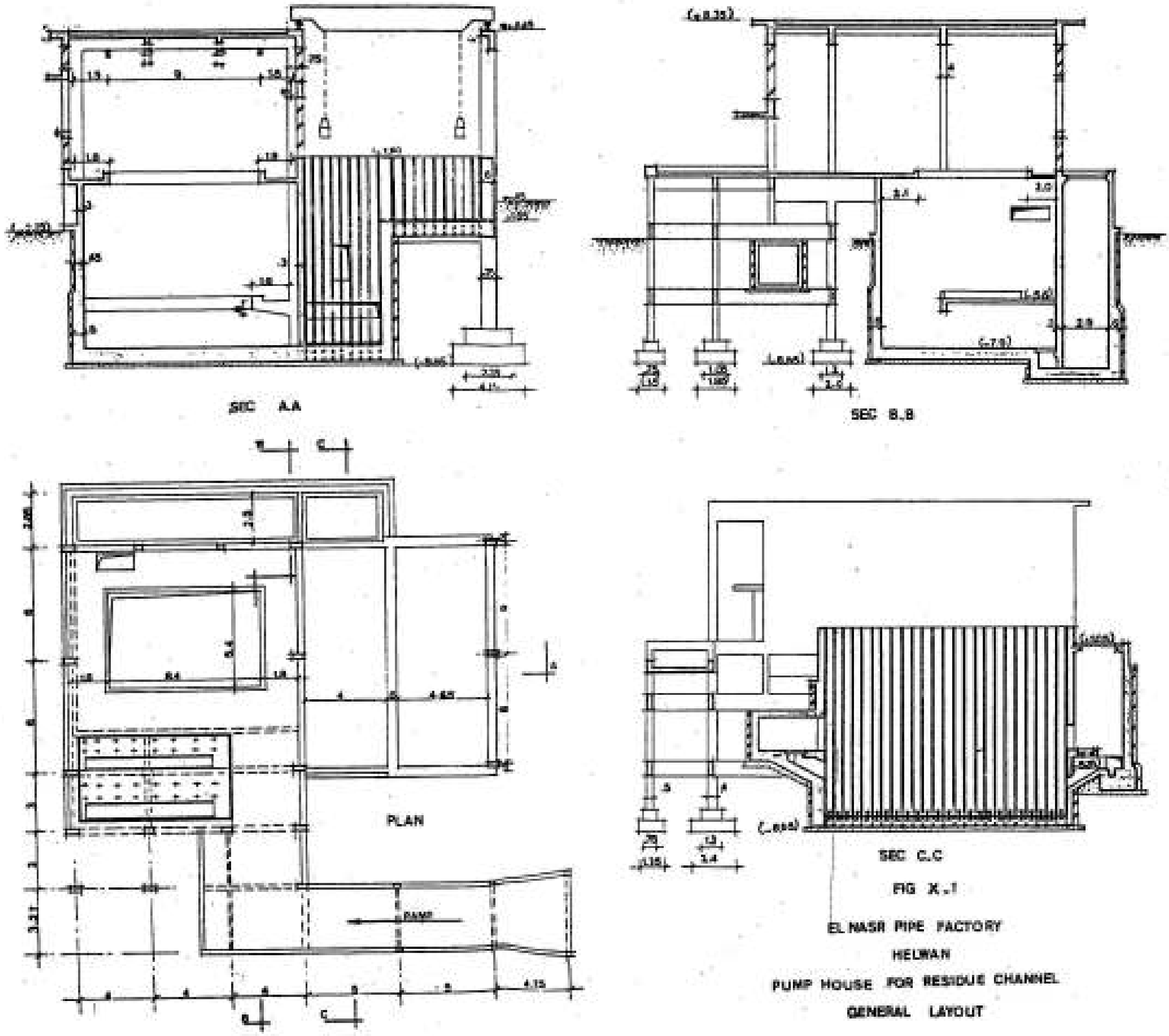


FIG X.1

EL NASR PIPE FACTORY

HELWAN

PUMP HOUSE FOR RESIDUE CHANNEL

GENERAL LAYOUT

X. P U M P   S T A T I O N S

Under-ground pump rooms, tanks and channels are amongst the main elements of pump stations. The statical behavior of pump rooms is similar to that of under-ground tanks; all provisions, precautions and methods of design can be directly applied.

We give in the following examples some special pump stations and settling tanks constructed in the pipe factory at Helwan.

X.1 PUMP STATION FOR RESIDUE CHANNEL

Figure X-1 shows the general layout giving the main elements and dimensions of the station, namely:

1.) The main hall of the station ~ 12 ms wide, ~ 15 ms long and ~ 7 ms high. The roof of the hall is a reinforced concrete solid slab supported on longitudinal secondary beams, 4.0 ms apart, and on cross frames 6 ms apart. The outside columns of the supporting frames extend to the ground level and are supported by a horisontal beam. Their vertical axes are chosen such that they coincide on that of the 60 cms reinforced concrete wall of the pump room. It was however not possible to sup port the 75 cm columns of the other side on the 30 cm thick walls underneath, so it appeared necessary to extend them to the floor of the pump room. The hall is provided with a mono-rail crane supported on the main frames; it is used for mounting and mentainance of the pumps. The horizontal thrust of the frame has been safely resisted by the floor foot path slab of the hall existing in its direction.

2.) The underground pump room is located underneath the station having the same width, ~ 12 ms, but ~ 10 ms long only; its floor is ~ 6 ms below

mean ground level. It includes the pumps which pump the clear water in the attached 2.5 ms wide tank (refer to section B-B) back to the main factory to be re-used. It is directly connected to the following two main elements:

3.) The steel-filing residue container, ~ 4 ms wide, 10 ms long and ~ 10 ms deep (refer to fig. X-1, plan, sec. A-A and sec. C-C) receives a mix of steel-filing residues and water trnsferred to it through the channel appearing in sec. B-B left with its inlet appearing in sec. C-C left. The steel-filing, being heavy, settles at the bottom of the container and the water at the top is transferred to the chamber appearing in sec. C-C right.

4.) The under-ground tank, 2.5 ms wide and 12 ms long (refer to fig. X-1, plan and sec. B-B right) is used to collect the clear water of the process. From there , the water is pumped for re-use.

5.) The steel-filing residues collected in the container given in (3) are transferred by the crane girder shown in sec. A-A right to a higher container 4.65 ms wide and 3.35 ms deep. From there, they are removed from the site by cars.

The soil at the site is good incompressible cemented coarse sand at the level of the floor slab of the pump room, the attached container and clear water tank. For this reason, the three elements are directly supported on the soil, all other elements , elevated container, channels, etc. , are supported on columns with isolated footings resting on the same layer.

In order to protect the containers including the steel-filing against the shocks of the bucket of the crane girder during the lifting operations, they are provided with steel rails fixed to their inner surface (refer to sections A-A and C-C). The vertical and horizontal rails are well anchored in the floor slab and wall so that they share in resisting the tensile stresses on their inner surfaces.

The details of reinforcements of the main two sections of the under-ground elements are shown in fig. X-2.

X.2 SETTLING TANK OF STEEL FILING

Figure X-3 shows the general layout and main dimensions of the tank. The soil at the site is composed of fill and loose materials to a depth of 5.5 ms from ground level underlaid by good incompressible cemented coarse sand. The weight of the filling material being relatively big, ~ 6 t/m^3, it was decided to replace the fill and loose materials by stabilized sand (100 kg cement $/m^3$ sand) and to construct the tank directly on this layer. Its area is chosen such that the extra bearing stress at level -5.50 is within the allowed value of 1.5 kg/cm^2 and with its center of gravity coinciding on that of the full tank. In order to confine the stabilized area , so that no horizontal displacement is liable to take place, it was surounded to its full hieght by a masonry wall 40 cms thick.

Due to the heavy loads acting on the deep part of the tank, the concrete dimensions of its elements are big relative to those of the lightly loaded shallower part. In order to get a tank of approximately the same rigidity, the floor of the shallow part is provided with longitudinal and cross stiffening beams.

For the same reasons stated in the previous example, the deeper part of the tank is provided with steel rails fixed to the inner face of the walls and floor.

The details of reinforcements of three main sections are shown in figure X-4 .

It is generally recommended to arrange longitudinal reinforcements at the top and bottom of walls to resist the eventual tensile stresses due to temperature changes or differential settelments. Such reinforcements can be seen in figures X-2 , 4 and 6

X.3 MAIN PUMP STATION

The function of this station is to supply the different factory halls, workshops, ... ect. of the plant by the water necassary for the different processes of the industry. The general layout and dimensions

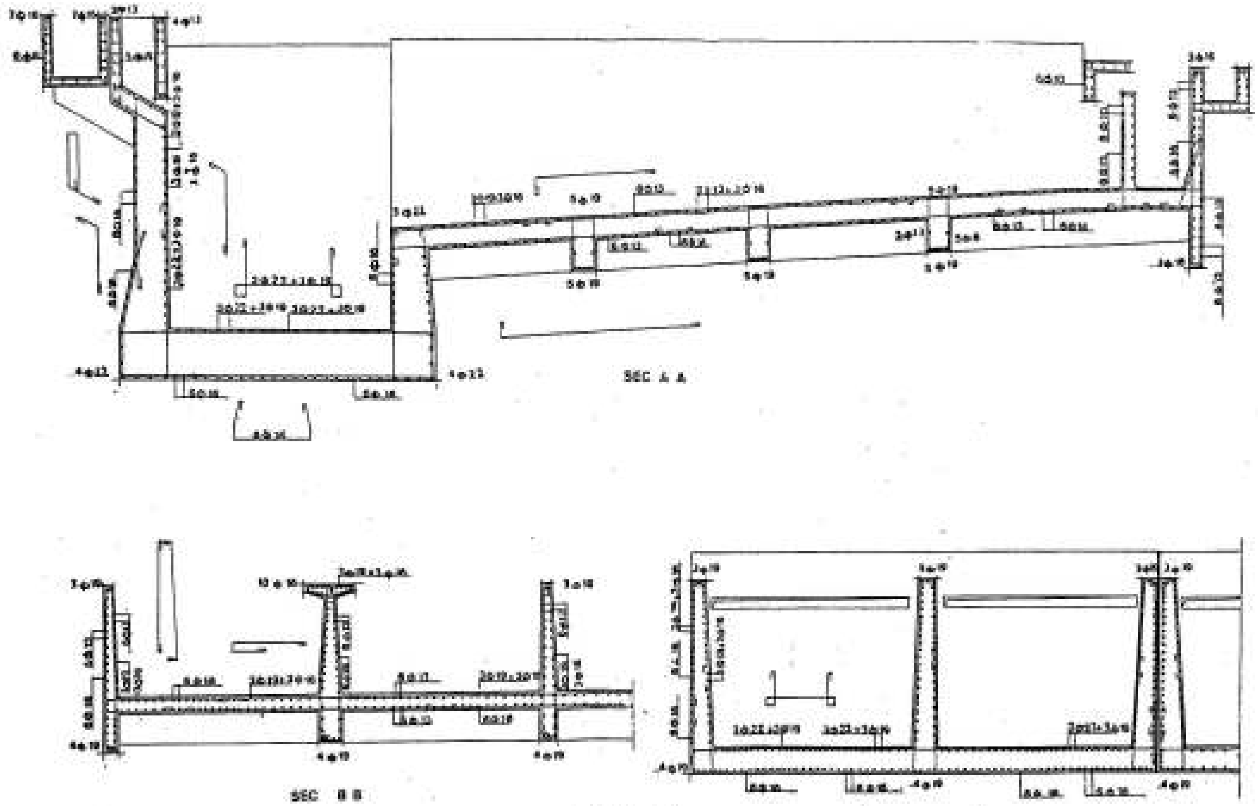


FIG. X-4

EL NASR PIPE FACTORY

HELWAN

SETTLING TANK OF STEEL FILING

DETAILS OF REINFORCEMENTS

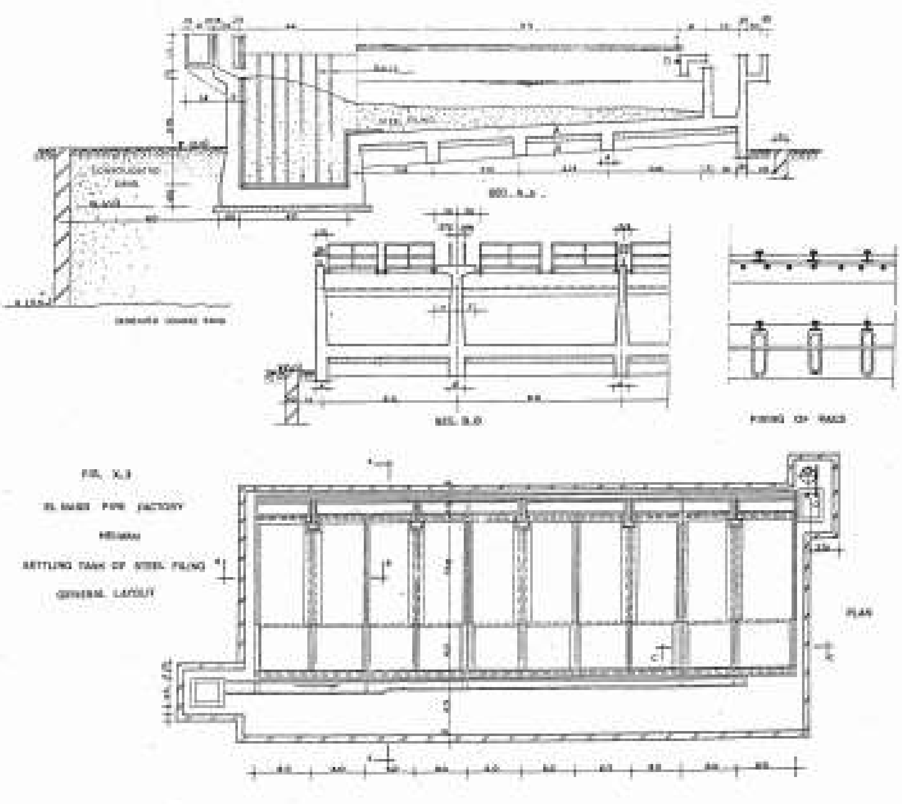


FIG. X.3

EL NASR PIPE FACTORY

HELWAN

SETTLING TANK OF STEEL PILING

GENERAL LAYOUT

of the main supporting elements are shown in figures X-5 and 6.

The sumps between axes A and B receive the clear water from the main sources of water - supply; while each of the tanks between axes B and C supplies certain sectors of the plant by the water required for the industry and receives the remaining part again, generally in a hot state. The water in these tanks is cooled by adding clear cold water from the sumps equal to the amount lost in the industry before being pumped again in a new cycle.

The pumps exist in the pump halls lying between axes D and E and the middle part between axes E and F.

The sumps, water tanks and pump halls are 8.5ms below the floor level of the pump hall.

The side areas between axes E and F are used for storing. The cable ducts exist in a mezzanine floor at level .38between the same axes.

The roof of the main hall between axes D and E is a reinforced concrete solid slab 10 cms thick and 9.40 ms above floor level. It is supported on simple girders 12 ms span and 3 ms apart; while the roof of the hall between axes E and F lies at 5.20 ms from floor level and is similarly supported on simple girders ~ 9.2 ms span.

The natural ground lies at level -7.9 and the under ground water contains relatively high percentage of sulfates (HO_3) which attack the concrete. The soil in the site at the ground water level and below being of medium sand, it was decided to choose the bottom of the foundation few cms above the ground water and to design them for a bearing stress of 1.5 kg/cm^2. The roads and passages around the station are planed to be at level - 2.0 so that the ramp and retaining walls shown in figures X-5 and 6 were essential.

The details of reinforcements of the sumps, tanks and retaining walls are shown in figure X-7

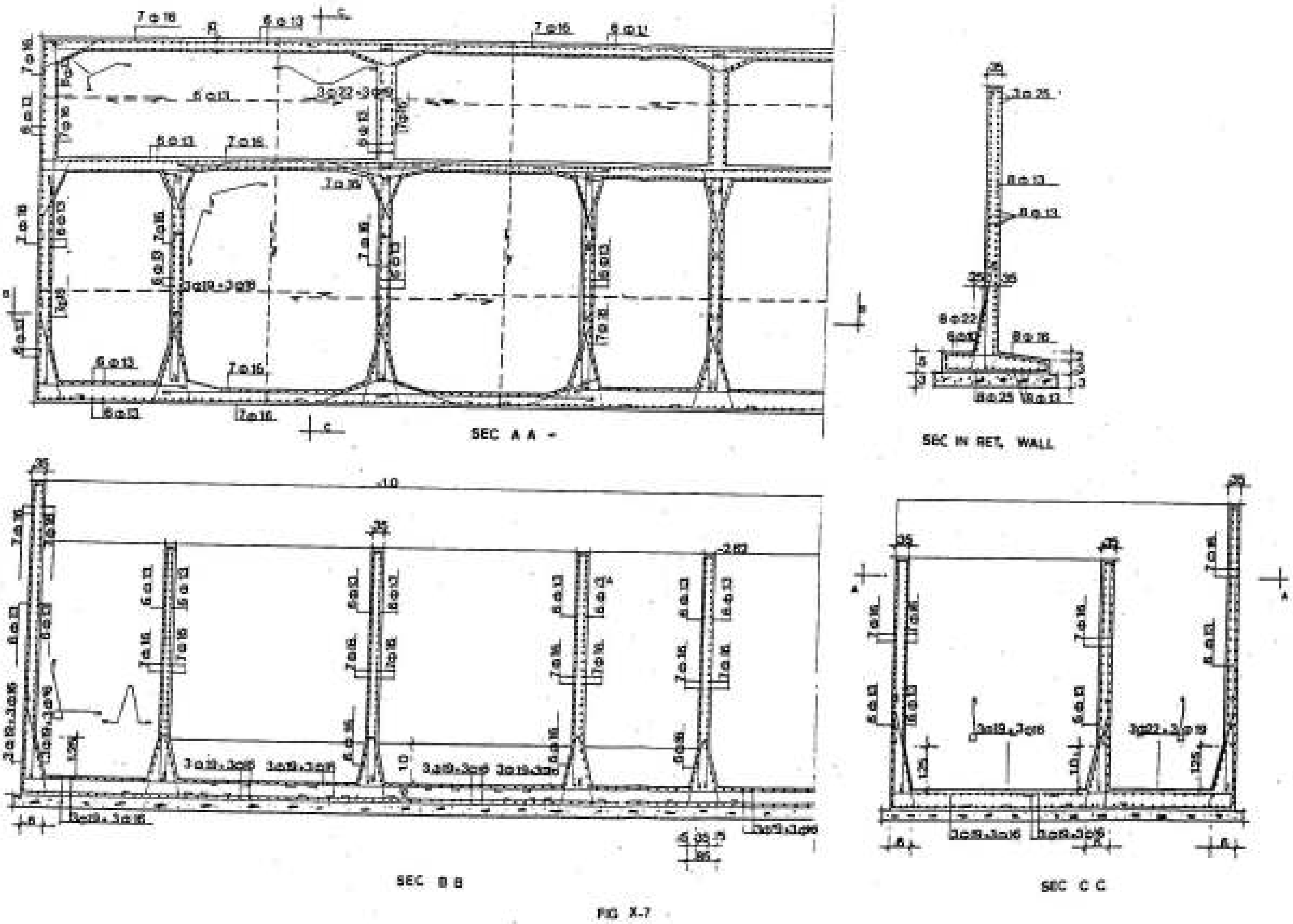


FIG X-7

EL NASR PIPE FACTORY

HELWAN

MAIN PUMP STATION

DETAILS OF REINFORCEMENTS

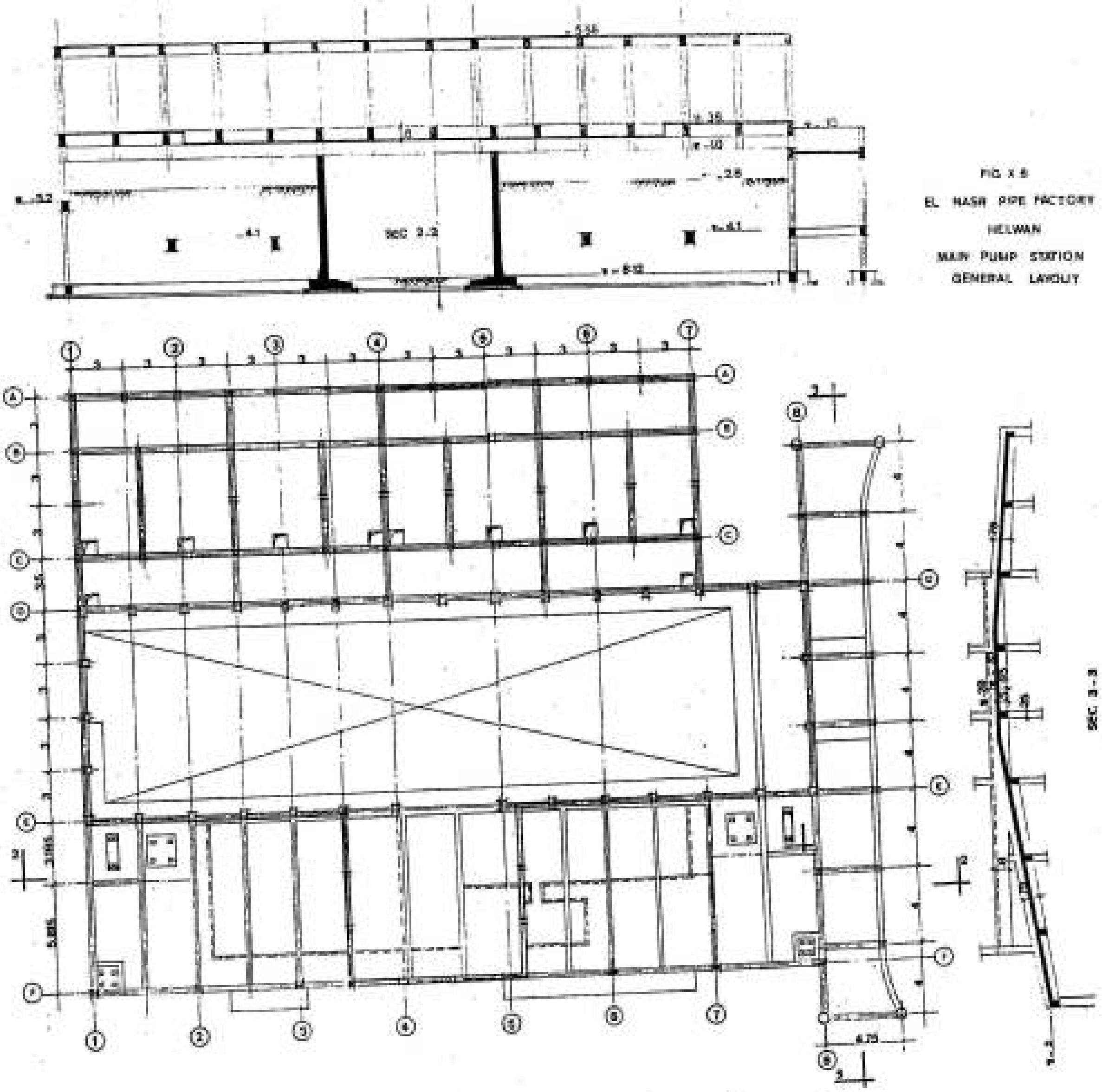


FIG X 8
EL NASR PIPE FACTORY
HELWAN
MAIN PUMP STATION
GENERAL LAYOUT

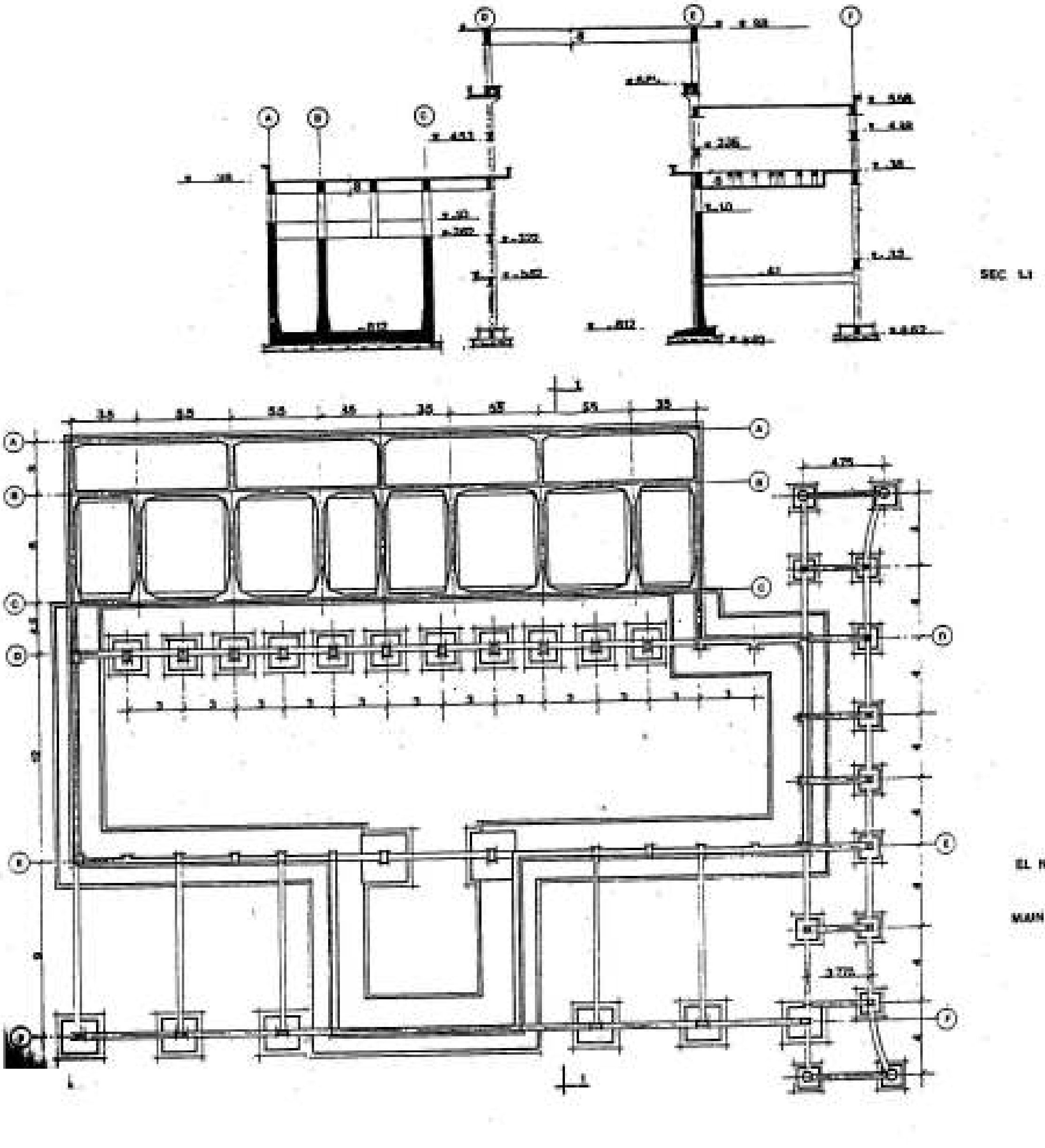


FIG X-5
EL NASR PIPE FACTORY
HELWAN
MAIN PUMP STATION
FOUNDATION

XI. COMPLEMENTARY DESIGNS

XI.1. WALLS ACTING AS DEEP BEAMS

a) Introduction :

In elevated tanks, silos, bunkers and pump houses, the walls may act as beams spanning between the supporting columns. If the depth of such walls is big, they may act as deep beams and the stress distribution can differ much from that of ordinary slender beams.

According to Navier, the stress - distribution in the homogenous, elastic, slender beams is assumed linear, and, the stresses are proportional to their distance from the neutral axis which passes through the center of gravity of the section. (Fig. XI.1).

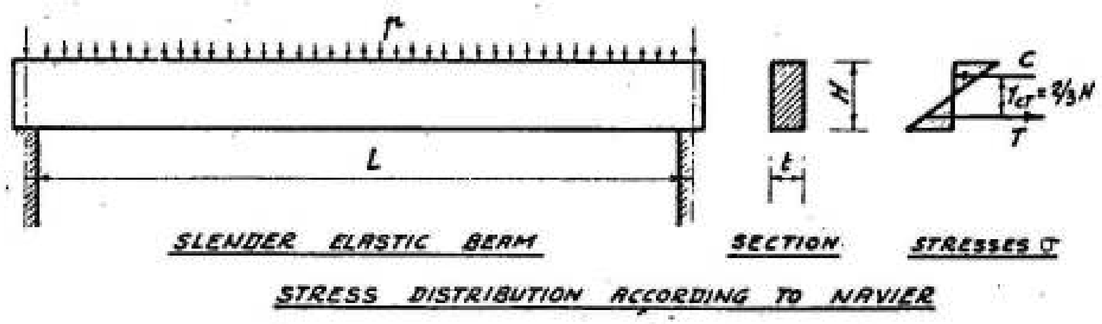


Fig. XI-1

This assumption is valid only in slender beams where the ratio of depth H to span L is smaller than 1 : 2 in simple beams and 2 : 5 in continuous beams; beyond these limits the stress distribution is not linear and the arm of internal forces Y_{CT} is much smaller than according to Navier.

Fig. XI.2 shows the stress distribution at the middle and over the supports of a continuous beam subject to uniform load p at its top surface and having a depth H equal to span L .

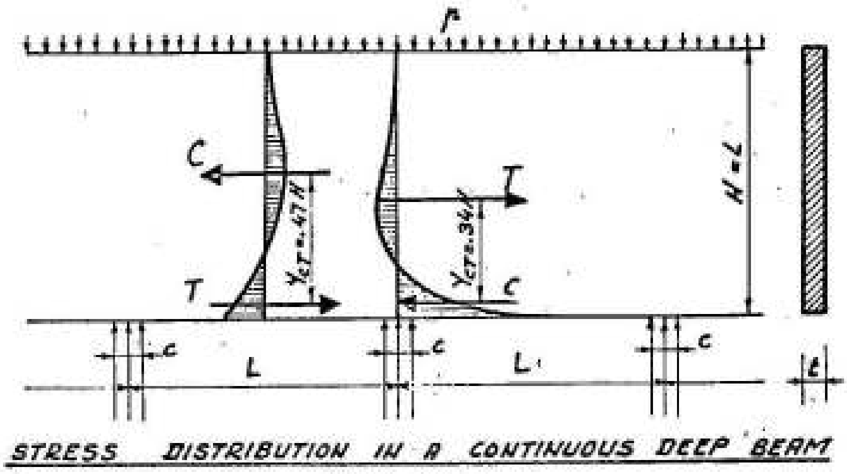


Fig. XI-2

b) Fundamental Equations of Stress Distribution :

In figure XI.3 we give the stresses acting on an elemental area dx . dy in the middle plane of a deep beam subject to uniform load p.

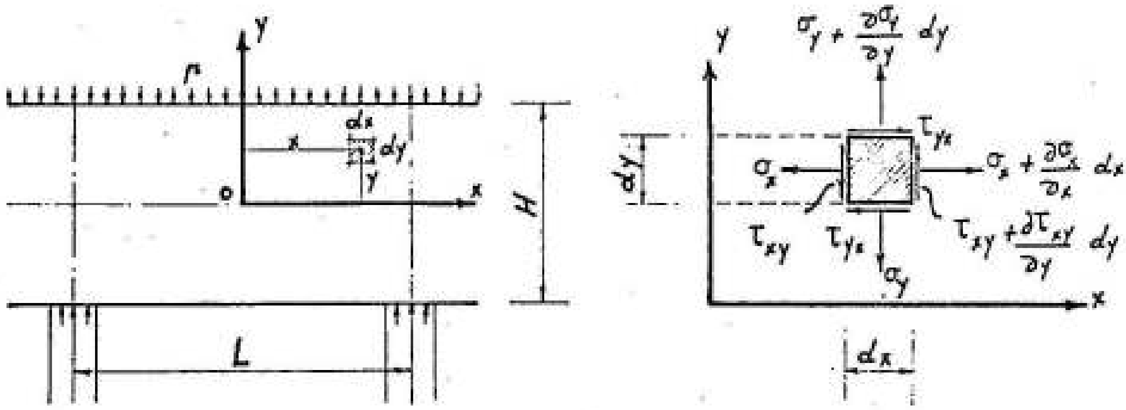


Fig. XI-3

According to the mathematical theory of elasticity the stresses must satisfy the condition :

$$\frac{\partial^4 F}{\partial x^4} + 2 \frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = 0$$

in which F (x , y) is the Airy biharmonic stress function, whose derivatives give the stresses in the form :

$$\sigma_x = \frac{\partial^2 F}{\partial y^2} \qquad \sigma_y = \frac{\partial^2 F}{\partial x^2} \qquad \tau_{xy} = \tau_{yx} = -\frac{\partial^2 F}{\partial x\ \partial y}$$

The given basic equation of the stress function includes no elasticity constants (E , G and $\nu = \frac{1}{m}$). It can be applied for every homogeneous isotropic elastic material that follows Hook's law. It can also be applied to reinforced concrete beams before the formation of tension cracks (Stage I). After the formation of cracks, the tension reinforcements must be sufficient to resist all the tensile stresses in the section ; it is recommended to arrange these reinforcements according to the tension trajectories. The calculation according to the theory of elastic beams gives sufficient safety as far as the amount of the tension steel is concerned, it is however essential to make good anchorage for the tension steel at the supports.

The Airy function can be imagined as the elastic surface of an elastically restrained flat plate under certain edge conditions . In this manner , the displacement of this surface is a measure for the normal stresses σ_x and σ_y and its twist is a measure for the shear stresses τ_{xy} .

The basic equation used for solving deep beams is as shown a linear partial differential equation of the fourth degree , its integration constants must satisfy the edge conditions of the case under consideration.

The solution of this differential equation was done by Dischinger(1) using the Fourier -Series, the results of his mathematical investigation were presented by the American Portland Cement Association(2) . Bay(3) and recently El-Darwish(4) have used the method of differences . Chow(5) and others solved some special problems of deep beams. Theimer(6) has given a valuable series of tables and curves for the design of reinforced concrete deep beams. A theoretical investigation has been recently published by Worch[7]. We give in the following some of the important results of these investigations as they may be required for tank problems[8] .

---

(1) F. Dischinger " Beitrag Zur Theorie der Halbscheibe und des Wandartiger Balkens " . Publications of the International Association of Bridge and Structural Engineering. Zürich, Switzerland, Vol. 1, 1932 pp. 69-93 .

(2) " Design of Deep Girders " Pamflet No. St. 66, Concrete Information, Structural Bureau, Portland Cement Association, Chicago, III.

(3) H. Bay " Wandartiger Träger und Bogenscheibe " Published by Konrad Wittwer Stuttgart, 1960 .

(4) I.A. El Darwish " Stresses in Simply Supported Deep Beams " . Bulletin of the Faculty of Engineering, Alexandria University , Vol. V. 1966 .

(5) Chow, Li., Conway, H. D. , and Winter , G., " Stresses in Deep Beams " . Transactions, A.S.C.E. , Vol. 118 , 1953 , p. 686 .

(6) O. F. Theimer " Hilfstafeln zur Berechnung Wandartiger Stahlbetonträger " . Published by Wilhelm Ernst & Sohn . Berlin, 1963

(7) G. Worch. " Elastische Scheiben " Beton Kalender 1967 . Vol. II. pp. 1.128 .

(8) Leonhardt : Wandartiger Träger
Deutscher Ausschuss fur Beton und Stahlbeton .

c) Simply Supported Deep Beams :

Notations : Fig. XI.4

Span of beam	L
Height of beam	H
Wall thickness	t
Breadth of support	c
Support width	t_s
Total length of beam	L_t
Load intensity	p/t

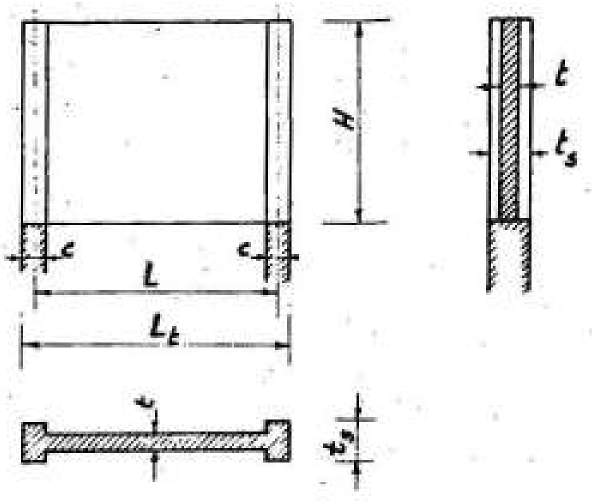


Fig. XI-4

Types of Loading : Fig. XI.5

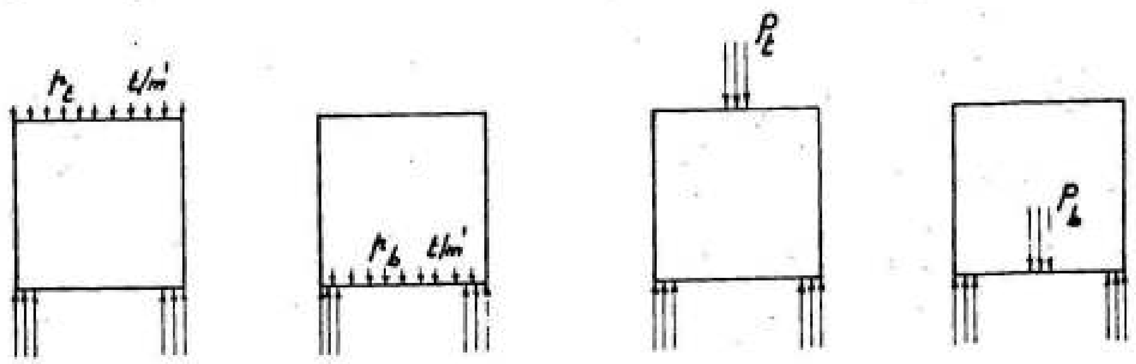


Fig. XI-5

Stress Distribution :

The stress distribution depends on : $\frac{H}{L}$, $\frac{c}{L}$, type (uniform or concentrated) and position (top or bottom) of loading , as well as $\frac{t_s}{t}$.

Figure XI.6 shows the stress distribution in a simply supported beam subject to uniformly distributed load p acting on its top surface for $\frac{c}{L} = 1/10$, $t_s = t$ and various values of $\frac{H}{L}$ in stage I .

N o t e s :

1) Stress distribution is linear for $H < \frac{L}{2}$ and curved for $H \geqslant \frac{L}{2}$

2) For $\frac{L}{2} < H < L$ the magnitude of the compressive stresses decreases with increasing depth.

3) No stresses are resisted by the upper part of a deep beam above $H = L$.

4) The max. tensile stress at the lower fiber is for $H = \frac{L}{2}$ equal to 1.5 times and for $H \doteq L$ equal to 2.2 times the values according to Navier.

5) The arm of the internal forces $Y_{CT} = 0.62 - 0.78\ H$ i.e. its magnitude as a factor of H does not vary much.

6) If the load p acts at the bottom, the normal stresses σ_x and the shear stresses τ are not much affected , while the compressive stresses σ_y are much reduced and high tensile stresses are created at the lower zones of the beam as shown in fig. XI.8 .

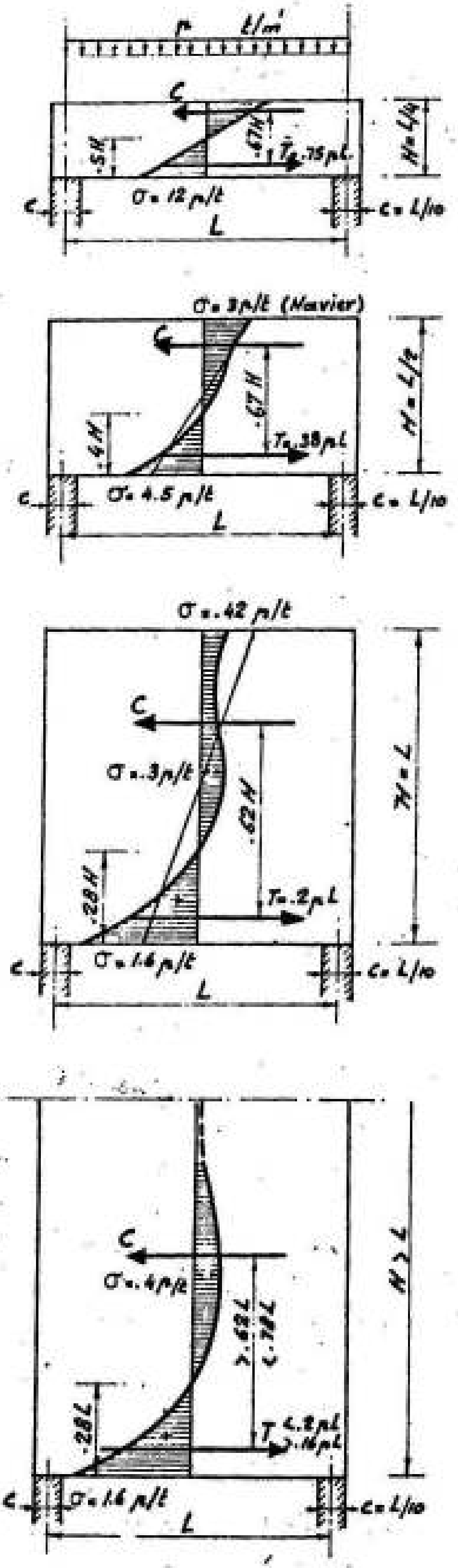


Fig. XI-6

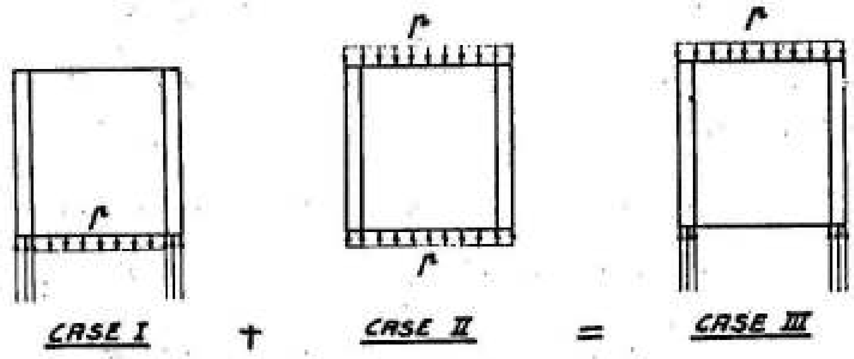


Fig. XI-7

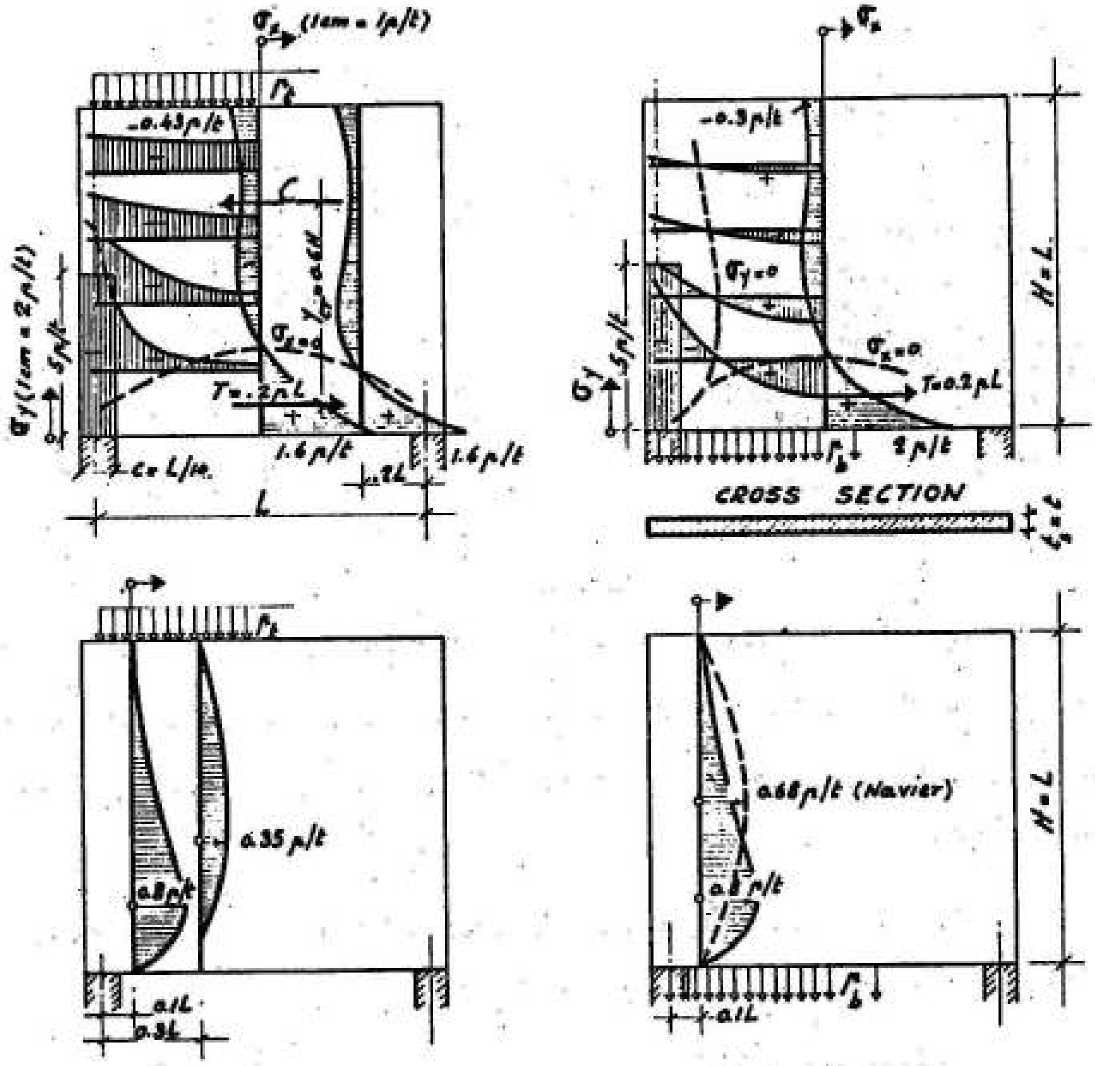
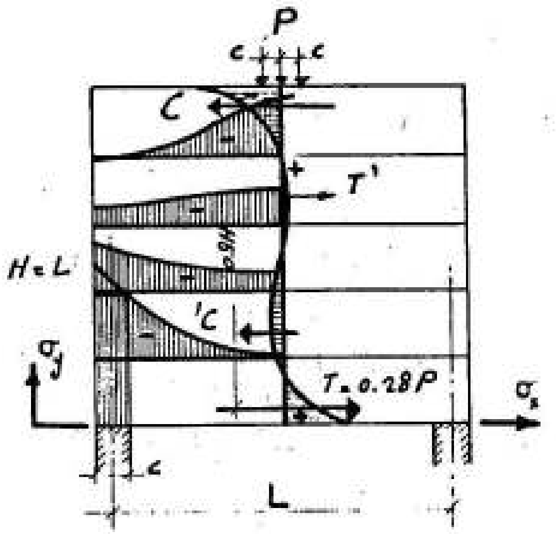


Fig. XI-8

The distribution of normal stresses in simple deep beams subject to concentrated loads acting at the middle of top surface is given in figure XI.9

In deep beams subject to concentrated loads acting at the lower surface, the stresses are distributed in a depth equal to L only , bigger depths are not effective .



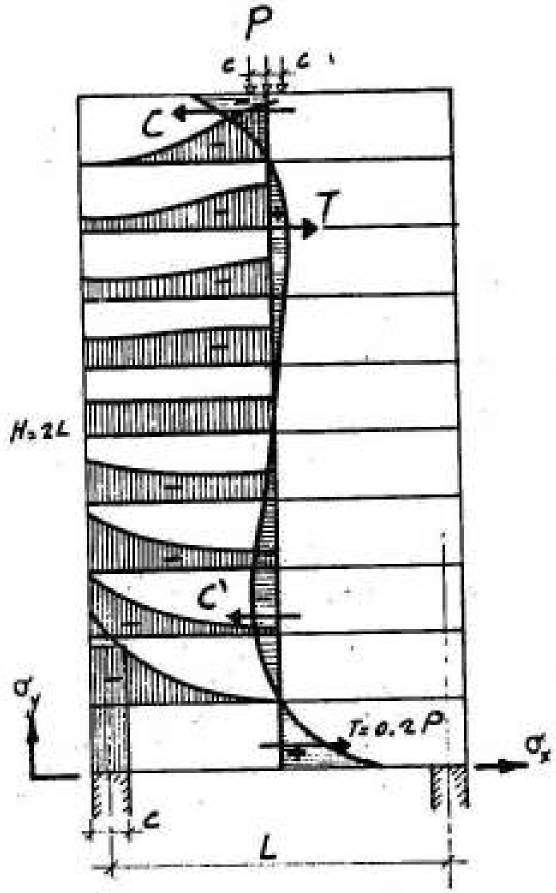


Fig. XI-9

The total tension T is the same whether the load P acts at top or bottom . Big tensile stresses σ_y , especially in tho lower part of the beam are created and must be resisted by vertical reinforcements.

Stiffening the edges of the beam affects the stresses in a manner similar to that of uniform loads.

d) <u>Continuous Deep Beams</u> :

The stress distribution at the center lines of the spans and over the supports of continuous deep beams subject to uniform loads acting on their top surface is shown in figure XI.10

It has to be noticed that the stress distribution at the middle of the spans is similar to that of simple beams but with smaller values due to continuity ; whereas at the supports , the arm of internal forces is relatively small and the compressive stresses at the lower fiber are high and may govern the design.

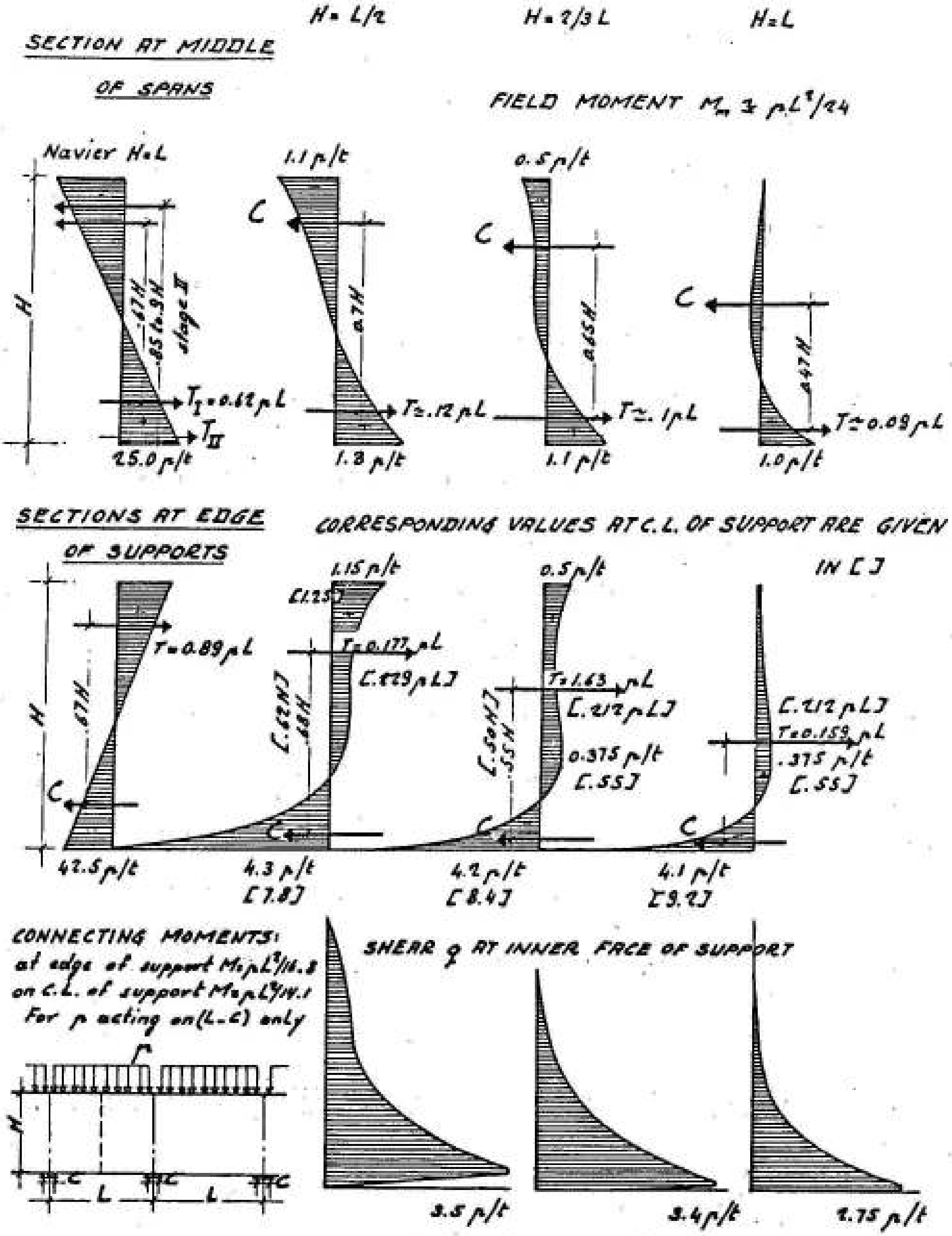
It has further been found that the connecting moments in continuous deep beams is smaller than that of slender beams of the same spans due to the high compressive strains that are liable to be developed at the supports ; and as a result of this fact, the field moments are bigger than those of slender beams.

Figure XI.11 gives the stress distribution at the center lines of continuous deep beams subject to concentrated loads at the top or bottom surfaces . The stresses over the center lines of the supports have the same values but with opposite signs.

e) <u>Guide Lines for the Design of Reinforced Concrete Deep Beams</u> :

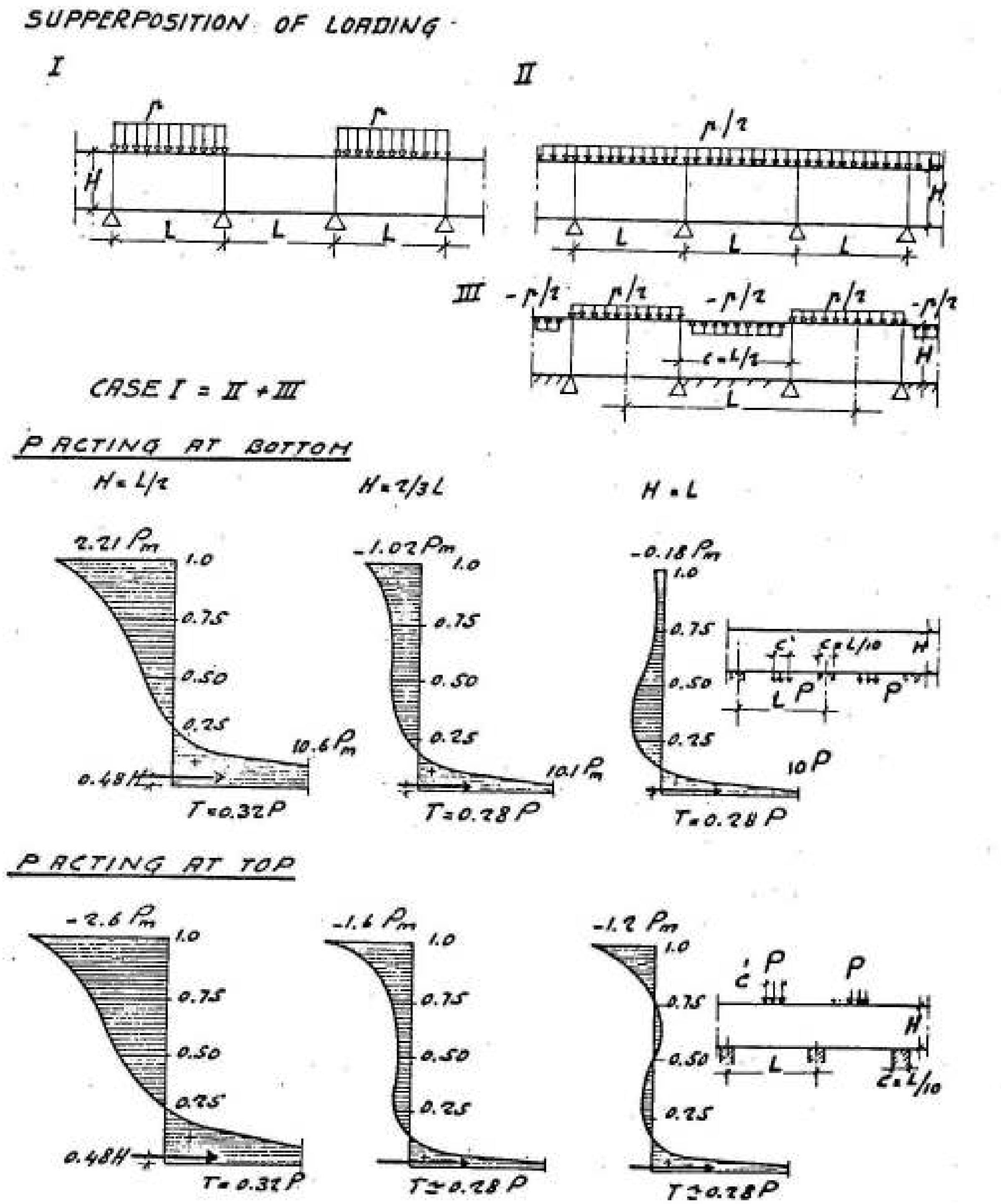
<u>Introduction</u> :

Experiments have shown that the theory of elastic deep beams can be applied to reinforced concrete before the formation of tension cracks (Stage I) . After the formation of cracks, which genearlly take place under working loads, the real stresses differ



NORMAL AND SHEAR STRESSES IN DEEP CONTINUOUS BEAM SUBJECT TO UNIFORM LOADS

Fig. XI-10



STRESSES AT CENTER LINE OF SUPPORT HAVE THE SAME VALUES WITH OPPOSITE SIGN

STRESSES AT THE MIDDLE OF THE SPANS OF CONTINUOUS DEEP BEAMS SUBJECT TO CONCENTRATED LOADS

Fig. XI-11

much from the theoretical values. The arm of the internal forces is increased and the stresses in the steel will be low if its magnitude is determined according to the elastic theory. On the other hand, the inclined principal compressive stresses at the supports are higher than the values determined by the elastic theory and they may be critical in thin heavily loaded deep beams. By the increase of the width of cracks under heavy loads, the compressive bending stresses increase by a ratio bigger than the increase of the load, but they generally do not govern the failure load.

Due to the big moment of inertia of the cross-sections of deep beams, any movement of the supports causes high internal stresses and is not to be neglected.

The compressive stresses at the middle of the spans is generally low and need not to be checked . The beam must however bu sufficiently thick (> 12 cms). that it does not buckle, otherwise stiffeners or compression flanges are to be arranged. In thin deep beams, stiffners at the supports are essential, they must be introcuded to a sufficient height from the bottom surface of the beam.

The tension steel is generally determined such that it resists the full tensile force calculated according to the elastic theory of deep beams in stage I.

The shear stresses vary much along the depth of the beam and are generally low so that no special bent bars or stirrups are needed for this purpose.

Determination of Tension Reinforcements

Simply supported deep beam subject to uniform loads

The maximum tension at the bottom of the beam is given by :

$$\max.T = \frac{\max.\ M_o}{Y_{CT}}$$

in which

$Y_{CT} \simeq 0.6\ H$ for $H \leqslant L$ and $Y_{CT} \simeq 0.6\ L$ for $H > L$

so that

$$\max.\ T = \frac{\max. M_o}{0.6\ H} \quad " \quad \text{and} \quad \max. T = \frac{\max. M_o}{0.6\ L} \quad "$$

In case of uniformly distributed load p/m' :

$$\max.\ M_o = p\,\frac{L^2}{8}$$

so that

$$\max. T = 0.2\ p\ L\ \frac{L}{H} \quad \text{for} \quad H \leqslant L$$

and

$$\max. T = 0.2\ p\ L \quad \text{for} \quad H > L$$

The max. area of tension steel A_s is therefore :

$$A_s = \max.\ T\ /\ \sigma_s$$

It is recommended to extend the tension steel over the whole length of the span and to anchor it well at the supports. Fig. XI.12

For loads hung at the bottom, vertical reinforcements carrying the full reaction are to be introduced for the full height H over a length of 0.7 L .

<u>Concentrated Loads</u> :

Deep beams subject to concentrated loads may be calculated in the same way as those subject to uniform loads introducing the corresponding values of M_o . The tension can also be directly determined from the triangle of forces shown in figure XI.13.

<u>Continuous Deep Beams</u> :

It has been stated before that connecting moments in deep continuous beams are smaller, while field moments are bigger than those

UNIFORM LOAD

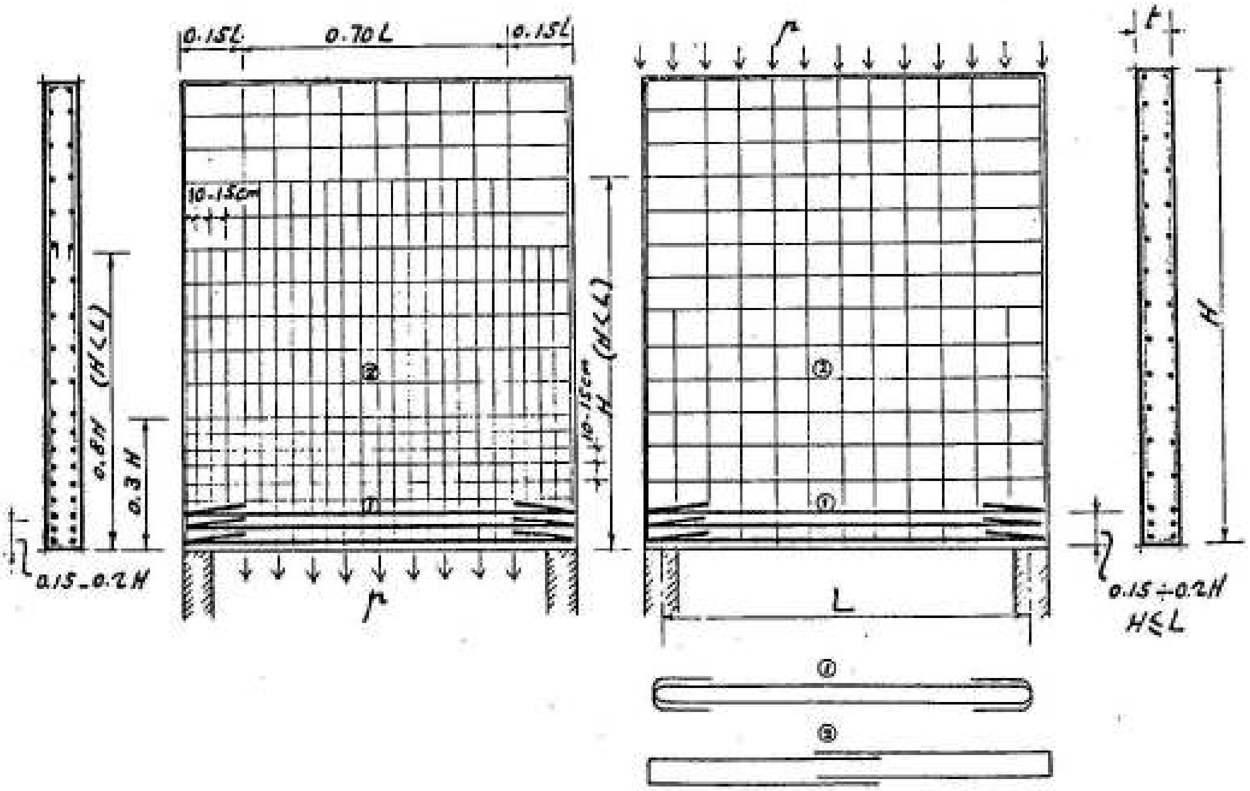


Fig. XI-12

CONCENTRATED LOAD

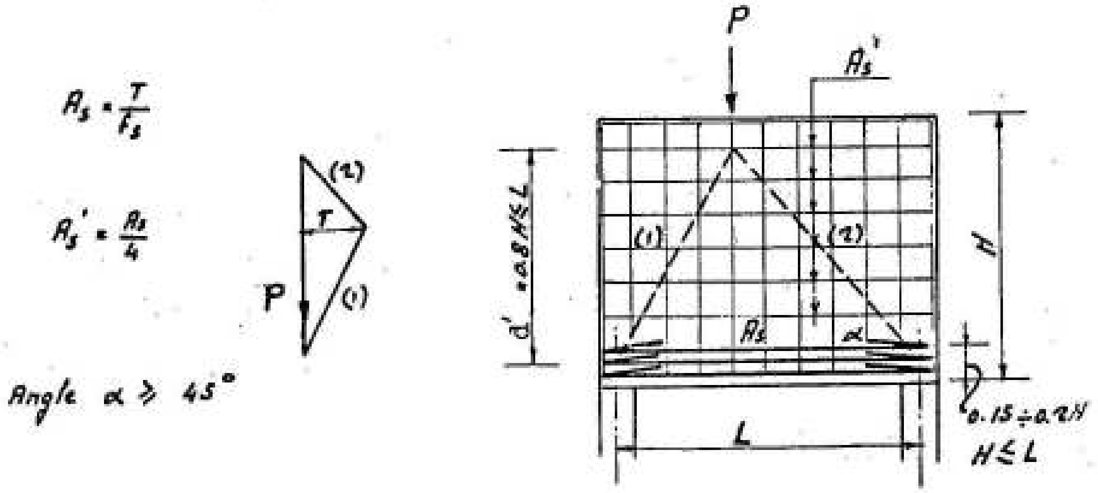


Fig. XI-13

of ordinary slender continuous beams. In order to make a simple , sufficiently safe, design taking this fact in consideration one may calculate the bending moments in the usual way according to the theory of elasticity as applied to normal slender beams and, to increase Y_{CT} at the supports and decrease it at the center lines in such a way as to give results conforming with the data extracted from the experiments in the following manner :

$Y_{CT} = 0.5\,H$ for $H \leqslant L$ and $Y_{CT} = 0.5\,L$ for $H > L$

The tension T being equal to $T = \dfrac{M}{Y_{CT}}$ then

for uniform load and $H \leqslant L$, we get :

At middle of outer spans : $T_m \simeq 0.18\ p\,L$

At middle of inner spans : $T_m \simeq 0.13\ p\,L$

At center line of inner support of outer span : $T_s \simeq 0.25\ p\,L$

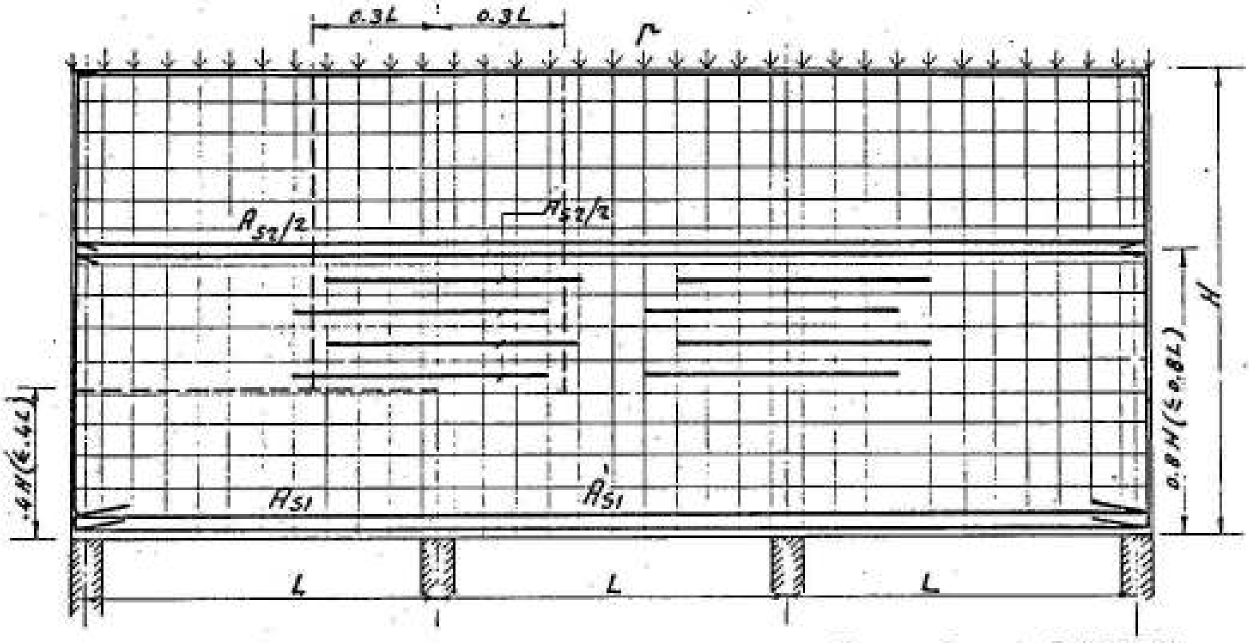
At center line of other inner supports : $T_s \simeq 0.20\ p\,L$

The steel reinforcement is given by $A_s = T / \sigma_s$

It is recommended to extend the full amount of the tension steel at the middle of the spans to the supports ; half the tension steel required at the center lines of the supports may be arranged over the full length of the adjoining spans and the other half to be extended to a distance equal to 0.3 L on each side of the center line of the supports as shown in figure XI.14 .

Loads hung at the lower surface of a deep continuous beam can be treated in the same way as given before for simple beams .

It is however essential to note that the governing design criterion for continuous deep beams is the stress conditions in the support regions (support reaction & principal compressive stresses),



DETAILS OF REINFORCEMENTS OF A CONTINUOUS DEEP BEAM SUBJECT TO UNIFORM LOADS AT TOP

Fig. XI-14

while for the reinforcements - particularly for the tension chord - constructive requirements (crack limitation & anchorage) are governing rather than the statical ones. Therefore no great accuracy is necessary for the design of chord reinforcement because on the one hand the cross section of the required steel area is anyhow small & on the other hand the opening of wide cracks has to be prevented over a high tensile zone. Thus it is sufficient in most cases to calculate the required tension reinforcements from the approximate formulae given before.

A danger of shear failure, which had to be resisted by stirrups or bentup bars, does not exist in case of girders with loading at top edge; a light orthogonal reinforcement mesh of vertical and horizontal stirrups, therefore, suffices. In cases of indirect loading or of hanging loads, an additional web reinforcement may be essential.

XI.2 DESIGN OF PYRAMID ROOFS :

Pyramid roofs* give in many cases a convenient solution for the cover of rectangular containers and pump houses.

Under the effect of vertical loads, a pyramid roof as shown in figure XI.15 is subject to inclined meridian forces T_1 acting along the triangular sides of the pyramid, and horizontal cross forces T_2. Both T_1 and T_2 are compression.

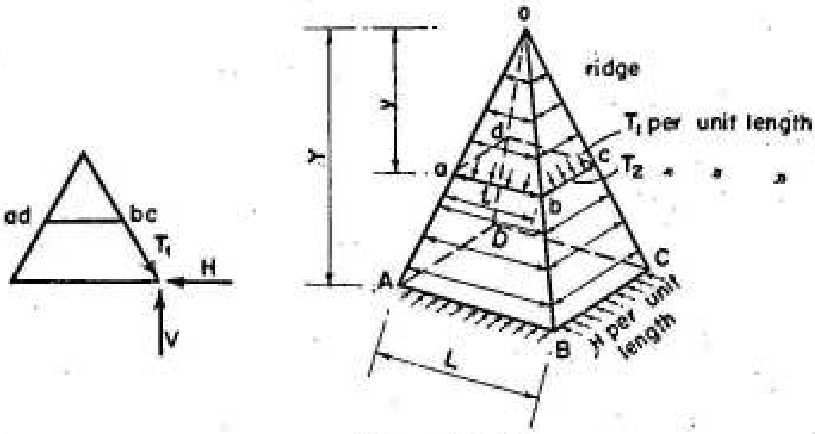


Fig. XI-15

In addition, the triangular plane surfaces of the pyramid are subject to bending moments both in the inclined and horizontal directions.

a) The Meridian Force :

The value of the meridian force T_1 at any intermediate depth y from the vertex O in a pyramid whose sides make an angle α with the vertical is obtained by equating the vertical components of the meridian forces T_1 to the vertical loads of the portion above.

Hence, if the pyramid weighs g per unit of area, the weight of the triangular portion above a b is given by (Fig. XI.16).

---

* Terington " Pyramid Roofs "

$$\tfrac{1}{2}\, g\, l\, y'$$

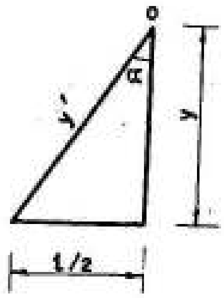


Fig. XI-16

in which :

$$l = 2\, y \tan \alpha$$

and

$$y' = y / \cos \alpha$$

y is the height of any triangular inclined surface between o and a b or b c ....

The vertical component of the meridian force T_1 along one side is given by :

$$T_1 \cdot l \cdot \cos \alpha$$

Therefore :

$$\tfrac{1}{2}\, g y' \, l \cdot y = T_1\, l \cos \alpha \qquad \text{or}$$

$$\tfrac{1}{2}\, g y / \cos \alpha = T_1 \cos \alpha \qquad \text{and}$$

$$\underline{T_1 = \tfrac{1}{2}\, g\, y / \cos^2 \alpha} \qquad \text{or} \qquad \underline{T_1 = g\, l / 4 \sin \alpha \cos \alpha}$$

These expressions give the magnitude of the meridian force per unit width at any intermediate level due to dead loads g per unit area.

For a superimposed load per unit horizontal area, we get :

$$p\, l^2 / 4 l = T_1 \cos \alpha \qquad \text{or}$$

$$\underline{T_1 = p\, l / 4 \cos \alpha}$$

b) <u>The Horizontal Thrust</u> :

The horizontal thrust T_2 exists at all intermediate levels . This may be demonstrated by considering an element strip of length ds cut from the structure by two horizontal planes, one a b c d at a depth y from the vertex and the other, e f g h at a depth y + dy from the vertex. (Fig. XI.17).

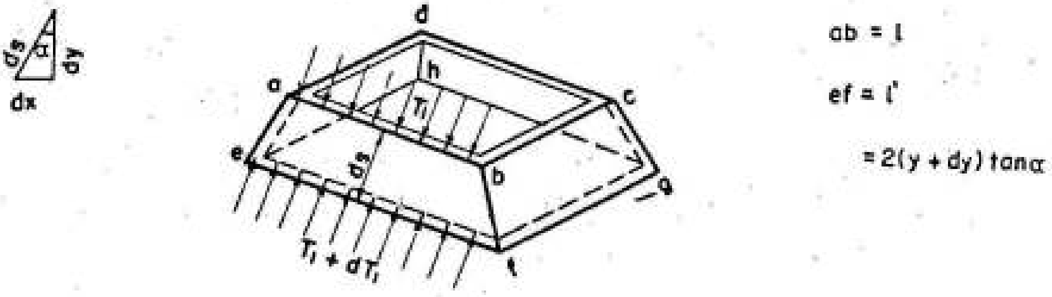


Fig. XI-17

The horizontal component of T_1 , which is $T_1 \sin\alpha$, causes outward thrust on the elemental portion ds , and the horizontal component of the supporting thrust $T_1 + d\,T_1$ causes inward thrust . The outward force per strip $ds = T_1 \sin\alpha\ l$

$$= T_1 \sin\alpha \,.\, 2\,y \tan\alpha$$

but $$T_1 = \frac{g\,y}{2\cos^2\alpha}$$

then the outward force per strip= $\dfrac{g\,y}{2} \cdot \dfrac{\sin\alpha}{\cos^2\alpha} \cdot 2\,y \tan\alpha$

The inward force due to $T_1 + dT_1$ per strip ds

$$= (T_1 + dT_1) \sin\alpha\ l'$$

$$= (T_1 + dT_1) \sin\alpha\ 2\,(y + dy) \tan\alpha$$

The net result of these two forces is an inward thrust over the width ds. This is resisted by two equal horizontal reactions in the planes at right angles. Let the magnitude of this resultant reaction be T_2 per unit width of strip. The horizontal compression induced in the adjacent planes is therefore :

$$T_2\ ds = (T_1 + dT_1) \sin\alpha\ (y + dy) \tan\alpha - T_1 \sin\alpha \,.\, y \tan\alpha \qquad \text{or}$$

$$\frac{T_2\ ds}{\cos\alpha} = d\,(T_1 \sin\alpha \,.\, y \tan\alpha)$$

For a dead load g/m^2 $$T_1 = \frac{g\,y}{2\cos^2\alpha}$$

then

$$\frac{T_2}{\cos\alpha} = \frac{d}{dy}\left(\frac{g\,y}{2\cos^2\alpha}\;y\sin\alpha\tan\alpha\right) \quad \text{but} \quad \tan\alpha = \frac{\sin\alpha}{\cos\alpha}$$

then

$$\frac{T_2}{\cos\alpha} = \frac{d}{dy}\left(\frac{g}{2}\;y^2\;\frac{\tan^2\alpha}{\cos\alpha}\right) \quad \text{and}$$

$$T_2 = \frac{g}{2}\;2\,y\tan^2\alpha \quad \text{or}$$

$$\underline{T_2 = g\,y\tan^2\alpha} \quad \text{or} \quad \underline{T_2 = \tfrac{1}{2}\,g\,l\,\tan\alpha}$$

These expressions give the magnitude of the cross horizontal compression T_2 at any depth y from the vertex due to dead loads g per unit area.

For a superimposed load p per unit horizontal area, we have:

$$T_1 = \frac{p\,l}{4\cos\alpha} \quad \text{but} \quad l = 2\,y\tan\alpha \quad \text{then}$$

$$T_1 = \frac{p\,2\,y\tan\alpha}{4\cos\alpha} = \frac{p\,y\tan\alpha}{2\cos\alpha}$$

Therefore

$$\frac{T_2}{\cos\alpha} = \frac{d}{dy}\left(\frac{p\,y\tan\alpha}{2\cos\alpha}\,.\,y\sin\alpha\,.\,\tan\alpha\right) \quad \text{or}$$

$$T_2 = \frac{d}{dy}\left(\tfrac{1}{2}\,p\,y^2\tan^2\alpha\,.\,\sin\alpha\right) \quad \text{i.e.}$$

$$\underline{T_2 = p\,y\tan^2\alpha\;.\sin\alpha} \quad \text{or} \quad \underline{T_2 = \tfrac{1}{2}\,p\,l\,\tan\alpha\sin\alpha}$$

c) <u>The bending Moments</u> :

The bending moments in the different surfaces of a pyramid depend on the actual proportions of each triangle forming a panel as they influence the manner in which the panels bend.

In an equilateral triangle the bending effect is symmetrical and analogous to the conditions existing in a square panel.

In an acute-angled triangle, as would occur in a sharply pointed pyramid, the most of the bending is taken horizontally from ridged to ridge i.e. analogous to one way slabs in the horizontal direction.

At the opposite extreme is the case of a panel of an obtuse angled triangle in which the base is very long in comparison with the sides. In this case, the most of the bending is taken in the sloping direction of the slab.

It will be noted that the moments compared in these three cases are those that occur horizontally and in the sloping direction of the slab. Actually in any triangular panel, the distribution of maximum moments is undoubtedly across the corners as indicated in figure XI.18 a particularly in triangular panels which approximate to equilateral proportions. From the point of view of the moments, therefore, the reinforcements might more resonably be disposed along the diagonals shown in figure (a).


(a) (b)

Fig. XI-18

However, it is generally more convenient to reinforce the slab in two directions at right angles, that is, horizontally and up and down the slab as indicated in figure (b). Hence, it is necessary to estimate the moments and direct thrusts in these two directions at the point of maximum deflection, which is approximately situated at the center of pressure".

The exact mathematical analysis of bending moments in triangular slabs is too complicated for practical design, but a fairly close estimate of the effective span may be made with sufficient accuracy.

For the purpose of the calculation this effective span may be taken as the diameter of the inscribed circle which touches all the sides figure XI.19 and for convenience the center of this circle is referred to as the " center of pressure ".

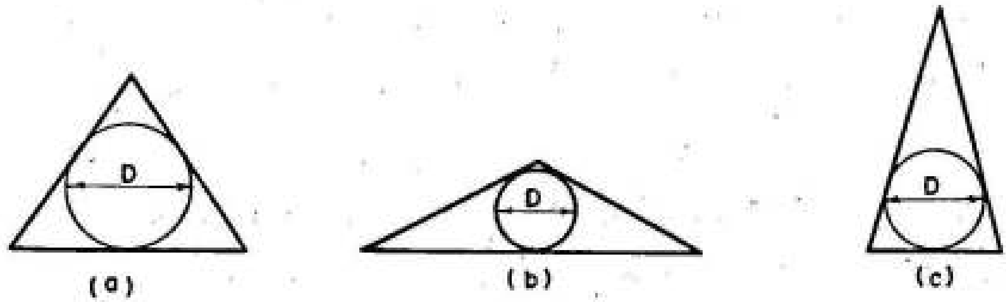


Fig. XI-19

To find the bending moments to be allowed for, it is convenient to consider three conditions :

1) A panel approximating to an equilateral triangle,
2) A panel with an obtuse angle at the vertex,
3) A panel with an acute angle at the vertex.

Case I :

The conditions at the supports are an indeterminate fixity of the slab at the lower edge along the supporting beam and continuity in the horizontal direction along the ridges forming the side supports. Comparing these conditions with a square slab spanning in two directions at right angles, it is a fundamental law that if two opposite edges are continuous supports and he other two edges are only partly fixed, the larger moment is taken in the direction of greater fixity. From this analogy, the moment in the horizontal direction will tend to be greater than the moment in the sloping direction. For a panel approximating to an equilateral triangle, the following moments may be taken to cover own weight and the effects of superimposed loading including wind, which actually causes a thrust on the wind ward side and a suction on the leeward side.

In the horizontal direction, in the span and over the ridges:

$$M = \pm w \sin \alpha \frac{D^2}{16}$$

In the sloping direction :

$$M = w \sin \alpha \frac{D^2}{18}$$

in which $w = g + p$

Case 2 :

Where a panel has an obtuse angle at the vertex, practically all the bending is taken in the sloping direction. The following bending moment may therefore be taken to cover effects of superimposed loading and own weight :

$$M = w \sin \alpha \frac{D^2}{10}$$

in the span and over the ridges.

In the horizontal direction, distribution steel, equal to about 20 % of the main reinforcement is required.

Case 3 :

In the other extreme case, where a panel has a particularly acute angle at the vertr, practically all the moment is taken in the horizontal direction and

$$M = \pm w \sin \alpha \frac{D^2}{12}$$

in the span and over the ridges.

In the sloping direction, distribution steel, equal to about 20% of the main reinforcement is required.

d) Forces at Bottom Edge :

The lower edge of the pyramid is subject to a vertical reaction V per meter equal to :

$$V = g\frac{Y'}{2} + p\frac{L}{4}$$

and a horizontal thrust H giving tension in each of the lower edges equal to T where

$$T = \frac{H\,L}{2}$$

in which

$$H = V \tan\alpha$$

For a dead load g, we get :

$$T = \frac{H\,L}{2} = V\tan\alpha\,\frac{L}{2} = g\frac{Y'}{2}\tan\alpha\,\frac{L}{2}$$

But $Y' = \dfrac{L}{2\sin\alpha}$ then ,

$$T = \frac{g\,L}{4\sin\alpha}\cdot\frac{\sin\alpha}{\cos\alpha}\cdot\frac{L}{2} \qquad \text{i.e.}$$

$$T = \frac{g\,L^2}{8\cos\alpha}$$

and for a superimposed load p, we get :

$$T = \frac{H\,L}{2} = V\tan\alpha\,\frac{L}{2} = p\frac{L}{4}\tan\alpha\,\frac{L}{2} \qquad \text{i.e.}$$

$$T = p\,\frac{L^2}{8\cos\alpha}\sin\alpha$$

Horizontal bending is also induced along the lower edges, and the magnitude of the moments is :

In the span $\quad M = \dfrac{H\,L^2}{24}$

$$M = \frac{L^3}{96 \cos \alpha} (g + p \sin \alpha)$$

and at the corners $M = \frac{H L^2}{12}$ or

$$M = \frac{L^3}{48 \cos \alpha} (g + p \sin \alpha)$$

It is however possible to consider each side of the pyramid as a deep beam subject to a load equal to the maximum meridian force at the base of the pyramid. The steel at the bottom edge should be sufficient to resist the tension of the deep beam $T = \frac{M}{Y_{CT}}$ plus the tension due to the reaction of the load on the two sides normal to that under consideration and given by $T = \frac{H L}{2}$

e) Example :

It is required to cover a tank as that shown in figure VII.30 by a pyramid roof of the form shown in figure XI.20

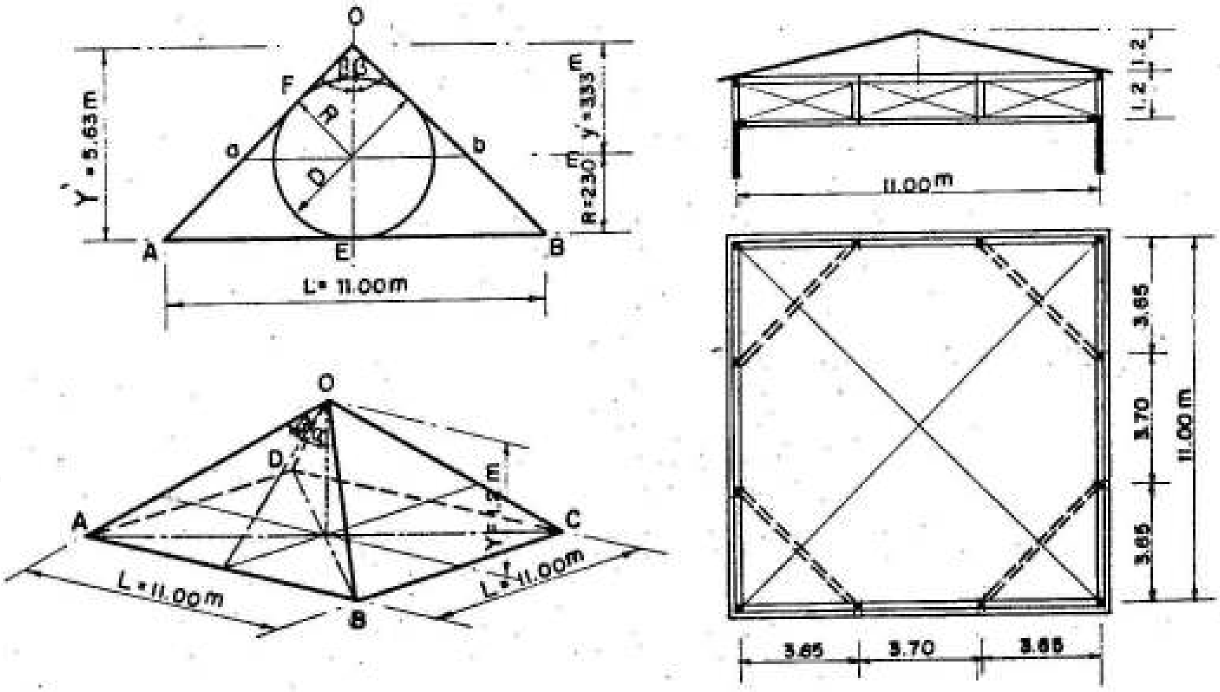


Fig. XI-20

<u>Data</u> :

Base L = 11.00 m Height Y = 1.20 m.

Thickness of pyramid slab t = 10 cms.

Total dead + Superimposed loads $w = 350\ kg/m^2$ surface

<u>Solution</u> :

Length $\overline{OE} = Y' = \sqrt{1.2^2 + 5.5^2} = \sqrt{31.69} = 5.63$ ms

$\sin\alpha = \frac{5.5}{5.63} = .978$, $\cos\alpha = \frac{1.2}{5.63} = .213$, $\tan\alpha = \frac{5.5}{1.2} = 4.58$

Length of ridge $= \overline{OA} = \sqrt{5.63^2 + 5.5^2} = \sqrt{61.94} = 7.85$ ms

$\overline{AF} = \overline{AE} = 5.5$ m $\quad \frac{R}{\overline{OF}} = \tan\beta$

$\overline{OF} = \overline{OA} - \overline{AF} = 7.85 - 5.5 = 2.35$ ms

$\tan\beta = \frac{\overline{AE}}{Y'} = \frac{5.5}{5.63} = 0.978$

Therefore :

$R = D/2 = \overline{OF}\tan\beta = 2.35 \times 0.978 = 2.30$ ms i.e.

$D = 4.6$ ms and

the height $y' = 5.63 - 2.30 = 3.33$ ms i.e.

the height of the vertex of the pyramid from the center of pressure :

$y = y'\cos\alpha = 3.33 \times 0.213 = 0.71$ ms

The average meridian force T_1 across section a b :

$T_1 = \frac{1}{2}\frac{w\,y}{\cos^2\alpha} = \frac{1}{2}\frac{350 \times 0.71}{0.213^2} = \underline{2740}$ kg/m

The bending moment in the inclined and horizontal directions at section a b :

$M = w\sin\alpha\frac{D^2}{16} = 350 \times 0.978 \times \frac{4.6^2}{16} = \underline{450}$ kg/m

The horizontal cross compression force T_2 at section a b

$T_2 = w\,y\tan^2\alpha = 350 \times 0.71 \times 4.58^2 = \underline{5220}$ kg/m

T_2 can further be calculated from the relation :

$T_2 = \frac{1}{2} w\, l \tan \alpha$

knowing that $l/L = y'/Y'$ then $l = 11 \times \frac{3.33}{5.63} = 6.5$ m and

we get :

$T_2 = \frac{1}{2} \times 350 \times 6.5 \times 4.85 \qquad = \underline{5220}$ kg/m

The max. meridian force at A B

$\max T_1 = \frac{1}{2} \frac{w\,Y}{\cos^2\alpha} = \frac{1}{2} \frac{350 \times 1.2}{0.213^2} \qquad = \underline{4620}$ kg/m

The max. horizontal cross compressive force T_2 at section A B :

$\max T_2 = \frac{1}{2} w\, L \tan\alpha = \frac{1}{2} \times 350 \times 11 \times 4.85 = \underline{8300}$ kg/m

The vertical load on the lower horizontal beam :

$V = \frac{1}{2} w\, Y' \qquad = \frac{1}{2} \times 350 \times 5.63 \qquad = \underline{985}$ kg/m

The horizontal load / m :

$H = V \tan \alpha \qquad = 985 \times 4.58 \qquad = \underline{4510}$ kg/m

The beam at the bottom of the pyramid is supported by vertical posts arranged at the four corners and at the third points of each side and horizontal diagonal ties arranged at the third points as shown in figure XI.20. The max. bending moment in the vertical beam :

$M_{max} \simeq \frac{1}{12} V \left(\frac{L}{3}\right)^2 = \frac{1}{12} \times 985 \left(\frac{11}{3}\right)^2 \qquad = \underline{1100}$ kgm

The load per post

$P = V\, L/3 \qquad = 985 \times 11/3 \qquad = \underline{3600}$ kgs

The max. bending moment in the horizontal beam :

$M_{max} \simeq \frac{1}{12} H \left(\frac{L}{3}\right)^2 = \frac{1}{12} \times 4510 \left(\frac{11}{3}\right)^2 \qquad = \underline{5050}$ kgm

The tension in the outside panels of the horizontal beam :

$T = \frac{1}{2} H \left(\frac{L}{3}\right) \qquad = \frac{1}{2} \times 4510 \times \frac{11}{3} \qquad = \underline{8270}$ kgs

The tension in the diagonal tie

$T = H \left(\frac{L}{3}\right)\sqrt{2} = 4510 \times \frac{11}{3} \times \sqrt{2} \qquad = \underline{23400}$ kgs

The tension in the intermediate panel of the horizontal beam :

$$T = 3 \times 8270 = 25000 \text{ kgs.}$$

It can be seen from the previous investigation that the slab of the pyramid is subjected to max. horizontal compressive stresses at its base while the bottom edge beam is subjected to tensile stresses. The corresponding difference of strain will cause shearing forces at the joint between the pyramid slab and the edge beam, and bending moments of the sense shown in figure IV.25 will be developed in the inclined direction of the slab. It is therefore recommended in flat pyramid roofs of relatively big dimensions to increase the thickness of the slab at its bottom edge and to reinforce it with top reinforcements of the order of the main steel used in the pyramid slab.

XI.3. TEMPERATURE STRESSES IN SILO WALLS

If the temperature of the stored material inside a silo-cell is increased, the walls are subject to temperature stresses. Assume that the temperature is T_1 in the inner face, T_2 in the outer face, and that the temperature decreases uniformly from inner to outer face, $T_1 - T_2$ being denoted as ΔT. Figure XI.21 shows a segment of a circular silo wall in two positions, one before and one after a uniform increase in temperature. The original length of the arc of the wall has been increased, but an increase that is uniform throughout will not create any stresses so long as the ring is supposed to be free and unrestrained at its edges. It is the temperature differential only, ΔT, which creates stresses.

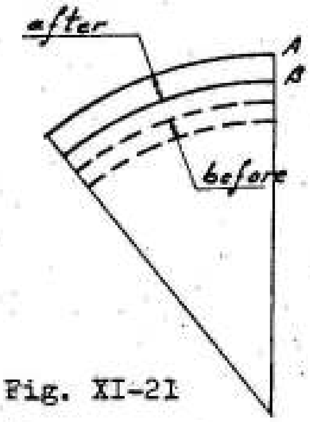


Fig. XI-21

The inner fibers being hotter tend to expand more than the outer fibers, so if the segment is cut loose from the adjacent portions of the wall, point A in figure XI.22 will move to A', B will move to B', and section A B, which represents the stressless condition due to a uniform temperature change throughout, will move to a new position A' B'. Actually the movements from A to A' and B to B' are prevented since the circle must remain a circle and stresses will be created that are proportional to the horizontal distances between A B and A' B'.

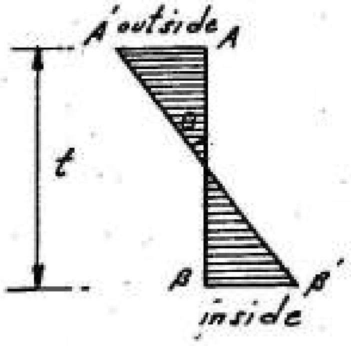


Fig. XI-22

It is clear that A A' = B B' = movement due to a temperature change of $\Delta T/2$ or when α is the coefficient of expansion, that A A' = B B' $= \frac{1}{2} \Delta T \alpha$ per unit length of arc and

$$\theta = \frac{A\,A'}{\frac{1}{2}\,t} = \frac{\alpha \,.\, \Delta T}{t}$$

In a homogeneous section, the moment M required to produce an angle change θ in an element of unit length may be written as

$$M = E\,I\,\theta$$

Eliminating θ gives

$$M = \frac{E\,I\,\alpha \,.\, \Delta T}{t}$$

The stresses in the extreme fibers created by M are :

$$\sigma = \frac{M}{I} \,.\, \frac{t}{2} = 1/2\, E\, \alpha \,.\, \Delta T$$

The stress distribution across the cross-section is as indicated in figure XI.22. The stresses are numerically equal at the two faces but have opposite signs. Note that the equation applies to uncracked sections only, and that this procedure of stress calculation is to be considered merely as a method by which the problem can be approached.

The variables E and I in the equations are uncertain quantities. E may vary from 100 000 to 300 000 kg/cm^2 , and I may also vary considerably because of deviations from the assumption of linear relation between stress and strain. Finally, if the concrete cracks, M can no longer be set equal to $E\,I\,\theta$, nor σ equal to $\frac{M}{I} \cdot \frac{t}{2}$. as a result, the equation $\sigma = 1/2 \, . \, E\,\alpha . \, \Delta T$ is to be regarded as merely indicative rather than formally correct.

The value of α may be taken 0.00001 and for the purpose of this problem one may chose $E = 100\,000$ kg/cm^2 . so that $E\,\alpha = 100\,000 \times 0.00001 = 1$. Knowing further that $I = 100 \frac{t^3}{12}$ we get :

$M = \frac{100\, t^2 \, . \, \Delta T}{12}$ in kg cms when t is in cms and

$\underline{M = t^2 \, . \Delta T/12}$ in kg ms. when t is in cms and

$\underline{\sigma = \Delta T/2}$ in kgs/cm^2

The value of ΔT is the difference between the temperature in the two surfaces of the concrete which may be computed from the temperature of the stored material and the outside air. When the flow of heat is uniform from the inside to the outside of the wall section in figure XI.23, the temperature difference, $\Delta T = T_1 - T_2$, is smaller than the difference $T_i - T_o$, between the inside liquid and the outside air.

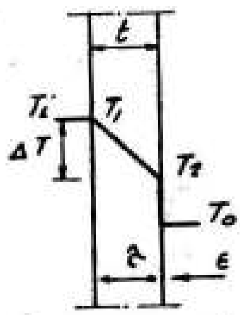


Fig. XI-23

Standard text books give

$$\Delta T = (T_i - T_o) \frac{t/\lambda_o}{1/\lambda}$$

$$\frac{1}{\lambda} = \frac{t}{\lambda_o} + \frac{t_1}{\lambda_1} + \frac{t_2}{\lambda_2} + \ldots\ldots\ldots + \frac{1}{\epsilon}$$

λ_o = coefficient of conductivity of gravel concrete = 0.67

$\lambda_1 , \lambda_2 \ldots$ = " " " " insulating layers

= 0.05 for cork, 0.40 for brick walls, 0.6 for air.. etc.

ϵ = outside surface coefficient = 10 t = thickness of reinforced concrete wall in meters

t_1 , t_2 ... = " " insulating layers in meters.

Thus for an uninsulated reinforced concrete wall

$$\Delta T = (T_i - T_o) \frac{\frac{t}{0.67}}{\frac{t}{0.67} + \frac{1}{10}} = (T_i - T_o) \frac{t}{t + 0.067}$$

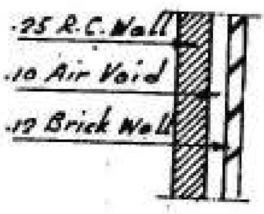
Assuming t = 0.25 ms, then

$$\Delta T = 0.8 (T_i - T_o)$$

If the wall were insulated by a 12 cms brick wall arranged as shown in figure XI.24, we get

$$\frac{1}{\lambda} = \frac{0.25}{0.67} + \frac{0.10}{0.60} + \frac{0.12}{0.40} + \frac{1}{10} = 0.670$$

$$\Delta T = \frac{T_i - T_o}{0.67} \cdot \frac{0.25}{0.67} = 0.557 (T_i - T_o)$$



If the air void were replaced by 5 cms cork, we get :

Fig. XI-24

$$\frac{1}{\lambda} = \frac{0.25}{0.67} + \frac{0.05}{0.05} + \frac{0.12}{0.40} + \frac{1}{10} = 1.503$$

$$\Delta T = \frac{T_i - T_o}{1.503} \cdot \frac{0.25}{0.67} = 0.25 (T_i - T_o)$$

Consider a circular silo wall 25 cms thick storing a material with a maximum temperature of 50°C while the temperature of the outside air is 20°C, then :

$$\Delta T = 0.8 (50 - 20) = 24°C$$

The corresponding bending moment due to this temperature change is :

$$M = t^2 \Delta T /12 = 25^2 \times 24/12 = 1250 \text{ kgm}$$

While the temperature stresses are

$$\sigma = \pm \Delta T / 2 = \pm 12 \text{ kg/cm}^2$$

 352

These stresses are tension in the outside and compression in the inside face. If the uniformly distributed ring tension is 15 kg/cm^2 ; the combined stress will be :

Outside fiber 15 + 12 = 27 kg/cm^2 (tension)
Inside fiber 15 - 12 = 3 kg/cm^2 (tension)

In reality, too much significance should not be attached to the temperature stress computed from the equation derwed. The stress equation is developed from the strain equation A A' = 1/2 α ΔT, based on the assumption that stress is proportional to strain. This assumption is rather inaccurate for the case under discussion. The inaccuracy may be rectified to some extend by using a relatively low value for E_c, such as E_c = 100 000 kg/cm^2 which has been used.

As computed in the example, a temperature differential of 24°C gives a stress of 12 kg/cm^2 in the extreme fiber. This is probably more than the concrete can take in addition to the regular ring tension stress without cracking on the colder surface. The temperature stress may be reduced by means of insulation, which serves to decrease the temperature differential, or additional horizontal reinforcement may be provided closer to the colder surface. A proceedure will be illustrated for determination of temperature steel. It is not based upon a rigorous mathematical analysis but will be helpful as a guide and as an aid to engineering judgement.

In the given example, the area of horizontal steel at the colder face computed as for a cracked section is given by the relation :

$$A_s = M/y_{CT}\,\sigma_s$$

For $y_{CT} \approx \frac{7}{8}$ d, where d = the theoretical depth of the section and σ_s = 1400 kg/cm^2 , then

$$A_s = M/1200\ d$$

For M in kgm and d in ms, A_{s_2} will be in cm^2 . Accordingly
A_s = 1250 / 1200 x 0.22 = <u>4.75</u> cm^2
This area is in addition to the regular ring steel at the outer face of the wall.

APPENDIX

Tables of Trigonometric and Hyperbolic Functions.

x	sin x	cos x	sinh x	cosh x
0	0	1	0	1
.001	0.001	1.000	0.0010	1.0000
.002	0.002	1.000	0.0020	1.0000
.003	0.003	1.000	0.0030	1.0000
.004	0.004	1.000	0.0040	1.0000
.005	0.005	1.000	0.0050	1.0000
.006	0.006	1.000	0.0060	1.0000
.007	0.007	1.000	0.0070	1.0000
.008	0.008	1.000	0.0080	1.0000
.009	0.009	1.000	0.0090	1.0000
.010	0.010	1.000	0.0100	1.0000
.011	0.011	1.000	0.0110	1.0001
.012	0.012	1.000	0.0120	1.0001
.013	0.013	1.000	0.0130	1.0001
.014	0.014	1.000	0.0140	1.0001
.015	0.015	1.000	0.0150	1.0001
.016	0.016	1.000	0.0160	1.0001
.017	0.017	1.000	0.0170	1.0001
.018	0.018	1.000	0.0180	1.0002
.019	0.019	1.000	0.0190	1.0002
.020	0.020	1.000	0.0200	1.0002
.030	0.030	1.000	0.0300	1.0004
.040	0.040	0.999	0.0400	1.0008
.050	0.050	0.999	0.0500	1.0012
.060	0.060	0.998	0.0600	1.0018
.070	0.070	0.998	0.0701	1.0024
.080	0.080	0.997	0.0801	1.0032
.090	0.090	0.996	0.0901	1.0040
.100	0.100	0.995	0.1002	1.0050
.110	0.110	0.994	0.1102	1.0061
.120	0.120	0.993	0.1203	1.0072
.130	0.130	0.992	0.1304	1.0085
.140	0.140	0.990	0.1405	1.0098
.150	0.149	0.989	0.1506	1.0113
.160	0.159	0.987	0.1607	1.0128
.170	0.169	0.986	0.1708	1.0145
.180	0.179	0.984	0.1810	1.0162
.190	0.189	0.982	0.1912	1.0181

x	sin x	cos x	sinh x	cosh x
0.20	0.199	0.980	0.2013	1.0201
0.21	0.208	0.978	0.2116	1.0221
0.22	0.218	0.976	0.2218	1.0243
0.23	0.228	0.974	0.2320	1.0266
0.24	0.238	0.971	0.2423	1.0289
0.25	0.247	0.969	0.2526	1.0314
0.26	0.257	0.966	0.2629	1.0340
0.27	0.267	0.964	0.2733	1.0367
0.28	0.276	0.961	0.2837	1.0395
0.29	0.286	0.958	0.2941	1.0424
0.30	0.296	0.955	0.3045	1.0453
0.31	0.305	0.952	0.3150	1.0484
0.32	0.315	0.949	0.3255	1.0516
0.33	0.324	0.946	0.3360	1.0550
0.34	0.334	0.943	0.3466	1.0584
0.35	0.343	0.939	0.3572	1.0619
0.36	0.352	0.936	0.3678	1.0655
0.37	0.362	0.932	0.3785	1.0692
0.38	0.371	0.929	0.3892	1.0731
0.39	0.380	0.925	0.4000	1.0770
0.40	0.389	0.921	0.4108	1.0811
0.41	0.399	0.917	0.4216	1.0852
0.42	0.408	0.913	0.4325	1.0895
0.43	0.417	0.909	0.4434	1.0939
0.44	0.426	0.905	0.4543	1.0984
0.45	0.435	0.900	0.4653	1.1030
0.46	0.444	0.896	0.4764	1.1077
0.47	0.453	0.892	0.4875	1.1125
0.48	0.462	0.887	0.4986	1.1174
0.49	0.471	0.882	0.5098	1.1225
0.50	0.479	0.878	0.5211	1.1276
0.51	0.488	0.873	0.5324	1.1329
0.52	0.497	0.868	0.5438	1.1383
0.53	0.506	0.863	0.5552	1.1438
0.54	0.514	0.858	0.5666	1.1494
0.55	0.523	0.853	0.5782	1.1551
0.56	0.531	0.847	0.5897	1.1609
0.57	0.540	0.842	0.6014	1.1669
0.58	0.548	0.837	0.6131	1.730
0.59	0.556	0.831	0.6248	1.1792

x	sin x	cos x	sinh x	cosh x
0.60	0.565	0.825	0.6366	1.1855
0.61	0.573	0.820	0.6485	1.1919
0.62	0.581	0.814	0.6605	1.1984
0.63	0.589	0.808	0.6725	1.2051
0.64	0.597	0.802	0.6846	1.2119
0.65	0.605	0.796	0.6968	1.2188
0.66	0.613	0.790	0.7090	1.2258
0.67	0.621	0.784	0.7213	1.2330
0.68	0.629	0.778	0.7336	1.2402
0.69	0.637	0.771	0.7461	1.2476
0.70	0.644	0.765	0.7586	1.2552
0.71	0.652	0.758	0.7712	1.2628
0.72	0.659	0.752	0.7838	1.2706
0.73	0.667	0.745	0.7966	1.2785
0.74	0.674	0.739	0.8094	1.2865
0.75	0.682	0.732	0.8223	1.2947
0.76	0.689	0.725	0.8353	1.303
0.77	0.696	0.718	0.8484	1.3114
0.78	0.703	0.711	0.8615	1.3199
0.79	0.710	0.704	0.8748	1.3286
0.80	0.717	0.697	0.8881	1.3374
0.81	0.724	0.690	0.9015	1.3464
0.82	0.731	0.682	0.9150	1.3555
0.83	0.738	0.675	0.9286	1.3647
0.84	0.745	0.668	0.9423	1.3740
0.85	0.751	0.660	0.9561	1.3835
0.86	0.758	0.652	0.9700	1.3932
0.87	0.764	0.645	0.9840	1.4029
0.88	0.771	0.637	0.9981	1.4128
0.89	0.777	0.630	1.0122	1.4229
0.90	0.783	0.621	1.0265	1.4331
0.91	0.790	0.614	1.0409	1.4434
0.92	0.796	0.606	1.0554	1.4539
0.93	0.802	0.598	1.0700	1.4645
0.94	0.808	0.590	1.0847	1.4753
0.95	0.813	0.582	1.0995	1.4862
0.96	0.819	0.574	1.1144	1.4973
0.97	0.825	0.565	1.1294	1.5085
0.98	0.831	0.557	1.1446	1.5200
0.99	0.836	0.549	1.1598	1.5314

x	sin x	cos x	sinh x	cosh x
1.00	0.842	0.540	1.1752	1.5431
1.01	0.847	0.532	1.1907	1.5549
1.02	0.852	0.523	1.2063	1.5669
1.03	0.857	0.515	1.2220	1.5790
1.04	0.862	0.506	1.2379	1.5913
1.05	0.867	0.498	1.2539	1.6038
1.06	0.872	0.489	1.2700	1.6164
1.07	0.877	0.480	1.2862	1.6292
1.08	0.882	0.471	1.3025	1.6421
1.09	0.887	0.463	1.3190	1.6552
1.10	0.891	0.454	1.3356	1.6685
1.11	0.896	0.445	1.3524	1.6820
1.12	0.900	0.436	1.3693	1.6956
1.13	0.904	0.427	1.3863	1.7093
1.14	0.909	0.418	1.4035	1.7233
1.15	0.913	0.409	1.4208	1.7374
1.16	0.917	0.399	1.4382	1.7517
1.17	0.921	0.390	1.4558	1.7662
1.18	0.925	0.381	1.4736	1.7808
1.19	0.928	0.372	1.4914	1.7956
1.20	0.932	0.362	1.5095	1.8107
1.21	0.936	0.353	1.5276	1.8258
1.22	0.939	0.344	1.5460	1.8412
1.23	0.943	0.334	1.5645	1.8568
1.24	0.946	0.325	1.5831	1.8725
1.25	0.949	0.315	1.6019	1.8884
1.26	0.952	0.306	1.6209	1.9045
1.27	0.955	0.296	1.6400	1.9208
1.28	0.958	0.287	1.6593	1.9373
1.29	0.961	0.277	1.7688	1.9540
1.30	0.964	0.268	1.6984	1.9709
1.31	0.966	0.258	1.7182	1.9880
1.32	0.969	0.248	1.7381	2.0053
1.33	0.971	0.239	1.7583	2.0228
1.34	0.974	0.229	1.7786	2.0404
1.35	0.976	0.219	1.7991	2.0583
1.36	0.978	0.209	1.8198	2.0764
1.37	0.980	0.199	1.8406	2.0947
1.38	0.982	0.190	1.8617	2.1132
1.39	0.984	0.180	1.8829	2.1320

x	sin x	cos x	sinh x	cosh x
1.40	0.985	0.170	1.9043	2.1509
1.41	0.987	0.160	1.9259	2.1700
1.42	0.989	0.150	1.9477	2.1894
1.43	0.990	0.140	1.9697	2.2090
1.44	0.992	0.130	1.9919	2.2288
1.45	0.993	0.121	2.0143	2.2488
1.46	0.994	0.111	2.0369	2.2691
1.47	0.995	0.110	2.0598	2.2896
1.48	0.996	0.091	2.0826	2.3103
1.49	0.997	0.081	2.1059	2.3312
1.50	0.998	0.071	2.1293	2.3524
1.51	0.998	0.061	2.1529	2.3738
1.52	0.999	0.051	2.1768	2.3955
1.53	0.999	0.041	2.2008	2.4174
1.54	1.000	0.031	2.2251	2.4395
1.55	1.000	0.021	2.2496	2.4619
1.56	1.000	0.011	2.2743	2.4815
1.57	1.000	0.001	2.2993	2.5074
1.58	1.000	-.009	2.3245	2.5305
1.59	1.000	-.019	2.3499	2.5538
1.60	1.000	-.029	2.3756	2.5775
1.61	0.999	-.039	2.4015	2.6014
1.62	0.999	-.049	2.4276	2.6255
1.63	0.998	-.059	2.4540	2.6499
1.64	0.998	-.069	2.4806	2.6746
1.65	0.997	-.079	2.5075	2.6995
1.66	0.996	-.089	2.5346	2.7247
1.67	0.995	-.099	2.5620	2.7502
1.68	0.994	-.109	2.5896	2.7760
1.69	0.993	-.119	2.6175	2.8020
1.70	0.992	-.129	2.6456	2.8283
1.71	0.990	-.139	2.6740	2.8549
1.72	0.989	-.149	2.7027	2.8818
1.73	0.987	-.159	2.7317	2.9090
1.74	0.986	-.168	2.7609	2.9364
1.75	0.984	-.178	2.7904	2.9642
1.76	0.982	-.188	2.8202	2.9922
1.77	0.980	-.198	2.8503	3.0206
1.78	0.978	-.208	2.8806	3.0492
1.79	0.976	-.217	2.9112	3.0782

x	sin x	cos x	sinh x	cosh x
1.80	0.974	-.227	2.9422	3.1075
1.81	0.972	-.237	2.9734	3.1370
1.82	0.969	-.247	3.0049	3.1669
1.83	0.967	-.256	3.0367	3.1972
1.84	0.964	-.266	3.0689	3.2277
1.85	0.961	-.276	3.1013	3.2585
1.86	0.959	-.285	3.1340	3.2897
1.87	0.956	-.295	3.1671	3.3212
1.88	0.953	-.304	3.2005	3.3530
1.89	0.950	-.314	3.2342	3.3852
1.90	0.946	-.323	3.2682	3.4177
1.91	0.943	-.333	3.3025	3.4506
1.92	0.940	-.342	3.3372	3.4838
1.93	0.936	-.352	3.3722	3.5173
1.94	0.933	-.361	3.4075	3.5512
1.95	0.929	-.370	3.4432	3.5855
1.96	0.925	-.379	3.4792	3.6201
1.97	0.921	-.389	3.5156	3.6551
1.98	0.917	-.399	3.5523	3.6904
1.99	0.913	-.407	3.5894	3.7261
2.00	0.909	-.416	3.6269	3.7622
2.01	0.905	-.425	3.6647	3.7986
2.02	0.901	-.434	3.7028	3.8355
2.03	0.897	-.443	3.7414	3.8727
2.04	0.892	-.452	3.7803	3.9103
2.05	0.887	-.461	3.8196	3.9483
2.06	0.883	-.470	3.8593	3.9876
2.07	0.878	-.479	3.8993	4.0255
2.08	0.873	-.488	3.9398	4.0647
2.09	0.868	-.496	3.9806	4.1043
2.10	0.863	-.505	4.0219	4.1443
2.11	0.858	-.513	4.0635	4.1847
2.12	0.853	-.522	4.1056	4.2256
2.13	0.848	-.531	4.1480	4.2668
2.14	0.842	-.540	4.1909	4.3086
2.15	0.837	-.547	4.2342	4.3507
2.16	0.831	-.556	4.2779	4.3932
2.17	0.826	-.564	4.3220	4.4362
2.18	0.820	-.572	4.3666	4.4796
2.19	0.814	-.580	4.4116	4.5236

x	sin x	cos x	sinh x	cosh x	x	sin x	cos x	sinh x	cosh x
2.20	0.809	-.589	4.4571	4.5679	2.60	0.516	-.857	6.6947	6.7690
2.21	0.803	-.597	4.5030	4.6127	2.61	0.507	-.862	6.7628	6.8363
2.22	0.797	-.605	4.5494	4.6580	2.62	0.498	-.867	6.8315	6.9043
2.23	0.791	-.613	4.5962	4.7037	2.63	0.490	-.872	6.9008	6.9729
2.24	0.784	-.620	4.6434	4.7499	2.64	0.481	-.877	6.9709	7.0423
2.25	0.778	-.628	4.6912	4.7966	2.65	0.472	-.882	7.0417	7.1123
2.26	0.771	-.636	4.7394	4.8437	2.66	0.463	-.886	7.1132	7.1831
2.27	0.765	-.644	4.7880	4.8914	2.67	0.454	-.891	7.1854	7.2546
2.28	0.759	-.651	4.8372	4.9395	2.68	0.445	-.895	7.2583	7.3268
2.29	0.752	-.659	4.8868	4.9881	2.69	0.436	-.900	7.3319	7.3998
2.30	0.746	-.666	4.9370	5.0372	2.70	0.427	-.904	7.4063	7.4735
2.31	0.739	-.674	4.9876	5.0868	2.71	0.418	-.908	7.4814	7.5479
2.32	0.732	-.681	5.0387	5.1370	2.72	0.409	-.912	7.5572	7.6231
2.33	0.725	-.688	5.0903	5.1876	2.73	0.400	-.917	7.6338	7.6990
2.34	0.719	-.696	5.1424	5.2388	2.74	0.391	-.920	7.7112	7.7758
2.35	0.712	-.703	5.1951	5.2905	2.75	0.382	-.924	7.7894	7.8533
2.36	0.704	-.710	5.2483	5.3427	2.76	0.372	-.928	7.8683	7.9316
2.37	0.697	-.717	5.3020	5.3954	2.77	0.363	-.932	7.9480	8.0106
2.38	0.690	-.724	5.3562	5.4487	2.78	0.354	-.935	8.0285	8.0905
2.39	0.683	-.731	5.4109	5.5026	2.79	0.344	-.939	8.1098	8.1712
2.40	0.676	-.737	5.4662	5.5570	2.80	0.335	-.942	8.1919	8.2527
2.41	0.668	-.744	5.5221	5.6119	2.81	0.326	-.946	8.2749	8.3351
2.42	0.661	-.751	5.5785	5.6674	2.82	0.316	-.949	8.3586	8.4182
2.43	0.653	-.757	5.6354	5.7235	2.83	0.307	-.952	8.4432	8.5022
2.44	0.645	-.764	5.6929	5.7801	2.84	0.297	-.955	8.5287	8.5871
2.45	0.638	-.770	5.7510	5.8373	2.85	0.288	-.958	8.6150	8.6728
2.46	0.630	-.777	5.8097	5.8951	2.86	0.278	-.961	8.7021	8.7594
2.47	0.622	-.783	5.8689	5.9535	2.87	0.268	-.963	8.7902	8.8469
2.48	0.614	-.789	5.9288	6.0125	2.88	0.259	-.966	8.8791	8.9352
2.49	0.606	-.795	5.9892	6.0721	2.89	0.249	-.969	8.9689	9.0244
2.50	0.598	-.801	6.0502	6.1323	2.90	0.239	-.971	9.0596	9.1146
2.51	0.590	-.807	6.1118	6.1931	2.91	0.230	-.973	9.1512	9.2056
2.52	0.582	-.813	6.1741	6.2545	2.92	0.220	-.976	9.2437	9.2976
2.53	0.574	-.819	6.2369	6.3166	2.93	0.210	-.978	9.3371	9.3905
2.54	0.566	-.824	6.3004	6.3793	2.94	0.200	-.980	9.4315	9.4844
2.55	0.558	-.830	6.3645	6.4426	2.95	0.190	-.982	9.5268	9.5792
2.56	0.549	-.836	6.4293	6.5066	2.96	0.181	-.984	9.6231	9.6749
2.57	0.541	-.841	6.4946	6.5712	2.97	0.171	-.985	9.7203	9.7716
2.58	0.533	-.846	6.5607	6.6365	2.98	0.161	-.987	9.8185	9.8693
2.59	0.524	-.852	6.6274	6.7024	2.99	0.151	-.989	9.9177	9.9680

x	sin x	cos x	sinh x	cosh x	x	sin x	mos x	sinh x	cosh x
3.00	0.141	-.990	10.018	10.068	3.40	-.256	-.967	14.965	14.999
3.01	0.131	-.991	10.119	10.168	3.41	-.265	-.964	15.116	15.149
3.02	0.121	-.993	10.221	10.270	3.42	-.275	-.962	15.268	15.301
3.03	0.111	-.994	10.325	10.373	3.43	-.284	-.959	15.422	15.455
3.04	0.101	-.995	10.429	10.477	3.44	-.294	-.956	15.577	15.610
3.05	0.092	-.996	10.534	10.581	3.45	-.304	-.953	15.734	15.766
3.06	0.082	-.997	10.640	10.687	3.46	-.313	-.950	15.893	15.924
3.07	0.072	-.997	10.748	10.794	3.47	-.323	-.947	16.053	16.084
3.08	0.062	-.998	10.856	10.902	3.48	-.332	-.943	16.215	16.245
3.09	0.052	-.999	10.966	11.011	3.49	-.341	-.940	16.378	16.408
3.10	0.042	-.999	11.076	11.122	3.50	-.351	-.937	16.543	16.573
3.11	0.032	-1.00	11.188	11.233	3.51	-.360	-.933	16.709	16.739
3.12	0.022	-1.00	11.301	11.345	3.52	-.369	-.930	16.877	16.907
3.13	0.012	-1.00	11.415	11.459	3.53	-.379	-.926	17.047	17.077
3.14	0.002	-1.00	11.530	11.574	3.54	-.388	-.922	17.219	17.248
3.15	-.008	-1.00	11.647	11.690	3.55	-.397	-.918	17.392	17.421
3.16	-.018	-1.00	11.764	11.807	3.56	-.406	-.914	17.567	17.596
3.17	-.028	-1.00	11.883	11.925	3.57	-.415	-.910	17.744	17.772
3.18	-.038	-.999	12.003	12.044	3.58	-.426	-.905	17.923	17.951
3.19	-.048	-.999	12.124	12.165	3.59	-.434	-.901	18.103	18.131
3.20	-.058	-.998	12.246	12.287	3.60	-.443	-.897	18.286	18.313
3.21	-.068	-.998	12.369	12.410	3.61	-.452	-.892	18.470	18.497
3.22	-.078	-.997	12.494	12.534	3.62	-.460	-.888	18.655	18.682
3.23	-.088	-.996	12.620	12.660	3.63	-.469	-.883	18.843	18.870
3.24	-.098	-.995	12.747	12.786	3.64	-.478	-.878	19.033	19.059
3.25	-.108	-.994	12.876	12.915	3.65	-.487	-.874	19.224	19.250
3.26	-.118	-.993	13.006	13.044	3.66	-.496	-.869	19.418	19.444
3.27	-.128	-.992	13.138	13.175	3.67	-.502	-.864	19.613	19.639
3.28	-.138	-.990	13.269	13.307	3.68	-.513	-.859	19.311	19.836
3.29	-.148	-.989	13.403	13.440	3.69	-.521	-.853	20.010	20.035
3.30	-.158	-.988	13.548	13.575	3.70	-.530	-.848	20.211	20.236
3.31	-.168	-.986	13.674	13.711	3.71	-.538	-.843	20.414	20.439
3.32	-.178	-.984	13.812	13.848	3.72	-.547	-.837	20.620	20.644
3.33	-.187	-.982	13.951	13.987	3.73	-.555	-.832	20.828	20.852
3.34	-.197	-.980	14.092	14.127	3.74	-.563	-.826	21.037	21.061
3.35	-.207	-.978	14.234	14.269	3.75	-.572	-.821	21.249	21.272
3.36	-.217	-.976	14.377	14.412	3.76	-.580	-.815	21.463	21.486
3.37	-.226	-.974	14.522	14.557	3.77	-.588	-.809	21.679	21.702
3.39	-.236	-.972	14.668	14.702	3.78	-.596	-.803	21.897	21.919
3.39	-.246	-.969	14.816	14.850	3.79	-.604	-.797	22.117	22.140

Printed and bound in Egypt by Zamzam Presses